***Solution Section* 3.3 – Gram-Schmidt Process**

***Exercise***

Use the Gram-Schmidt process to find an orthonormal basis for the subspaces of .



***Solution***





































***Exercise***

Use the Gram-Schmidt process to find an orthonormal basis for the subspaces of .



***Solution***





























***Exercise***

Use the Gram-Schmidt process to find an orthonormal basis for the subspaces of .



***Solution***

 



















 

























 



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***Solution***









 





































***Exercise***

Use the Gram-Schmidt process to find an orthonormal basis for the subspaces of .



***Solution***

















































***Exercise***

Use the Gram-Schmidt process to find an *orthonormal* basis for the subspaces of .



***Solution***



















































***Exercise***

Use the Gram-Schmidt process to find an *orthonormal* basis for the subspaces of .



***Solution***



















































***Exercise***

Use the Gram-Schmidt process to find an *orthonormal* basis for the subspaces of .



***Solution***











.







































































***Exercise***

Use the Gram-Schmidt process to find an ***orthogonal*** basis for the subspaces of .



***Solution***



















































***Exercise***

Use the Gram-Schmidt process to find an ***orthogonal*** basis for the subspaces of .



***Solution***













































***Exercise***

Use the Gram-Schmidt process to find an ***orthogonal*** basis for the subspaces of .



***Solution***













































***Exercise***

Use the Gram-Schmidt process to find an ***orthogonal*** basis for the subspaces of .



***Solution***













































***Exercise***

Use the Gram-Schmidt process to find an ***orthogonal*** basis for the subspaces of .



***Solution***















































***Exercise***

Use the Gram-Schmidt process to find an ***orthogonal*** basis for the subspaces of .



***Solution***













































***Exercise***

Use the Gram-Schmidt process to find an ***orthogonal*** basis for the subspaces of .



***Solution***







































































***Exercise***

Use the Gram-Schmidt process to find an ***orthogonal*** basis for the subspaces of .



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***Exercise***

Use the Gram-Schmidt process to find an ***orthogonal*** basis for the subspaces of .



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***Exercise***

Use the Gram-Schmidt process to find an ***orthogonal*** basis for the subspaces of .



***Solution***













































***Exercise***

Use the Gram-Schmidt process to find an ***orthogonal*** basis for the subspaces of .



***Solution***















































***Exercise***

Use the Gram-Schmidt process to find an ***orthogonal*** basis for the subspaces of .



***Solution***







































































***Exercise***

Find the *QR*−decomposition of

|  |  |  |
| --- | --- | --- |
|  |  |  |

***Solution***

1. Since , The matrix is invertible



















































The *QR*-decomposition of the matrix is



1. The column vectors of are: 









































1. Since the column vectors  are linearly independent, so has a *QR*−decomposition.



















































The *QR−*decomposition of the matrix is



1. Since ,

The matrix is invertible, so it has a *QR*-decomposition.

































































































The *QR*−decomposition of the matrix is



1. 





The matrix is linearly dependent, so ***doesn’t*** have a *QR*−decomposition.

***Exercise***

Verify that the Cauchy-Schwarz inequality holds for the given vectors using the Euclidean inner product



***Solution***















***Exercise***

Apply the Gram-Schmidt ***orthonormalization*** process in  spanned by the functions, using the inner product 

***Solution***

Let 





























The orthogonal basis is 























The orthonormal basis is 

***Exercise***

Apply the Gram-Schmidt ***orthonormalization*** process in  spanned by the functions, using the inner product 

***Solution***

Let 

























































The orthogonal basis is 



































The orthonormal basis is 

***Exercise***

Apply the Gram-Schmidt ***orthonormalization*** process in  spanned by the functions, using the inner product 

***Solution***

Let 















































The orthogonal basis is 

























The orthonormal basis is 

***Exercise***

Apply the Gram-Schmidt ***orthonormalization*** process in  spanned by the functions, using the inner product 

***Solution***

Let 















































The orthogonal basis is 





















The orthonormal basis is 

***Exercise***

Apply the Gram-Schmidt ***orthonormalization*** process in  spanned by the functions, using the inner product 

***Solution***

Let 











 





















 









 











The orthogonal basis is 





















The orthonormal basis is 

***Exercise***

Apply the Gram-Schmidt ***orthonormalization*** process in  spanned by the functions, using the inner product 

***Solution***

Let 











 



















 







 











The orthogonal basis is 





















The orthonormal basis is 

***Exercise***

For , define the inner product over  as



1. If  is a unit vector in ?
2. Find an orthonormal basis for the subspace spanned by .
3. Complete the basis in part (*b*) to an orthonormal basis for  with respect to the inner product.
4. Is



Also, an inner product for 

1. Find a pair of vectors  and  such that



1. Is the basis found in part (*c*) are orthonormal basis for  with respect to the inner product in part (*d*)?

***Solution***

1. 











Therefore, when  is ***not*** a unit vector in 

1. Let 















































The orthonormal basis is 

1. Since  in 

Then, let 















































The orthonormal basis is 

1. 

Let 































































































The orthonormal basis is 

Therefore,  is an inner product for 

1. Let assume: 









 ***√***





 ***√***

1. The orthonormal basis in part (*c*)  are ***not*** the same as

the orthonormal basis in part (*d*) 