***Solution Section* 3.4 – Orthogonal Matrices**

***Exercise***

Show that the matrix is orthogonal 

***Solution***











∴ ***A*** is an orthogonal

***Exercise***

Show that the matrix is orthogonal 

***Solution***











∴ ***A*** is an orthogonal.

***Exercise***

Determine if the matrix is orthogonal. For those that is orthogonal find the inverse



***Solution***





∴ is orthogonal with inverse 

***Exercise***

Determine if the matrix is orthogonal. For those that is orthogonal find the inverse



***Solution***





∴ is orthogonal with inverse  (It is a standard matrix for a rotation of 45°)

***Exercise***

Determine if the matrix is orthogonal. For those that is orthogonal find the inverse



***Solution***





∴ is orthogonal with inverse 

***Exercise***

Determine if the matrix is orthogonal. For those that is orthogonal find the inverse



***Solution***





∴ is orthogonal with an inverse 

***Exercise***

Determine if the matrix is orthogonal. For those that is orthogonal find the inverse



***Solution***





∴ is not an orthogonal

***Exercise***

Determine if the matrix is orthogonal. For those that is orthogonal find the inverse



***Solution***





***Or***  ∴***A*** is ***not*** orthogonal

***Exercise***

Determine if the matrix is orthogonal. For those that is orthogonal find the inverse



***Solution***





**∴** is orthogonal with inverse 

***Exercise***

Determine if the matrix is orthogonal. For those that is orthogonal find the inverse



***Solution***





**∴** is orthogonal with inverse 

***Exercise***

Determine if the matrix is orthogonal. For those that is orthogonal find the inverse



***Solution***





**∴**  is orthogonal with inverse 

***Exercise***

Determine if the matrix is orthogonal. For those that is orthogonal find the inverse



***Solution***







***Or***





**∴** The matrix is ***not*** an orthogonal

***Exercise***

Find a last column so that the resulting matrix is orthogonal



***Solution***













Let 









***Exercise***

Determine if the given matrix is orthogonal. If it is, find its inverse



***Solution***

  













The given matrix is ***not*** orthogonal

***Exercise***

Prove that if *A* is orthogonal, then  is orthogonal.

***Solution***

Since *A* is orthogonal then  and 

Then   is orthogonal

Another word, since *A* is orthogonal, then both column and row vectors of *A* form an orthonormal set.

 is just *A* with its row and column vectors are swapped.

The column vectors of  (which are the row vectors of *A*) and row vectors of  (which are the column vectors of *A*) form orthonormal sets, therefore  is orthogonal

***Exercise***

Prove that if *A* is orthogonal, then  is orthogonal

***Solution***

Since *A* is orthogonal then  and 

 



**∴**  is orthogonal

***Exercise***

Prove that if *A* and *B* are orthogonal, then  is orthogonal

***Solution***

Since *A* is orthogonal then 

and *B* is orthogonal then 







**∴**  is orthogonal

***Exercise***

Let *Q* be an  orthogonal matrix, and let *A* be an  matrix.

Show that 

***Solution***



 Since *Q* is an orthogonal matrix 



 ***√***

***Exercise***

Let 

1. Is matrix *A* an orthogonal matrix?
2. Let *B* be the matrix obtained by normalizing each row of *A*, find *B*.
3. Is *B* an orthogonal matrix?
4. Are the columns of *B* orthogonal?

***Solution***

1. 



∴ is ***not*** an orthogonal

1. 













1. *Yes*, since the rows are orthogonal with unit vectors.





1. *Yes*, since the rows of *B* form an orthonormal set of vectors.

Then, the column of *B* must form an orthonormal set.





















