***Solution Section* 4.2 – General Linear Transformations**

***Exercise***

The matrix  gives a shearing transformation .

What happens to (1, 0) and (2, 0) on the *x*-axis.

What happens to the points on the vertical lines *x* = 0 and *x* = *a*?.

***Solution***

The points (1, 0) and (2, 0) on the *x*-axis transform by *T* to (1, 3) and (2, 6). The horizontal *x*-axis transforms to the straight line with slope 3 (going through (0, 0) of course). The points on the *y*-axis are not moved because *T*(0, *y*) – (0, *y*). The *y* -axis is the line of eigenvectors of *T* with λ = 1.

The vertical line *x* = *a* is moved up by 3*a*, since 3*a* is added to the *y* component. This is ***shearing***. Vertical lines slide higher as you go from left to right.

***Exercise***

A nonlinear transformation *T* is invertible if every  in the output space comes from exactly one x in the input space.  always has exactly one solution. Which of these transformation (on real numbers  is invertible and what is ? None are linear, not even . When you solve , you are inverting *T*:



***Solution***

is not invertible because

 and  has no solution.

is not invertible because

 has no solution.

 is invertible.

The solutions to 

 is invertible.

The solutions to 

 is invertible.

The solutions to 

***Exercise***

If *S* and *T* are linear transformations, is  linear or quadratic?

1. If  and , then  or ?
2.  and  combine into



***Solution***

1.  

1. 



It is quadratic.

***Exercise***

Find the range and kernel (like the column space and nullspace) of *T*:

1. 
2. 
3. 
4. 

***Solution***

1. Range is the line *y* = 0, Kernel is the line *x = y* in the *xy* plane.
2. Range is the *xy* plane, Kernel is the complementary line in .
3. Range is the point (0, 0), Kernel is plane
4. Range is the line *x = y* in the *xy* plane, Kernel is the line *x* = 0.

***Exercise***

*M* is any 2 by 2 matrix and . The transformation *T* is defined by . What rules of matrix multiplication show that *T* is linear?

***Solution***

The distribution law and the association law for multiplication give the linearity







***Exercise***

Which of these transformations satisfy  and which satisfy ?

1. 
2. 
3. 
4. = largest component of .

***Solution***

1. This is scaling the vector into a normal vector. This it is impossible that we get additivity, because the sums of normal vectors don’t have to be normal. For example *T*(0, 1) and *T*(1, 0) for instance. However, true to its name this does have the scaling property. For ***c*** value, this value will be canceled from  and .
2. This satisfies both. One immediate way to see that it is matrix multiplication by [1, 1, 1], which is a linear operation and thus satisfies both properties.
3. This satisfies both. This a matrix multiplication by 
4. Doesn’t satisfy additivity [(0, 1) and (1, 0) still work]. Scaling doesn’t work either, if we scale by -1 we now pick out the negative of the smallest component, which doesn’t have to be related in any way to the largest component.

***Exercise***

Consider the basis for , where  and let  be the linear transformation for which



Find a formula for , and then use that formula to compute 

***Solution***

Assume: 





















***Exercise***

Consider the basis for , where  and let  be the linear transformation for which



Find a formula for , and then use that formula to compute 

***Solution***

Assume: 







































***Exercise***

let  be vectors in a vector space *V*, and let  be the linear transformation for which

.

Find 

***Solution***









***Exercise***

Let  be the linear operation given by the formula 

Which of the following vectors are in 



***Solution***

1. 









This is a consistent system, therefore (1, −4) is in 

1. 







This is an inconsistent system, therefore (5, 0) is not in 

1. 









This is a consistent system, therefore (−3, 12) is in 

***Exercise***

Let  be the linear operation given by the formula 

Which of the following vectors are in 



***Solution***

1. 



Therefore (5, 10) is in 

1. 



Therefore (3, 2) is not in 

1. 



Therefore (1, 1) is not in 

***Exercise***

Let  be the linear operation given by the formula



Which of the following vectors are in 



***Solution***

1. 















This is a consistent system, therefore (0, 0, 6) is in 

1. 















This is a consistent system, therefore (1, 3, 0) is in 

1. 















This is a consistent system, therefore (2, 4, 1) is in 

***Exercise***

Let  be the linear operation given by the formula



Which of the following vectors are in 



***Solution***

1. 



Therefore, (3, −8, 2, 0) is in 

1. 

Therefore, (0, 0, 0, 1) is ***not*** in 

1. 



Therefore, (0, −4, 1, 0) is ***not*** in 

***Exercise***

Determine if the given function *T* is a linear transformation



***Solution***

Let  and 













Function *T* is NOT a linear transformation.

***Exercise***

Determine if the given function *T* is a linear transformation



***Solution***

Let  and 







 ***√***











 ***√***

Since  and , then function *T* is a linear transformation.

***Exercise***

Determine if the given function *T* is a linear transformation where *A* is fixed  matrix



***Solution***













Function *T* is a linear transformation

***Exercise***

Determine if the given function *T* is a linear transformation. Also give the domain and range of *T*; if *T* is linear, find the *A* such 



***Solution***

Let 













The function *T* is ***not*** a linear transformation.

Domain: 

***Exercise***

Determine if the given function *T* is a linear transformation. Also, give the domain and range of *T*; if *T* is linear, find the *A* such .



***Solution***

Let 





















Since  and , then function *T* is a linear transformation.

***Domain***: 







***Exercise***

Determine if the given function *T* is a linear transformation. Also, give the domain and range of *T*; if *T* is linear, find the *A* such .



***Solution***

Let 





















Since  and , then function *T* is a linear transformation.

***Domain***: 







***Exercise***

Determine if the given function *T* is a linear transformation. Also give the domain and range of *T*; if *T* is linear, find the *A* such 



***Solution***

Let 



















Since  and , then function *T* is a linear transformation.

***Domain***: 







***Exercise***

Determine if the given function *T* is a linear transformation. Also give the domain and range of *T*; if *T* is linear, find the *A* such 



***Solution***

Let 























Since  and , then function *T* is a linear transformation.

***Domain***: 







***Exercise***

Determine if the given function *T* is a linear transformation. Also give the domain and range of *T*; if *T* is linear, find the *A* such 



***Solution***

Let 























Since  and , then function *T* is a linear transformation.

***Domain***: 







***Exercise***

Determine if the given function *T* is a linear transformation. Also give the domain and range of *T*; if *T* is linear, find the *A* such 



***Solution***

Let 























Since  and , then function *T* is a linear transformation.

***Domain***: 







***Exercise***

Determine if the given function *T* is a linear transformation. Also give the domain and range of *T*; if *T* is linear, find the *A* such 



***Solution***

Let 























Since  and , then function *T* is a linear transformation.

***Domain***: 







***Exercise***

Determine if the given function *T* is a linear transformation. Also give the domain and range of *T*; if *T* is linear, find the *A* such 



***Solution***

Let 





















Since  and , then function *T* is a linear transformation.

***Domain***: 



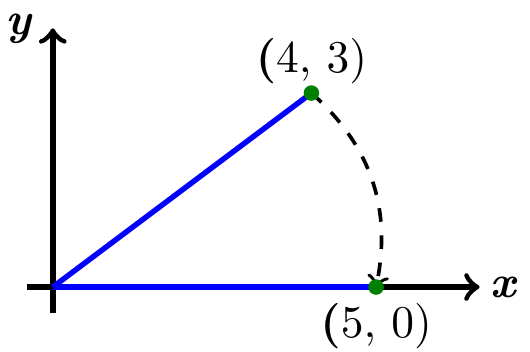




***Exercise***

A Givens rotation is a linear transformation from  used in computer to create a zero entry in a vector (usually a column of a matrix). The standard matrix of a Givens rotation in  has the form





A Givens rotation in 

Find *a* and *b* that  is rotated into .

***Solution***



















