***Solution Section* 4.5 – Diagonalization**

***Exercise***

The Lucas numbers are like Fibonacci numbers except they start with . Following the rule . The next Lucas numbers are 4, 7, 11, 18. Show that the Lucas number .

***Solution***

Let 

the rule 

becomes .









The characteristic equation is  and the solutions are



For 







For 







The linear combination:













The solution:







***Exercise***

Find all eigenvector matrices *S* that diagonalize *A* (rank 1) to give :



What is ? Which matrices *B* commute with *A* (so that *AB* = *BA*)

***Solution***

Since *A* has rank 1, its nullspace is a two-dimensional plane. Any vector with  solves  . So λ = 0 is an eigenvalue with multiplicity 2. There are two independent eigenvectors. The other eigenvalues must be λ = 3 because the trace *A* is 1 + 1 + 1 = 3.







The eigenvalues are 

For 





For 





The possible matrices *S*:



and



.

The powers  come:



and



If *AB* = *BA*, all the column and row of ***B*** must be the same.

One possible ***B*** is***A*** itself, since , ***B*** is any linear combination of permutation matrices.

***Exercise***

Determine whether the matrix is diagonalizable 

***Solution***







The eigenvalues are: 

For 









The inverse doesn’t exist.

Therefore, the matrix *A* is not diagonalizable.

***Exercise***

Determine whether the matrix is diagonalizable 

***Solution***









The only eigenvalue: 

For 





 (***linearly dependent***)





The inverse doesn’t exist.

Therefore, the matrix *A* is not diagonalizable.

***Exercise***

Determine whether the matrix is diagonalizable 

***Solution***















The eigenvalues are: 

For 







***Exercise***

Determine whether the matrix is diagonalizable 

***Solution***

Since the matrix is upper triangular with diagonal entries 2, 2, 3, and 3, the eigenvalues are 2 and 3 (each multiplicity of 2)

For 







 has dimension 1.

For 









The matrix is not diagonalizable since it has only 2 distinct eigenvectors. Note that showing that the geometric multiplicity of either eigenvalue is less than its algebraic multiplicity is sufficient to show that the matrix is not diagonalizable.

***Exercise***

Determine if the matrices are diagonalizable. If so, find a matrix ***P*** that diagonalizes *A* and determine 



***Solution***





The eigenvalues are: 

For 





Therefore, the eigenvector: 

For 





Therefore, the eigenvector: 

The eigenvectors matrix form:











***Exercise***

Determine if the matrices are diagonalizable. If so, find a matrix *P* that diagonalizes *A* and determine 



***Solution***





The eigenvalues are: 

For 





Therefore, the eigenvector: 

For 





Therefore, the eigenvector: 

The ***eigenvector*** ***matrix*** is given by:









 ***√***

***Exercise***

Determine if the matrices are diagonalizable. If so, find a matrix ***P*** that diagonalizes *A* and determine 



***Solution***

Upper triangular; the eigenvalues are the main diagonal entries.

The eigenvalues are: 

For 





Therefore, the eigenvector: 

Since, the eigenvalues are repeated, then the matrix *A* is ***not*** diagonalizable.

***Exercise***

Determine if the matrices are diagonalizable. If so, find a matrix ***P*** that diagonalizes *A* and determine 



***Solution***







The eigenvalues are: 

For 





Therefore, the eigenvector: 

Since, the eigenvalues are repeated, then the matrix *A* is ***not*** diagonalizable.

***Exercise***

Determine if the matrices are diagonalizable. If so, find a matrix ***P*** that diagonalizes *A* and determine 



***Solution***







The eigenvalues are: 

For 





Therefore, the eigenvector: 

For 





Therefore, the eigenvector: 

The ***eigenvector*** ***matrix*** is given by:











 ***√***

***Exercise***

Determine if the matrices are diagonalizable. If so, find a matrix ***P*** that diagonalizes *A* and determine 



***Solution***











The eigenvalues are: 

For 



Let 

Therefore, the eigenvector: 

Let 

Therefore, the eigenvector: 

For 















Therefore, the eigenvector: 

The ***eigenvector*** ***matrix*** is given by:

















 ***√***

***Exercise***

Determine if the matrices are diagonalizable. If so, find a matrix ***P*** that diagonalizes *A* and determine 



***Solution***













The eigenvalues are: 

For 



Let 

Therefore, the eigenvector: 

Let 

Therefore, the eigenvector: 

For 













Therefore, the eigenvector: 

The ***eigenvector*** ***matrix*** is given by:



















 ***√***

***Exercise***

Determine if the matrices are diagonalizable. If so, find a matrix ***P*** that diagonalizes *A* and determine 



***Solution***















The eigenvalues are: 

For 



,,,,,,,,



Let 

Therefore, the eigenvector: 

Let 

Therefore, the eigenvector: 

For 













Therefore, the eigenvector: 

The ***eigenvector*** ***matrix*** is given by:



















 ***√***

***Exercise***

Determine if the matrices are diagonalizable. If so, find a matrix ***P*** that diagonalizes *A* and determine 



***Solution***





The eigenvalues are: 

For 





Therefore, the eigenvector: 

For 







Therefore, the eigenvector: 

The ***eigenvector*** ***matrix*** is given by:



















 ***√***

***Exercise***

Determine if the matrices are diagonalizable. If so, find a matrix ***P*** that diagonalizes *A* and determine 



***Solution***









The eigenvalues are: 

For 







Therefore, the eigenvector: 

For 



Let 

Therefore, the eigenvector: 

Let 

Therefore, the eigenvector: 

The ***eigenvector*** ***matrix*** is given by:



















 ***√***

***Exercise***

Determine if the matrices are diagonalizable. If so, find a matrix ***P*** that diagonalizes *A* and determine 



***Solution***















The eigenvalues are: 

For 











Therefore, the eigenvector: 

Since the only real eigenvalue  which has only a one-dimensional eigenspace.

Therefore, the given matrix A is ***not diagonalizable*** over real numbers.

***Exercise***

Determine if the matrices are diagonalizable. If so, find a matrix ***P*** that diagonalizes *A* and determine 



***Solution***

Since the given matrix is a lower triangular, then

The eigenvalues are: 

For 





Therefore, the eigenvector: 

Since the eigenvalue  has only one−dimensional eigenspace.

Therefore, the given matrix *A* is ***not diagonalizable*** over real numbers.

***Exercise***

Determine if the matrices are diagonalizable. If so, find a matrix ***P*** that diagonalizes *A* and determine 



***Solution***











The eigenvalues are: 

For 











Therefore, the eigenvector: 

For 











Therefore, the eigenvector: 

For 









Therefore, the eigenvector: 

The ***eigenvector*** ***matrix*** is given by:



















 ***√***

***Exercise***

Determine if the matrices are diagonalizable. If so, find a matrix ***P*** that diagonalizes *A* and determine 



***Solution***











The eigenvalues are: 

For 







For 







The ***eigenvector*** ***matrix*** is:















 ***√***

***Exercise***

Determine if the matrices are diagonalizable. If so, find a matrix ***P*** that diagonalizes *A* and determine 



***Solution***





The eigenvalues are: 

For 







The ***eigenvector*** ***matrix*** is:



Since the one eigenvector has no−dimensional eigenspace.

Therefore, the given matrix *A* is ***not diagonalizable***

***Exercise***

Determine if the matrices are diagonalizable. If so, find a matrix ***P*** that diagonalizes *A* and determine 



***Solution***













The eigenvalues are: 

For 













For 













The ***eigenvector*** ***matrix*** is:



 doesn’t exist, one column with zero entries.

Since the eigenvalue  has no−dimensional eigenspace.

Therefore, the given matrix *A* is ***not diagonalizable*** (repeated eigenvalues)

***Exercise***

Determine if the matrices are diagonalizable. If so, find a matrix ***P*** that diagonalizes *A* and determine 



***Solution***

Since the matrix *A* is a lower triangular, then the eigenvalues are the entries values of the main diagonal.

The eigenvalues are: 

For 







For 







The ***eigenvector*** ***matrix*** is:



 doesn’t exist, one row with zero entries

Since the 2 eigenvectors have only the same one−dimensional eigenspace.

Therefore, the given matrix *A* is ***not diagonalizable***

***Exercise***

Determine if the matrices are diagonalizable. If so, find a matrix ***P*** that diagonalizes *A* and determine 



***Solution***

Since the matrix *A* is an upper triangular, then the eigenvalues are: 

For 



 



For 







The ***eigenvector*** ***matrix*** is:















 ***√***

***Exercise***

Determine if the matrices are diagonalizable. If so, find a matrix ***P*** that diagonalizes *A* and determine 



***Solution***

Since the matrix *A* is an upper triangular, then the eigenvalues are: 



For 













For 









For 









The ***eigenvector*** ***matrix*** is:





















 ***√***

***Exercise***

Determine if the matrices are diagonalizable. If so, find a matrix ***P*** that diagonalizes *A* and determine 



***Solution***

Since the matrix *A* is a lower triangular, then the eigenvalues are: 



For 







For 







Since the eigenvalue  has only one−dimensional eigenspace.

Therefore, the matrix *A* is *not diagonalizable*.

***Exercise***

The 4 by 4 triangular Pascal matrix and its inverse (alternating diagonals) are



Check that  and  have the same eigenvalues. Find a diagonal matrix D with alternating signs that gives , so  is similar to . Show that  with columns of alternating signs is its own inverse.

Since  and  are similar they have the same Jordan form *J*. Find *J* by checking the number of independent eigenvectors of  with λ = 1.

***Solution***

The triangular matrices  and  both have λ = 1, 1, 1, 1 on their main diagonals. Choose *D* with alternating 1 and −1 on its diagonal. *D* equals :









***Check***:

Changing signs in rows 1 and 3 of , and columns 1 and 3, produces the four negative entries in . Multiply row *i* by  and column *j* by , which gives the alternating diagonals.

Then  has columns with alternating signs and equals its own inverse!







 has only one line of eigenvectors  with λ = 1. The rank of  is certainly 3. So its Jordan form *J* has only one block (also with λ = 1):

 and  are somehow similar to Jordan’s 

***Exercise***

These Jordan matrices have eigenvalues 0, 0, 0, 0. They have two eigenvectors (one from each block). But the block sizes don’t match and they are not similar:



For any matrix *M* compare *JM* with *MK*. If they are equal show that *M* is not invertible. Then  is Impossible; *J* is not similar to *K*.

***Solution***

Let , then





If *JM* = *MK* then 

Which in particular means that the second row is either a multiple of the fourth row, or the fourth row is all 0’s. In either of these cases *M* is not invertible.

Suppose that *J* were similar to *K*. Then there would be some invertible matrix *M* such that . But we just showed that in this case *M* is never invertible (contradiction). Thus, *J* is not similar to *K*.

***Exercise***

If ***x*** is in the nullspace of *A* show that  is in the nullspace of .

The nullspaces of *A* and  have the same (vectors) (basis) (dimension)

***Solution***





So, any vector in  is a linear combination of those in , hence is contained in it. That is, the two vector spaces consist of the same vectors.

***Exercise***

Prove that  is always similar to A (λ′s are the same):

1. For one Jordan block , find  so that .
2. For any *J* with blocks , build  from blocks so that .
3. For any : Show that  is similar to  and so to J and so to *A*.

***Solution***

1. For one Jordan block , then



So, *J* is similar to 

1. For any *J* with block , that satisfies 

Let  be the block-diagonal matrix consisting of the  along the diagonal. Then









1. 

So is similar to , which is similar to *J*, which is similar to *A*, Thus any matrix is similar to its transpose.

***Exercise***

Why are these statements all true?

1. If *A* is similar to *B* then  is similar to .
2.  and  can be similar when *A* and *B* are not similar.
3.  is similar to 
4.  is not similar to 
5. If we exchange rows 1 and 2 of *A*, and then exchange columns 1 and 2 the eigenvalues stay the same. In this case *M* =?

***Solution***

1. If *A* is similar to *B* then  for some *M*. Then , so  is similar to .
2. Let , then  so they are similar but *A* is not similar to *B* because nothing but zero matrix.
3. 
4. They are not similar because the first matrix has a plane of eigenvectors for the eigenvalues 3, while the second only has a line.
5. In order to exchange two rows of *A* we multiply on the left by



In order to exchange two columns, we multiply on the right by the same *M*. As  the new matrix is similar to the old one, so the eigenvalues stay the same.

***Exercise***

If an *n* x *n* matrix *A* has all eigenvalues λ = 0 prove that  is the zero matrix.

***Solution***

Suppose that the Jordan Block has a size of ***i*** with eigenvalue 0. Then  will have a diagonal of 1’s two diagonals above the main diagonal and zeroes elsewhere.  will have a diagonal of 1’s three diagonals above the main diagonal and zeroes elsewhere. Therefore , since there is no diagonal ***i*** diagonals above the main diagonal. If A has all eigenvalues λ = 0 then A is similar to some matrix with Jordan block  with each  of size  and .

Each Jordan block will have eigenvalue of 0, so that , and thus 

As  is similar to a block-diagonal matrix with blocks  and each of these is 0 we know that .

Another way, if *A* has all eigenvalues 0 this means that the characteristic polynomial of A must be , as this is the only polynomial of degree *n* all of whose roots are 0. Thus  by the Cayley-Hamilton theorem.

***Exercise***

If *A* is similar to , must all the eigenvalues equal to 1 or −1?.

***Solution***

***No***



Thus  is similar to 

***Exercise***

Determine whether the *two matrices* are similar matrices 

***Solution***





; therefore, *A* and *B* are ***not*** similar

***Exercise***

Determine whether the *two matrices* are similar matrices 

***Solution***





; therefore, *A* and *B* are ***not*** similar

***Exercise***

Determine whether the *two matrices* are similar matrices 

***Solution***





; therefore, *A* and *B* are ***not*** similar

***Exercise***

Determine whether the *two matrices* are similar matrices 

***Solution***











Therefore, *A* and *B* are similar

***Exercise***

Determine whether the *two matrices* are similar matrices 

***Solution***











Therefore, *A* and *B* are similar

***Exercise***

Determine whether the *two matrices* are similar matrices 

***Solution***











Therefore, *A* and *B* are similar

***Exercise***

Determine whether the *two matrices* are similar matrices 

***Solution***











Therefore, *A* and *B* are ***not*** similar

***Exercise***

Determine whether the *two matrices* are similar matrices 

***Solution***











Therefore, *A* and *B* are ***not*** similar

***Exercise***

Determine whether the *two matrices* are similar matrices 

***Solution***











Therefore, *A* and *B* are similar

***Exercise***

Prove that two similar matrices have the same determinant. Explain geometrically why this is reasonable.

***Solution***

Suppose that 

Then  











***Geometric Explanation***: The determinant tells us what Factor area changes when using a linear transformation. This “factor” doesn’t care about the particular basis you use.

***Exercise***

Prove that two similar matrices have the same characteristic polynomial and thus the same eigenvalues. Explain geometrically why this is reasonable.

***Solution***

Suppose that 

Then the characteristic polynomial is equal to .











 



***Geometric Explanation***: At least in terms of the eigenvalues, these values are numbers λ such that there exists a vector  such that the linear transformation *T* satisfies .

***Exercise***

Suppose that *A* is a matrix. Suppose that the linear transformation associated to *A* has two linearly independent eigenvectors. Prove that *A* is similar to a diagonal matrix.

***Solution***

Let *T* be the linear transformation associated with *A*. Consider the basis  of the 2 linearly independent eigenvectors of *A* where  the eigenvalues associated with. Then,



Let *T* be a matrix with respect to the basis , then we obtain the matrix 

This completes the proof because *A* is similar to this diagonal matrix by definition.

***Exercise***

Prove that if *A* is a  matrix that has two distinct eigenvalues, then *A* is similar to a diagonal matrix.

***Solution***

Suppose *A* has 2 distinct eigenvalues .

Let  be an eigenvector for .

Suppose that  are not linearly independent, thus they are scalar multiples of each other.

So, there exists  such that . Then









 



So, that 

But then  which contradicts the initial assumption.

Thus  are linearly independent then 

Let *T* be a matrix with respect to the basis , then we obtain the matrix 

This completes the proof because *A* is similar to this diagonal matrix by definition.

***Exercise***

Suppose that the characteristic polynomial of a matrix has a double root. Is it true that the matrix has two linearly independent eigenvectors? Consider the example . Is it true that matrices with equal characteristic polynomial are necessarily similar?

***Solution***





The characteristic polynomial:  which has a double root .





Therefore, the eigenvectors are vectors of the form  which can transform to 

Thus, matrices whose characteristic polynomials have a double root do not necessarily have 2 linear independent.

Let , then the characteristic polynomial:  which has a double root . But they are not similar. The eigenvector is the  vector.

The linear transformation associated to the second matrix send every vector to . Thus the 2 matrices can’t represent the same linear transformation.

Thus, matrices with equal characteristic polynomial are not necessarily similar.

***Exercise***

Show that the given matrix is not diagonalizable. 

***Solution***



Since the determinant is 0, the inverse doesn’t exist.

Therefore, the matrix is not diagonalizable

***Exercise***

Determine if the given matrix is diagonalizable. If, so, find matrices S and  such that the given matrix equals 

|  |  |
| --- | --- |
|  |  |

***Solution***

1. 





For 





Therefore, the eigenvector: 

For 





Therefore, the eigenvector: 













and 









1. 









The given matrix is not diagonalizable, since the eigenvalues are not distinct.

***Exercise***

*A* is a  matrix with *two* eigenvalues. One eigenspace is *three*−dimensional, and the other eigenspace is *two*−dimensional. Is *A* diagonalizable? Why?

***Solution***

Since  matrix *A* has two eigenvalues with one of the eigenvalues has three linearly independent eigenvectors in the *three*−dimensional and the other eigenvalue has two linearly independent eigenvectors in the *two*−dimensional.

Therefore, since all the ***five*** eigenvectors are linearly independent eigenvectors, that implies that the  matrix *A* is diagonalizable.

***Exercise***

*A* is a  matrix with *two* eigenvalues. Each eigenspace is *one*−dimensional. Is *A* diagonalizable? Why?

***Solution***

The given  matrix *A* has two eigenvalues that implies one of the eigenvalues is repeated value.

Since the eigenvectors are in *one*−dimensional, the repeated eigenvalue will result with two eigenvectors linearly dependent.

Therefore, the given  matrix *A* is ***not*** diagonalizable

***Exercise***

*A* is a  matrix with *three* eigenvalues. One eigenspace is *one*−dimensional, and one of the other eigenspace is *two*−dimensional. Is it possible that *A* is *not* diagonalizable? Justify your answer?

***Solution***

The given  matrix *A* has three eigenvalues that implies one of the eigenvalues is repeated value.

However, one eigenspace is *one*−dimensional, and one of the other eigenspace is *two*−dimensional which include that these two eigenvectors are linearly independent.

Since, the other two distinct eigenvalues will result to the linearly independent eigenvectors.

That implies that all the eigenvectors are linearly independent.

Therefore, the given  matrix *A* is diagonalizable.

***Exercise***

*A* is a  matrix with *three* eigenvalues. One eigenspace is *two*−dimensional, and one of the other eigenspace is *three*−dimensional. Is it possible that *A* is *not* diagonalizable? Justify your answer?

***Solution***

The given  matrix *A* has three eigenvalues which results to 7 eigenvalues.

Since, one eigenspace is *two*−dimensional, and one of the other eigenspace is *three*−dimensional that will result to 5 linearly independent eigenvectors for that two eigenvalues.

If the third eigenvalue is repeated with *one*−dimensional, it will result to linearly dependent eigenvectors.

Therefore, the given  matrix *A* is ***not*** diagonalizable

***Exercise***

Show that if *A* is diagonalizable and invertible, then so is .

***Solution***

Since *A* is invertible, then:



And *A* is diagonalizable:











Since *D* is diagonal then  is diagonal matrix.

Therefore,  is diagonalizable

***Exercise***

Show that if *A* has *n* linearly independent eigenvectors, then so does .

***Solution***

If *A* has *n* linearly independent eigenvectors, then *A* is diagonalizable.

By the diagonalizable theorem 



 Since *D* is diagonal then 

 Assume that 



Therefore,  is diagonalizable with the columns *Q* are *n* linearly independent eigenvectors

***Exercise***

A factorization  is not unique. Demonstrate this for the matrix . With  , find a matrix  such that 

***Solution***







The eigenvalues are: 

For 





Therefore, the eigenvector: 

For 





Therefore, the eigenvector: 

The ***eigenvector*** ***matrix*** is given by: 

Which implies to: 









 ***√***

However, if we multiply the eigenvector  with 2, it will result  that implies to: 











 ***√***



Therefore, that is shows that matrix *A* has many different factorizations.

***Exercise***

Construct a nonzero  matrix that is invertible but not diagonalizable.

***Solution***

For a invertible matrix *A*, the eigenvalues must be nonzero and determinant of *A* is not equal to zero.

Let assume 



Matrix A is invertible

Since the matrix *A* is an upper triangular then the eigenvalues are the main diagonal entries



For the matrix *A* to be not diagonalizable when the eigenvectors are linearly dependents or in one−dimensional.

If we have a repeated eigenvalue that it will result in *one*−dimensional, that it will result that .





For 





The eigenvectors are: 

Therefore, the matrix *A* to be not diagonalizable since the eigenvectors are linearly dependent in *one*−dimensional

***Example***: 

***Exercise***

Construct a nonzero  matrix that is diagonalizable but not invertible.

***Solution***

Any  matrix with 2 distinct eigenvalues is diagonalizable.

Any  matrix is not invertible when determinant is zero, or either one row or one column is equal to zero.

If one of the eigenvalues is zero, then the matrix is not invertible.

Let assume 

The eigenvalues are: 

For 





The eigenvectors are: 

For 





The eigenvectors are: 

The ***eigenvector*** ***matrix*** is given by:











 ***√***

Therefore, the result proves that is diagonalizable but not invertible

***More Example***: 

***Exercise***

What are the matrices that are similar to themselves only?

***Solution***

Any matrix to be similar to itself if only if the similar formula  








One of the matrices that are similar is a scalars matrices .

***Exercise***

For any scalars *a, b*, and *c*, show that



are similar.

Moreover, if, then *A* has two zero eigenvalues.

***Solution***













Since , then the matrices *A*, *B*, and *C* are similars.









Given that 









Since , then















So,





The eigenvalues are: 

Since, then *A* has ***two zero*** eigenvalues

***Exercise***

For positive integer , compute 

***Solution***

Let 







The eigenvalues are: 

For 







For 







The eigenvectors matrix:





















***Exercise***

For positive integer , compute 

***Solution***

Let 

Since it is an upper triangular, then

The eigenvalues are: 

For 







Since the eigenvalues are repeated and the eigenvectors are one-dimensional, therefore the matrix is not diagonalizable.

To compute 













***Exercise***

For positive integer , compute 

***Solution***

Since the eigenvalues  are repeated then it is not diagonalizable, which it will result the matrix doesn’t have linearly independent eigenvectors.









Therefore;

If 

Otherwise 

***Exercise***

For positive integer , compute 

***Solution***

Let 





The eigenvalues are: 

For 







The given matrix is not diagonalizable, since the matrix doesn’t have linearly independent eigenvectors.

















When







***Exercise***

Let . Show that  is similar to *A* fro every positive integer *k*. It is true more generally for any matrix with all eigenvalues equal to 1.

***Solution***

Since it is an upper triangular, then

The eigenvalues are: 

For 





Since the eigenvalues are repeated and the eigenvectors are one-dimensional which are not linearly independent, therefore the matrix is not diagonalizable.















Since it is an upper triangular, then

The eigenvalues are: 

Therefore,  is similar to *A* fro every positive integer *k*.

Let *A* be  matrix with upper triangular and one’s in the main diagonal, which implies that all eigenvalues equal to 1. If we use Jordan block, then each  block is similar to *A*.

***Exercise***

Can a matrix be similar to two different diagonal matrices?

***Solution***

The matrix can be similar to two different diagonal matrices as long the size is greater or equal to 3.

And they the same eigenvalues by changing the entries in the main diagonal.

*Example*:



***Exercise***

Prove that if *A* is diagonalizable, then  is diagonalizable.

***Solution***

If *A* is diagonalizable, then by the diagonalizable theorem





 Since *D* is diagonal then 

 Assume that 



Therefore,  is diagonalizable with the columns *Q* are *n* linearly independent eigenvectors

***Exercise***

Prove that if the eigenvalues of a diagonalizable matrix *A* are all , then the matrix is equal to its inverse.

***Solution***

Since the matrix *A* is diagonalizable with eigenvalues are , then the diagonal matrix *D* has  entries along the main diagonal.

So, 

Matrix *A* is diagonalizable that implies to 







 ***√***

Therefore, the matrix is equal to its inverse

***Exercise***

Prove that if *A* is diagonalizable with *n* real eigenvalues , then 

***Solution***

If *A* is diagonalizable with *n* real eigenvalues  and *D* is diagonal with the eigenvalues as entries, then













***Exercise***

If *x* is a real number, then we can define  by the series



In similar way, If *X* is a square matrix, then we can define  by the series



Evaluate , where *X* is the indicated square matrix.

|  |  |
| --- | --- |
|  |  |

***Solution***

1. 









Where, 

1. 

The eigenvalues are: 

For 





For 





The eigenvectors matrix:

























Given that: 

1. 























1. 





The eigenvalues are: 

For 





For 





The eigenvectors matrix:























Where, 

