***Linear Algebra Glossary***

***Adjacency matrix of a graph***: Square matrix with  when there is an edge from node ***i*** to node ***j***; otherwise  for an undirected graph.

***Affine Transformation***:  linear transformation plus shift.

***Back substitution***: Upper triangular systems are solved in reverse order .

***Basis for V***: Independent vectors  whose linear combinations give every ***v*** in **V**. A vector space has many bases.

***Block matrix***: A matrix can be partitioned into matrix blocks, by cuts between rows and/or between columns, ***Block multiplication*** of ***AB*** is allowed if the block shapes permit (the columns of ***A*** and rows of ***B*** must be matching blocks).

***Cayley-Hamilton Theorem***: .

***Change of basis matrix M:*** The old basis vectors  are combinations  of the new basis vectors. The coordinates of  are related by . (For *n* = 2 set .)

***Characteristic equation:*** . The *n* roots are the eigenvalues of ***A***.

***Cholesky factorization:***  for positive eigenvalues of ***A***.

***Circulant matric C:*** Constant diagonals wrap around as in cyclic shift *S*. Every Circulant is . *C****x*** = ***convolution*** ***c***\****x***. Eigenvectors in *F*.

***Cofactor:*** Remove row ***i*** and column ***j***; multiply the determinant by 

***Column picture of Ax = b:*** The vector ***b*** becomes a combination of the columns of ***A***. The system is solvable only when ***b*** is in the column space *C*(***A***).

***Column space*** : consists of all linear combinations of the columns. The combinations are all possible vectors .

***Commuting matrices:*** If diagonalizable, they share ***n*** eigenvectors.

***Companion matrix:*** Put  in row *n* and put *n* – 1 1’s along diagonal 1. Then 

***Complete solution:***  to . 

***Complex conjugate:***  for any complex number . Then 

***Covariance matrix* ∑*:*** When random variables  have mean = average value = 0, their covariances  are the averages of . With means , the matrix ∑ = mean of  is positive (semi) definite; it is diagonal if the  are independent.

***Cramer’s Rule for Ax = b: *** has ***b*** replacing column ***j*** of *A*, and .

***Cross product :*** Vector perpendicular to ***u*** and ***v***, length  = parallelogram area, computed as the “determinant” of 

***Diagonal matrix D:*** . ***Block diagonal***: zero outside square blocks .

***Dimension of a vector space:*** dim(***V***) = number of vectors in any basis for ***V***.

***Dot Product: ***. Complex dot product is ******. Perpendicular vectors have zero dot product. (row *i* of *A*)**.**(column *j* of *B*)

***Echelon matrix U:*** The first nonzero entry (the pivot) in each row comes after the pivot in the previous row. All zero rows come last.

***Eigenvalue λ and eigenvector x:***  so 

***Elimination:*** A sequence of row operations that reduces ***A*** to an upper triangular *U* or to the reduced form *R* = ***rref***(*A*). Then *A = LU* with multipliers in *L*, or *PA = LU* with row exchanges in *P*, or *EA = R* with an invertible *E*.

***Factorization:*** *A = LU*. If elimination takes *A* to *U* without row exchanges, then the lower triangular *L* with multipliers  brings *U* back to *A*.

***Fibonacci numbers***: 0, 1, 1, 2, 3, 5, … satisfy . Growth rate  is the largest eigenvalue of the Fibonacci matrix 

***Four Fundamental subspaces of A*** .

***Free Columns of A:*** Columns without pivots; combinations of earlier columns.

***Free variables:*** Column *i* has no pivot in elimination. We can give the *n – r* free variables any values, the *Ax = b* determines the *r* pivot variables (if solvable!).

***Full column rank*** *r = n*. Independent columns, , no free variables.

***Full row rank*** *r = m*. Independent rows, at least one solution to , column space is all of . Full rank means full column rank or full column rank or full row rank.

***Fundamental Theorem***: the nullspace *N*(*A*) and row space  are orthogonal complements (perpendicular subspaces of with dimensions *r* and *n – r*) from *Ax* = **0**. Applied to , the column space ***C***(***A***) is the orthogonal complement of .

***Independent vectors***: . No combination  vector unless all . If the v’s are the columns of A, the only solution to *Ax* = **0** is ***x* = 0**.

***Least squares solution ***: The vector ****** that minimizes the error  solves ******. Then  is orthogonal to all columns of ***A***.

***Length*** : Square root of  (Pythagoras in *n* dimensions).

***Linear combination***  . Vector addition and scalar multiplication.

***Linear Transformation T***: Each vector v in the input space transforms to *T*(***v***) in the output space, and linearity requires .

***Linearly dependent*** . A combination other than all  gives 

***Linearly independent*** when the only solution to  is . ***No other combination  of the columns gives the zero vector***.

***Nullspace*** of *A* consists of all solutins to . These solution vectors *x* are in . The Nullspace containing all solutions is denoted by .  is the nullspace of *A*,  (Can also be called ***Kernel*** of *A*)

***Particular solution *** Any solution to ; often****** has free variables = 0.

***Permutation matrix P***. There are ***n*!** orders of 1, …, *n*; the **n!** *P*’s have the rows of I in those orders. *PA* puts the rows of *A* in the same order. *P* is a product of row exchanges ; *P* is *even* or *odd* (det*P* = 1 or -1) based on the number of exchanges.

***Pivot columns of A***: Columns that contain pivots after row reduction; not combinations of earlier columns. The pivot columns are a basis for the column space.

***Pivot d***: The diagonal entry (first nonzero) when a row is used in elimination.

***Rank*** of a matrix *A* (***m*** by ***n***) is the number of ***nonzero rows*** in the row-reduced echelon form of *A*. (it is the number of pivot). 

***R****educed* ***R****ow* ***E****chelon* ***F****orm* (***rref***): is a matrix (*R*) with each pivot column has only one nonzero entry (the pivots which is always 1).

***Row space* ** = all combinations of rows of *A*. Column vectors by convention.

***Schwarz inequality*** : Then 

***Singular matrix A*:** A square matrix that has no inverse: .

***Spanning set***  ***for V*:** Every vector in V is a combination of .

***Subspace:*** of a vector space is a set of vectors (including 0) that satisfies two requirements: if *v* and *w* are vectors in the subspace and *c* is any scalar, then  is in the subspace and  is in the subspace

***Symmetric matrix A:*** The transpose is , and .  is also symmetric. All matrices of the form ,  and  are symmetric. Symmetric matrices have real eigenvalues in and orthonormal eigenvectors in *Q*.

***Trace of A:*** = sum of diagonal entries = sum of eigenvalues of ***A***. .

***Transpose matrix :*** Entries .  is *n* by *m*,  is square, symmetric, positive semi-definite. The transposes of *AB* and  are 