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***Limit***

**End Behavior and *Asymptotes* of Rational Functions**

Let  be a rational function.

1. If the degree of numerator is less than of denominator (*n* < *m*) ⇒ *y* = 0

***Horizontal Asymptote*** (**HA**)

1. If the degree of numerator is equal of denominator (*n* = *m*) ⇒ ***Horizontal Asymptote*** (**HA**)
2. If the degree of numerator is greater than of denominator (*n* > *m*) ⇒ No ***Horizontal Asymptote***

***Vertical Asymptote* - *Think Domain***

***Average rate of change***: 

***Sandwich Theorem***  

***Precise Definition of a Limit***

Let  be defined on an open interval about , except possibly at  itself. We say that **the limit of  as *x* approaches is the number *L***, and write: 

If, for every number , there exists a corresponding number  such that for all *x*,



***One-Sided Limits***

If the approach is from the *right*, the limit is a ***right-hand limit***. 

If for every number *ε* > 0 there exists a corresponding number *δ* > 0 such that for all *x*



If the approach is from the *left*, the limit is a ***left-hand limit***. 

If for every number *ε* > 0 there exists a corresponding number *δ* > 0 such that for all *x*



***Continuity*:**

Let *c* be a number in the interval (*a*, *b*), and let *f* be a function whose domain contains the interval (*a*, *b*). The function *f*  is continuous at the point *c* if the following conditions are true.

1. is defined
2. 
3. 

***Interior point***: A function  is **continuous at an interior point *c*** of its domain if



***Endpoint***: A function  is **continuous at a left point *a*** or is **continuous at a right point *b*** of its domain if



***Intermediate Value Theorem***

We call a solution of the equation  a ***root*** of the equation or zero of the function *f*. The Intermediate Value Theorem said that if ***f*** is continuous, then any interval on which *f* changes sign contains a zero of the function.