***Multivariable***



***Distance***: 

***Midpoint***: 

***Sphere Equation***: 

***Equation of a line in space***: 



***Length* (**or ***Norm*** or ***Magnitude*)**: 

***Dot* (*Inner*) *Product***: 



***Angle*** ***between 2 vectors***: 





***Projection matrix*** 

*Length* 

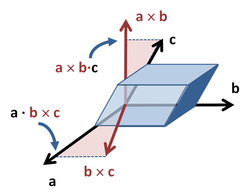
*Error* 

* Two vectors are ***parallel*** *iff* they are scalar multiples of each other.
* Two vectors *A* and *B* are ***orthogonal*** (*perpendicular*) *iff* 
* A set of vectors  are ***linearly dependent*** iff there are scalars  not all zero such that 

They are ***linearly independent*** if no such collection of scalars exist.

***Properties of Vector Product***

*  is a vector perpendicular to both *A* and *B*.
* 
* 
* 
*  for any scalar *c*.
* The length of  is the ***area of the parallelogram*** spanned by *A* and *B*. This area is 
* If *A* or *B* is 0 or *A* is parallel to *B*, then  is the vector 0.

***Volume*** of the Parallelepiped is





***Schwarz Inequality*:**

***Determinant***: 



***Cross Product***: 







***Cross Product Properties***

1.  reverses rows 2 and 3 in the determinant so it is equals 
2. The cross product  is perpendicular to ***u***, then 
3. The cross product  is perpendicular to ***v***, then 
4. The cross product of any vector with itself (two equal rows) is .
5. 
6. Lagrange’s identity: 





***Work***:  ***Volume***: 

***Planes and Surfaces in Space***

The plane through  normal to :

Vector Equation: 

Component equation: 



***Vector-valued Functions and Motion in Space***

Let  be a vector function.

Then  is the velocity vector and  is the speed.

The acceleration vector is .

The unit tangent vector is  and the length of  from  to  is 

***Lines in Space***

A vector equation of the line through  parallel to  is 

, for 

Parametric equations for this line are:

 for 

Principal Unit Normal: 

***Curvature***: 

Radius of ***Curvature***: 

Tangential and normal scalar components of acceleration:



***Quadratic Surfaces***

***Ellipsoid:*** 

***Elliptic Paraboloid:*** 

***Elliptic Cone:*** 

***Hyperboloid of one sheet:*** 

***Hyperboloid of two sheets:*** 

***Hyperbolic Paraboloid:*** 

***Integration in Vector Fields***

***Line Integrals***

To integrate a continuous function  over a curve *C*:

* 
* 

Where 

**F** conservative on *D*  for some potential function *ϕ*

 over closed paths *C* in *D*

 is independent of path for *C* in *D*.

 on *D*.

***Work***: 

***Circulation*** of *F* on *C*: 

***Flux*** of *F* across *C* = 

Where  and **n** is outward pointing normal along *C*.

***Conservative Vector Field***

***F*** is conservative if  for some function . If ***F*** is conservative, then 

is independent of path and 





***Fundamental Theorem of Calculus***

|  |  |
| --- | --- |
| ***Fundamental Theorem of Calculus*** |  |
| ***Fundamental Theorem of Line Integrals*** |  |

***Green’s Theorem***







Circulation: 

Flux: 

Divergence: 

***Stokes’ Theorem***



***Surface Integrals***



Unit normal to the surface: 

***Area***: Area of surface S = 



***Formulas from Vector Calculus***

Assume , where  are differentiable on a region *D* of .

***Gradient***: 

***Curl***: 





***Divergence***: 







***Multiple Integrals***

***Double Integrals as Volumes***

When  is a positive function over a region *R* in the *xy*-plane, we may interpret the double integral of *f* over *R* as the volume of the 3-dimensional solid region over the *xy*-plane bounded below by *R* and above the surface .

This volume can be evaluated by computing an iterated integral

Let  be continuous on a region *R*.

1. If *R* is defined by , with  and  continuous on [*a, b*], then



1. If *R* is defined by , with  and  continuous on [*c, d*], then



***Area***: 



***Triple Integrals***





***Cylindrical Coordinates* (*r, θ, z*)**

Equations Relating Rectangular (*x, y, z*) and Cylindrical (*r, θ, z*) Coordinates



***Spherical Coordinates* (*ρ, ϕ, θ***  **)**



Change of Variables Formula for Double Integrals



***Jacobian*** ***determinant***: 