Low Reynolds number gravitational settling of a sphere through a fluid-fluid interface: Modelling using a boundary integral method

Paul Jarvis*[1], Jon Blundy[1], Katharine Cashman[1], Herbert E Huppert[1,2], and Heidy Mader[1]

[1]School of Earth Sciences, University of Bristol, Wills Memorial Building, Queens Road, Bristol, BS8 1RJ, UK

[2]Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Wilberforce Road, Cambridge, CB3 0WA, UK

Abstract

1 Introduction

2 Theoretical Development

The problem being modelled is the low Reynolds number gravitational settling of a sphere towards an initialy horizontal interface separating two density stratified, immiscible, semi-infinte fluids (figure 1). The fluids are characterised by the velocity $\mathbf{u}_l(\mathbf{x})$ and pressure

*Corresponding author:paul.jarvis@bristol.co.uk

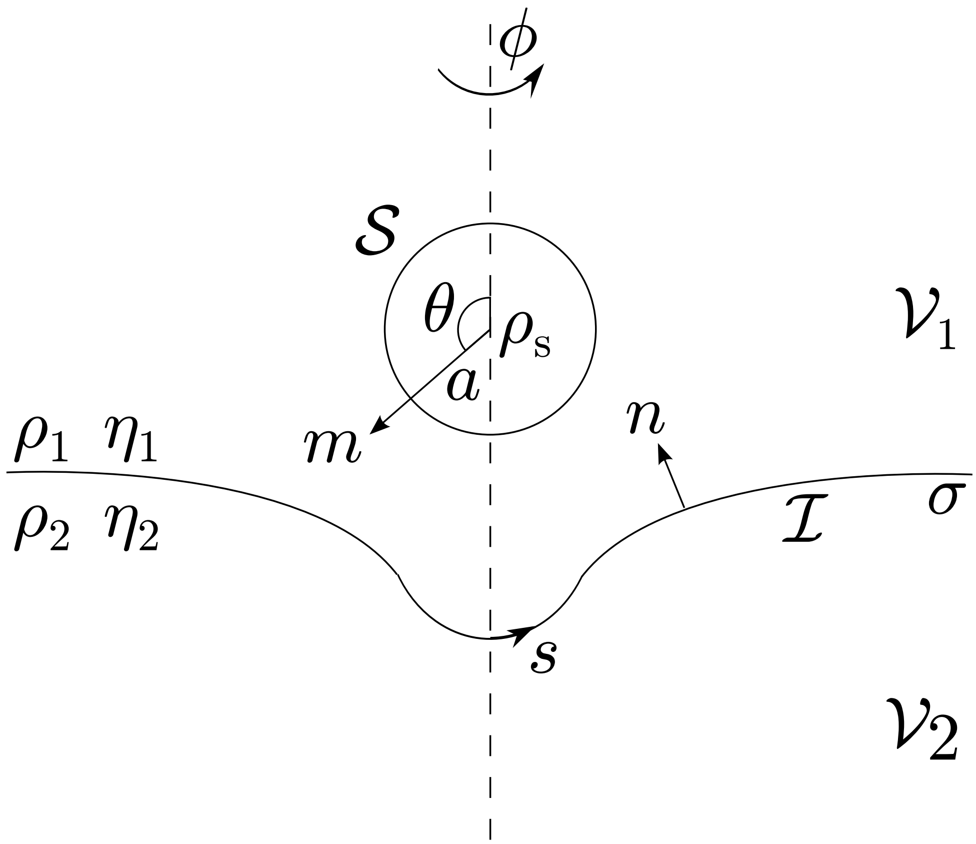

Figure 1: Diagrammatic representation of the system. A sphere falls under gravity, at low Reynolds number, towards an initially horizontal interface between two density stratified, immiscible semi-infinite fluids. See table 1 for definition of symbols.

$p_l(\mathbf{x})$ fields where $l = 1, 2$ denotes the fluid and $\mathbf{x}$ is a position vector. The dynamic pressure is defined as

$$p_{\mathrm{d},l}(\mathbf{x}) = p_l(\mathbf{x}) - \rho_l \mathbf{g} \cdot \mathbf{x}, \quad (1)$$

where $\mathbf{g}$ is acceleration due to gravity.

Table 1: Definition of symbols.

Symbol	Definition
a	Sphere radius
$\mathbf{g} = (-9.81\mathrm{m\ s}^{-2})\hat{\mathbf{z}}$	Acceleration due to gravity
$\mathcal{I}$	Surface of interface

$l = 1, 2$	Fluid label
m	Outward normal to sphere surface
n	Normal to interface (points into fluid 1)
$p_l(\mathbf{x})$	Pressure field of fluid l
$p_{\mathrm{d},l}(\mathbf{x})$	Dynamic pressure of fluid l
s	Arc length along interface measured from axis
$\mathcal{S}$	Surface of sphere
$\mathbf{u}_l(\mathbf{x})$	Velocity field of fluid l
$\mathcal{V}_l$	Volume of fluid l
$\mathbf{x}$	Position vector
$\hat{\mathbf{z}}$	Unit vector in the upward vertical direction
η_l	Viscosity of fluid l
θ	Polar angle with respect to sphere centre
ρ_l	Density of fluid l
ρ_{s}	Sphere density
σ	Interfacial Tension
ϕ	Azimuhtal angle with respect to axis of motion