

Time Domain Full Waveform Inversion involving Discontinuous Galerkin approximation

Waves 2019

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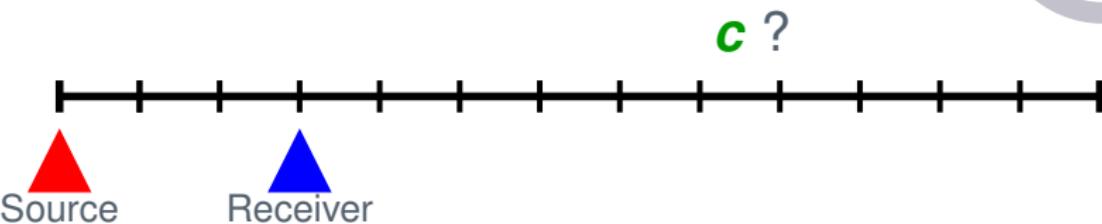
First year PhD Student
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Pau, FRANCE

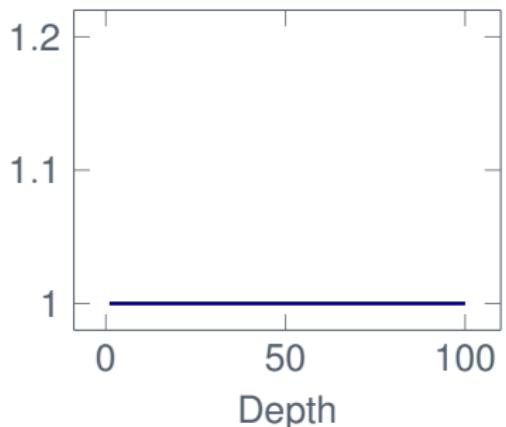


Outline

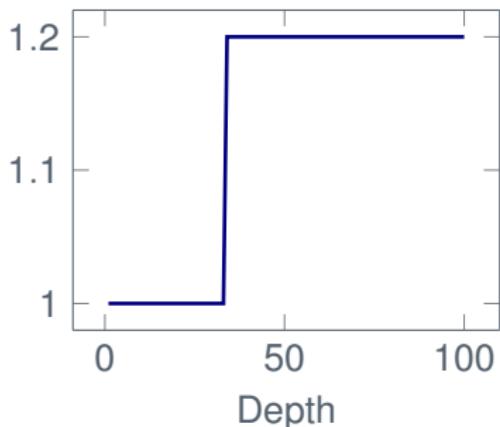
1D Preliminary tests



Initial c Model



Target c Model



1D Preliminary tests :

1D FWI :

- ▶ Lagrange / B-Bézier Operators
- ▶ RK4 / AB3 time-schemes

Gradient expression :

$$\nabla_{\mathbf{c}} \mathcal{J} = - \int_0^T \int_{\Omega} \frac{2}{\rho \mathbf{c}^3} \frac{\partial \mathbf{p}}{\partial t} \lambda_1 d\Omega dt$$

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Adjoint test passed with :

- ▶ With a canonical space inner-product
 $(\langle u, v \rangle_x = \sum_i u_i v_i)$
- ▶ With a M-space inner product
 $(\langle u, v \rangle_X^M = \langle Mu, v \rangle_x)$

```
./run
```

```
--- Adjoint test ----
```

```
inner product U/D 553123.57586755091
```

```
inner product G/Q 553123.57586756046
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Gradient expression :

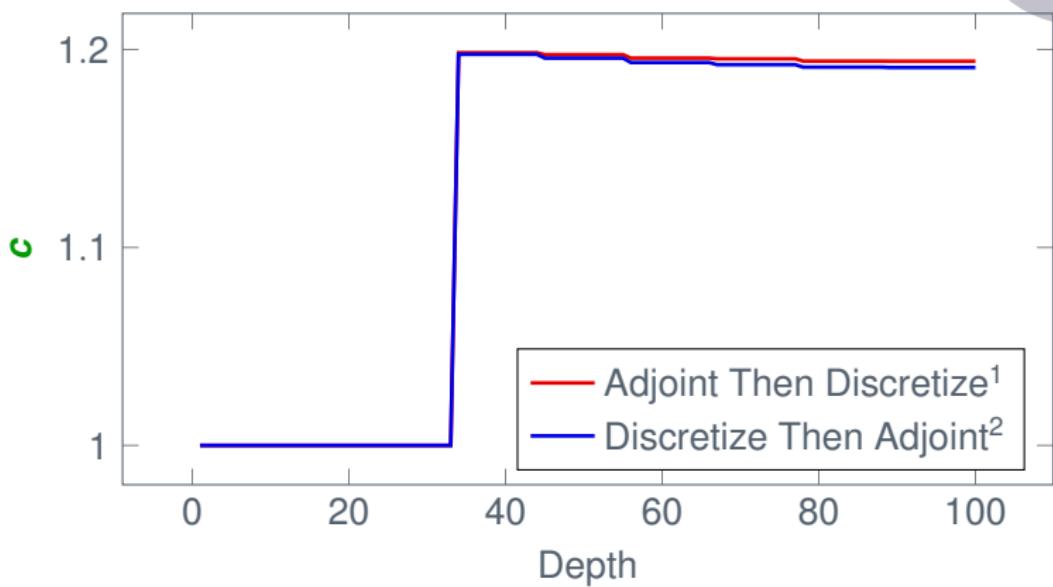
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```
./run
--- Adjoint test ----
inner product U/D 553123.57586755091
inner product G/Q 553123.57586756046

./run
--- Adjoint test ----
inner product U/D -75077.332007383695
inner product G/Q -75077.332007386358

./run
--- Adjoint test ----
inner product U/D 125669.89223600870
inner product G/Q 125669.89223600952
```

1D Velocity Model Reconstructions



c Model at the 100th FWI iteration

¹With Bernstein-Bézier elements

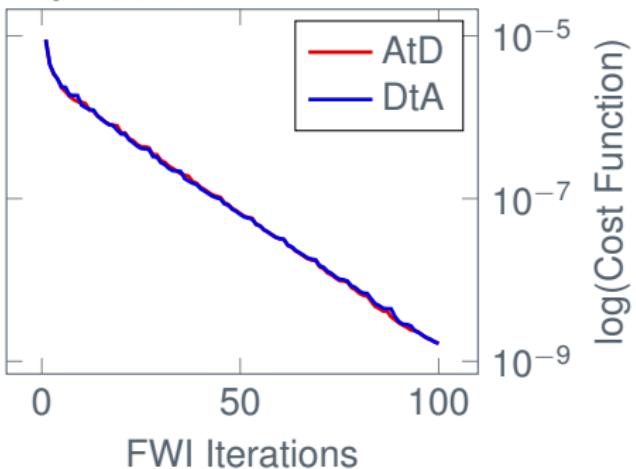
²With canonical scalar product

1D Velocity Model Reconstructions

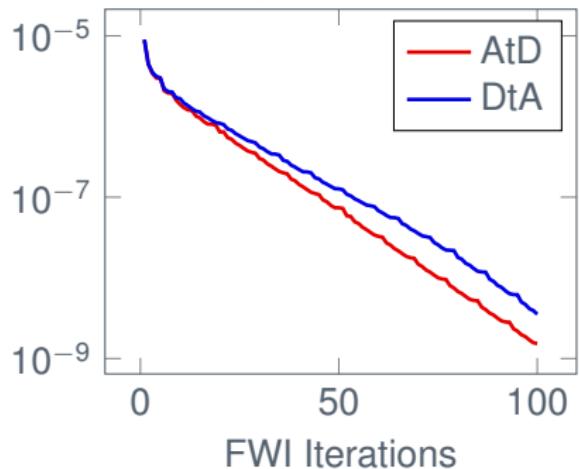


5

With RK4 :



With AB3 :

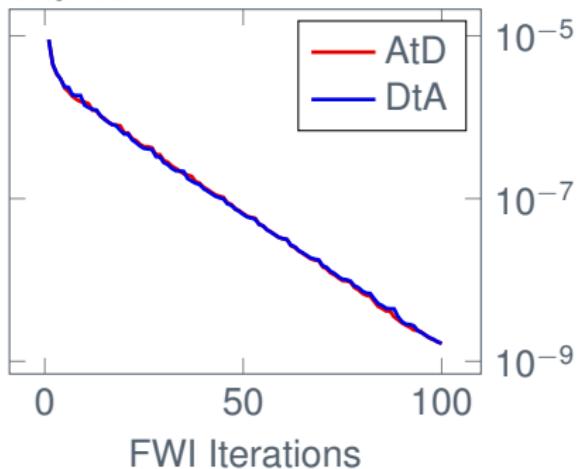


1D Velocity Model Reconstructions

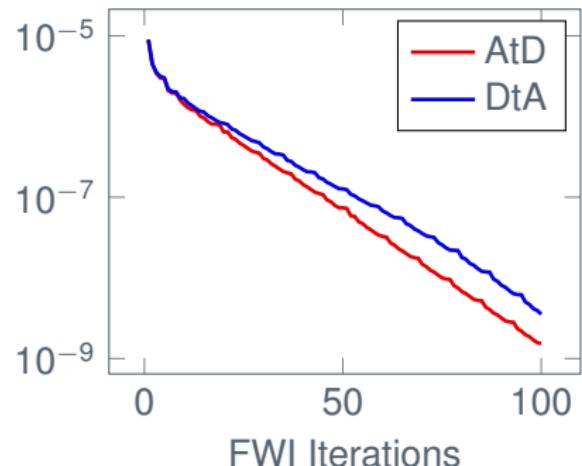


5

With RK4 :



With AB3 :



- ▶ For RK4 scheme : Similar convergency
- ▶ For AB3 scheme : **AtD** is slightly better than **DtA**
- ▶ The slope strongly depends on the optimizer -> Impossibility to conclude

2D Time Domain Reconstruction



2D FWI :

- ▶ Developped in Total environnement (DIP³)
- ▶ Nodal Space Operators (Lagrangian/Jacobian)
- ▶ Modal Space Operators (Bernstein-Bézier)
- ▶ Runge Kutta 2/4 and Adams Bashforth time-schemes

Discretize Then Adjoint strategy not implemented :

- ▶ Tremendous task in a complex industrial code

³<http://dip.inria.fr/>

2D Time Domain Reconstruction

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Gradient expression :

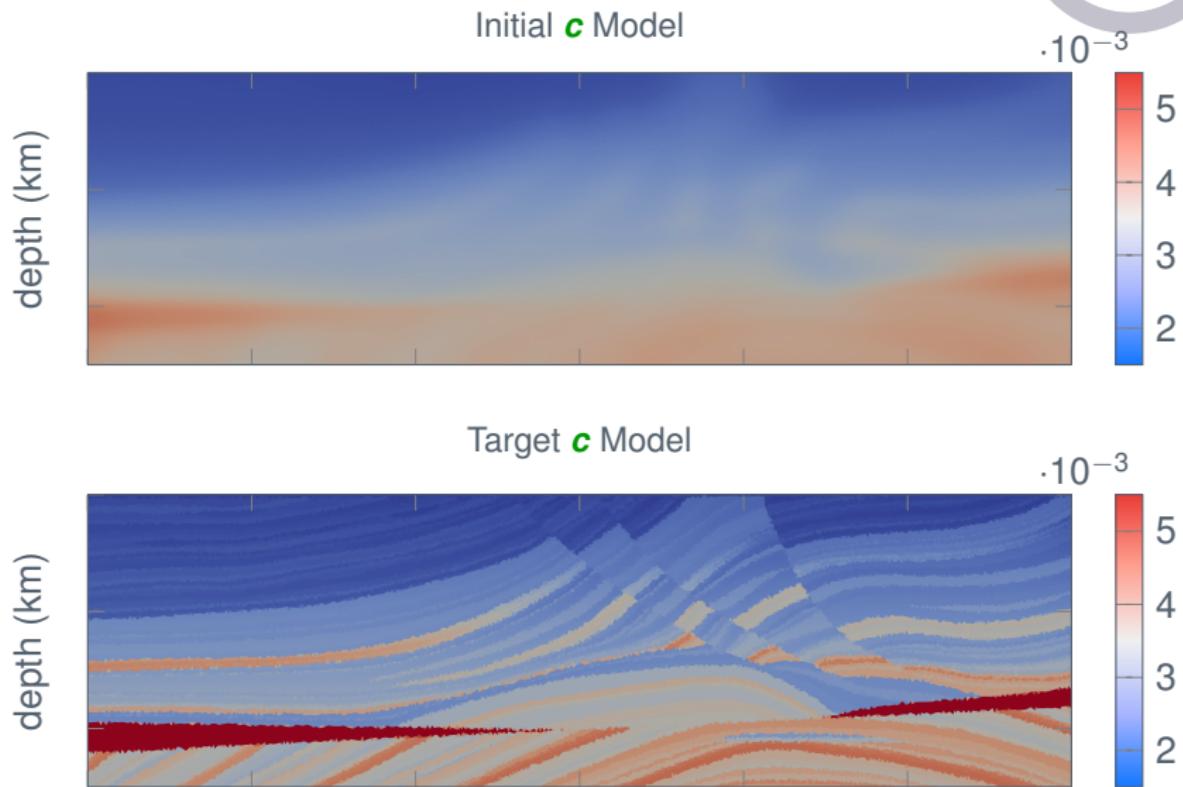
$$\nabla_{\frac{1}{\kappa}} \mathcal{J} = \int_0^T \int_{\Omega} \frac{\partial \mathbf{p}}{\partial t} \boldsymbol{\lambda}_1 d\Omega dt \quad \text{with : } \kappa = \rho \mathbf{c}^2$$

\mathbf{c} , ρ and κ Constant per elements

⁴<http://dip.inria.fr/>

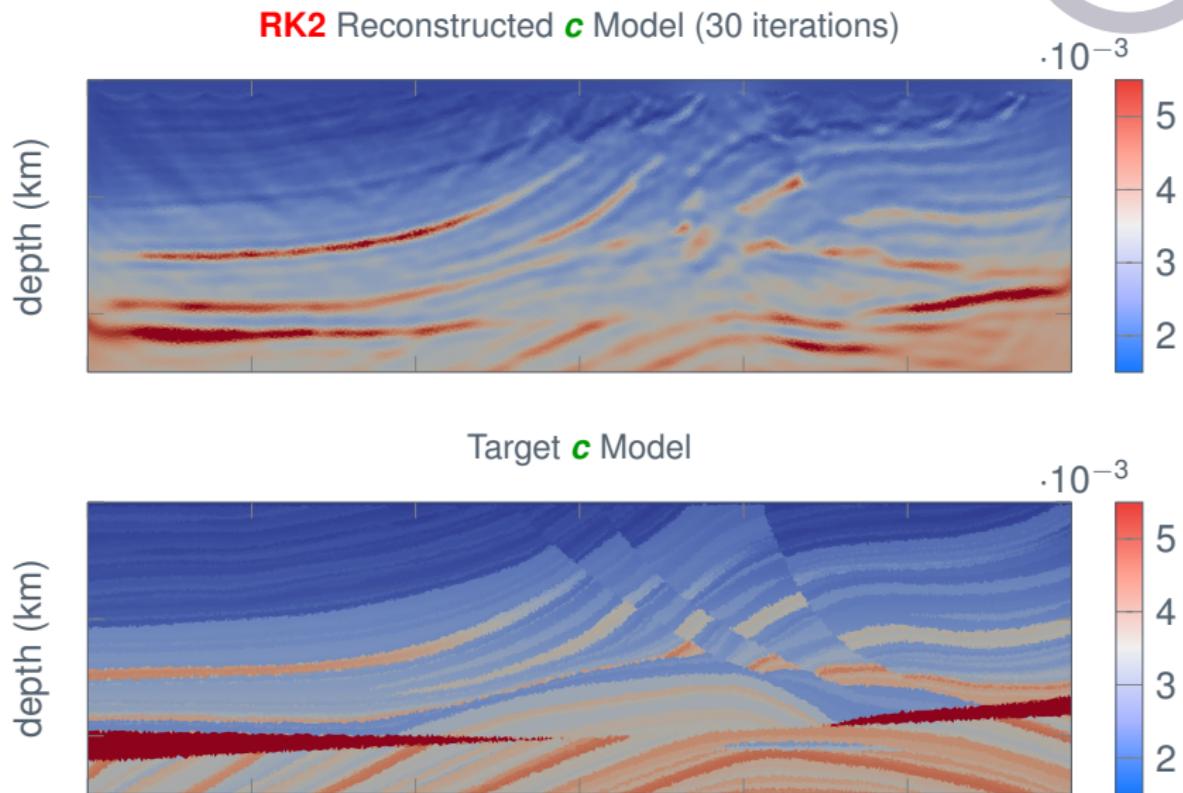
2D Time Domain FWI Reconstructions

Time-schemes comparison



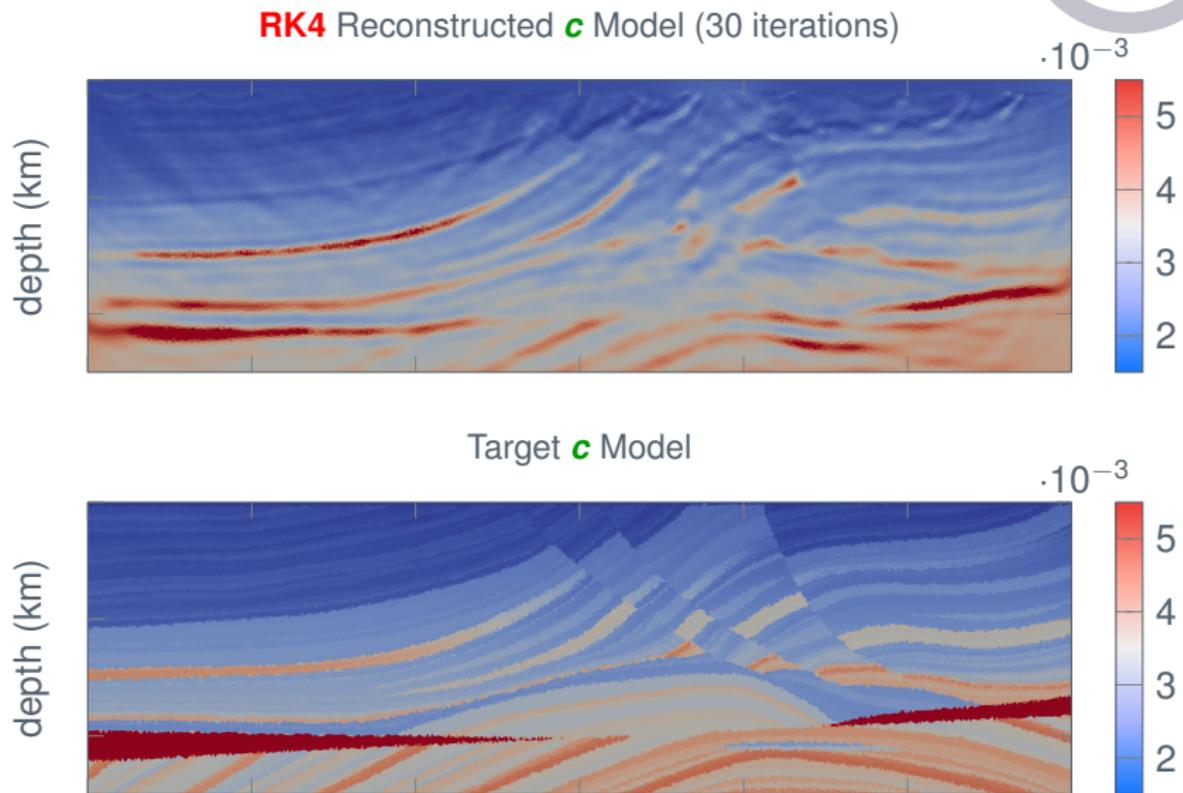
2D Time Domain FWI Reconstructions

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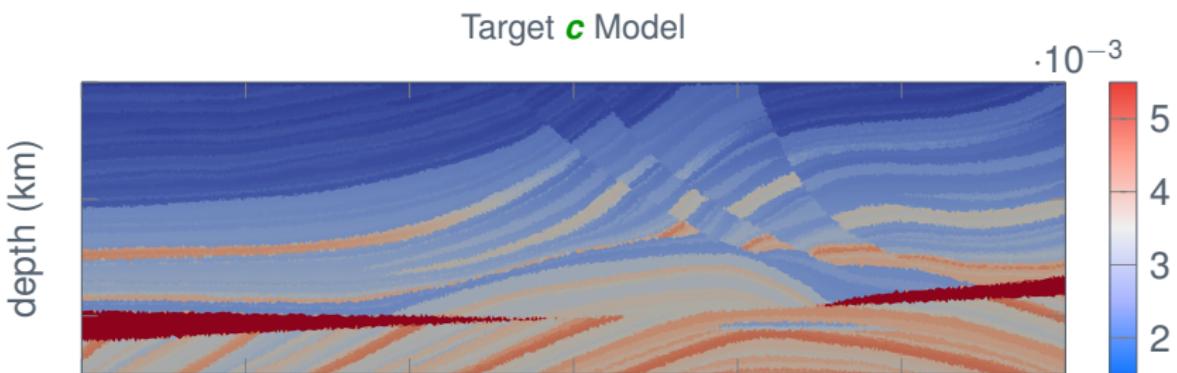
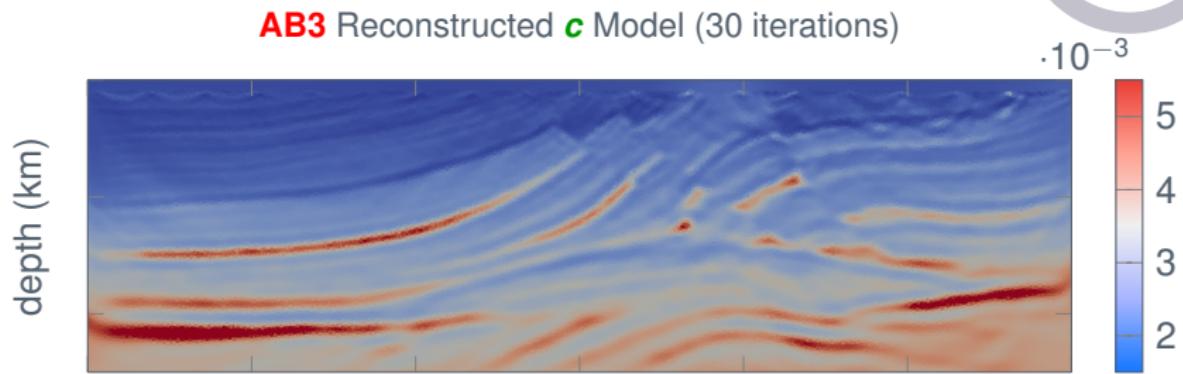
2D Time Domain FWI Reconstructions

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2D Time Domain FWI Reconstructions

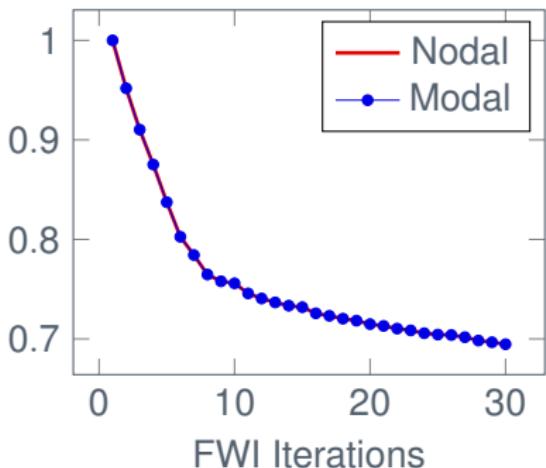
Nodal/Modal Comparison



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- ▶ 47k P1 elements
- ▶ Time Scheme : AB3
- ▶ Constant ρ model ($\rho = 1$)
- ▶ 19 sources / 181 Receivers
- ▶ 30 iterations
- ▶ 120 cores
- ▶ Nodal computation time :
5h10
- ▶ Modal computation time :
7h10^[1]

Cost function evolution :

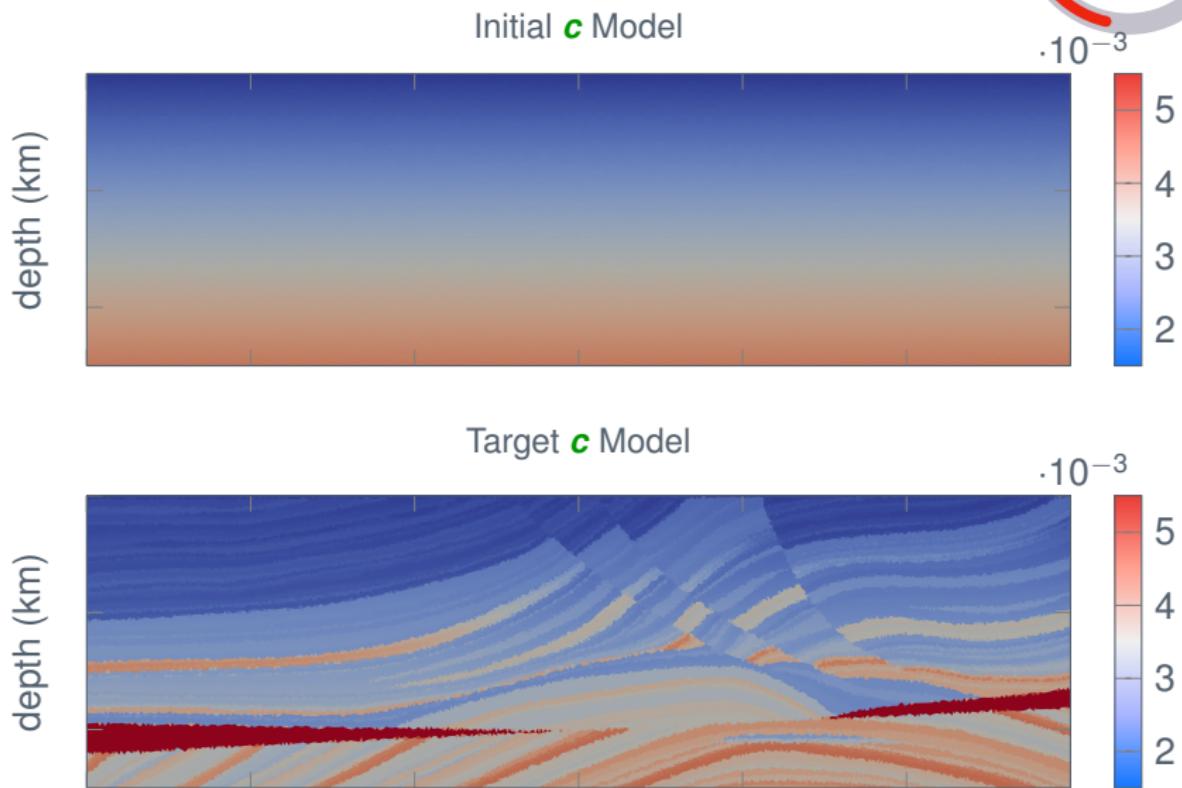


[1] Chan J. and Warburton T.

GPU-Accelerated Bernstein Bézier Discontinuous Galerkin Methods for Wave Problems
SIAM Journal on Scientific Computing 2017

2D Multiscale Reconstructions

Reconstruction with an initial smooth model



2D Multiscale Reconstructions

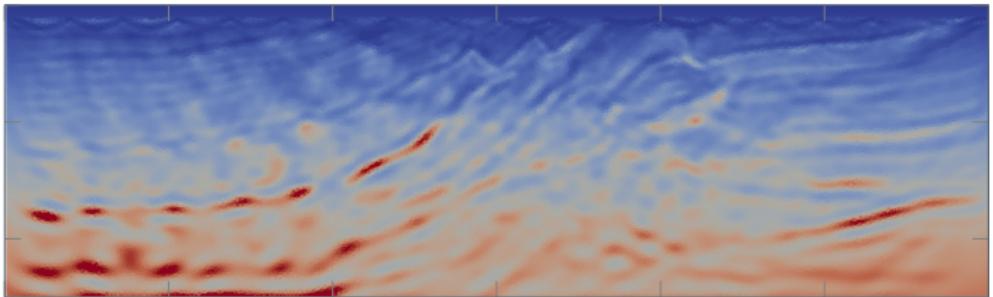
Reconstruction with an initial smooth model



depth (km)

Reconstructed model Model (30 iterations AB3)

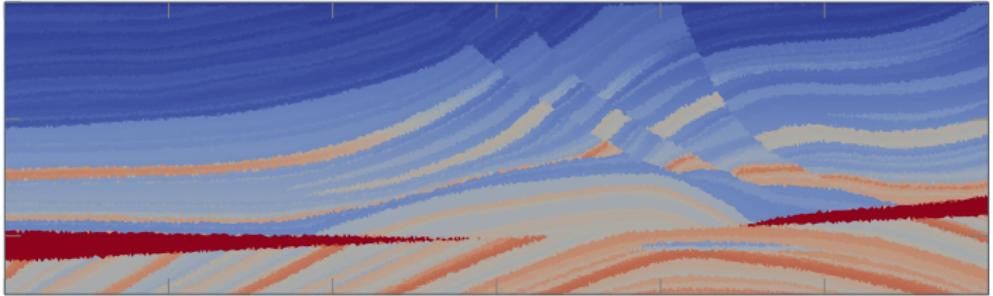
$\cdot 10^{-3}$



depth (km)

Target Model

$\cdot 10^{-3}$

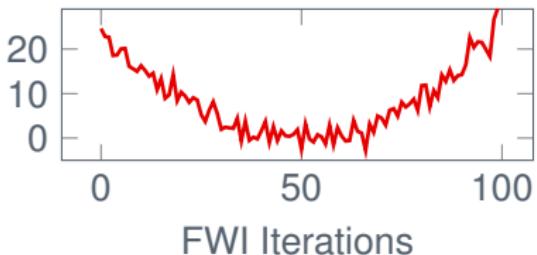


2D Multiscale Reconstructions

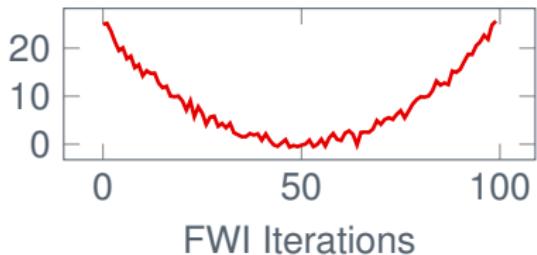
Multiscale Principle



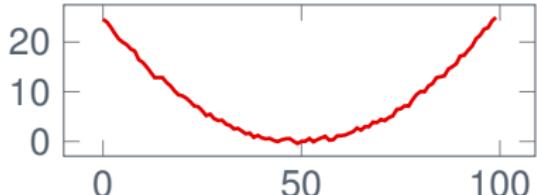
10



FWI Iterations



FWI Iterations



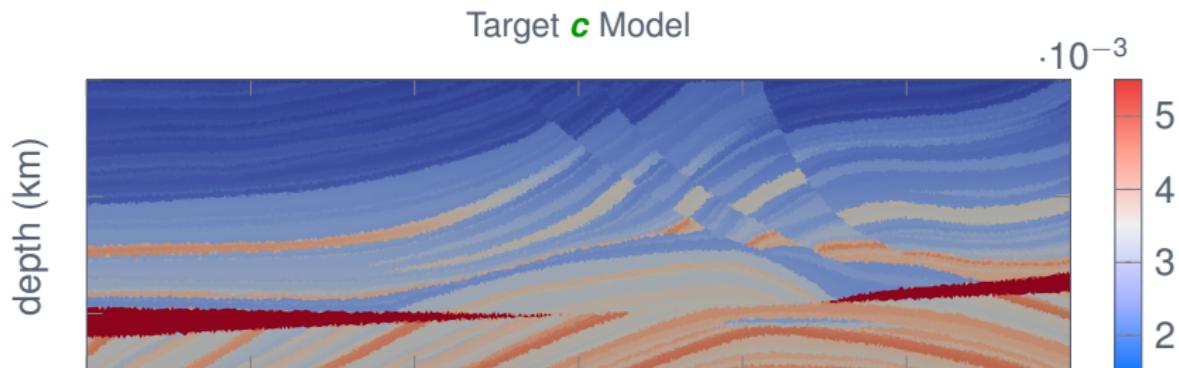
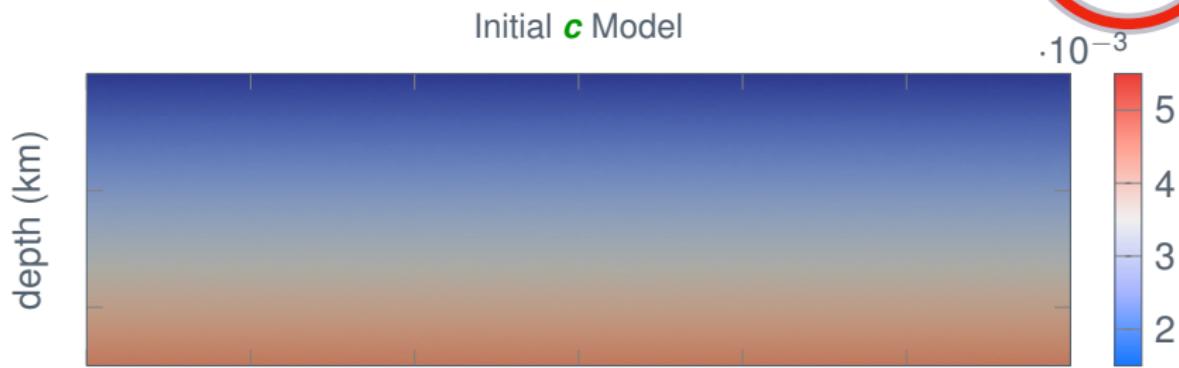
Time Domain FWI involving DG approximation

2D Multiscale Reconstructions

Reconstruction with an initial smooth model



11



2D Multiscale Reconstructions

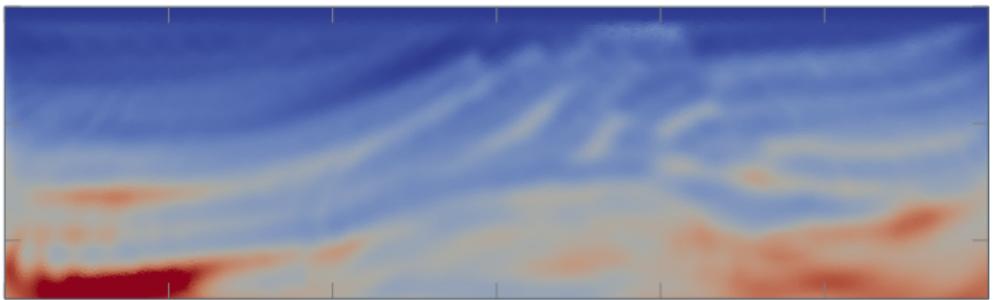
Reconstruction with an initial smooth model



11

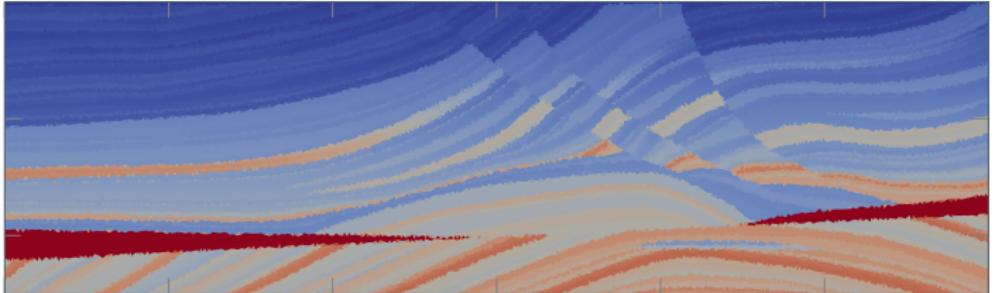
Reconstructed c Model with 1.0-2.5Hz filter

depth (km)



Target c Model

depth (km)



2D Multiscale Reconstructions

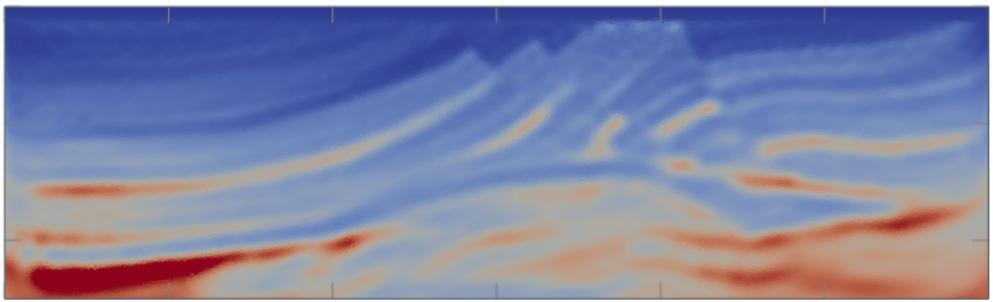
Reconstruction with an initial smooth model



11

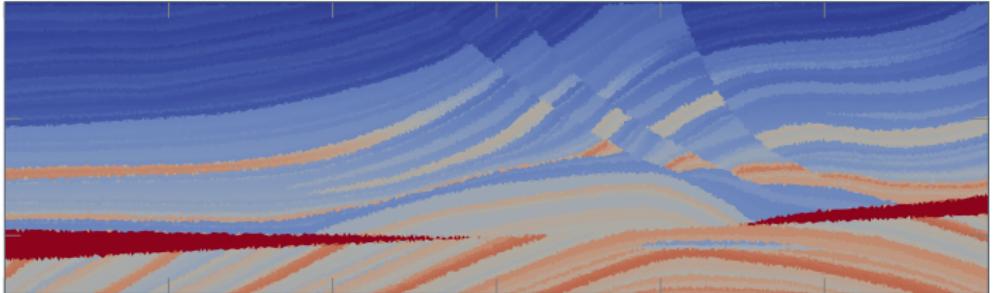
Reconstructed **c** Model with 1.0-7.5Hz filter

depth (km)



Target **c** Model

depth (km)



2D Multiscale Reconstructions

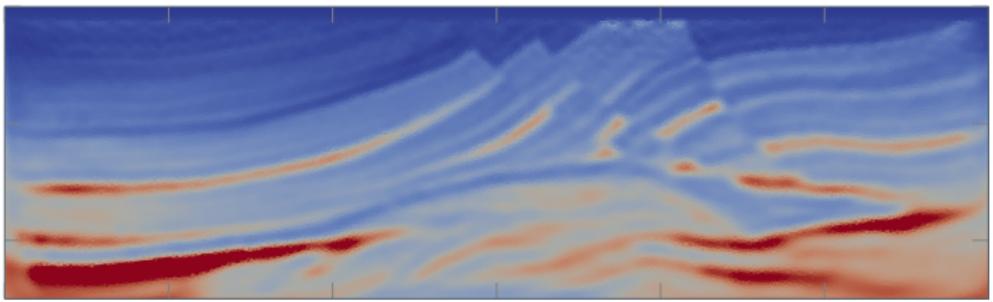
Reconstruction with an initial smooth model



11

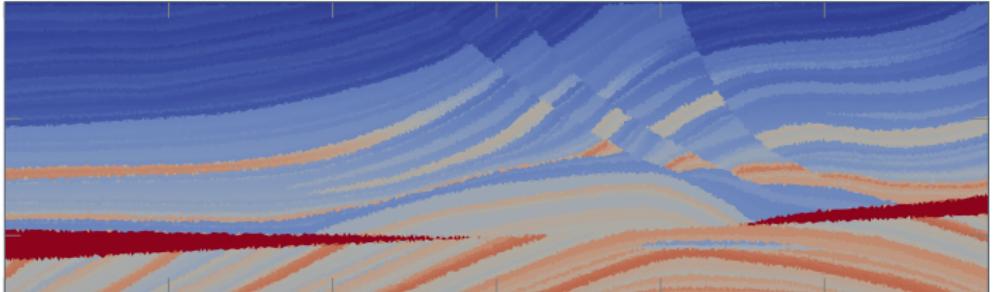
Reconstructed **c** Model with 1.0-10Hz filter

depth (km)



Target **c** Model

depth (km)



2D Multiscale Reconstructions

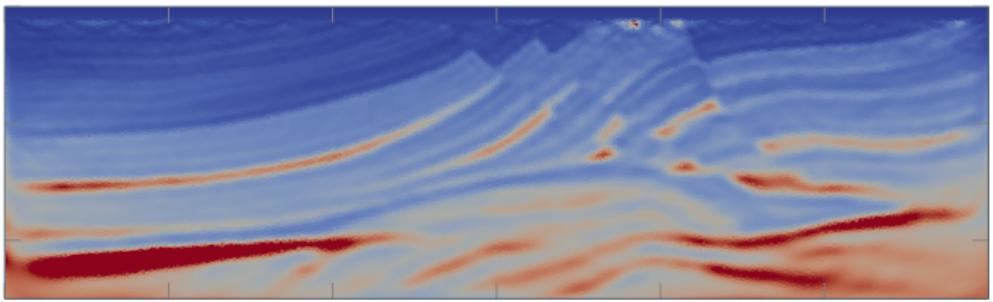
Reconstruction with an initial smooth model



11

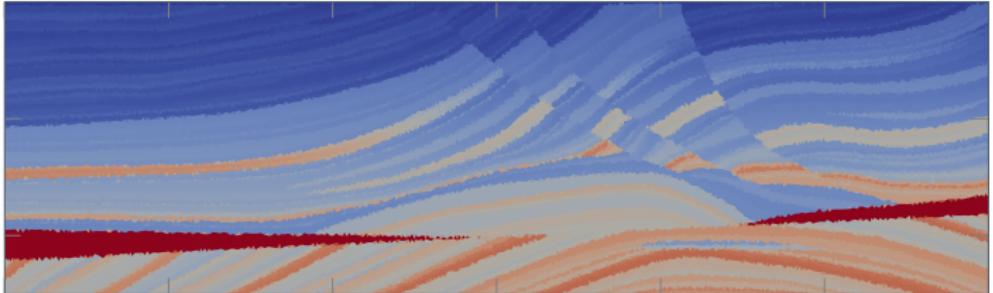
Reconstructed **c** Model with 1.0-15Hz filter

depth (km)



Target **c** Model

depth (km)



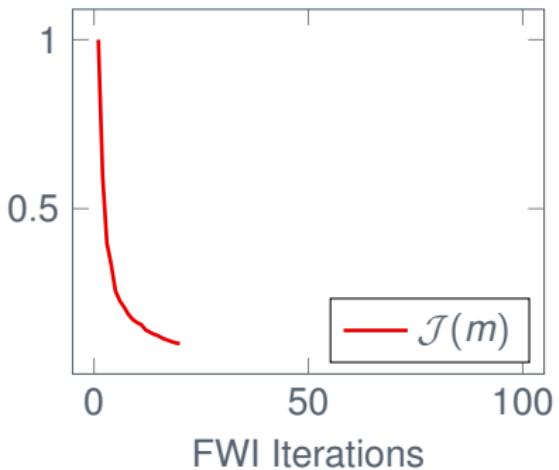
2D Multiscale Reconstructions



12

- ▶ 47k P1 elements
- ▶ Time Scheme : AB3
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- ▶ Computation time : 17h
- ▶ Frequencies : 1-2.5Hz

Cost function evolution :



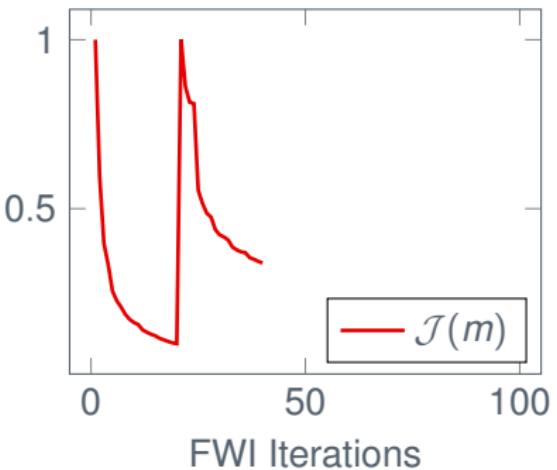
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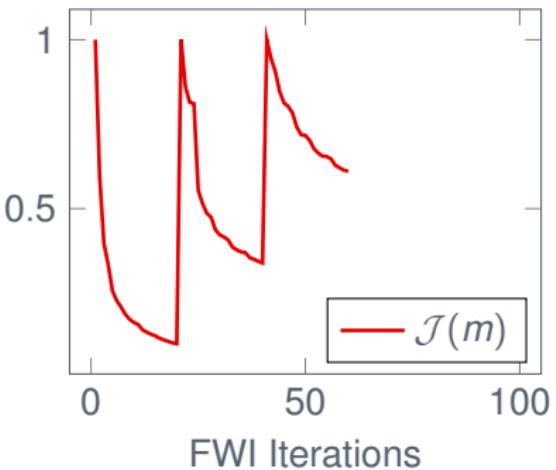
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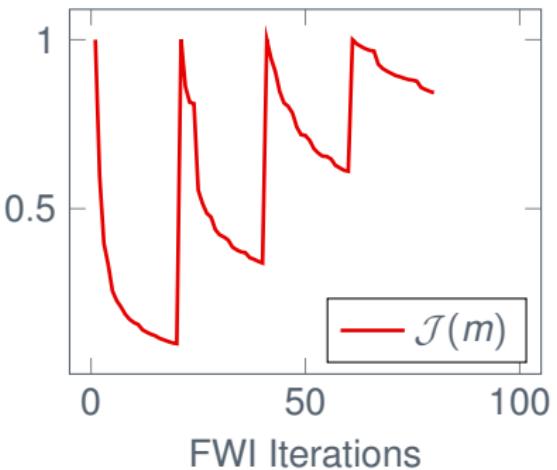
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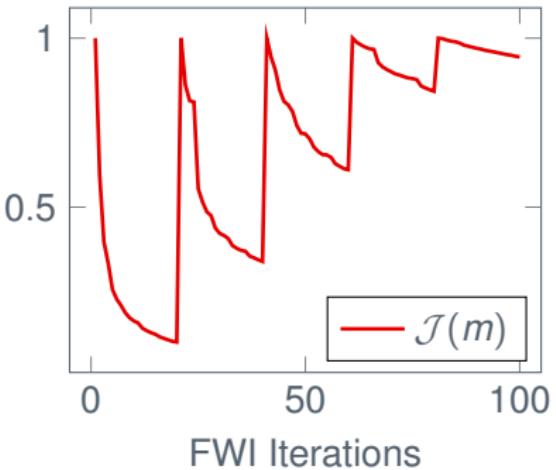
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1-5Hz / 1-7.5Hz / 1-10Hz /
1-15Hz

Cost function evolution :



Conclusion



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conclusion