Time Domain Full Waveform Inversion (FWI)

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Outline

PhD statred the 6^{th} of November 2017 :

- First main axis : The Direct Problem

- Second main axis : The Inverse Problem

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- First main axis : The Direct Problem
  - Coupling SEM & DG (Aurélien's thesis)
  - Familiarization with DG formulation and implementation
  - Comparison between Lagrange and Bernstein polynomial elements
- Second main axis : The Inverse Problem

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- Second main axis : The Inverse Problem
  - Adjoint state equation
  - Gradient formulation
  - Strategy for the implementation in Total environment

The Direct Problem
Discontinuous Galerkin formulation

First order Acoustic Wave equation discretisation :

Continuous Problem :

$$\begin{cases} \rho_0 \dfrac{\partial \vec{u}}{\partial_t} + \overrightarrow{grad}(p) = 0 \\ \dfrac{\partial p}{\partial_t} + \kappa_0 div(\vec{u}) = 0 \end{cases}$$

Discrete Problem :

$$\begin{cases} P^{n+\frac{1}{2}} = P^{n-\frac{1}{2}} + A_p U^n + RHS_p \\ U^{n+1} = U^n + A_u p^{n+\frac{1}{2}} + RHS_u \end{cases}$$

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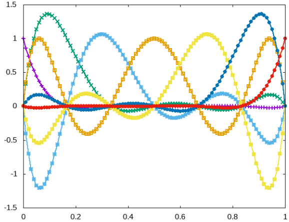

Figure: P[X^6] Lagrange basis on [0:1]

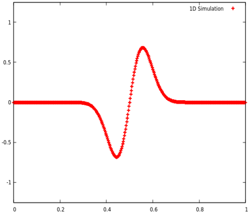

Figure: First 1D DG Simulation

Berstein formulation :

$$B_{ijkl}^{N} = C_{ijkl}^{N} \lambda_0^{i} \lambda_j^{j} \lambda_2^{k} \lambda_3^{l} \quad \text{with :} \quad C_{ijkl}^{N} = \frac{N!}{i!j!k!l!}$$

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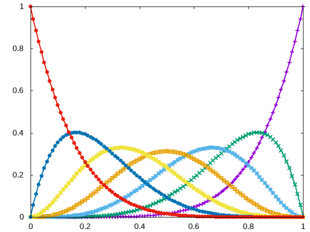

P[X^6] Berstein basis

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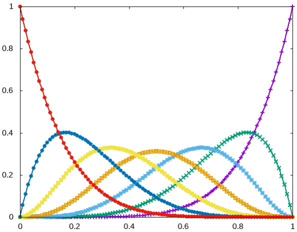
$$B^N_{ijkl} = C^N_{ijkl}\lambda_0^i\lambda_j^j\lambda_2^k\lambda_3^l \quad \text{with :} \; C^N_{ijkl} = \frac{N!}{i!j!k!l!}$$

Easy Derivative expression :

$$\frac{\partial B^N_\alpha}{\partial \lambda_p} = NB^{N-1}_{\alpha - e_p} \quad \text{with :} \; \alpha = (i, j, k, l)$$



$P[X^6]$ Berstein basis

Berstein formulation :

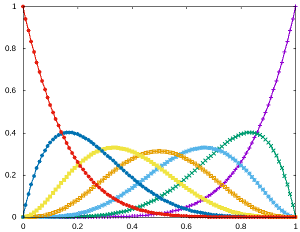
$$B^N_{ijkl} = C^N_{ijkl} \lambda_0^i \lambda_j^j \lambda_2^k \lambda_3^l \quad \text{with : } C^N_{ijkl} = \frac{N!}{i!j!k!l!}$$

Easy Derivative expression :

$$\frac{\partial B^N_\alpha}{\partial \lambda_p} = N B^{N-1}_{\alpha - e_p} \quad \text{with : } \alpha = (i, j, k, l)$$

Sparse Degree Elevation operator :

$$B^{N-1}_\alpha = \sum_{p=0}^{d} \frac{\alpha_p + 1}{N} B^N_{\alpha + e_p}$$



$P[X^6]$ Berstein basis

Berstein formulation :

$$B^N_{ijkl} = C^N_{ijkl}\lambda_0^i\lambda_j^j\lambda_2^k\lambda_3^l \quad \text{with :} \quad C^N_{ijkl} = \frac{N!}{i!j!k!l!}$$

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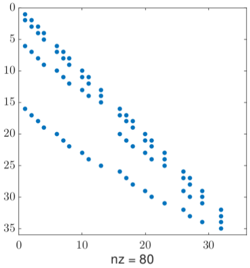

3D D_0 Profil, from J. Chan & T. Warburton [1]

First order Acoustic Wave equation discretisation :
Discrete Problem :

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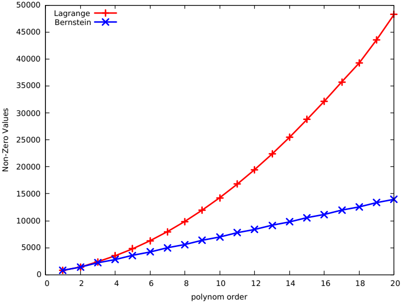
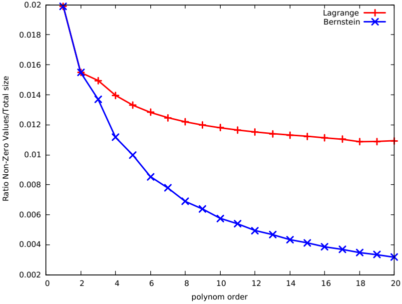


A_p/A_u NZVs as a function of the order



A_p/A_u NZVs Sparse $(NZVs/Size)$

Inverse Problem

FWI Gradient formulation

$$\begin{cases} M_\alpha \partial_t^2 u - \Delta u - f = 0 \text{ on } \Omega \\ u|_{t=0} = 0 \\ \partial_t u|_{t=0} = 0 \\ u|_{\Gamma_1} = 0 \text{ on } \Gamma_1 \\ \partial_t u|_{\Gamma_2} + c\nabla u|_{\Gamma_2}.\vec{n} = 0 \text{ on } \Gamma_2 \\ f = \text{boundaries+sources signal} \end{cases}$$

$$\begin{cases} M_\alpha \partial_t^2 \lambda - \Delta \lambda - f' = 0 \text{ on } \Omega \\ \lambda|_{t=T} = 0 \\ \partial_t \lambda|_{t=0} = 0 \\ \lambda|_{\Gamma_1} = 0 \text{ on } \Gamma_1 \\ \partial_t \lambda|_{\Gamma_2} + c\nabla \lambda|_{\Gamma_2}.\vec{n} = 0 \text{ on } \Gamma_2 \\ f' = -R^* R(u(T-t,(\alpha)) - d) \end{cases}$$

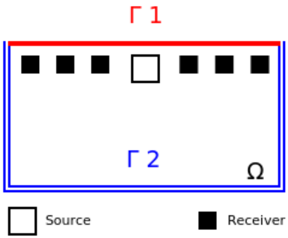


Figure: Domain Ω

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$$\partial_\alpha J(u,\alpha) = \int_0^T (< \partial_\alpha (M_\alpha) \partial_t^2 u(t), \lambda(T-t) >_\Omega) dt$$

FWI gradient [2] :

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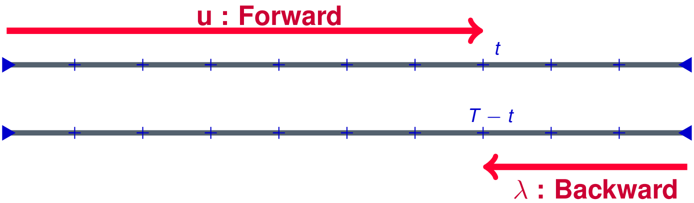
RTM Imaging Condition [3]:

$$Im = \int_0^T (< u(t), \lambda(T-t) >_\Omega) dt$$



Conclusion

- Direct Problem familiarization
  - Discontinuous Galerkin
  - Bernstein polynomials properties and assets

Conclusion

- Direct Problem familiarization
  - Discontinuous Galerkin
  - Bernstein polynomials properties and assets
- Inverse Problem
  - Theoretical approach
  - First implementation via RTM in order to accustom with Total environment
  - Then FWI in collaboration with Total engineers

Bibliography

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