State Estimation - Assignment 2

Parham Jafary

June 5, 2023

Abstract

In this assignment, our objective is to perform position estimation using a Kalman Filter (KF) and an Extended Kalman Filter (EKF) for a linear and a non-linear system respectively. Our system is a simplified robot and we have measurement models instead of sensors. Randomness (noise) was added both to the robot's motion model (representing the actuator noise) and to the measurement model (representing the sensor noise). The code is written in Python language and the PyGame library is used for simulation and visualization purposes.

1 Kalman Filter

1.1 Question

In this question we are given the 2D robot model below and we are asked to simulate it and perform state estimation using a Kalman Filter.

$$\begin{aligned} \dot{x} = \frac{r}{2}(u_r + u_l) + w_x \ , \ \dot{y} = \frac{r}{2}(u_r + u_l) + w_y \\ w_x = N(0, 0.1) \ , \ w_y = N(0, 0.15) \ , \ r = 0.1 \end{aligned} \tag{1}$$

the speed of the wheel is constant and equal to $0.1m/s$ and our initial values for x, y, and covariance matrix are:

$$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad , \quad P_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{2}$$

and our measurement is given by:

$$\begin{aligned} z = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} r_x \\ r_y \end{bmatrix} \\ r_x = N(0, 0.05) \\ r_y = N(0, 0.075) \end{aligned} \tag{3}$$

1.2 Solution

The overall picture for this question is that we have a robot model, a sensor model, Gaussian noise for the actuators, and Gaussian nose for the sensor and the objective is to create a KF to estimate the position of the robot. In order to do so, we need to form the position equations and the KF algorithm, the required steps are described below. The first step is the motion model; we have the $\dot{x}, \dot{y}$ therefore we need to integrate to calculate the x, y. Since we have constant speed the integration can be done by multiplying the $\dot{x}, \dot{y}$ by δt . Using Eq. 1, we can write the state of our robot in matrix form as follows:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} r\delta t/2 & r\delta t/2 \\ r\delta t/2 & r\delta t/2 \end{bmatrix} \begin{bmatrix} u_r \\ u_l \end{bmatrix} + \begin{bmatrix} w_x \delta t \\ w_y \delta t \end{bmatrix} \tag{4}$$

Now that we have x, y we need to discretize the Eq. 4 to make it suitable for coding purposes. We also need to add the previous position to the equation.

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} r\delta t/2 & r\delta t/2 \\ r\delta t/2 & r\delta t/2 \end{bmatrix} \begin{bmatrix} u_r \\ u_l \end{bmatrix} + \begin{bmatrix} w_x^t \delta t \\ w_y^t \delta t \end{bmatrix} \tag{5}$$

Since the speed of the wheels is constant, we can solve for out control inputs and find them to be $u_r = u_l = 1$. To plot the true position, we can omit the noise in the Eq. 5

The next step is the sensor or the so called measurement model. Discretizing Eq. 3 gives our measurment model and since the measurement does not rely on the the previous data, the equation stays exactly the same but with a t index.

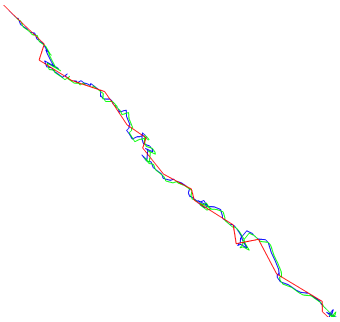
The KF algorithm as suggested in the Probabilistic Robotics reference is:

$$\begin{aligned} \bar{\mu}_t &= A_t \mu_{t-1} + B_t u_t \\ \bar{\Sigma}_t &= A_t \Sigma_{t-1} A_t^T + R_t \\ K_t &= \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1} \\ \mu_t &= \bar{\mu} + K_t (z_t - C_t \bar{\mu}_t) \\ \Sigma_t &= (I - K_t C_t) \bar{\Sigma}_t \end{aligned} \tag{6}$$

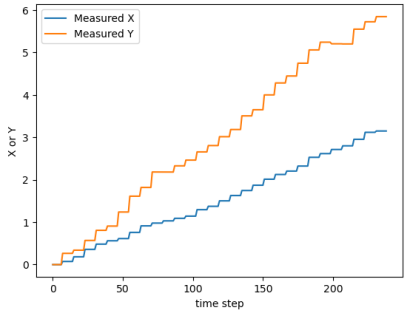
based on the values given in the question we have:

$$\begin{aligned} A_t &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad , \quad B_t = \begin{bmatrix} r\delta t/2 & r\delta t/2 \\ r\delta t/2 & r\delta t/2 \end{bmatrix} \quad , \quad R_t = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.15 \end{bmatrix} \\ C_t &= \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \quad , \quad Q_t = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.75 \end{bmatrix} \end{aligned} \tag{7}$$

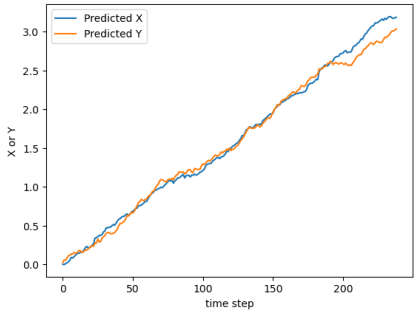
Simulating the system with the above value yields the following figures for the predicted trajectory, measured position, predicted position, and calculated covariance (the video clip is uploaded at GitHub).



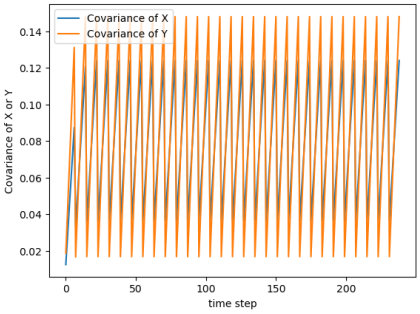
(a) Predicted, Measured, and True trajectories



(b) Measured Position



(c) Predicted Position



(d) Calculated Covariance

Figure 1: Results from Kalman Filter

In Fig. 1-a, we can see the predicted, true, and measured trajectories, in blue, green, and red colors respectively. In sub-figure a) we can see the measured x, y positions and in sub-figure c) we can see the predicted x, y positions. Sub-figure d) shows the computed covariance matrices at each time step. We can observe that as new data comes, the covariance values change, causing the Kalman gain to change that leads to an updated prediction that is more accurate.

2 Extended Kalman Filter

2.1 Question

Repeat the previous assignment, this time with a classic motion model and range observations made from a landmark located at M = [10,10]. L is the distance between the wheel, known as wheelbase, and is 0.3m. Program the robot such that it loops around point M

$$\begin{aligned}
&\dot{x} = \frac{r}{2}(u_r + u_l)\cos\theta + w_x \ , \ \dot{y} = \frac{r}{2}(u_r + u_l)\sin\theta + w_y \ , \ \dot{\theta} = \frac{r}{L}(u_r - u_L) \\
&u_\omega = \frac{1}{2}(u_r + u_l) \ , \ u_\psi = (u_r - u_l) \\
&\dot{x} = r u_\omega \cos\theta + w_\omega \ , \ \dot{y} = r u_\omega \sin\theta + w_\omega \ , \ \dot{\theta} = \frac{r}{L}u_\psi + w_\psi \\
&w_\psi = N(0, 0.01) \ , \ w_\omega = N(0, 0.1) \ , \ r = 0.1
\end{aligned} \tag{8}$$

(a) Compute the EKF with the linear measurement model in the previous assignment.
(b) Compute the EKF with range/bearing measurements of point M. Assume range noise is N(0,0.1) and bearing noise is N(0,0.01). Range is in meters, and bearing is in radians. Visualize the measurements as well.

2.2 Solution

We are asked to make the robot to go around point M around. The procedure is the same as part 1, and we start by discretizing, integrating the state variables, and writing the state equations in matrix format:

$$\begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} = \begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{bmatrix} + \begin{bmatrix} r\cos\theta\delta t & 0 \\ r\sin\theta\delta t & 0 \\ 0 & \frac{r}{L}\delta t \end{bmatrix} \begin{bmatrix} u_\omega^t \\ u_\psi^t \end{bmatrix} + \begin{bmatrix} w_\omega^t \delta t \\ w_\omega^t \delta t \\ w_\psi^t \delta t \end{bmatrix} \tag{9}$$

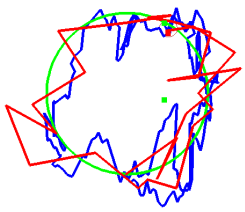
We want to program the robot to stay on a circular trajectory around point $M = (10, 10)$, and the starting position is $x_0 = (0, 0)$ therefore, left and right wheels' control input should be adjusted to keep a distance of 10 from M.

For part a, since the sensor's model is the same as question 1, the rest of the algorithm is the same but the matrices are different as shown in Eq. 10.

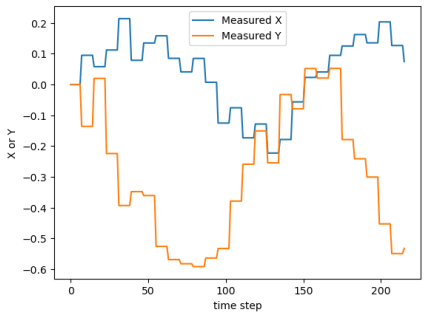
$$\begin{aligned}
A_t &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad , \quad B_t = \begin{bmatrix} r\delta t\cos\theta & 0 \\ r\delta t\sin\theta & 0 \\ 0 & r\delta t/L \end{bmatrix} \quad , \quad R_t = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.01 \end{bmatrix} \\
C_t &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad , \quad Q_t = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.75 \end{bmatrix}
\end{aligned} \tag{10}$$

The results from the simulation are illustrated in Fig.2.

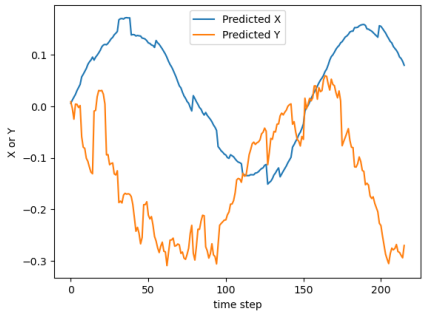
Same as question 1, when a new measurement arrives, the covariance changes and causes the prediction to change and become more accurate.



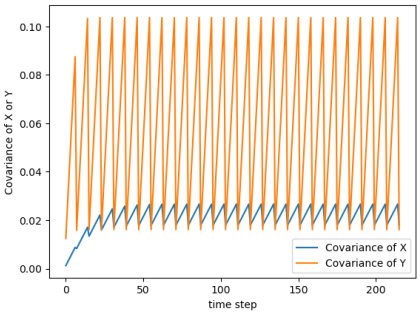
(a) Predicted, Measured, and True trajectories



(b) Measured Position



(c) Predicted Position



(d) Calculated Covariance

Figure 2: Results from Kalman Filter

For part b, we need to convert the measured data to a range and bearing measurement, which is basically a polar representation of the robot's position with origin located at point M. Then, we need to update the control algorithm with the range and bearing measurement. The conversion formula are given in the equation below:

$$\begin{aligned} \rho &= \sqrt{(x - x_M)^2 + (y - y_M)^2} \\ \theta &= \arctan\,(y - y_M)/(x - x_M) \end{aligned} \tag{11}$$