CKForms - Manual for version 2.x

## Notation and convention

This instruction descibes the functionality associated with the following article ([Bochenski, Jastrzebski, and Tralle 2021](#ref-BJT2021))

We use the notation of the theory of real Lie algebras from CoReLG Package, ([Dietrich, Faccin, and Graaf 2014](#ref-CoReLG)).

## Function for real Lie algebras

RealRank(g)

The input is a real Lie algebra (as an Lie algebra object). The output is the real rank of (the dimension of the Cartan subalgebra of ).

AHypRank(g)

The input is a real Lie algebra (as an Lie algebra object). The output is the a-hyperbolic rank of .

Example:

gap> g:=RealFormById("A",5,6);  
<Lie algebra of dimension 35 over SqrtField>  
gap> NameRealForm(g);  
"sl(6,R)"  
gap> RealRank(g);  
5  
gap> AHypRank(g);  
3

## Main procedure

As in Theorem 6 in ([Bochenski, Jastrzebski, and Tralle 2021](#ref-BJT2021)) we are checking three conditions:

* - Calabi–Markus phenomenon,
* -
* -
* - none of the above conditions is met

gap> CheckRankConditions("A",5,6);  
g=sl(6,R) | real rank(g)=5 | a-hyp rank(g)=3  
----------------------------  
#1: h=sl(3,R)+sl(3,R) + a torus of 1 non-compact dimensions | real rank(h)=5 | ahyp rank(h)=2  
 | L0-true | L1-false | L2-false | L3-false  
----  
#2: h=sl(3,C) + a torus of 1 compact dimensions | real rank(h)=2 | ahyp rank(h)=1  
 | L0-false | L1-false | L2-true | L3-false  
----  
#3: h=sl(2,R)+sl(4,R) + a torus of 1 non-compact dimensions | real rank(h)=5 | ahyp rank(h)=3  
 | L0-true | L1-true | L2-false | L3-false  
----  
#4: h=sl(5,R) + a torus of 1 non-compact dimensions | real rank(h)=5 | ahyp rank(h)=2  
 | L0-true | L1-false | L2-false | L3-false  
----  
#5: h=sl(3,R) | real rank(h)=2 | ahyp rank(h)=1  
 | L0-false | L1-false | L2-true | L3-false  
----  
#6: h=sl(2,R)+sl(3,R) | real rank(h)=3 | ahyp rank(h)=2  
 | L0-false | L1-false | L2-false | L3-true  
----  
#7: h=su(4) | real rank(h)=0 | ahyp rank(h)=0  
 | L0-false | L1-false | L2-true | L3-false  
----  
#8: h=su(2,2) | real rank(h)=2 | ahyp rank(h)=2  
 | L0-false | L1-false | L2-true | L3-false  
----  
#9: h=sl(2,H) | real rank(h)=1 | ahyp rank(h)=1  
 | L0-false | L1-false | L2-true | L3-false  
----  
#10: h=sl(4,R) | real rank(h)=3 | ahyp rank(h)=2  
 | L0-false | L1-false | L2-false | L3-true  
----  
#11: h=sp(3,R) | real rank(h)=3 | ahyp rank(h)=3  
 | L0-false | L1-true | L2-false | L3-false  
----

The next possible h are consecutive maximal subalgebras of g generated by the function MaximalReductiveSubalgebras from ([Dietrich, Faccin, and Graaf 2014](#ref-CoReLG)).

All calculations are being done in the “Database - v2” section.

Next if L3 is equal to true, then we check the condition for orbits as in Theorem 5 in ([Bochenski, Jastrzebski, and Tralle 2021](#ref-BJT2021)).

gap> CheckProperSL2RAction("A",5,6,6);  
proper

The last argument of the function CheckProperSL2RAction is an index of the maximal subalgebra (subalgs output MaximalReductiveSubalgebras).

## References

Bochenski, Maciej, Piotr Jastrzebski, and Aleksy Tralle. 2021. “Homogeneous Spaces of Real Simple Lie Groups with Proper Actions of Non Virtually Abelian Discrete Subgroups: A Calculational Approach.” <http://arxiv.org/abs/2106.05777>.

Dietrich, H., P. Faccin, and W. A. de Graaf. 2014. “CoReLG, Computation with Real Lie Groups, Version 1.20.” <http://users.monash.edu/~heikod/corelg/>.