Project: Reduced Basis Methods for Parametrized PDEs

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Models expressed as parametrized partial differential equations are ubiquitous throughout engineer- ing and the applied sciences as models for unsteady and steady heat and mass transfer, acoustics, solid and fluid mechanics, electromagnetics or problems of finance. In such models a number of input-parameters are used to characterize a particular problem and possible variations in its geo- metric configuration, physical properties, boundary conditions or source terms. The parametrized model implicitly connects these input parameters to outputs of interest of the model, e.g., a max- imum system temperature, an added mass coefficient, a crack stress intensity factor, an effective constitutive property, a waveguide transmission loss, a channel flowrate or a pressure drop, etc.

While the development of accurate computational tools to allow the solution of such problems clearly is of broad interest, we focus on problems in which the solution is sought for a large num- ber of parameters. Examples of typical applications of relevance are optimization, control, design, uncertainty quantification, real time query and others. In such cases it is not only the accuracy of the model that matters, but the computational efficiency of the model is likewise critical. Similar constraints emerge when real-time or near real-time responses are needed for rapid prototyping or computer animations relying on models of increasing physical accuracy, for instance.

In such situations, we need the ability to accurately and efficiently evaluate an output of interest when the input parameters are being varied. However, the complexity and computational cost associated with solving the full partial differential equation for each new parameter value rules out a direct approach. We must therefore seek a different approach that allows us to evaluate the desired output at minimal cost, yet without sacrificing the predictive accuracy of the complex model.

The goal of this project is to study the basic foundation for a class of methods, known as reduced basis methods, to accomplish this. As a convenient expository vehicle for the introduction of the methodology, we primarily consider the case of linear functional outputs of parametrized linear elliptic coercive partial differential equations. This class of problems, while relatively simple, is relevant to many important applications in transport (e.g., conduction and convection-diffusion, but also reaction) and continuum mechanics (e.g., linear elasticity, fluid mechanics). Furthermore, they serve as examples to follow when considering more complex applications.

The project will consist in the study of a selection of chapters of a monograph by J. Hesthaven and co-authors :

Certified Reduced Basis Methods for Parametrized Partial Differential Equations

We shall then consider the open problem of reduced basis to a class of transport equations : viscous Burgers equations, Airy equation, Korteweg de Vries equation.

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