1 Problems - Set 1

1.1 Kolmogorov axioms

Show that the definition of *frequentist* probability given in the lectures satisfies the three Kolmogorov axioms (I don't expect any very rigorous proofs). If you can also show this for the Bayesian definition, then have a go (but you don't need to in order to get the full marks)

1.2 No correlation does not mean independence

In the lectures, we said that two random variables which are independent will have a zero correlation coefficient.

1. Show that two continuous random variables X and Y, with $(X, Y) \sim f(X, Y)$ which are independent will have a 0 correlation coefficient.
2. Let X be a continuous random variable symmetrically distributed around 0 with a density function $f(X)$. Let $Y = X^2$. Show that despite the fact that Y and X are clearly dependent, their correlation coefficient is 0.

1.3 Moments of the Poisson distribution

In the lecture notes, we calculated the expectation and variance of the Poisson distribution with parameter λ are $E[k] = \lambda$ and $V(k) = \lambda$.

What is the third central moment of the Poisson distribution? You should show how you derive your answer.

1.4 Cauchy distribution

In the lectures, we showed how the sum of two Gaussian distributed random variables is itself Gaussian distributed. Suppose now that $X \sim \phi(X; 0, 1)$ and $Y \sim \phi(Y; 0, 1)$ are independent random variables. Show that the distribution of $U = \dfrac{X}{Y}$ is Cauchy, i.e that $p(U) = \dfrac{1}{\pi(1 + U^2)}$.

Hint: You should start by deriving the marginal distribution formula for the ratio of two independent random variables. Careful that the case $Y = 0$ will cause a problem, so split the marginal distribution into two cases, one for $Y > 0$ and one for $Y < 0$. The sum of these marginal distributions will be the total distribution.