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1. Abstract

Image compression is used to reduce the cost to save and transmit data over the internet and for storage purposes. For compressing images we use SVD which is a linear matrix transformation. The psycho visual redundancies (The information that is redundant or not useful) are eliminated to ensure compression without the loss of image quality. The Singular Value Decomposition represents image data in terms of number of eigen vectors depending upon the dimension of an image. The MSE (Mean squared error) and compression ratio are going to be used as thresholding parameters for reconstruction.

Keywords – SVD, Image Processing , Wavelet Transform, and Image Compression techniques.

2. Introduction

For compressing images we use SVD which is a linear matrix transformation. SVD stands for singular value decomposition.

Wavelet Transform (a variant of discrete cosine transform) uses wavelets instead of DCT block based algorithm and is suitable for image compression.This will be applied on a variety of images for experimentation.

SVD consists of decomposing an image matrix into a product of 3 matrices U,S and V(S being singular values matrix).

The equation used to decompose a matrix I into the 3 matrices is given below.

$$I = U \times S \times V^T = \sum_{i=1}^{n} \left(\sigma_i \times u_i \times v_i^T \right)$$

Where,

$$S = \begin{pmatrix} \sigma_i & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_n \end{pmatrix}$$

Even though DCT gives more energy compaction compared to SVD but normally SVD gives better compression results in images with high standard deviation(higher pixel density).

3. Objectives

Reducing the size of an image and aiming to increase the capacity storage of media (limited in capacity). Images of lower size can be transmitted faster over the network.

Reduction of redundancy to reduce the memory to save and transfer an image is the objective of image compression.

The technique that we are going to use for the purpose of image compression is SVD which is known as singular value decomposition. We achieve compression by minimizing the number of bits to represent an image.

4. Methodology

Image compression deals with reduction of irrelevant information or redundant information.

We focus on the

1. Psycho visual redundancy which is due to the limitation of the human visual system to interpret very fine details in an image. This is visually non-essential information.

2. Inter pixel redundancy is due to the similarities in the neighboring pixels.

3. Redundancy in coding. Here more bits are used in encoding the image data than actually required.

We'll make use of these redundancies to compress the image data. As compression essentially means to reduce the number of bits to represent an image.

The quality of the image compression is

generally measured with PSNR(peak-signal-to-noise ratio) and SSIM(structural similarity index).
We aim to show weaknesses in these metrics and make use of energy ratio as a metric to evaluate the quality of image compression.
Applying SVD does not result in reduction of the image size rather it remains the same. After applying SVD , only the singular values are retained and the rest others are removed which result in saved space of those removed values. The removal also follows the property of removing only those values which contain the least information. This ensures efficient compression with minimum loss of information and no image distortion .
We can take the first k values of these sorted(descending) singular values and the image quality is directly proportional to the value of k and inversely proportional to the compression quality.

Mean Squared Error can also be calculated for the compressed images using the following relation-

$$MSE = \frac{1}{m \times n} \sum_{i=1}^{m} \sum_{j=1}^{n} \left[I(i,j) - I_k(i,j) \right]^2$$

We aim to increase the compression ratio while minimizing the MSE as much as possible by introducing energy ratio.

5. Result

We do an exploratory analysis of values retained for the set of test images and compare the values of SSIM and E.

After applying SVD to the frames of the video, we get the following result, taking different values of k. fig. 8 shows the original frame. Lower value of k results in lower image quality which can be seen below

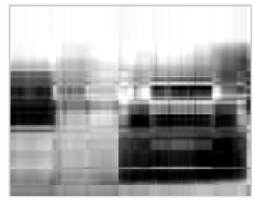
Fig. 1 k = 5

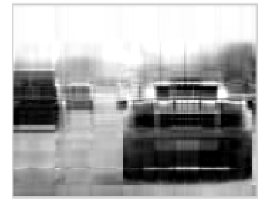
Fig. 2 k = 8


Fig. 3 k = 16


Fig. 4 k = 25


Fig. 5 k = 32


Fig. 6 k = 128


Fig. 7 k = 392


Fig. 8 Original Image

The given graph plots gives an understanding of the relationship between SSIM and E vs k. E gives output lying in the domain [0.90 , 1] for most of the test set

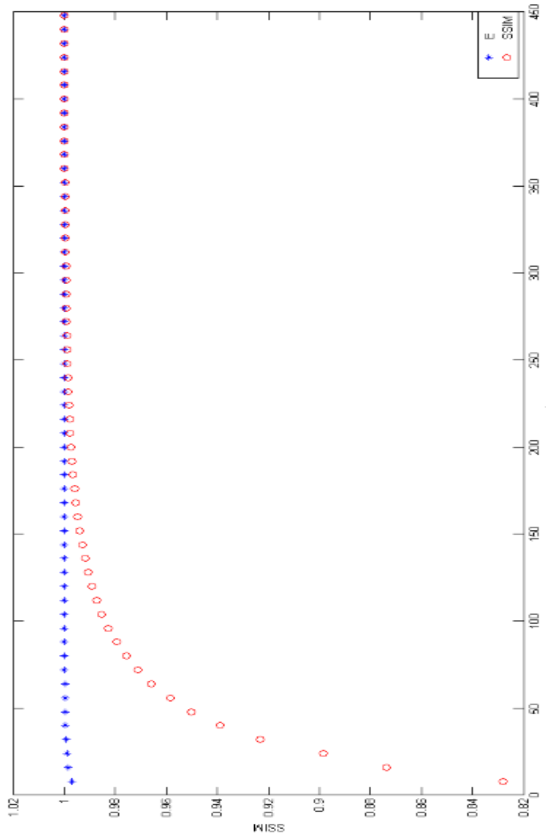

Fig. 2 Relationship of SSIM and E according to k

images and a small variation of E (energy ratio) differing to SSIM is noticed.
The given Figure shows that 99.9% restored energy is achievable with k>=40.

The figure also shows 3 different quality levels corresponding to 3 different areas (Fig 3, 4 and 5).

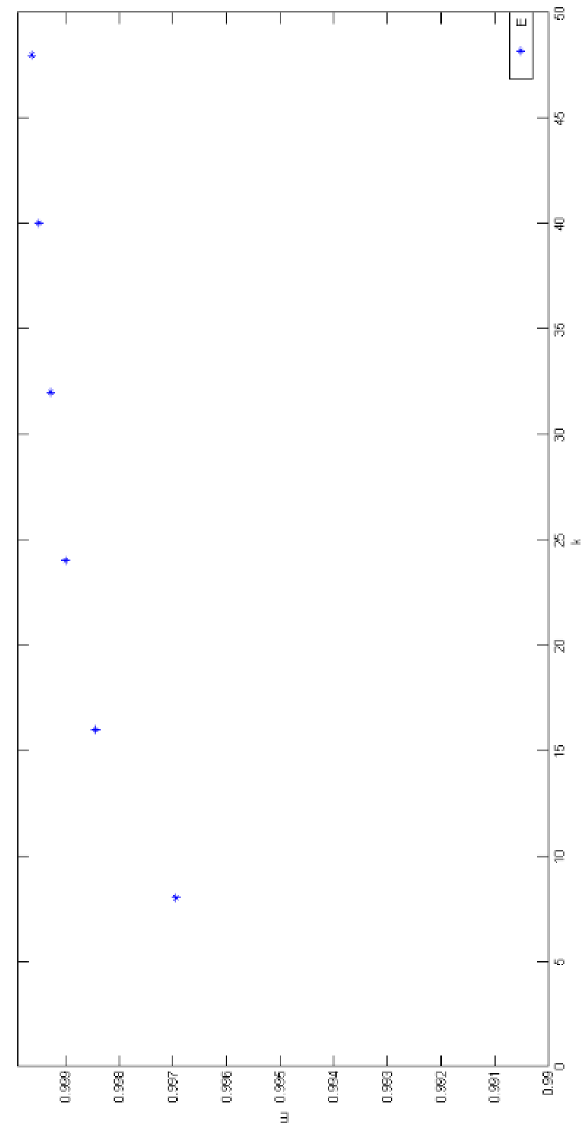

Fig 3 according to k variation in E (between 0.99 and 0.999) (AREA 1)

The area inside the figure gives the values of E in the domain [0.99 , 0.999].

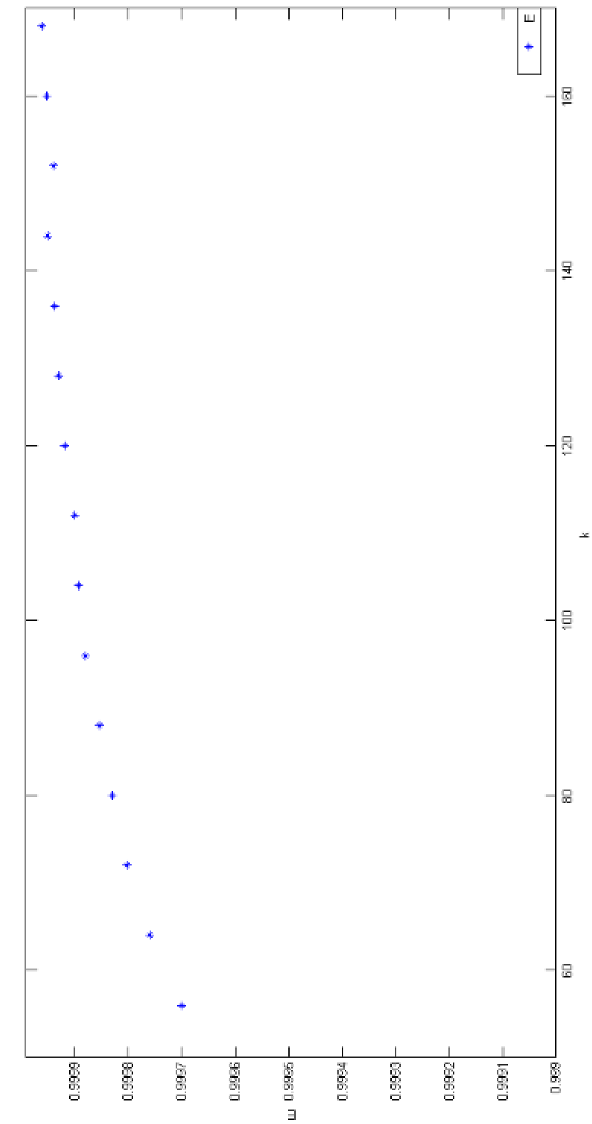

Fig 4 according to k variation E (between 0.999 and 0.999) (AREA 2)

The second area inside the figure gives the values of E in the domain [0.999 , 0.9999].

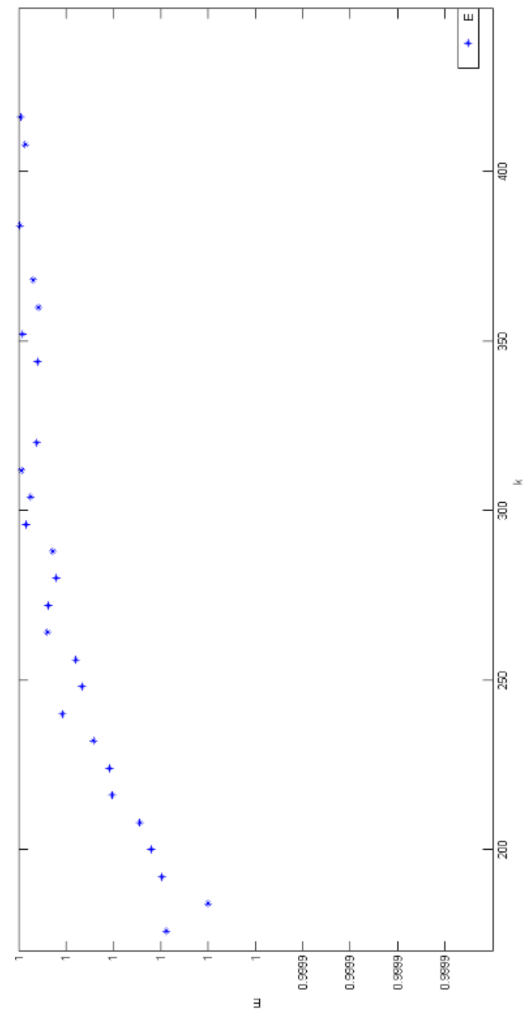

Fig-5 according to k variation in E(between 0.9999 and 1) (AREA 3)

The third area inside the figure gives the values of E in the domain [0.9999 , 1.0].
The following table gives an insight on the average values of PSNR, SSIM and E describing the 3 areas for all the test set images.

k	8 to 32	40 to 120	128 to 448
PSNR	27 dB to 34 dB	35 dB to 42 dB	43 dB to 98 dB
SSIM	0,82 to 0,93	0,94 to 0,98	0,98 to 1
E	99,39 to 99,85	99,9 to 99,98	99,99 to 100
Zone	99	999	9999
Appreciation	Poor quality	Good quality	Very good quality

Fig. Appreciation areas based on energy ratio

The image quality obtained for values of k in [8 , 32] is very poor.
The image quality is considerably better for the values of k in range [40 , 120].
We obtain very good quality for the values of k greater than 128.

The figure is showcasing the quality of image compressed with E = 99.9% and PSNR =35 dB. When we lower the value of k to a very low value. The quality of the image reduces very significantly and thus it is advised to try different values of k and find the one which best suits without much loss.

6. Conclusion

The given result leads to the conclusion that the advantage of SVD is it's less computational complexity and good compression result.
The degree of compression required by models and applications can be achieved by adjusting the values of k.
This results in varying degrees of compression depending on the number of eigenvalues chosen.
The image quality is directly proportional to the degree of compression.
Thus, a proper value of k is to be chosen to ensure proper compression to image quality balance.The chosen value then can be used for the whole scope of application for compression purposes.
The applications of SVD encompasses not only image compression but also noise reduction, face recognition and watermarking etc.

7. References

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