Selected Topics in CFD - list 9

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1.

Consider an initially circular vortex sheet in an infinite, incompressible, and inviscid 2D domain. The geometry is parameterized by $\xi \in [0, 2\pi)$ as:

$$\mathbf{x}(\xi, 0) = \begin{bmatrix} \sin(\xi) \\ \cos(\xi) \end{bmatrix}.$$

The initial strength of the sheet, understood as the tangential velocity jump across the interface, is:

$$\gamma(\xi) = \Delta u_\tau = \sin(\xi)$$

The flow is governed by the streamfunction-vorticity $(\psi - \omega)$ formulation. The velocity field $\mathbf{u}$ and vorticity $\boldsymbol{\omega}$ are related via:

$$\boldsymbol{\omega} = \nabla \times \mathbf{u}, \qquad \mathbf{u} = \nabla \times \boldsymbol{\psi}$$

This implies the vector Poisson equation for the streamfunction:

$$\nabla^2 \boldsymbol{\psi} = -\boldsymbol{\omega}.$$

The Green's function for the Poisson equation in an infinite 2D domain is:

$$G = \frac{1}{2\pi} \ln |\mathbf{x}|.$$

For numerical stability, we employ the regularized (vortex blob) kernel with smoothing parameter $\delta = 0.1$:

$$G_\delta = \frac{1}{2\pi} \ln \sqrt{|\mathbf{x}|^2 + \delta^2}$$

Discretize the vortex sheet and simulate its evolution in a Lagrangian framework.