Selected Topics in CFD - list 4

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1.

Solve the 1D diffusion equation:

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}, \qquad x \in [0, \pi]$$

using the following initial condition:

$$u(x)\big|_{t=0} = \cos(x) + 1$$

and two sets (cases) of the boundary conditions:

a) $\frac{\partial u}{\partial x}\Big|_{x=0} = \frac{\partial u}{\partial x}\Big|_{x=\pi} = 0$

b) $u\big|_{x=0} = 2, \qquad u\big|_{x=\pi} = 0$

- Determine a stable time step
- Check if the total u is conserved in the first case
- Represent the discretized laplacian as a matrix (despite drawbacks)

2.

Consider the analogical 2D case:

$$\frac{\partial u}{\partial t} = \alpha \nabla^2 u, \qquad x \in [0, \pi], \quad y \in [0, \pi]$$

$$u(x, y)\big|_{t=0} = \cos(x)\cos(y)$$

$$\frac{\partial u}{\partial n} = 0$$