Selected Topics in CFD - list 7

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1.

Implement the building blocks of a two-grid algorithm, including:

- Weighted Jacobi solver
- Restriction operator R using the full-weighting approach
- Prolongation operator P using linear interpolation

Propose a simple test for each component and verify your implementation.

2.

Consider the Poisson equation with Dirichlet boundary conditions:

$$Au = b$$

discretized on a fine grid with 17 nodes and a coarse grid with 9 nodes.

- How does the operator $A_c = RA_fP$ differ from A discretized manually on the coarse grid?
- Approximately how many iterations does the weighted Jacobi solver require on the fine grid and on the coarse grid if:
  - $u = \sin\left(\frac{\pi}{L}x\right)$, $x \in [-\pi, \pi]$, $b = Au$
  - $u = x$, $x \in [-\pi, \pi]$, $b = Au$

  assuming the initial guess is a perturbed solution: $u_0 = u + \sin\left(5\frac{\pi}{L}x\right)$, and RMS residual tolerance is $1e-12$. Explain your observations.

3.

Implement the two-grid algorithm. How many full iterations are required to solve $u = e^{-x^2}$, $x \in [-\pi, \pi]$, with $b = Au$ and $u_0 = 0$? Compare the results with the weighted Jacobi method on the fine grid.