Dynamic Behavior

In analyzing process dynamic and process control systems, it is important to know how the process responds to changes in the process inputs.

A number of standard types of input changes are widely used for two reasons:

1. They are representative of the types of changes that occur in plants.

2. They are easy to analyze mathematically.

1. Step Input

A sudden change in a process variable can be approximated by a step change of magnitude, *M*:

$$U_s = \begin{cases} 0 & t < 0 \\ M & t \geq 0 \end{cases} \qquad (5\text{-}4)$$

$$\mathrm{U_s}(s) = M / s \qquad (5-6)$$

The step change occurs at an arbitrary time denoted as $t = 0$.

- *Special Case:* If $M = 1$, we have a “unit step change”. We give it the symbol, $S(t)$.

- *Example of a step change:* A reactor feedstock is suddenly switched from one supply to another, causing sudden changes in feed concentration, flow, etc.

Example:

The heat input to the stirred-tank heating system in Chapter 2 is suddenly changed from 8000 to 10,000 kcal/hr by changing the electrical signal to the heater. Thus,

$$Q(t) = 8000 + 2000S(t), \qquad S(t) \text{ @unit step}$$

and

$$Q'(t) = Q - \bar{Q} = 2000S(t), \qquad \bar{Q} = 8000 \text{ kcal/hr}$$

2. Ramp Input

- Industrial processes often experience "drifting disturbances", that is, relatively slow changes up or down for some period of time.
- The rate of change is approximately constant.

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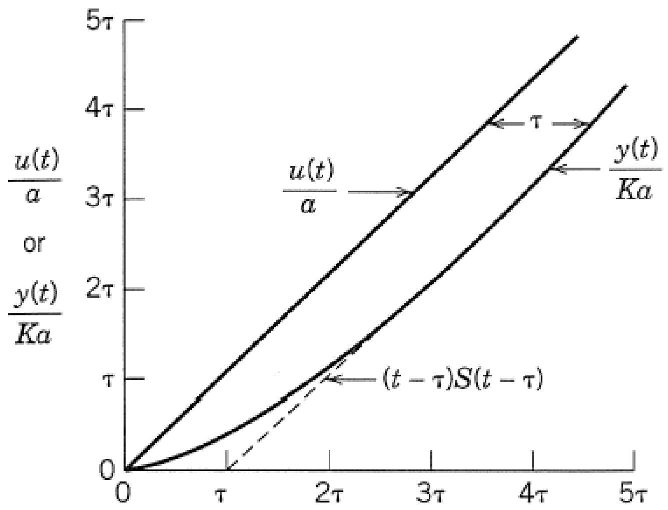
We can approximate a drifting disturbance by a *ramp input:*

$$U_R(t) = \begin{cases} 0 & t < 0 \\ at & t \geq 0 \end{cases} \qquad (5\text{-}7)$$

$$U_R(s) = a / s^2 \qquad (5-8)$$



Examples of ramp changes:

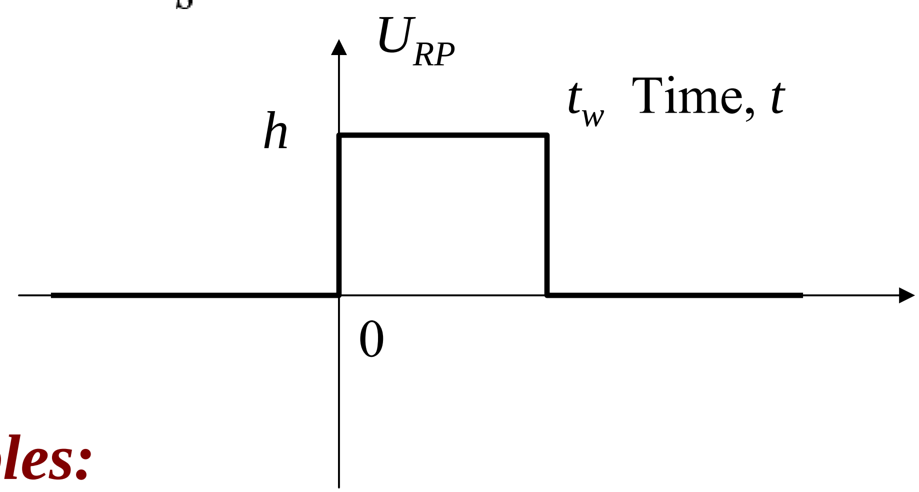
1. Ramp a setpoint to a new value. (Why not make a step change?)
2. Feed composition, heat exchanger fouling, catalyst activity, ambient temperature.

3. Rectangular Pulse

It represents a brief, sudden change in a process variable:

$$U_{RP}(t) = \begin{cases} 0 & \text{for} \quad t < 0 \\ h & \text{for} \quad 0 \le t < t_w \\ 0 & \text{for} \quad t \ge t_w \end{cases} \qquad (5\text{-}9)$$

$$\mathrm{U_{RP}}(s) = \frac{h}{s}\left[1 - e^{-t_w s}\right] \qquad (5-11)$$



Examples:

1. Reactor feed is shut off for one hour.
2. The fuel gas supply to a furnace is briefly interrupted.

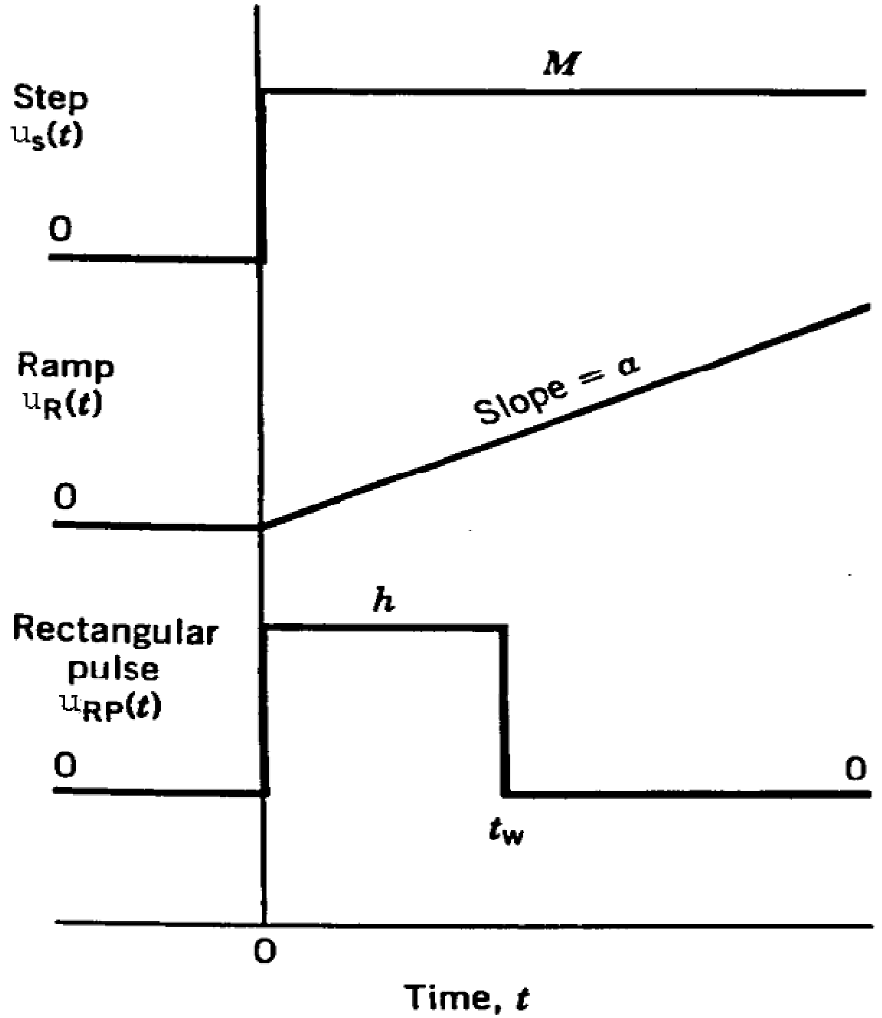
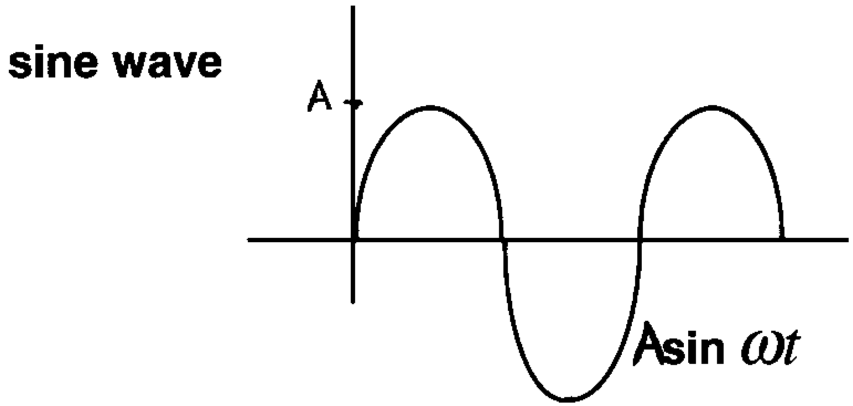


Figure 5.2. Three important examples of deterministic inputs.

Other Inputs

4. **Sinusoidal Input**



Processes are also subject to periodic, or cyclic, disturbances. They can be approximated by a sinusoidal disturbance:

$$U_{\sin}(t) \;@ \begin{cases} 0 & \text{for} \quad t < 0 \\ A\sin(\omega t) & \text{for} \quad t \geq 0 \end{cases} \qquad (5\text{-}14)$$

where: A = amplitude, ω = angular frequency

$$U_{\sin}(s) = \frac{A\omega}{s^2 + \omega^2}$$

Examples:

1. 24 hour variations in cooling water temperature.
2. 60-Hz electrical noise (in USA!)

For a sine input (1st order process)

$$U(s) = \frac{\omega}{s^2 + \omega^2}$$

output is...

$$Y(s) = \frac{K_p}{\tau s + 1} \cdot \frac{\omega}{s^2 + \omega^2} = \frac{\alpha_0}{\tau s + 1} + \frac{\alpha_1 s}{s^2 + \omega^2} + \frac{\alpha_2}{s^2 + \omega^2}$$

By partial fraction decomposition,

$$\alpha_0 = \frac{\omega K_p \tau^2}{\omega^2 \tau^2 + 1}$$

$$\alpha_1 = \frac{-\omega K_p \tau}{\omega^2 \tau^2 + 1}$$

$$\alpha_2 = \frac{\omega K_p}{\omega^2 \tau^2 + 1}$$

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Inverting,

this term dies out for large t

$$y(t)=\frac{K_p\omega\tau}{\omega^2\tau^2+1}e^{-t/\tau}+\frac{K_p}{\sqrt{\omega^2\tau^2+1}}\sin(\omega t+\phi)$$

$$\phi=-\arctan(\omega\tau)$$

note: ϕ is not a function of t but of τ and ω.

For large t, y(t) is also sinusoidal,
output sine is attenuated by…

$$\frac{1}{\sqrt{\omega^2\tau^2+1}}$$ (fast vs. slow ω)

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5. Impulse Input

- Here, $U_I(t) = \delta(t)$ *and* $U_I(s) = 1$
- It represents a short, transient disturbance.
- It is the limit of a rectangular pulse for $t_w \rightarrow 0$ and $h = 1/t_w$

Examples:

1. Electrical noise spike in a thermo-couple reading.
2. Injection of a tracer dye.

Here,

$$Y(s) = G(s) \qquad (1)$$

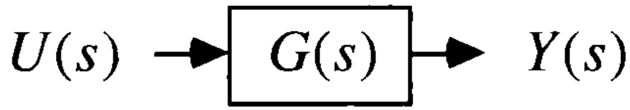
Second order process example, Example 4.2

$$y = T - \overline{T} \qquad \mathrm{u=Q-\overline{Q}} \qquad \mathrm{T_i} \ \ \text{fixed}$$

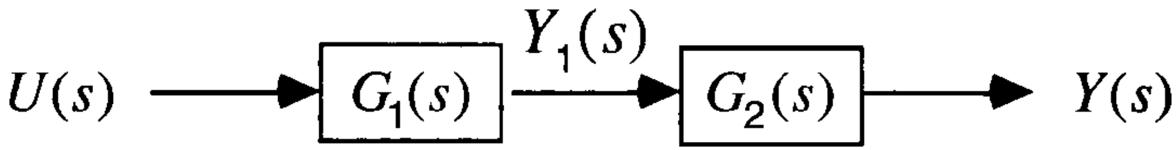
$$\frac{\mathrm{mm_eC_e}}{\mathrm{wh_eA_e}} \frac{d^2 y}{dt^2} + \left(\frac{\mathrm{m_eC_e}}{\mathrm{h_eA_e}} + \frac{\mathrm{m_eC_e}}{\mathrm{wC}} + \frac{m}{w} \right) \frac{dy}{dt} + y = \frac{1}{wC} \cdot u$$

note when $C_e \rightarrow 0$, obtain 1st order equation (simpler model)

Block Notation:



Composed of two first order subsystems (G_1 and G_2)



$$G(s) = \frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

2nd order ODE model (overdamped)

$$\tau = \sqrt{\tau_1 \tau_2}$$

$$G(s) = \frac{K}{\tau_1 \tau_2 s^2 + (\tau_1 + \tau_2)s + 1}$$

$$\zeta = \frac{\tau_1 + \tau_2}{2\sqrt{\tau_1 \tau_2}}$$

$\zeta > 1$ *overdamped*

$\zeta < 1$ *underdamped*

$\zeta = 1$ *critically damped*

roots: $\dfrac{-\zeta \pm \sqrt{\zeta^2 - 1}}{\tau}$

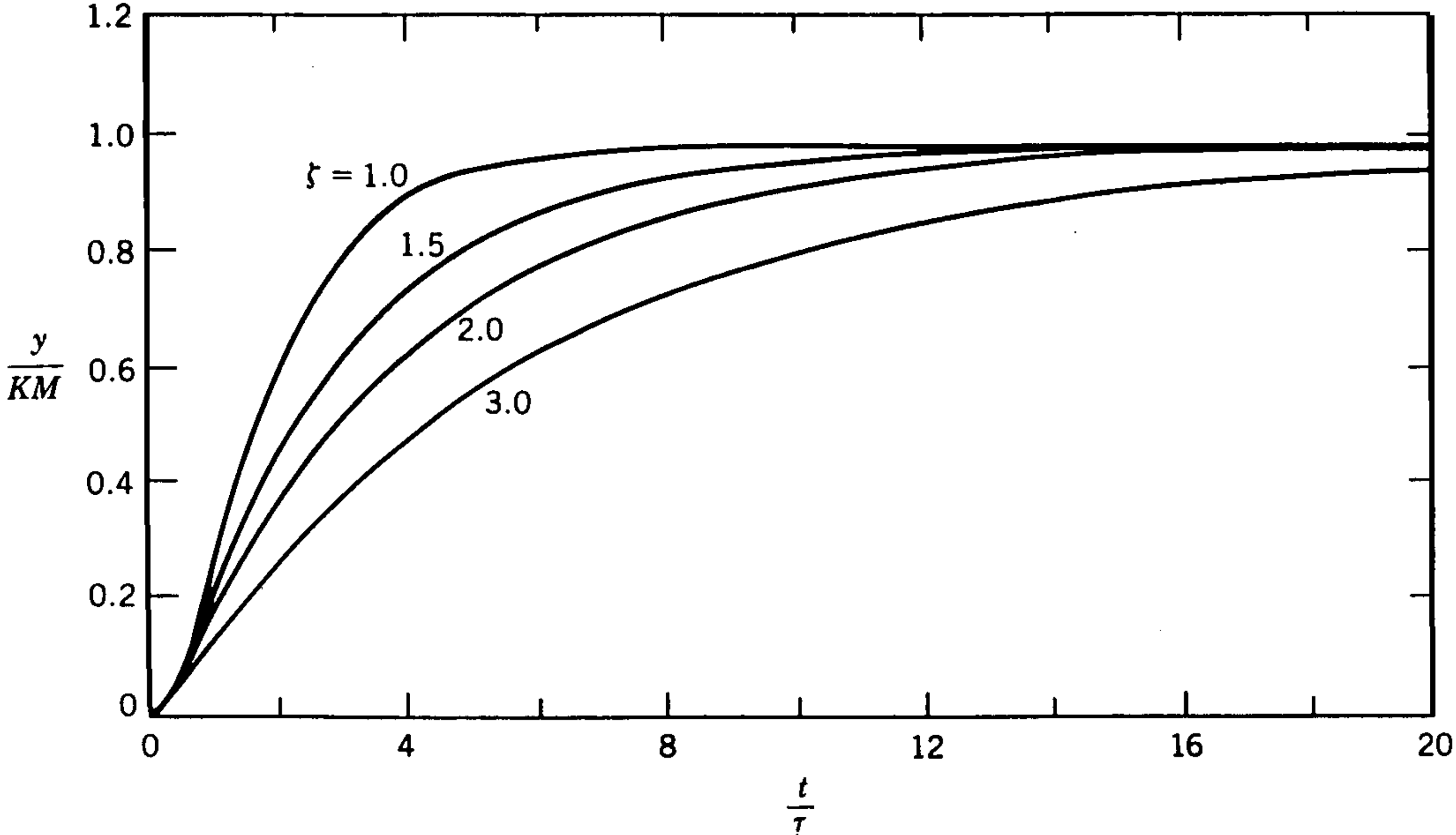


Figure 5.9. Step response of critically-damped and overdamped second-order processes.

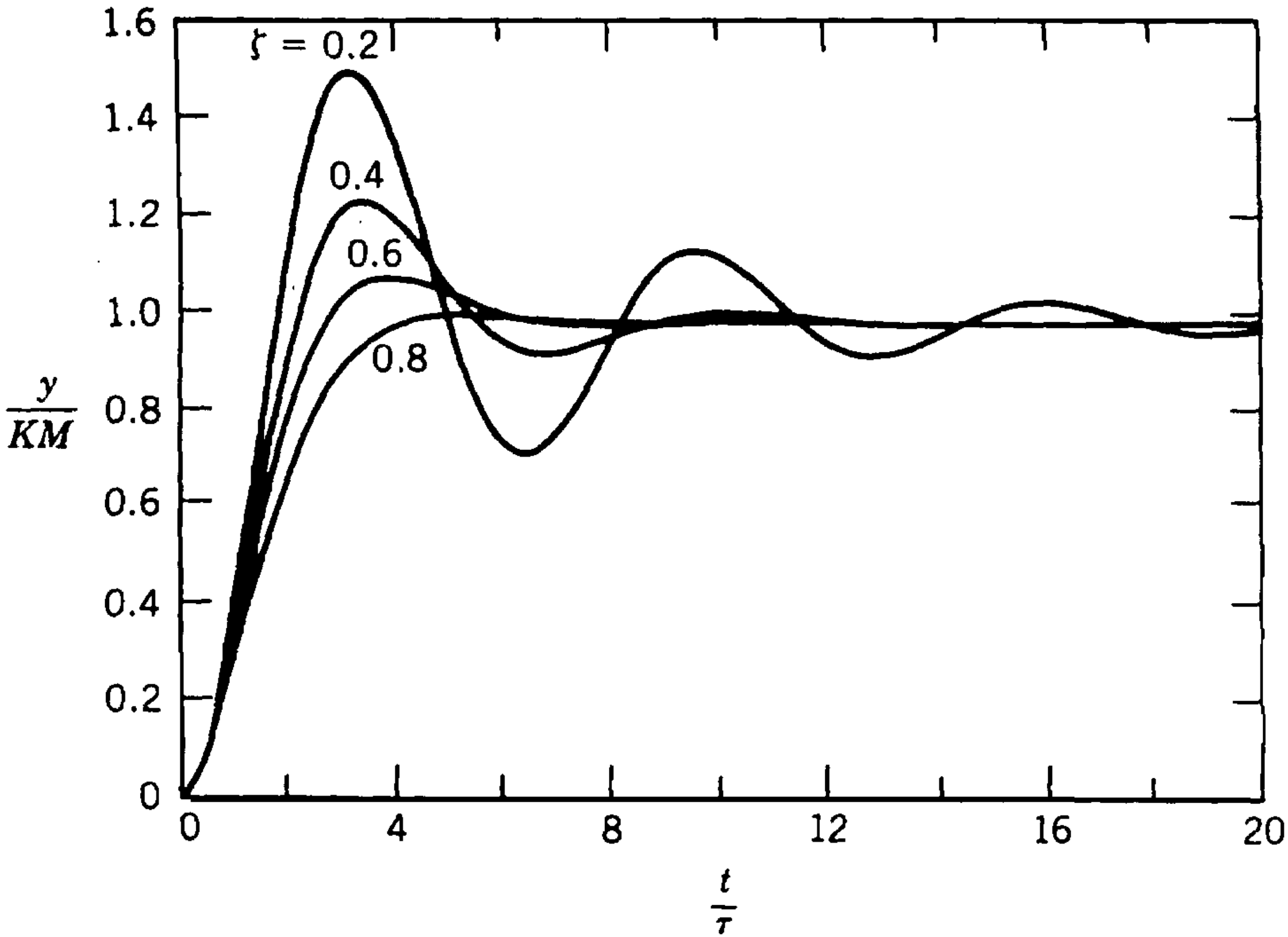


Figure 5.8. Step response of underdamped second-order processes.

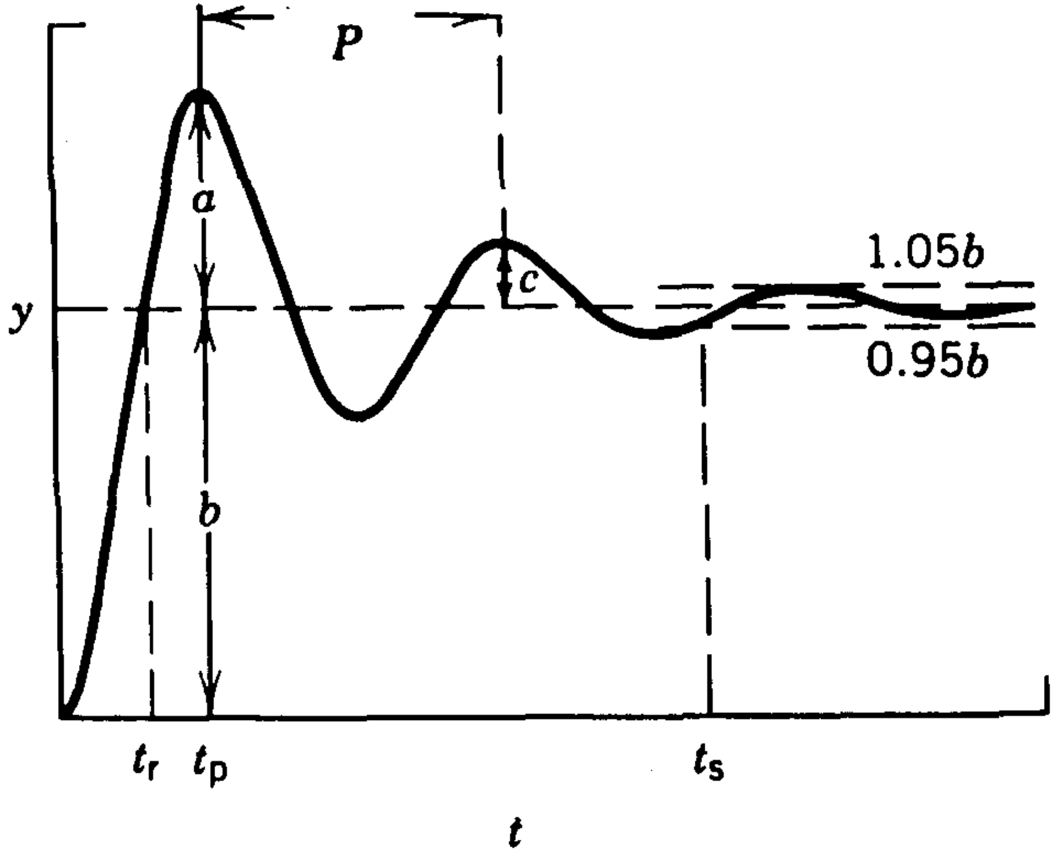


Figure 5.10. Performance characteristics for the step response of an underdamped process.

Second Order Step Change

a. Overshoot

$$\frac{a}{b} = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right)$$

b. time of first maximum

$$t_p = \frac{\pi\tau}{\sqrt{1-\zeta^2}}$$

c. decay ratio (successive maxima – not min.)

$$\frac{c}{a} = \exp\left(\frac{-2\pi\zeta}{\sqrt{1-\zeta^2}}\right) = \frac{a^2}{b^2}$$

d. period of oscillation

$$p = \frac{2\pi\tau}{\sqrt{1-\zeta^2}}$$

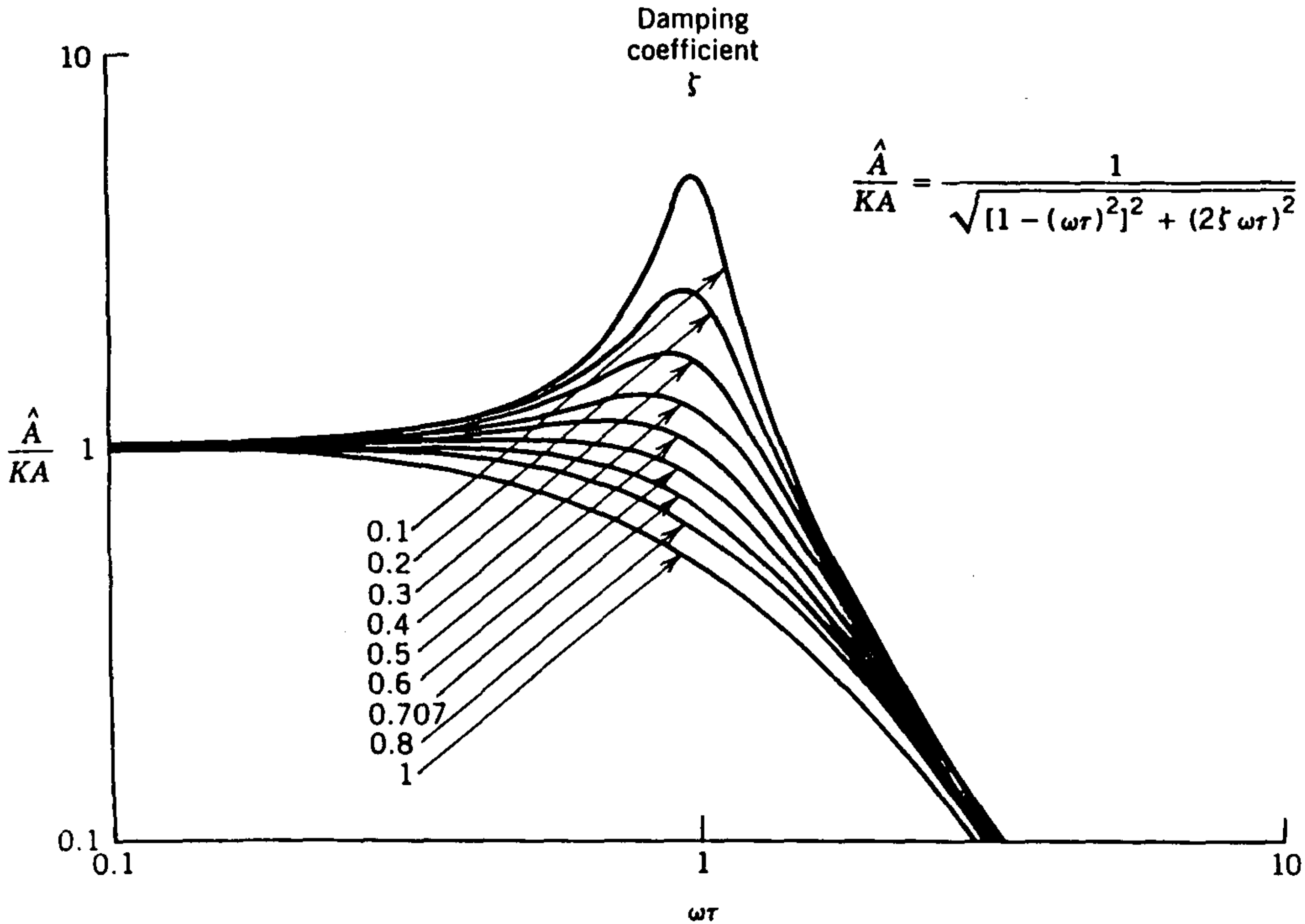


Figure 5.12. Sinusoidal response amplitude of a second-order system after exponential terms have become negligible.