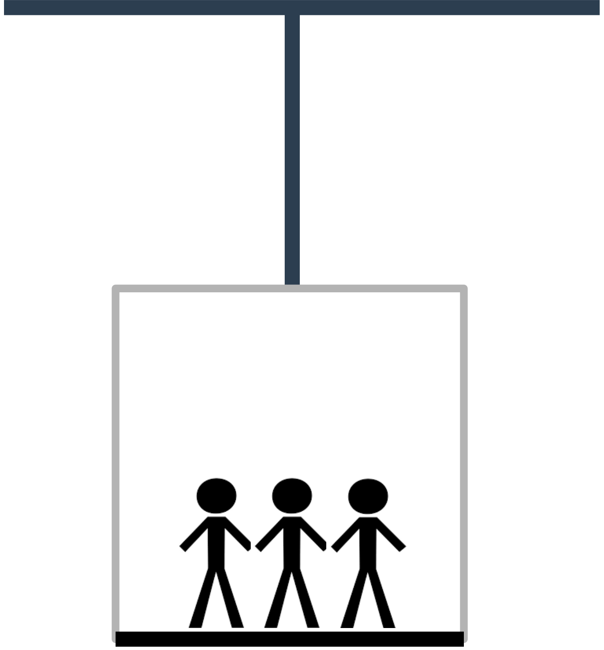
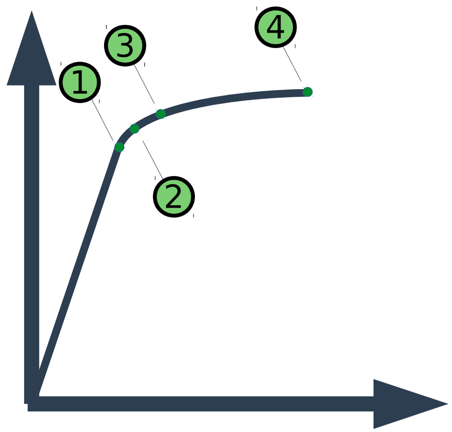
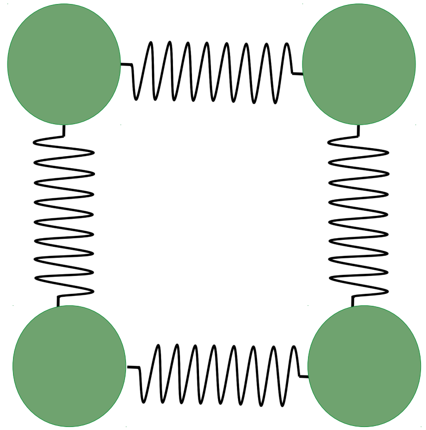
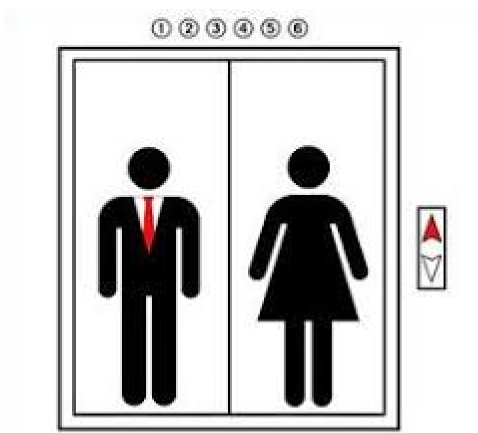


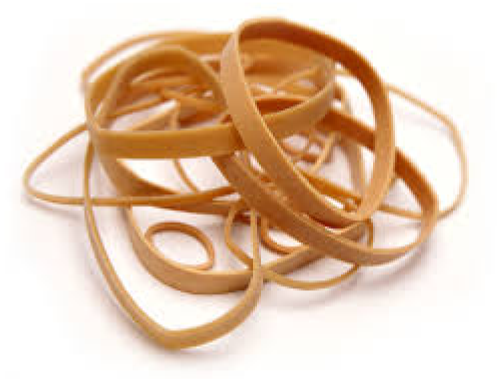


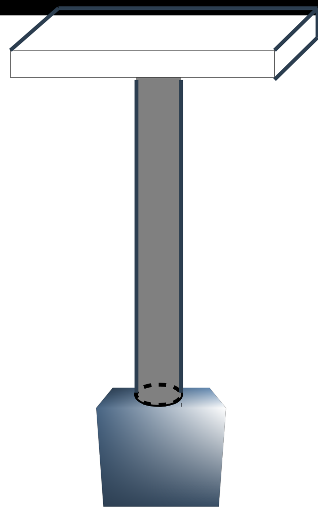


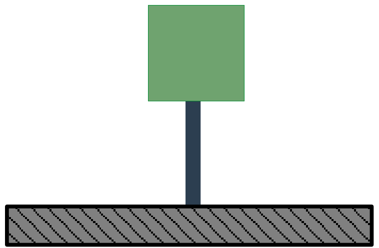
Topic: Properties of Matter

Sub-Topic: Elasticity









Class #1 Goals

- Understand definitions of stress and strain
- Understand the definition of elasticity
- Understand the strain vs stress plot for a elastic material

List of Related Variables

Property	Variable	SI Units
Pressure	P	$\frac{N}{m^2}$
Force	F	N
Extension	e	m
Length	l	m
Area	A	m^2
Work	W	$N \times m$

*
If you don't know these, write them down
*

Jump-Start Question

We know when a weight is added to a spring, there is an extension. (Fig A)

If a weight is attached to a metal wire, what is true? (Fig B)

Fig. (A)

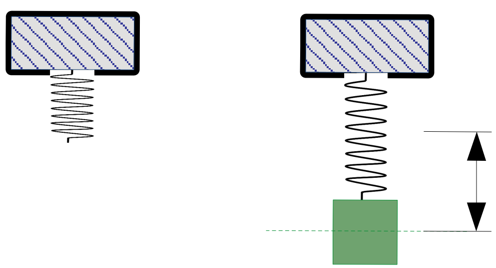
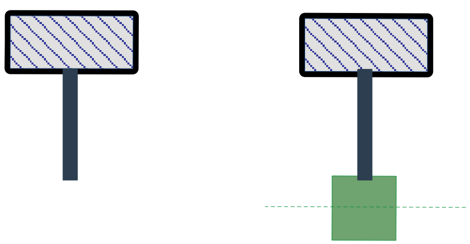


Fig. (B)



There is a non-zero extension

There is zero extension

Jump-Start Question

We know when a weight is added to a spring, there is an extension. (Fig A)

If a weight is attached to a metal wire, what is true? (Fig B)

Fig. (A)

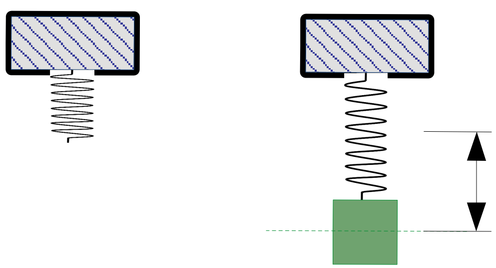
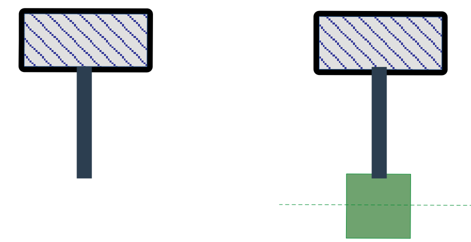


Fig. (B)

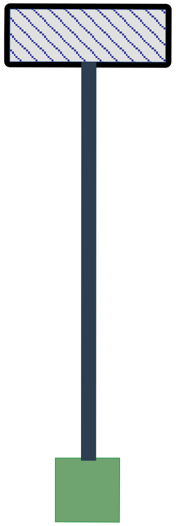


There is a non-zero extension

There is zero extension

Intro to Elasticity

When introduced to a force, any material produces an extension.



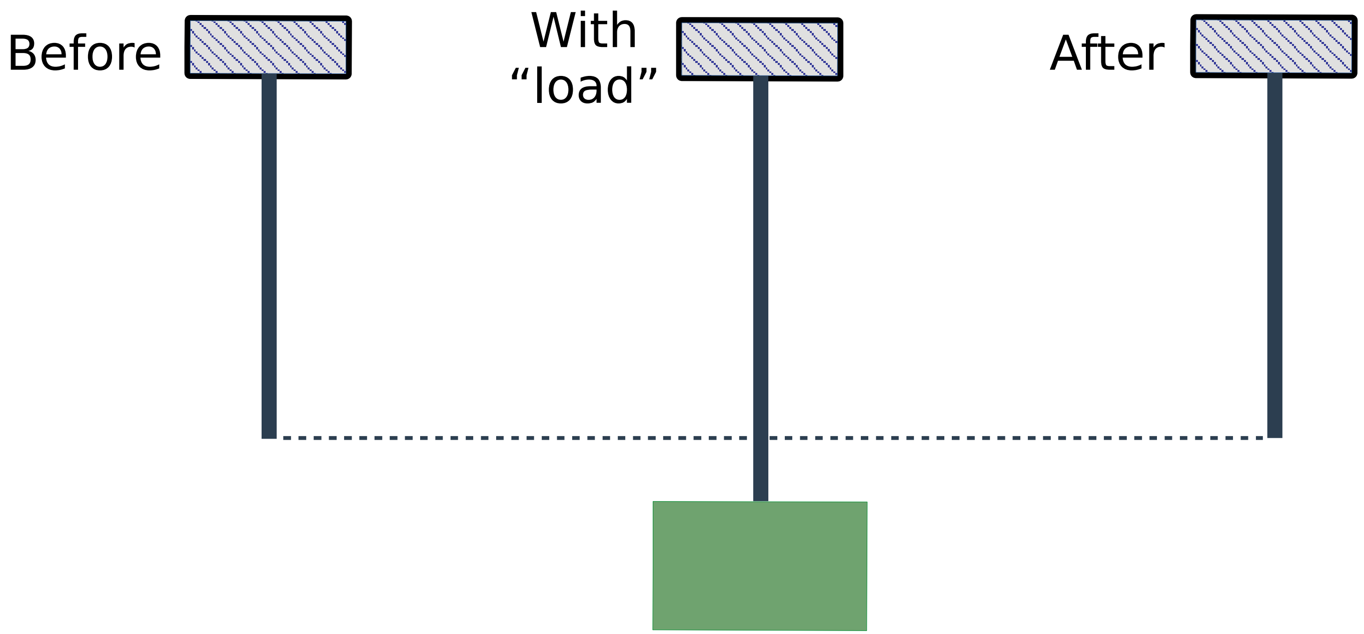
This extension can be <u>very small</u>

(example: Steel Wire with small weight attached)

Intro to Elasticity (pt. 2)

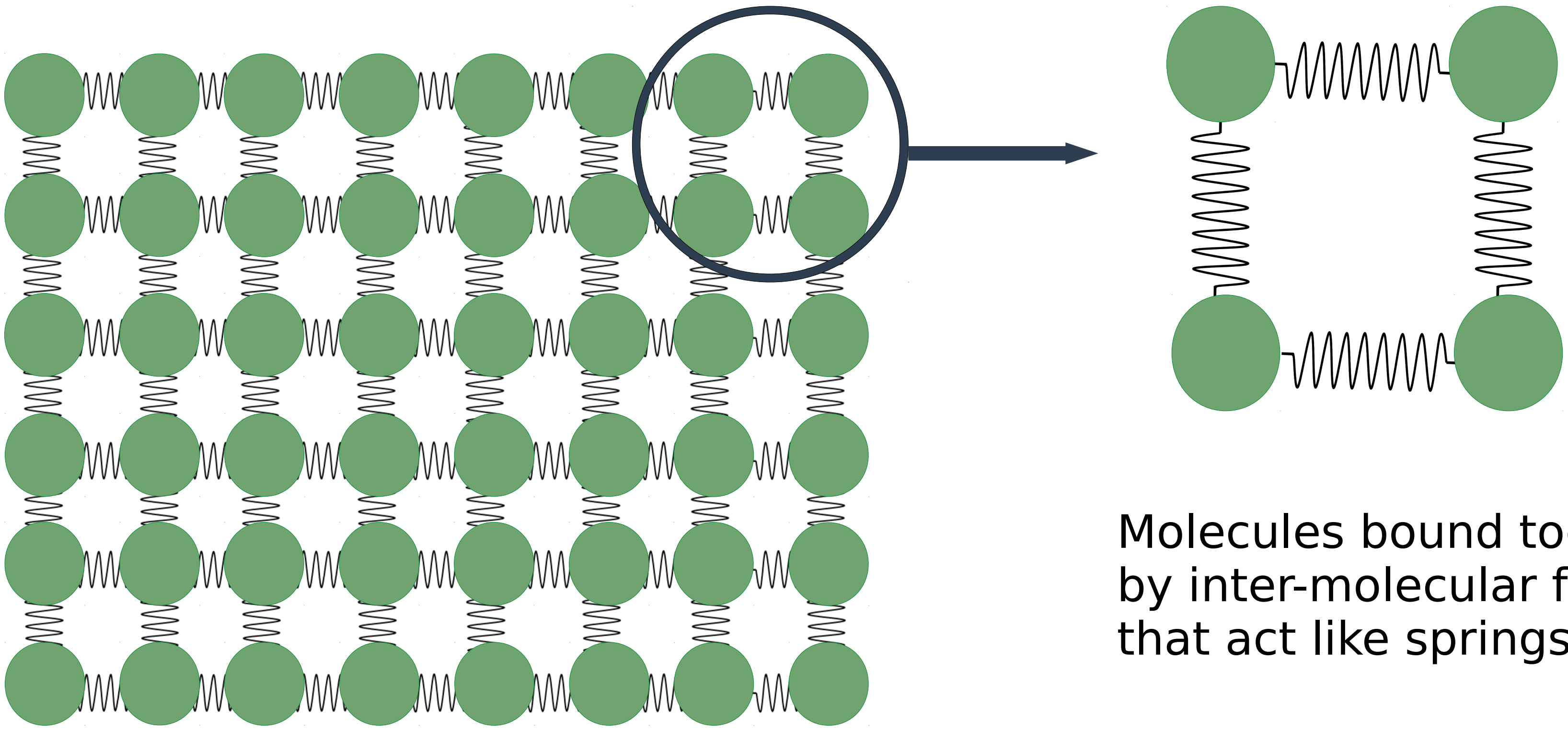
Elasticity

The tendency or property of a body to return to its original size or shape after it has been stretched or compressed. (once the load which has been deforming it is removed)



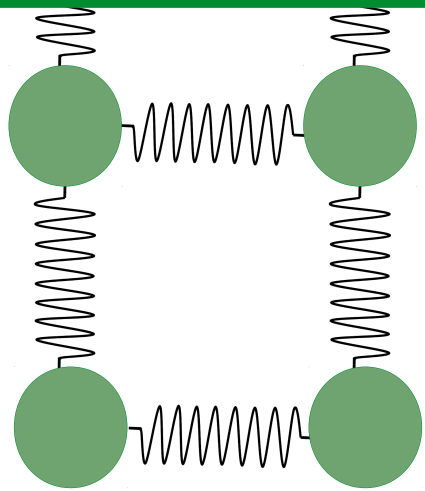
Molecular Explanation

Molecular composition of a solid

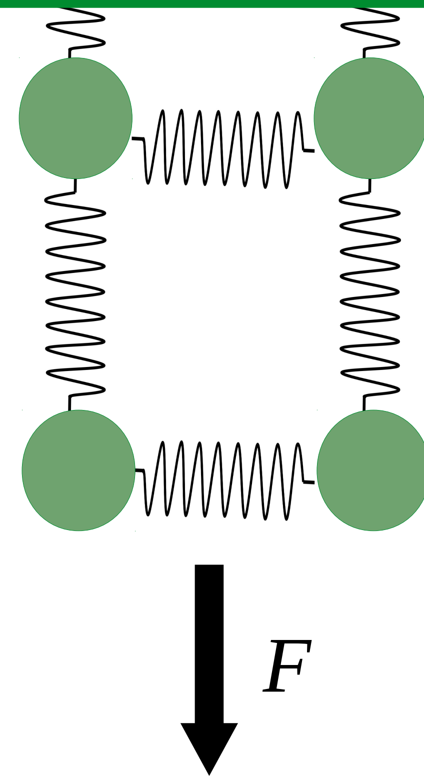


Molecules bound together by inter-molecular forces that act like springs

Force Applied



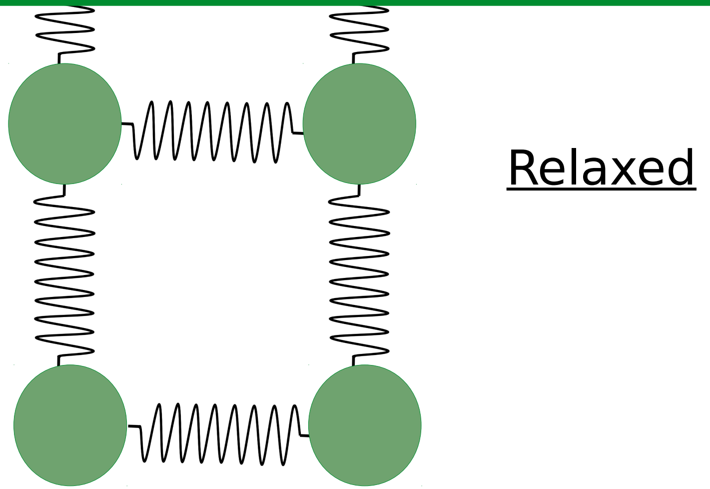
Relaxed

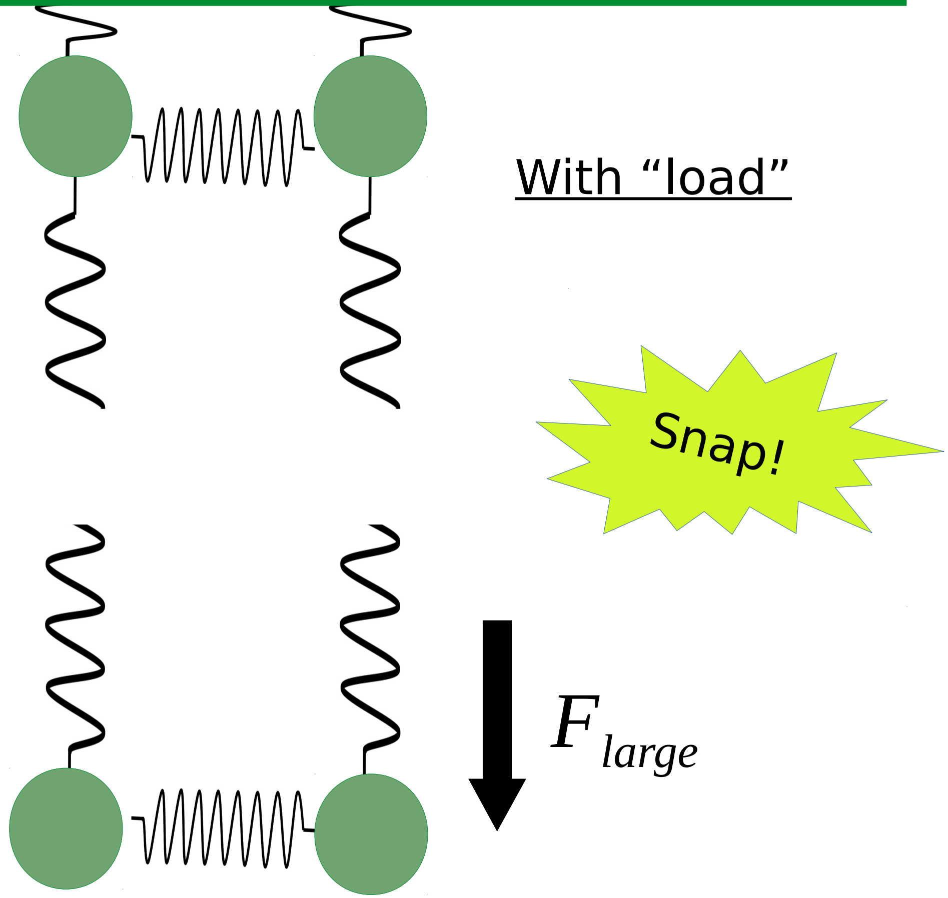


With “load”

When a force is added, spacing between molecules increase

Breaking Point



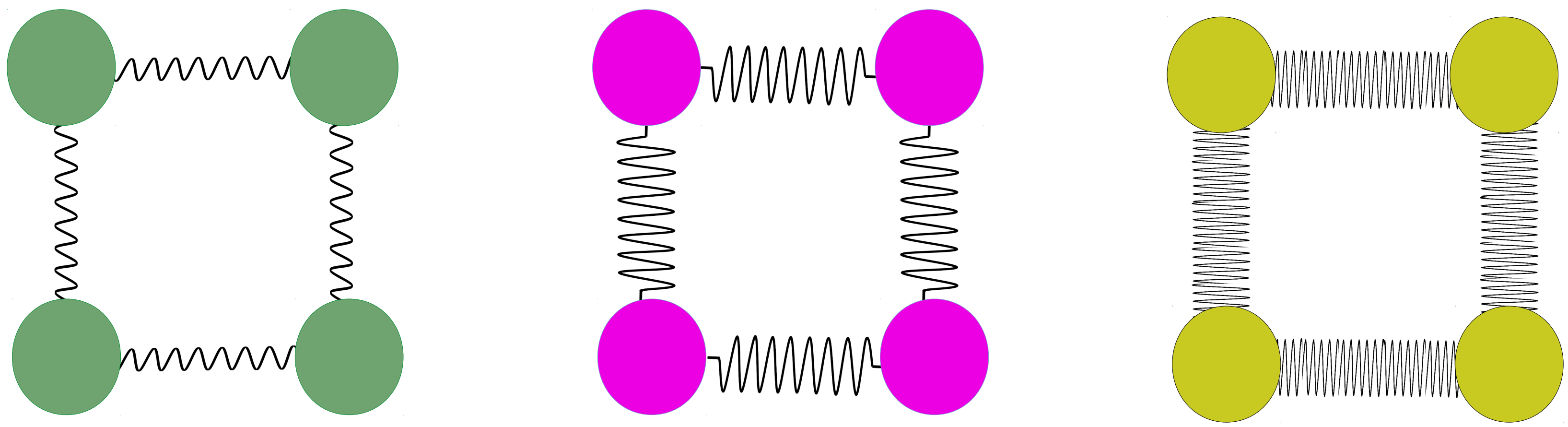


If enough force is applied, the molecular connections separate and the material breaks.

Materials

Different materials have different molecular structures. Some are more resistant to distortion.

distortion = change in shape



Tensile Stress & Strain

Property	Definition	Variable	Equation	SI Units
Tensile Stress	the force acting normally per unit cross-sectional area.	σ	$\sigma = \frac{F}{A}$	$\frac{N}{m^2}$
Tensile Strain	the ratio of the extension produced per unit original length.	ϵ	$\epsilon = \frac{e}{l}$	None!

Important note on tensile strain

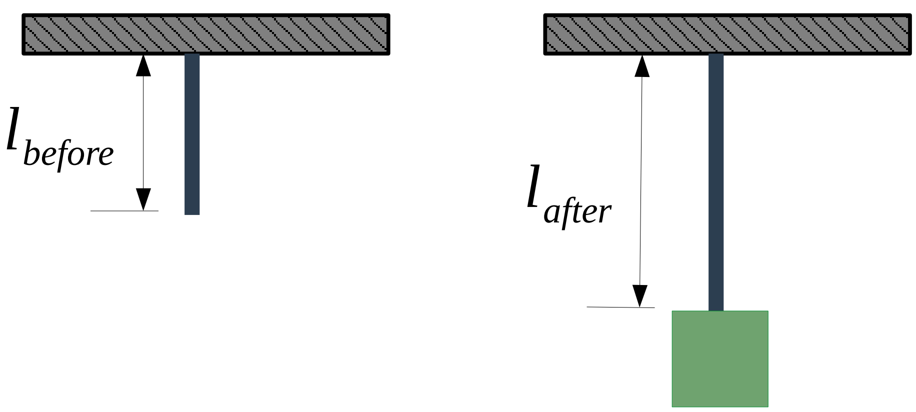
$$\text{Tensile Strain} = \epsilon = \frac{e}{l} = \frac{extension}{length}$$

$$e = l_{after} - l_{before}$$

Example #1:

A weight is added to suspended metal rod.

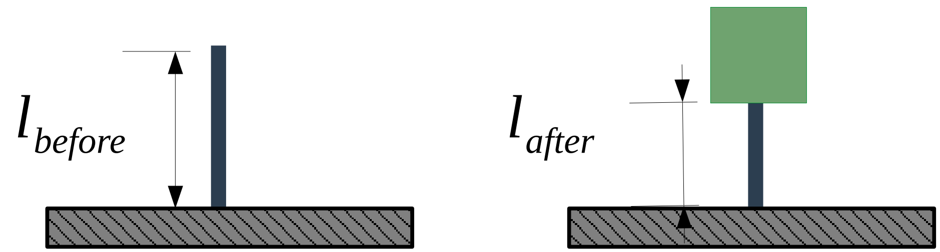
Extension here is positive!



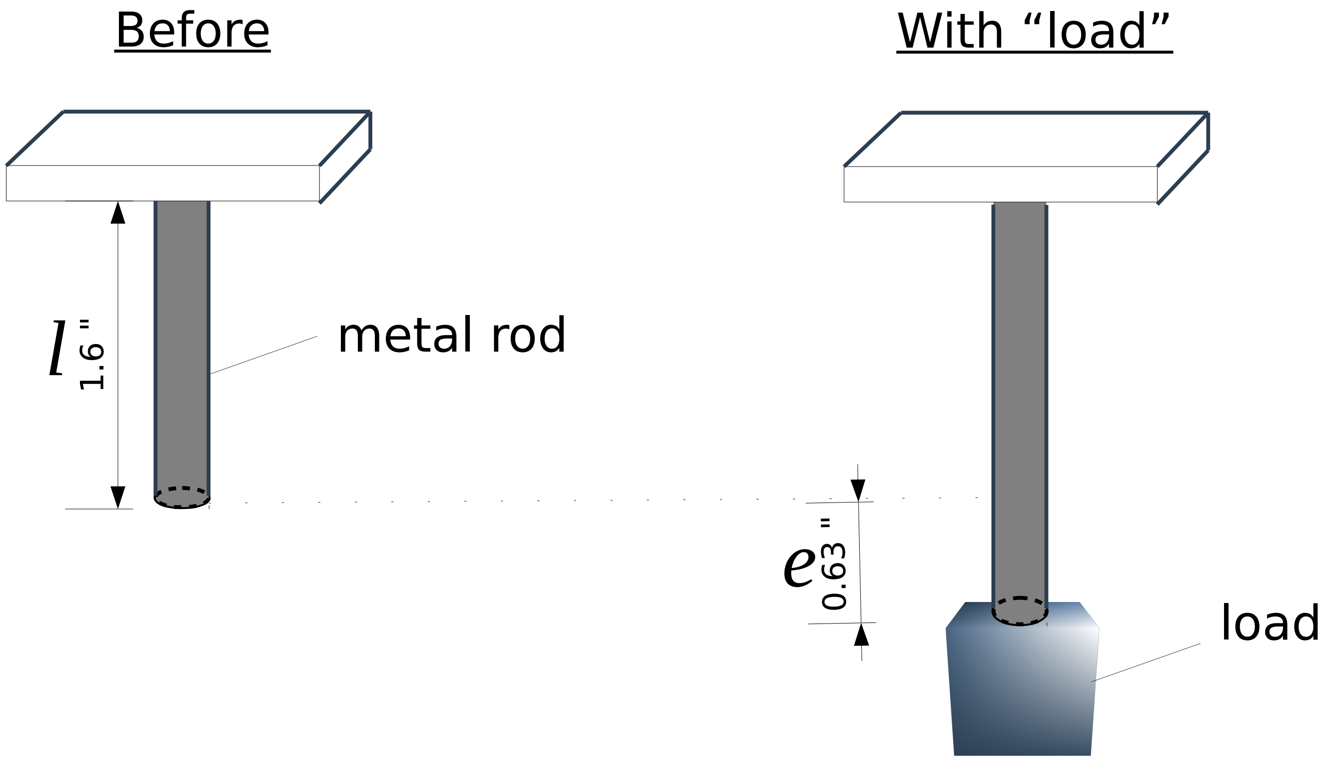
Example #2:

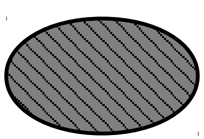
A weight is added on top of a vertical metal rod.

Extension here is negative!



3D visualization of Stress/Strain





$$= Area$$

$$m_{load} \times a_{gravity} = F$$

Example worked out

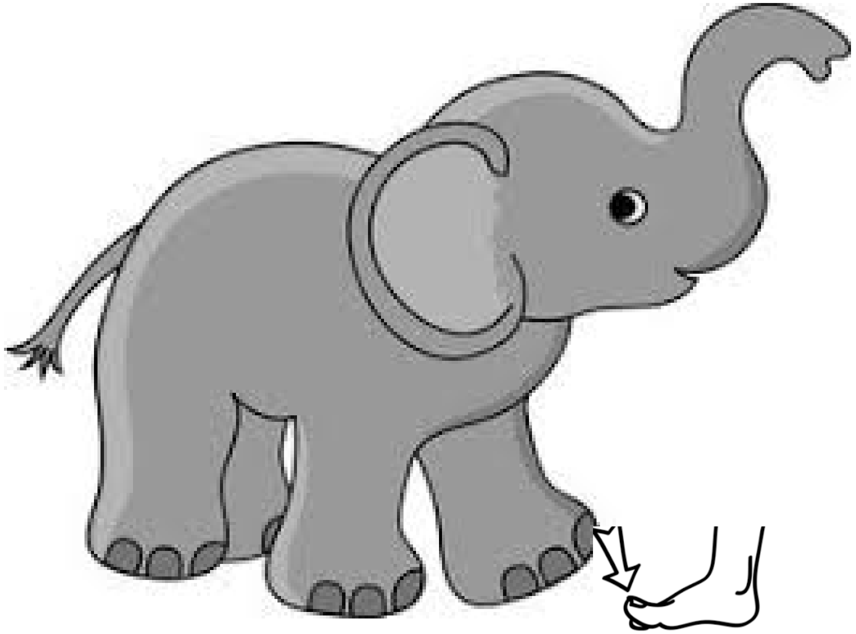
An elephant that weighs 6000kg steps on your toe (with the full weight of its body). Your toe has a cross sectional area of approximately 12cm² and a height (length) of 2cm.

What is the tensile stress on your toe?

$$49 \times 10^{6} \frac{N}{m^{2}}$$

After, your toe has a height of 5mm.
What was the tensile strain on your toe?

$$-0.75$$



Check for Understanding #1

A 5kg weight is attached to a cylindrical metal rod of 2cm in diameter and 1 meter in length. It stretches $5\mu m$. What is the tensile stress on the metal rod?

△ $\sigma = 155{,}971.84\,\dfrac{N}{m^2}$	▲ $\sigma = 38{,}992.96\,\dfrac{N}{m^2}$
○ $\sigma = 389.93\,\dfrac{N}{m^2}$	● $\sigma = 1{,}559.72\,\dfrac{N}{m^2}$

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Check for Understanding #2

A 5kg weight is attached to a cylindrical metal rod of 2cm in diameter and 1 meter in length. It stretches $5\mu m$. What is the tensile strain on the metal rod?

$*\ \mu = 10^{-6}$

△ 10^{-6}	▲ 10^{6}
○ 5×10^{-6}	● 5×10^{6}

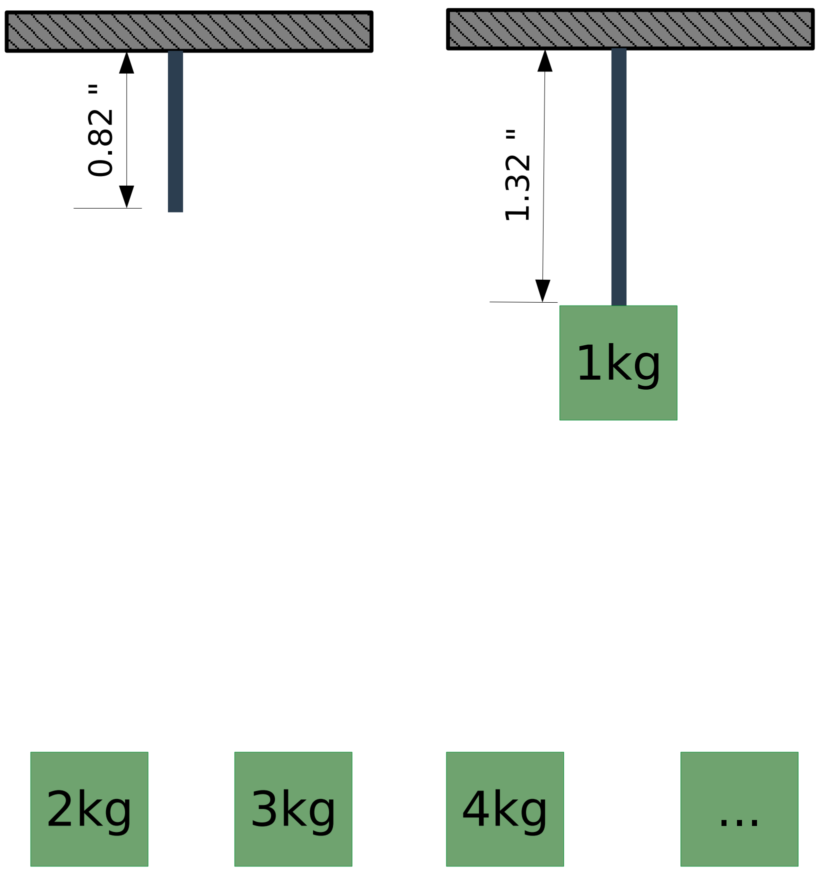
Check for Understanding #2

A 5kg weight is attached to a cylindrical metal rod of 2cm in diameter and 1 meter in length. It stretches $5\mu m$. What is the tensile strain on the metal rod?

$*\mu = 10^{-6}$

△ 10^{-6}	▲ 10^{6}
○ 5×10^{-6}	● 5×10^{6}

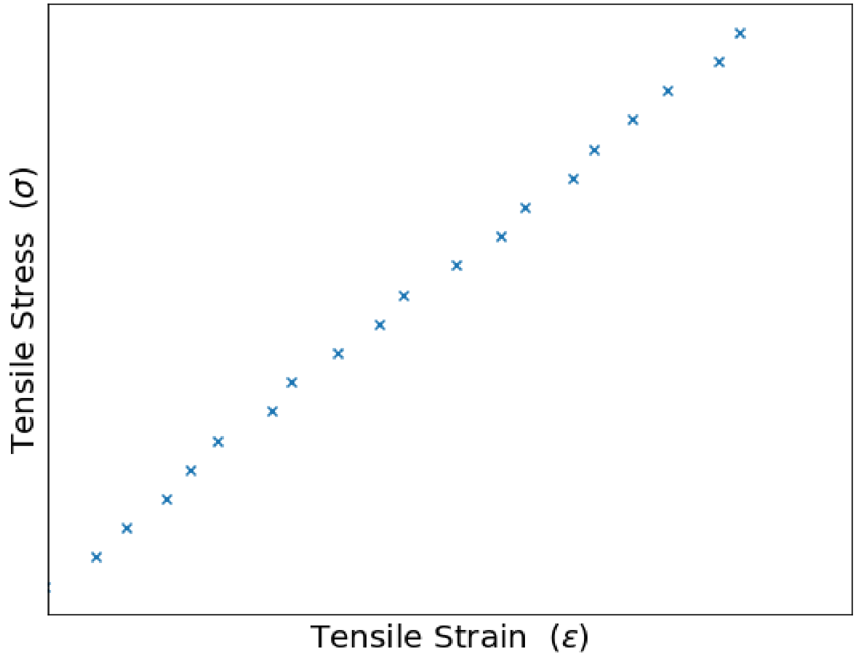
Plotting Stress and Strain



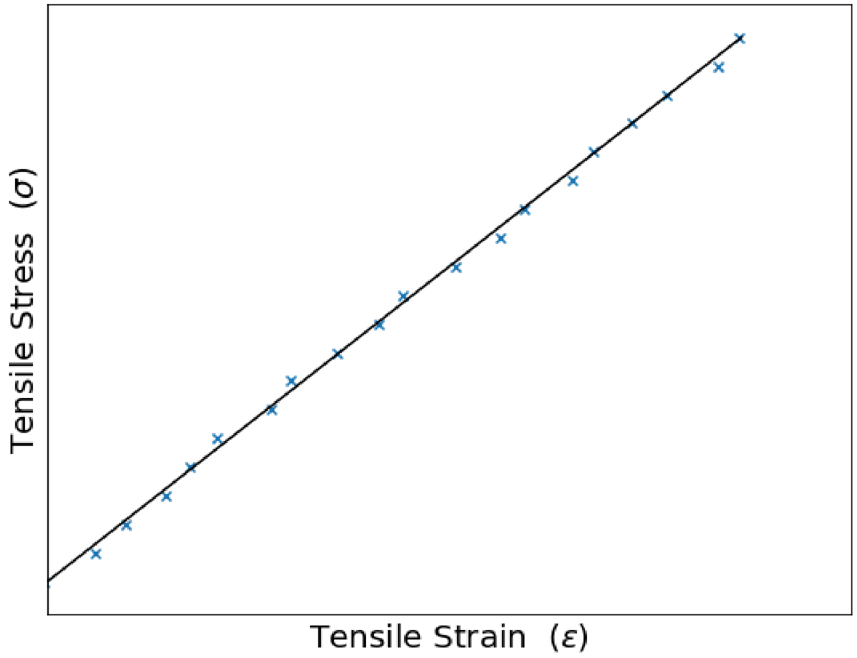
m	g	F	A	L	e	σ	ϵ
1	9.8	9.8	0.01	1	$1.0\mu m$	980	1×10^{-6}
2	9.8	19.6	0.01	1	$2.1\mu m$	1960	2.1×10^{-6}
3	9.8	29.4	0.01	1	$2.9\mu m$	2940	2.9×10^{-6}
4	9.8	39.2	0.01	1	$4.1\mu m$	3920	4.1×10^{-6}
...	...	...	...	...	...	...	...

Plotting Tensile Strain vs Stress

Plot the data!



It is linear!



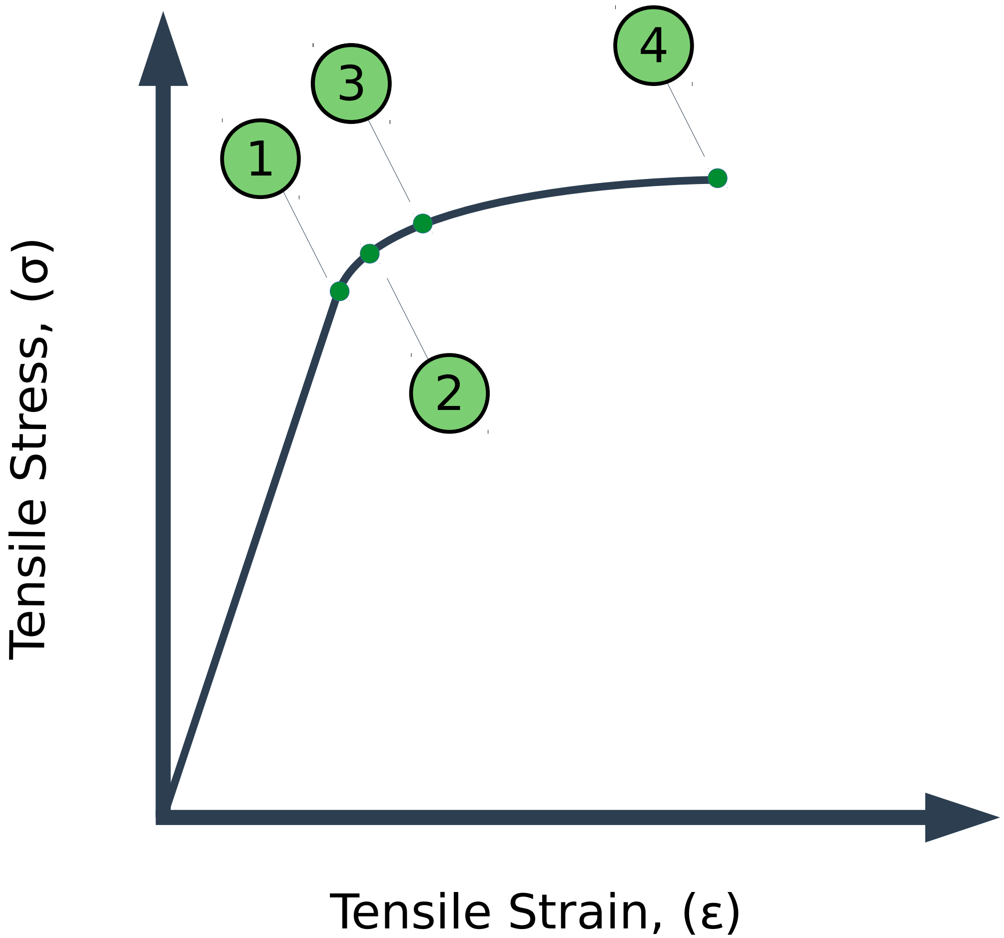
Hooke’s Law Definition

Hooke’s Law

Up to some maximum load (known as the limit of proportionality), the extension of a wire or material is proportional to the applied load or force.

Up to some maximum load, tensile strength and tensile strain are proportional.

Modulus of Elasticity

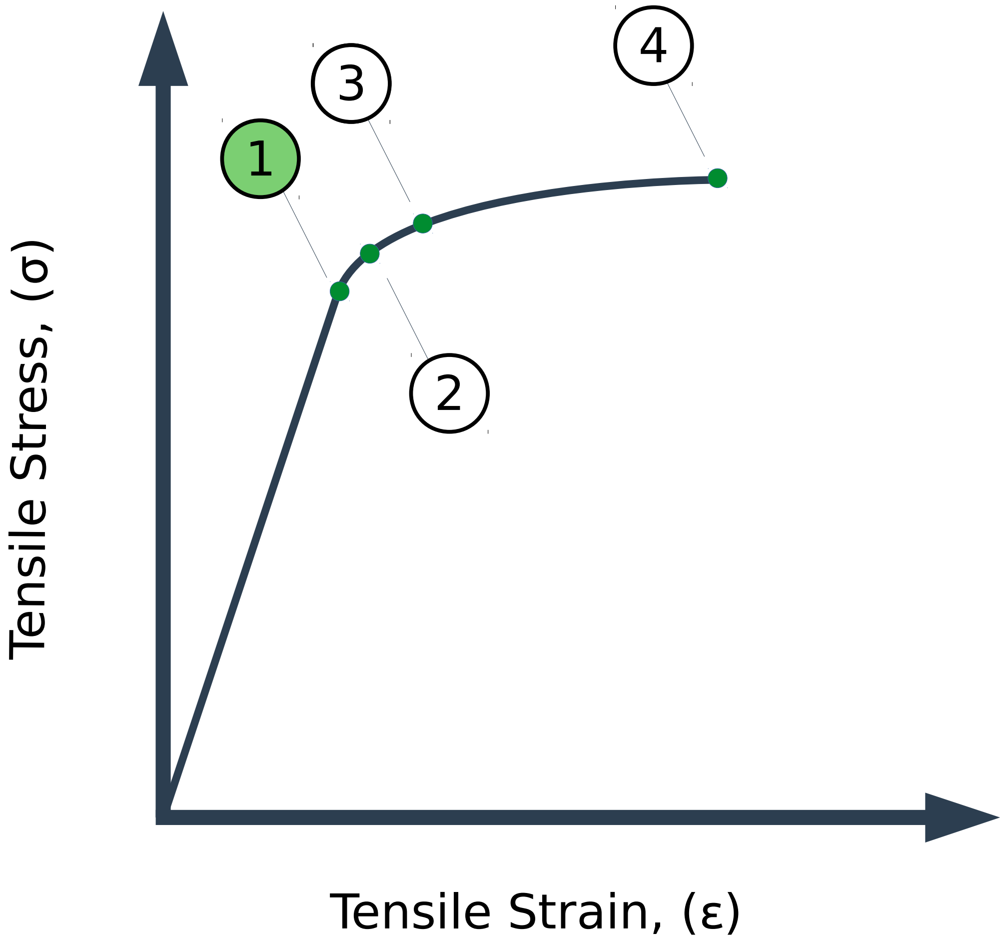


Definition

The Modulus of Elasticity describes the behavior of a material under a given stress.

- It is different for different materials.
- It contains four specific parts

Modulus of Elasticity (pt. 1)

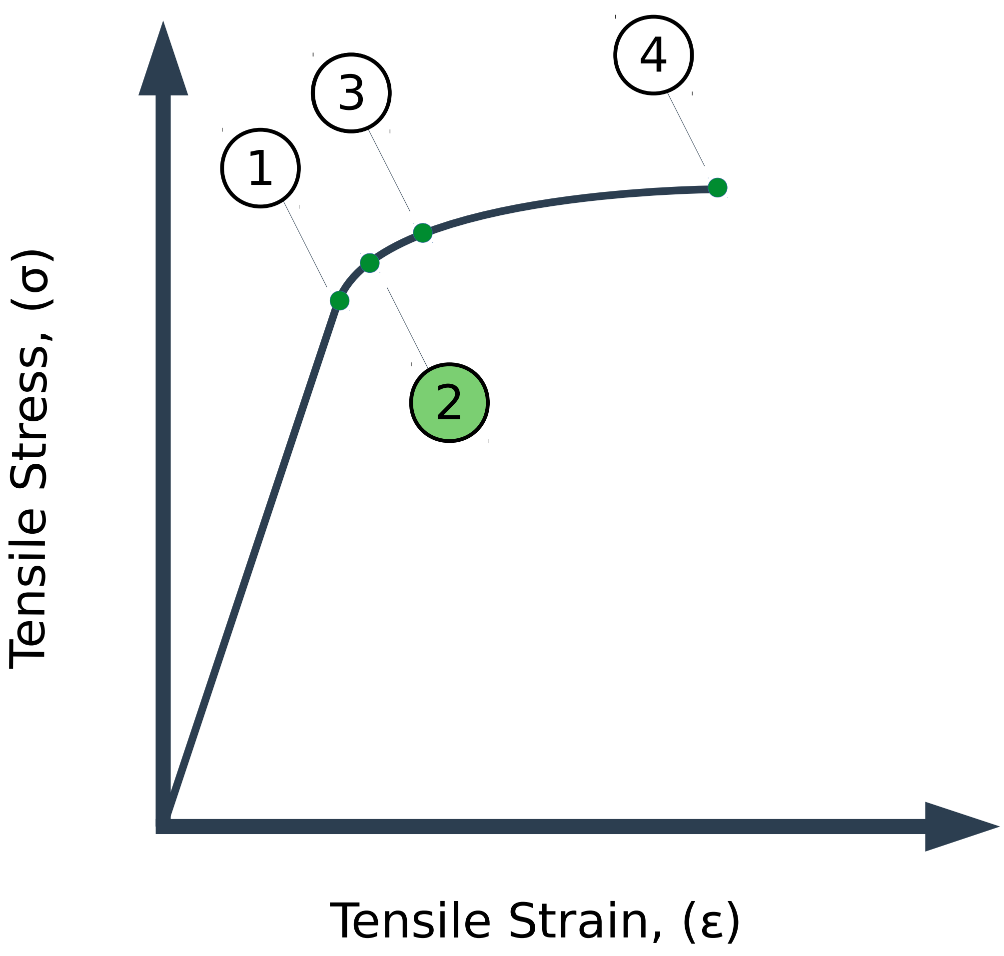


1 **Limit of Proportionality**

The limit of proportionality is the point at which tensile stress and tensile strain are no longer proportional.

After this point, Hooke's Law does not apply.

Modulus of Elasticity (pt. 2)

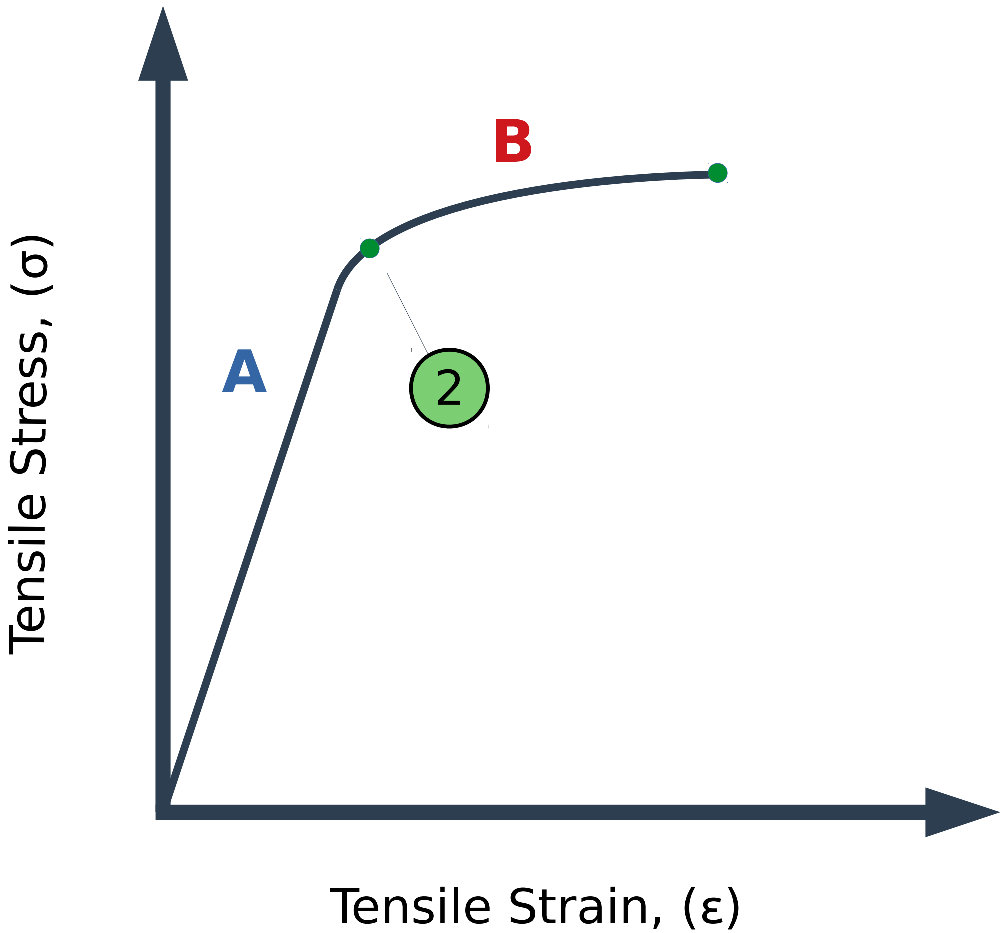


2 Elastic Limit

The elastic limit is the maximum stress up to which a material can exhibit the property of elasticity.

If a stress above elastic limit is applied to a material, the material is permanently deformed.

Modulus of Elasticity (pt. 2b)

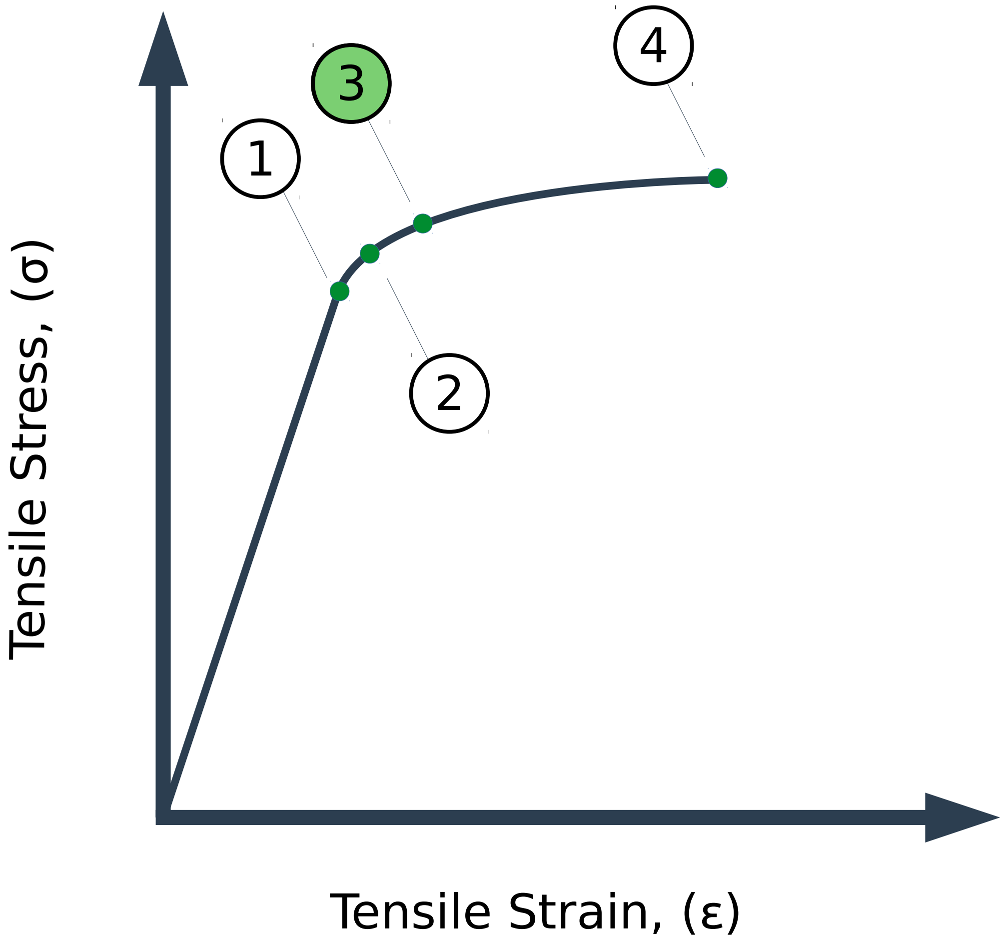


2 **Elastic Limit**

A: A material exhibits **elastic deformation** before the elastic limit.

B: A material exhibits **plastic deformation** after the elastic limit

Modulus of Elasticity (pt. 3)

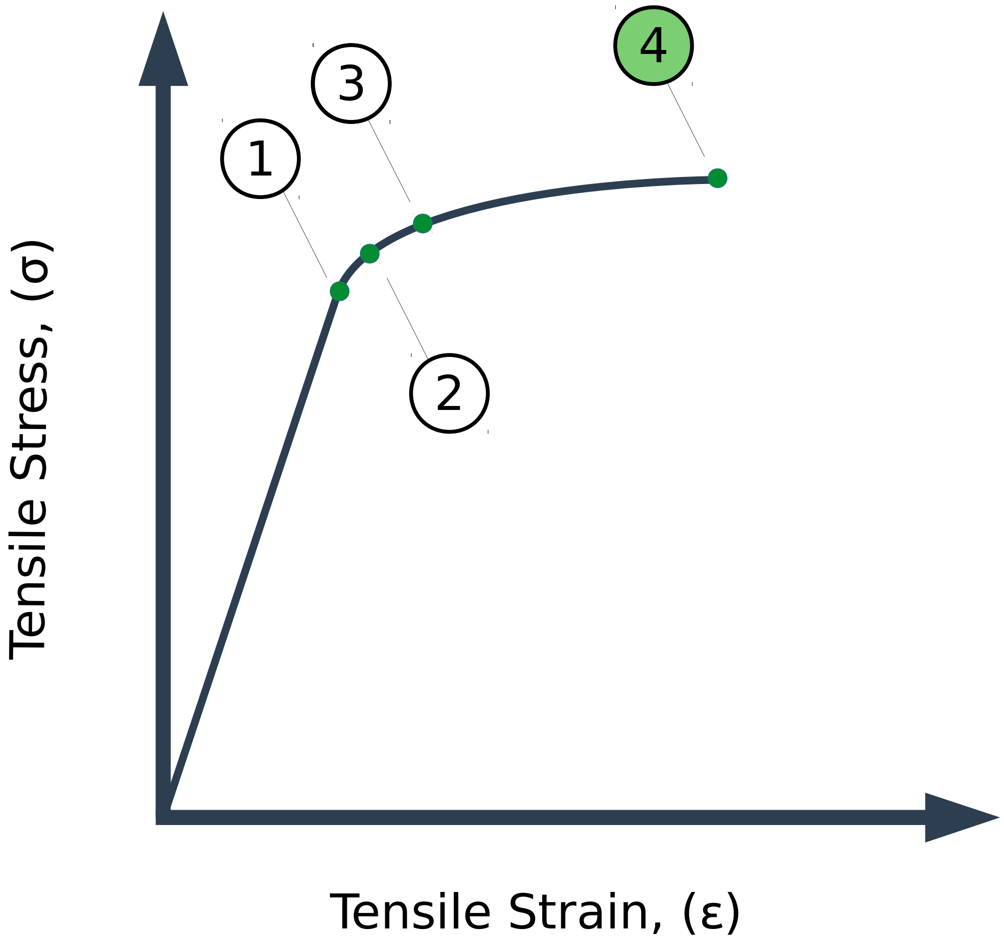


3 Yield Point

The yield point is the point where for a small increase in tensile stress, there is a marked increase in tensile strain.

At this point, the internal structure of a material has changed.

Modulus of Elasticity (pt. 4)



4 **Ultimate Tensile Strength (UTS)**

The ultimate tensile strength (UTS) of a material is the tensile stress at which it breaks.

Class #1 Goals Reviewed

- **Understand definitions of stress and strain**
- **Understand the definition of elasticity**
- **Understand the strain vs stress plot for a elastic material**

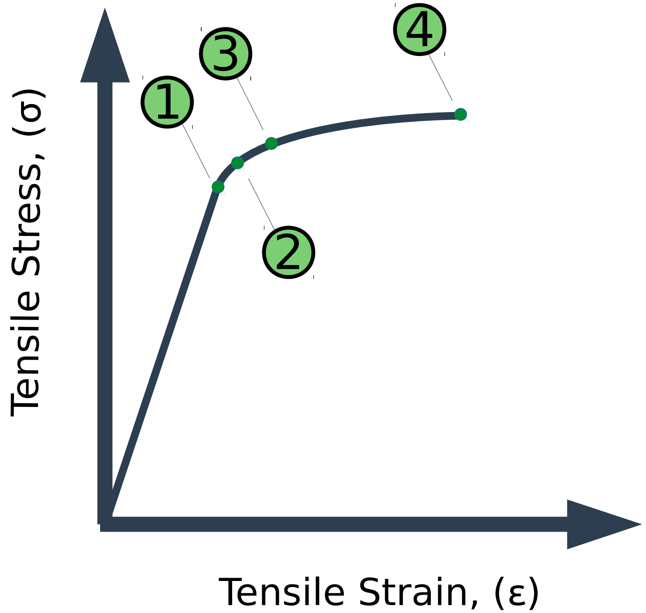
New Section!

Class #2 Goals

- **Gain a strong understanding of Modulus of Elasticity**
- **Learn/practice important vocabulary relating to elasticity.**
- **Review more complex tensile stress/strain examples**

Review Problem #1

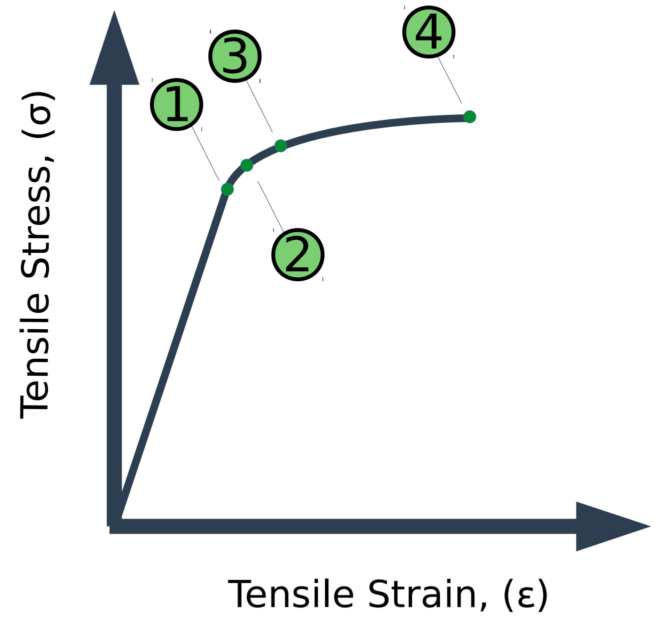
What does point 2 represent?



Limit of Proportionality	
Elastic Limit	Yield Point

Review Problem #1 Solution

What does point 2 represent?

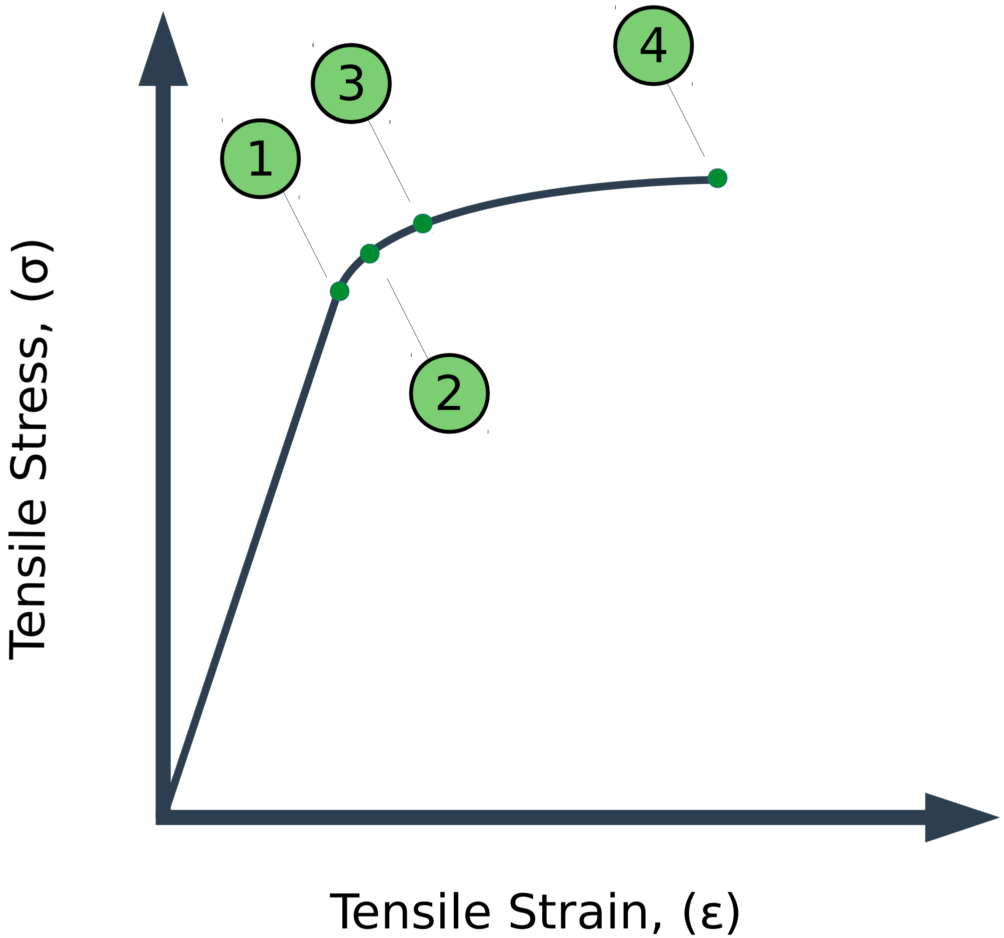


Limit of Proportionality

Elastic Limit

Yield Point

Modulus of Elasticity Review



Modulus of Elasticity

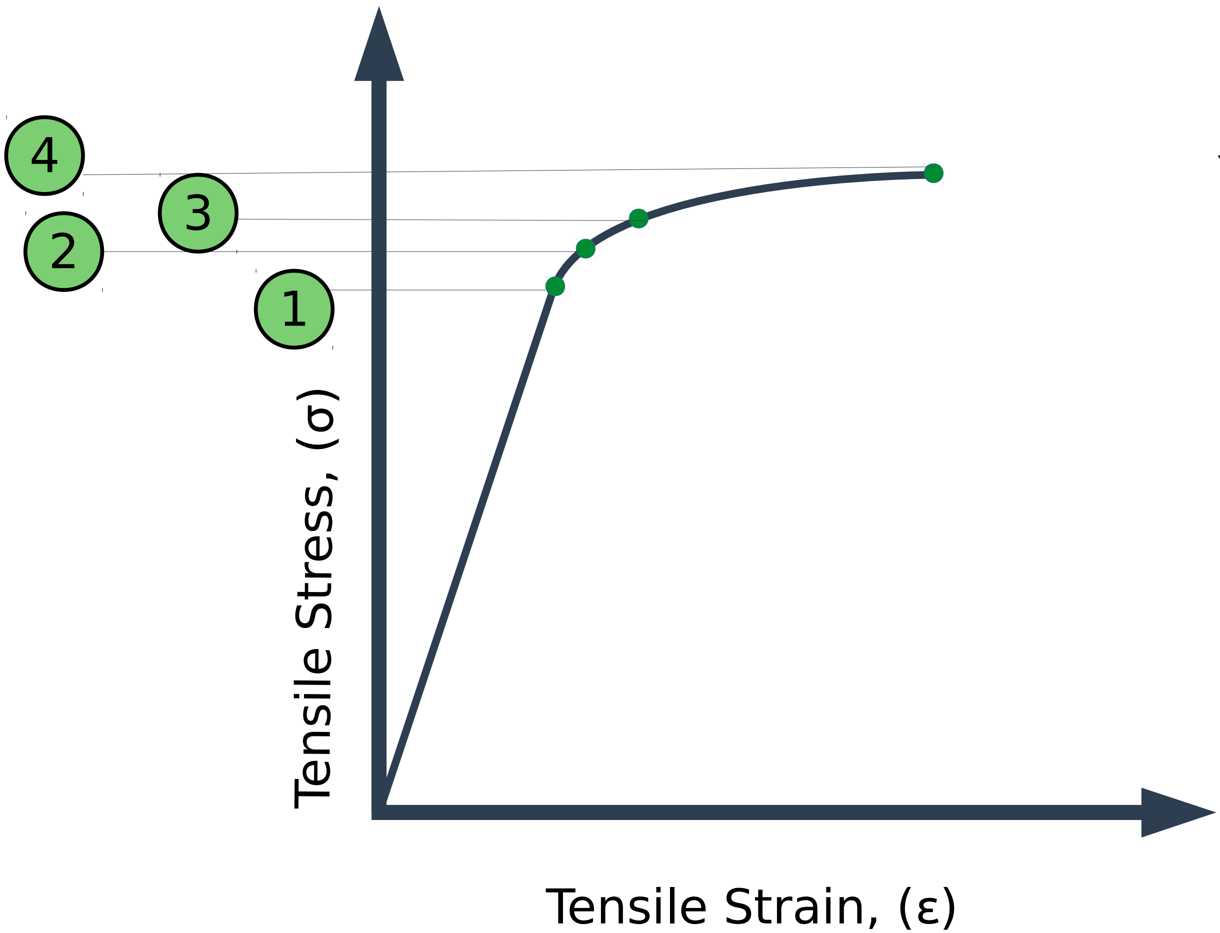
1) Limit of Proportionality

2) Elastic Limit

3) Yield Point

4) Ultimate Tensile Strength

Modulus of Elasticity Review



Limit of Proportionality
Elastic Limit
Yield Point
Ultimate Tensile Strength

***all measures of stress**

Units: N/m^2

Young’s Modulus Definition

Young’s Modulus of Elasticity

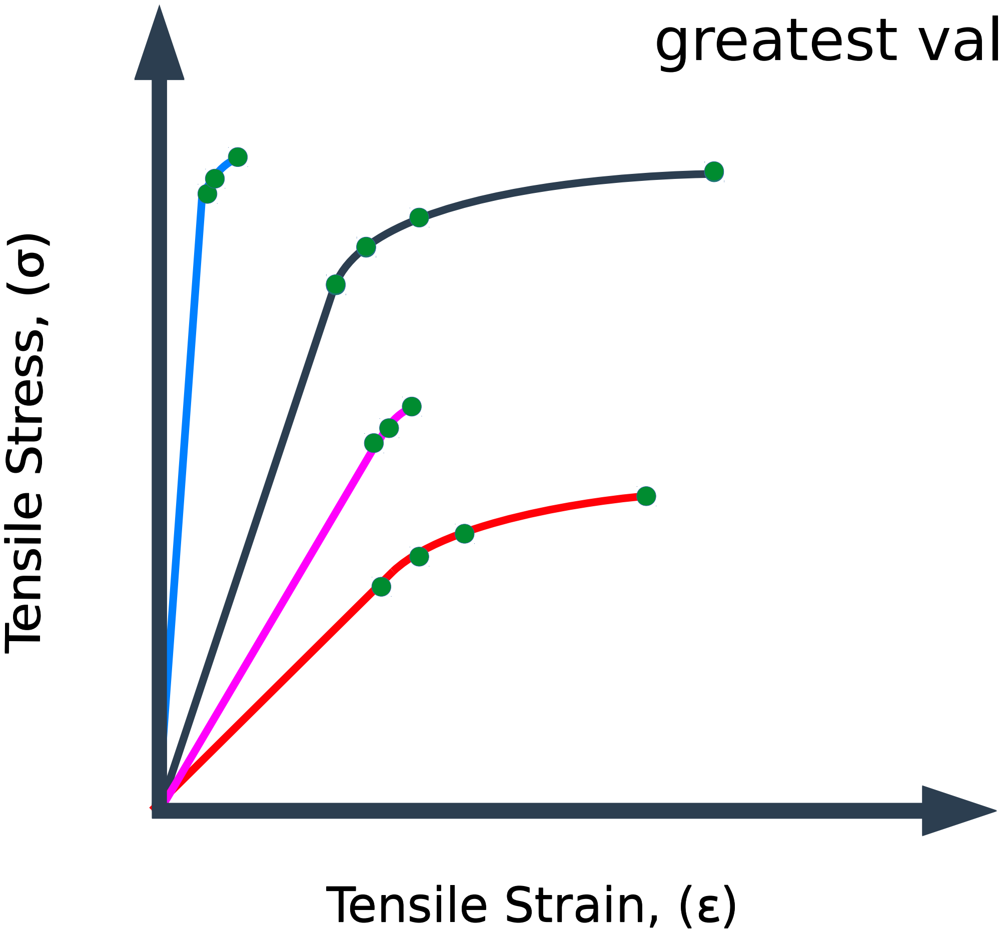
Young’s Modulus of elasticity is the ratio of the tensile stress to tensile strain measured in N/m^2.

Young’s Modulus → $E = \frac{Tensile\ Stress}{Tensile\ Strain} = \frac{F/A}{e/l} = \frac{Fl}{Ae}$

NOTE: The Young’s Modulus of elasticity is measured **before** the limit of proportionality. At this time, Hooke’s Law is still true.

Critical Thinking Practice #1

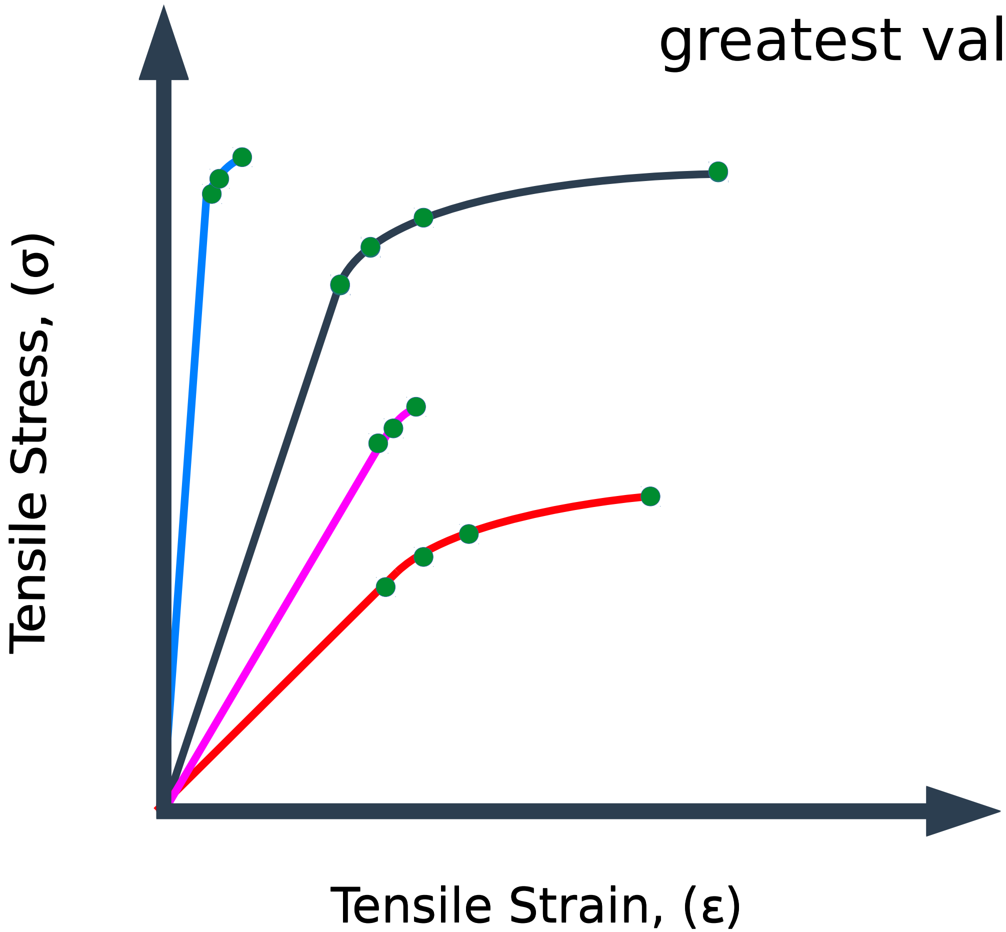
The plot below shows the modulus of elasticity for 4 materials. Which material has the greatest value of Young’s Modulus of Elasticity?



Material Red	**Material Pink**
Material Black	**Material Blue**

Critical Thinking Practice #1 Solution

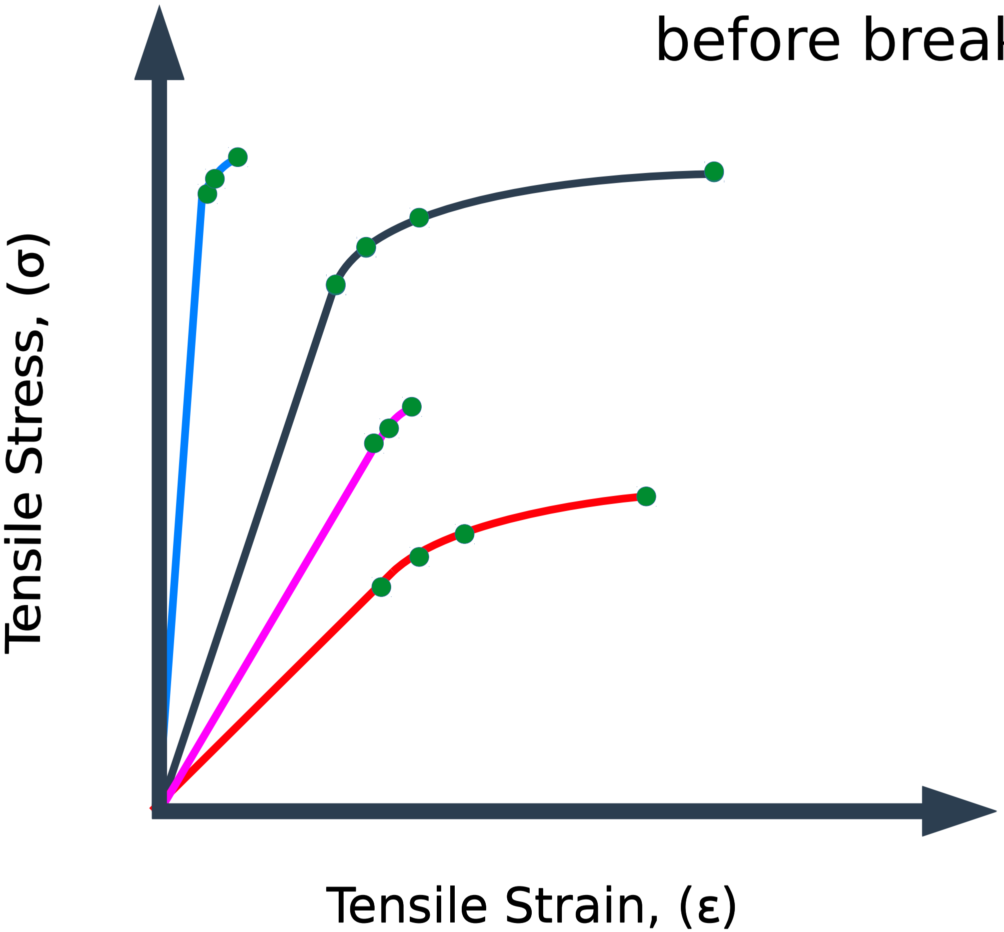
The plot below shows the modulus of elasticity for 4 materials. Which material has the greatest value of Young's Modulus of Elasticity?



△ **Material Red**	▲ **Material Pink**
○ **Material Black**	● **Material Blue**

Critical Thinking Practice #2

If all materials have an initial length of L=1m, which material has the largest length just before breaking?



Material Red	Material Pink
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Critical Thinking Practice #2 Solution

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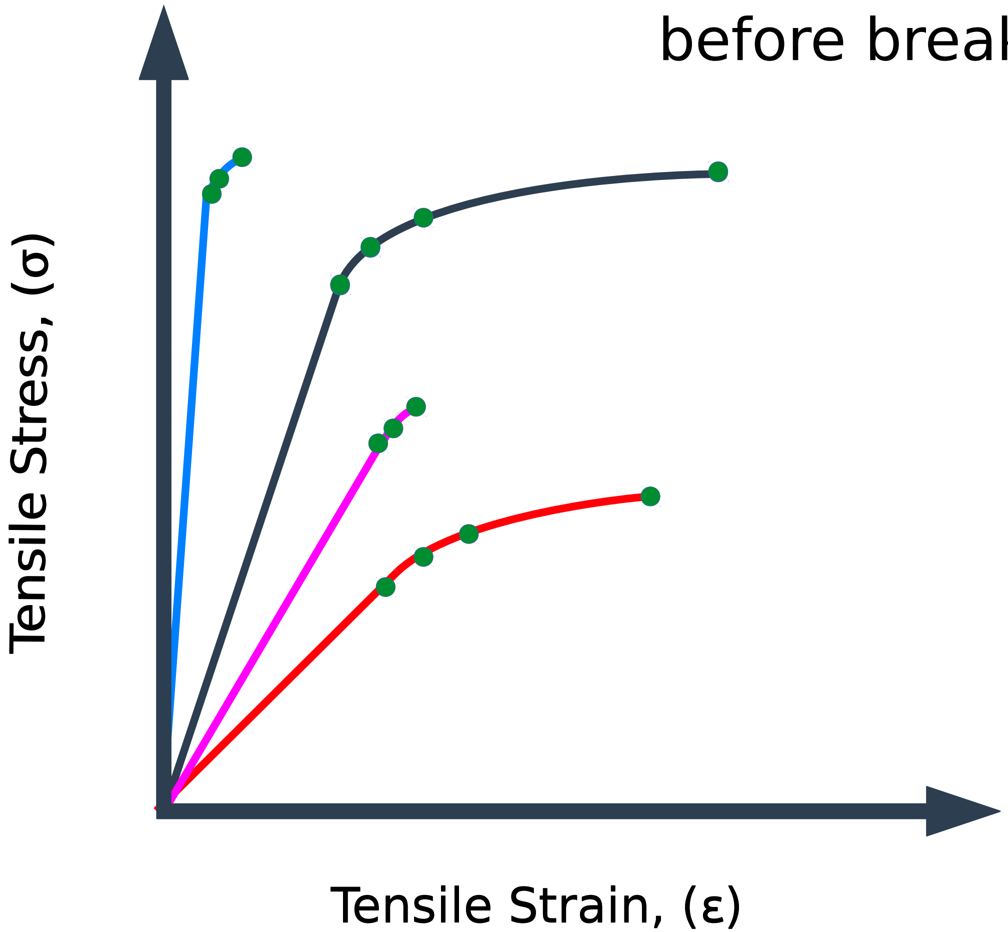


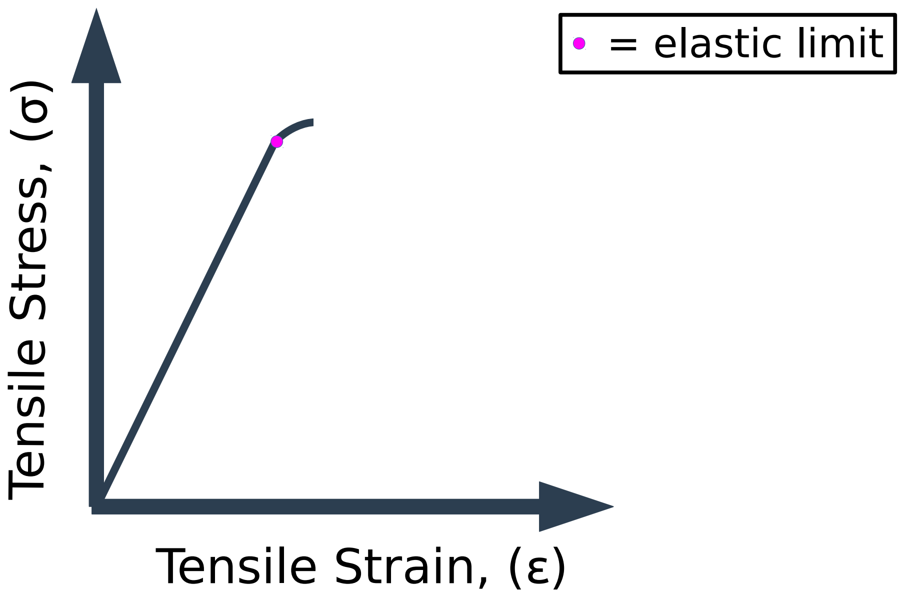
Δ Material Red	▲ Material Pink
O Material Black	● Material Blue

Brittle vs Ductile

Brittle Material

Undergoes little to no plastic deformation before breaking.

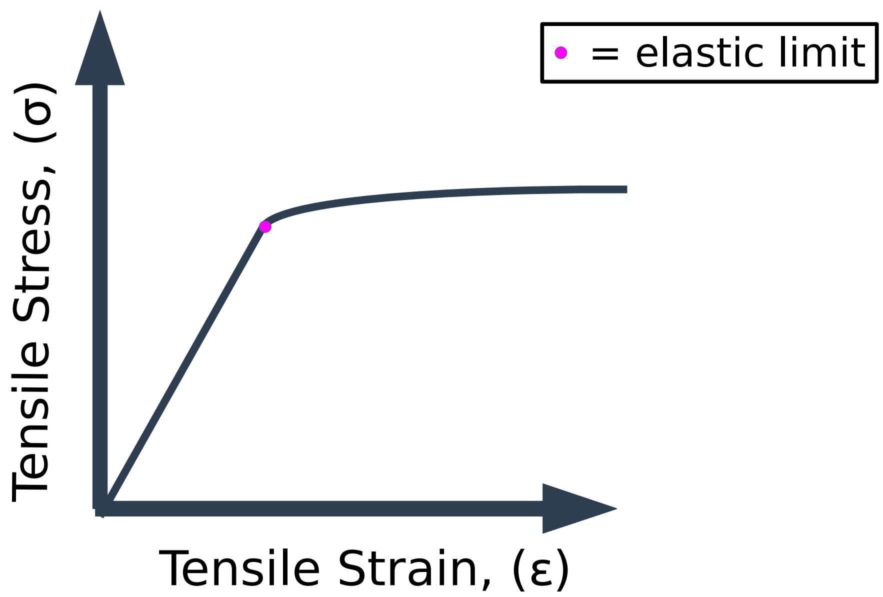
Ex: Glass, Steel, Ceramic



Ductile/Tough Material

Undergoes a great deal of plastic deformation before breaking.

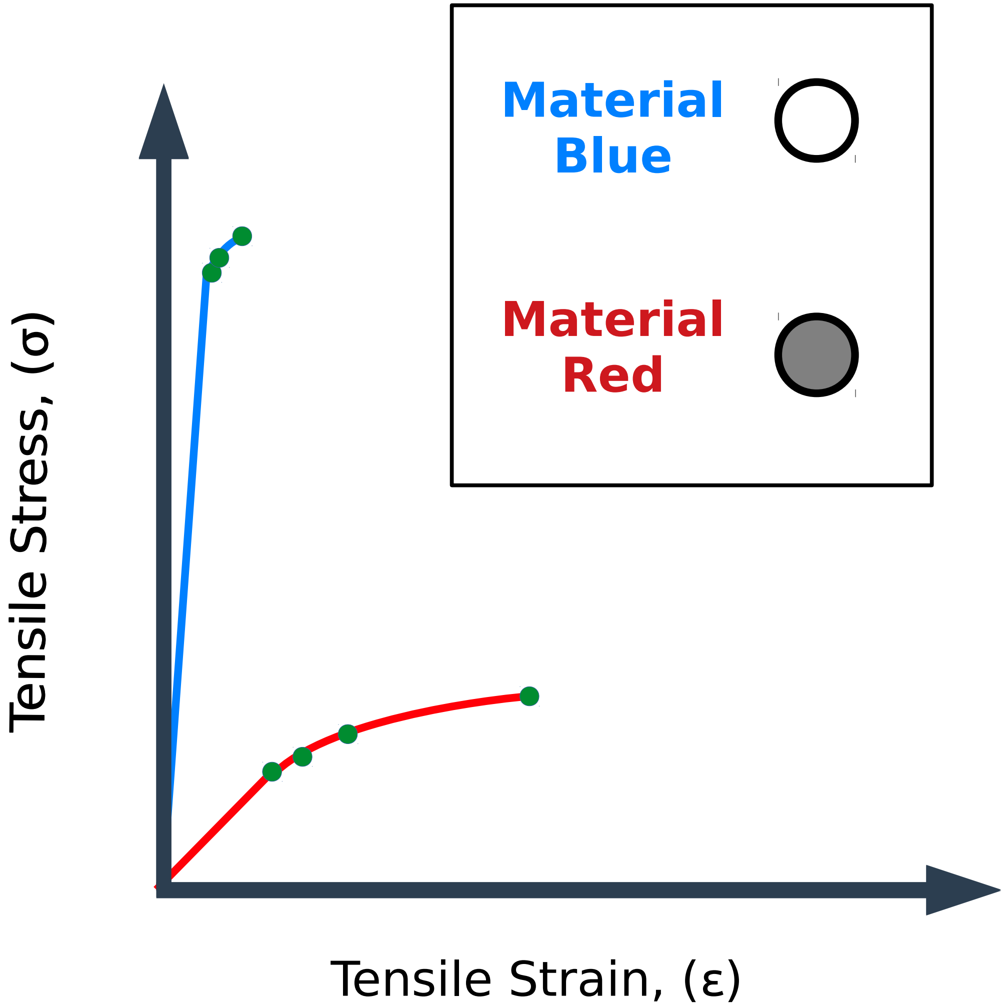
Ex: Copper



Other Vocabulary for Materials

Word	Description	Additional Notes
Strong	A large stress is needed to break it; not weak	UTS is large
Weak	A small stress is needed to break it; not strong	UTS is small
Elastic	Returns to unstretched form when stresses are removed.	Happens before the elastic limit
Plastic	Undergoes permanent deformation under stress	Happens after the elastic limit
Stiff	Small strains for large stresses; not stretchy	Young's Modulus is large
Stretchy	Large strains for large stresses; not stiff	Young's Modulus is small

Rapid Fire



Which material is stronger?

Which material is weaker?

Which material is more plastic?

Which material is more elastic?

Which material is more stiff?

Which material is more stretchy?

Which material is more brittle?

Which material is more ductile/tough?

Review Important Properties

Property	Definition	Variable	Equation	SI Units
Tensile Stress	the force acting normally per unit cross-sectional area.	σ	$\sigma = \frac{F}{A}$	$\frac{N}{m^2}$
Tensile Strain	the ratio of the extension produced per unit original length.	ϵ	$\epsilon = \frac{e}{l}$	None!
Young's Modulus	The ratio of the tensile stress to tensile strain	E	$E = \frac{\sigma}{\epsilon} = \frac{Fl}{eA}$	$\frac{N}{m^2}$

Practice Problem #1

A cylindrical wire with a length of $50\,cm$, cross sectional area of $3.14\times10^{-4}\,m^2$, and an applied load (force) of $100\,N$ extends by $2\,mm$. What is the Young's Modulus of elasticity for this material?

$E\approx1274\,\frac{N}{m^2}$	$E\approx7.96\times10^{9}\,\frac{N}{m^2}$
$E\approx12.74\,\frac{N}{m^2}$	$E\approx7.96\times10^{7}\,\frac{N}{m^2}$

Practice Problem #1 Solution

A cylindrical wire with a length of $50\,cm$, cross sectional area of $3.14\times10^{-4}\,m^2$, and an applied load (force) of $100\,N$ extends by $2\,mm$. What is the Young's Modulus of elasticity for this material?

$$E \approx 1274\,\frac{N}{m^2}$$

$$E \approx 7.96\times10^{9}\,\frac{N}{m^2}$$

$$E \approx 12.74\,\frac{N}{m^2}$$

$$E \approx 7.96\times10^{7}\,\frac{N}{m^2}$$

Practice Problem #2

A cylindrical wire with a Young's Modulus of $1.56\times10^{8}\,\frac{N}{m^{2}}$, a length of 2 meters, a diameter of 4cm has a load attached to it. The load has a mass of 10kg. What is the extension of the wire?

△ $e \approx 1cm$	▲ $e \approx 2.5mm$
○ $e \approx 1mm$	● $e \approx 0.25mm$

Practice Problem #2 Solution

A cylindrical wire with a Young's Modulus of $1.56\times10^{8}\frac{N}{m^{2}}$, a length of 2 meters, a diameter of 4cm has a load attached to it. The load has a mass of 10kg. What is the extension of the wire?

$e\approx 1cm$	$e\approx 2.5mm$
$e\approx 1mm$	$e\approx 0.25mm$

Class #2 Goals Reviewed

- Gain a strong understanding of Modulus of Elasticity
- Learn/practice important vocabulary relating to elasticity.
- Review more complex tensile stress/strain examples

New Section!

Class #3 Goals

- **Review elasticity concepts from before break**
- **Review more complex tensile stress/strain examples**
- **Understand work done / energy in stretching a wire**

Review Question #1

A 5kg weight is attached to a cylindrical metal rod of $0.1m^2$ in cross-sectional area and 1 meter in length. It stretches 5mm. What is the tensile stress on the metal rod?

△ $\sigma = 4.9\,\frac{N}{m^2}$	▲ $\sigma = 490\,\frac{N}{m^2}$
○ $\sigma = 50\,\frac{N}{m^2}$	● $\sigma = 0.5\,\frac{N}{m^2}$

Review Question #1 Solution

A 5kg weight is attached to a cylindrical metal rod of 0.1m² in cross-sectional area and 1 meter in length. It stretches 5mm. What is the tensile stress on the metal rod?

△ $\sigma = 4.9 \dfrac{N}{m^2}$	▲ $\sigma = 490 \dfrac{N}{m^2}$
○ $\sigma = 50 \dfrac{N}{m^2}$	● $\sigma = 0.5 \dfrac{N}{m^2}$

Review Question #2

A 5kg weight is attached to a cylindrical metal rod of $0.1m^2$ in cross-sectional area and 1 meter in length. It stretches 5mm. What is the tensile strain on the metal rod?

$\epsilon = 1.5$	$\epsilon = 0.005$
$\epsilon = 200$	$\epsilon = 500$

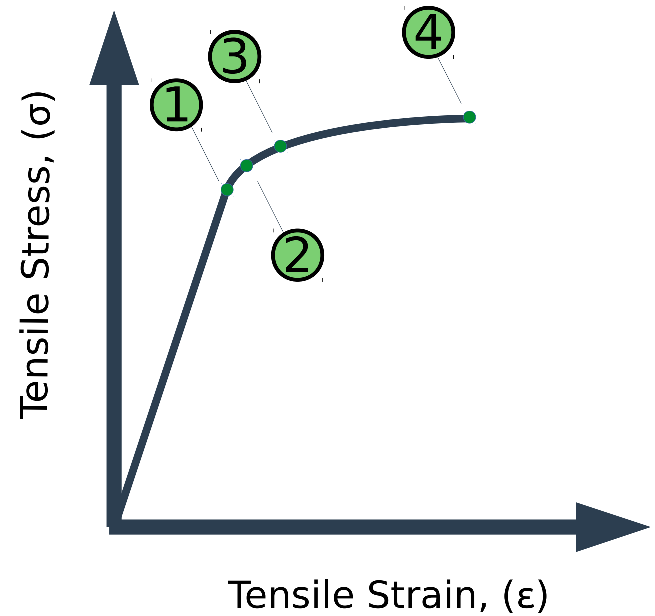
Review Question #2 Solution

A 5kg weight is attached to a cylindrical metal rod of 0.1m² in cross-sectional area and 1 meter in length. It stretches 5mm. What is the tensile strain on the metal rod?

$\epsilon = 1.5$	$\epsilon = 0.005$
$\epsilon = 200$	$\epsilon = 500$

Review Problem #3

What does point 4 represent?



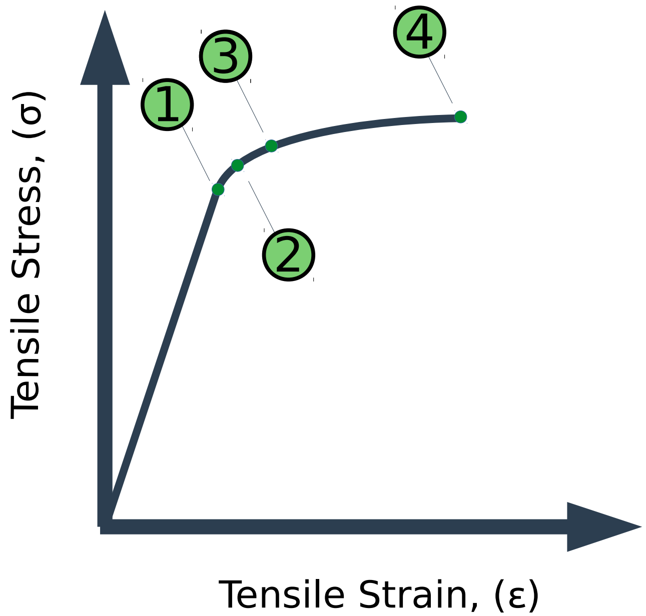
Limit of Proportionality

Elastic Limit

Ultimate Tensile Strength

Review Problem #3 Solution

What does point ④ represent?



Limit of Proportionality	
Elastic Limit	Ultimate Tensile Strength

Review Question #4

All four parts of the modulus of elasticity - limit of proportionality, elastic limit, yeild point, and ultimate tensile strength (UTS) – have what units?

$\frac{N}{m^2}$	N
m^3	$unitless$

Review Question #4 Solution

All four parts of the modulus of elasticity - limit of proportionality, elastic limit, yeild point, and ultimate tensile strength (UTS) – have what units?

△ $\frac{N}{m^2}$	▲ N
○ m^3	● $unitless$

Complex Problem Review

A lift is designed to hold a maximum of 10 people. The lift cage has a mass of 1000kg.

Useful Constants: $UTS_{steel} = 4 \times 10^8\, Nm^{-2}$, $m_{1\,person} = 70\, kg$

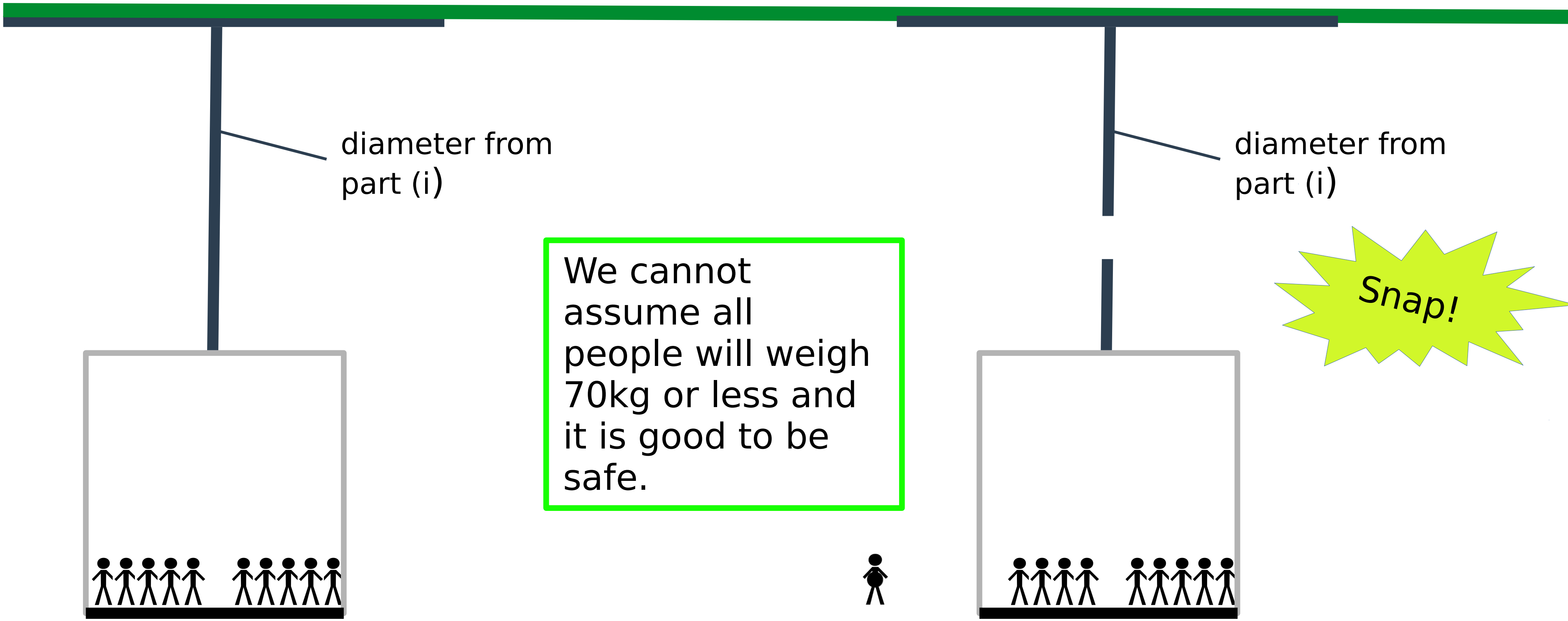
i. What minimum radius should the cable have in order to support the maximum load? $\boxed{r \approx 0.3641\, mm}$

ii. Why should the cable be thicker than solved for in the part (i) in practice?

Hooke's Law

Why should the cable be thicker than solved for in the part (i) in practice?

= average person   = fat person (more mass)



Practice Problem #3

Steel has a Ultimate Tensile Strength of $UTS_{steel} = 4 \times 10^8\, Nm^{-2}$ and a Young's Modulus of $E_{steel} = 2 \times 10^{11}\, Nm^{-2}$. What is the extension of an originally 2m steel beam just before it breaks?

△ $e = 2mm$	▲ $e = 1000\, m$
○ $e = 4\, mm$	● **None of these**

Practice Problem #3 Solution

Steel has a Ultimate Tensile Strength of $UTS_{steel} = 4 \times 10^{8}\, Nm^{-2}$ and a Young's Modulus of $E_{steel} = 2 \times 10^{11}\, Nm^{-2}$. What is the extension of an originally 2m steel beam just before it breaks?

△ $e = 2mm$	▲ $e = 1000\, m$
○ $e = 4\, mm$	● **None of these**

Work it out

Consider the amount of work done δW in moving a wire a small distance δx at constant force.

$$\delta W = F\,\delta x$$

Using calculus we find that:

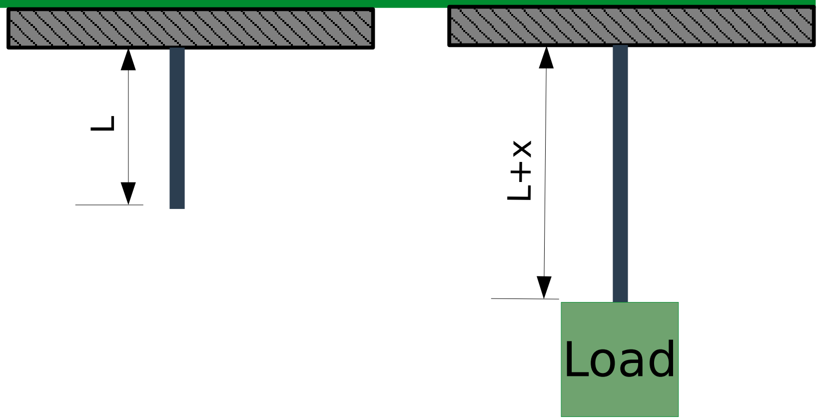
If Hooke's Law applies

$$\boxed{F = kx}$$

$$W = \int_0^x F\,dx$$

$$W = \int_0^x kx\,dx \quad \Rightarrow \quad \boxed{W = \frac{1}{2}kx^2 = \frac{1}{2}Fx}$$

Important Note!



$$W = \frac{1}{2} F x$$

"Stretching a wire a distance of x" → $x = e$

$$\boxed{W = \frac{1}{2} F e}$$

* you use this equation more often than the one with k

$$W = \frac{1}{2}ke^2 \qquad W = \frac{1}{2}Fe$$

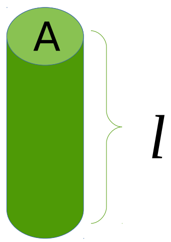
<u>This quantity is called:</u>

- Work done in stretching a wire
- Energy stored
- Elastic potential energy
- Strain energy

Same

Strain Energy per unit volume

For a wire: $W = \frac{1}{2} F e$, $V = A\,l$



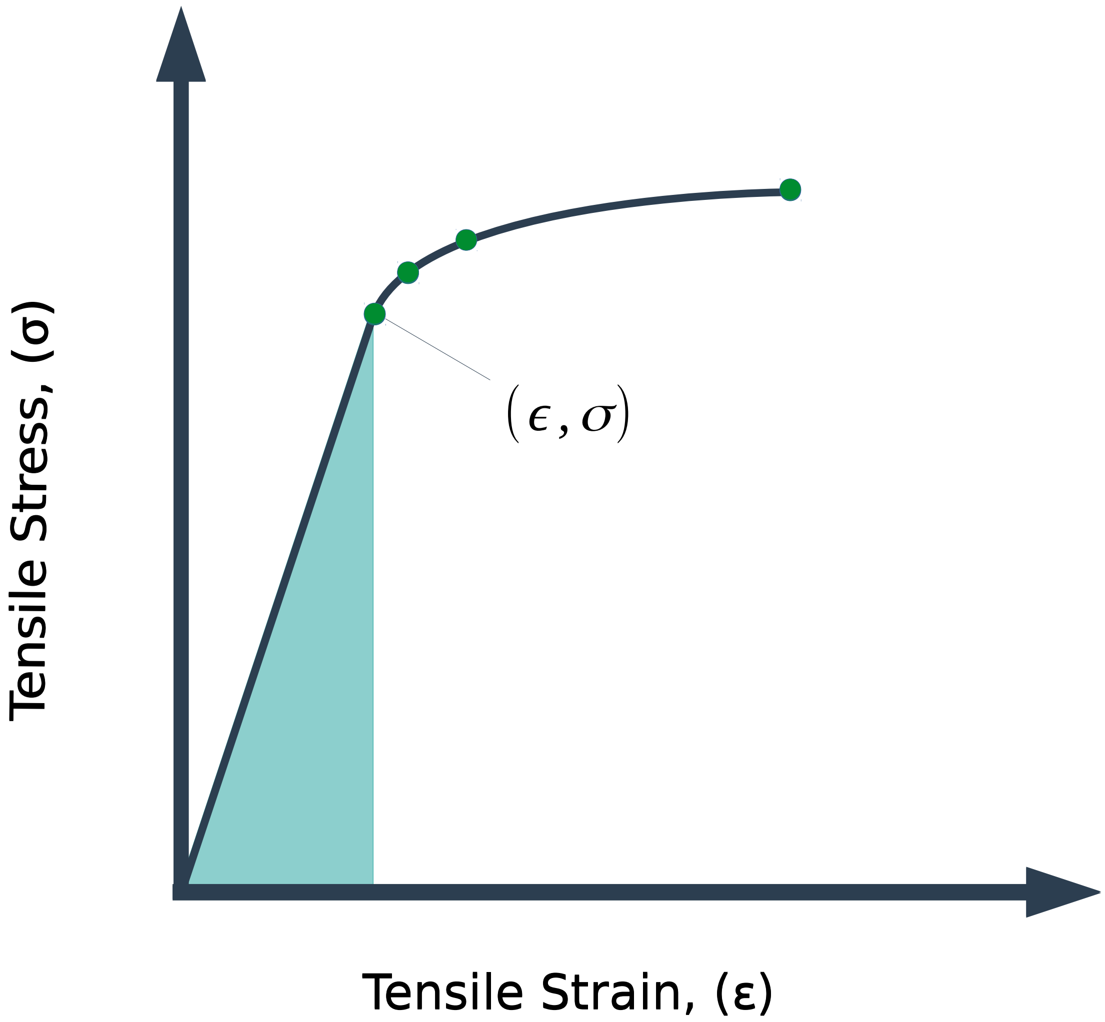
Tensile stress

$$\text{Strain energy per unit volume } = \frac{W}{V} = \frac{\frac{1}{2} F e}{A l} = \frac{1}{2} \times \boxed{\frac{F}{A}} \times \boxed{\frac{e}{l}}$$

Tensile strain

$$\boxed{\text{Strain energy per unit volume } = \frac{1}{2} \sigma \epsilon}$$

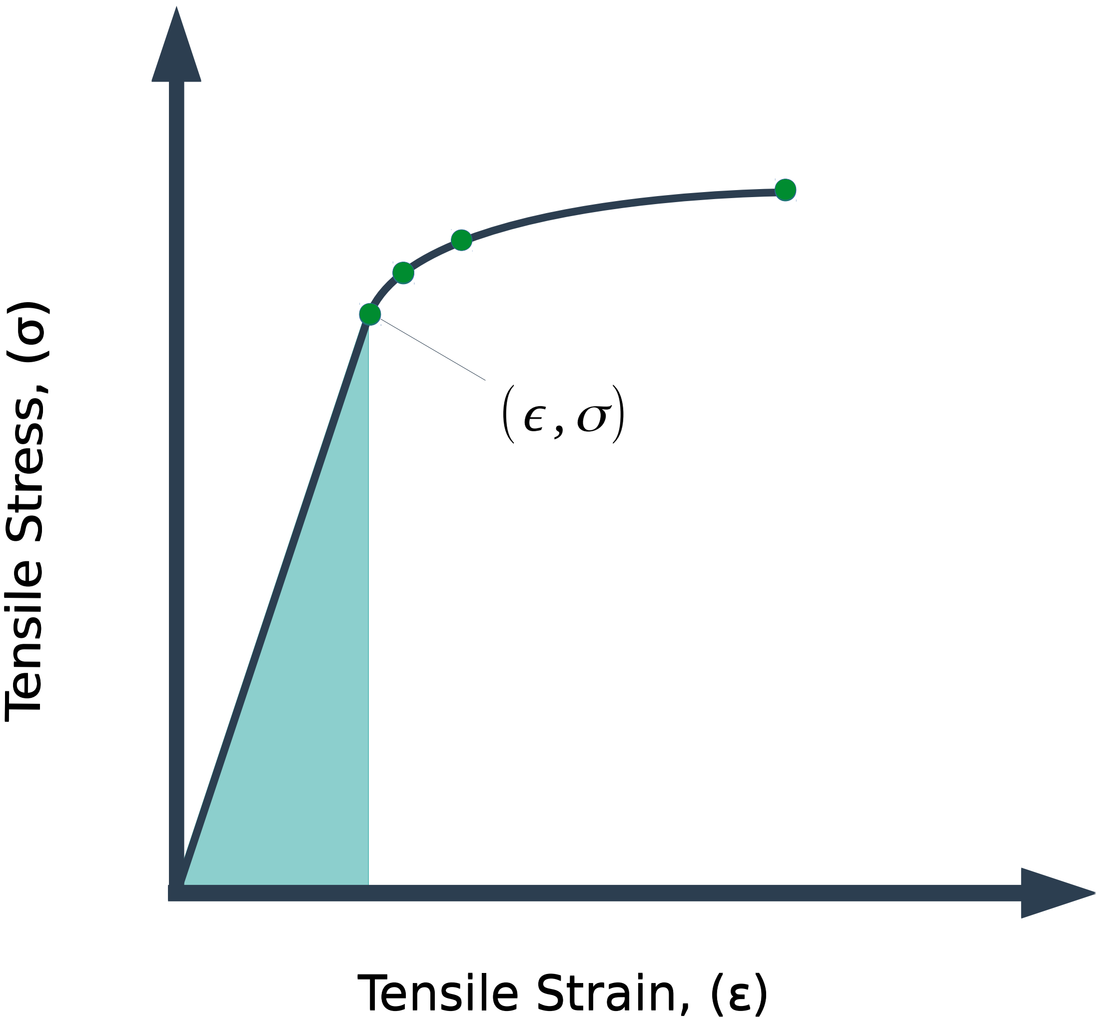
Critical Thinking Practice #1



What is the area of the shaded region?

△ $\epsilon\sigma$	▲ $\frac{1}{2}\epsilon\sigma$
○ $(\epsilon\sigma)^2$	● $\frac{1}{2}(\epsilon\sigma)^2$

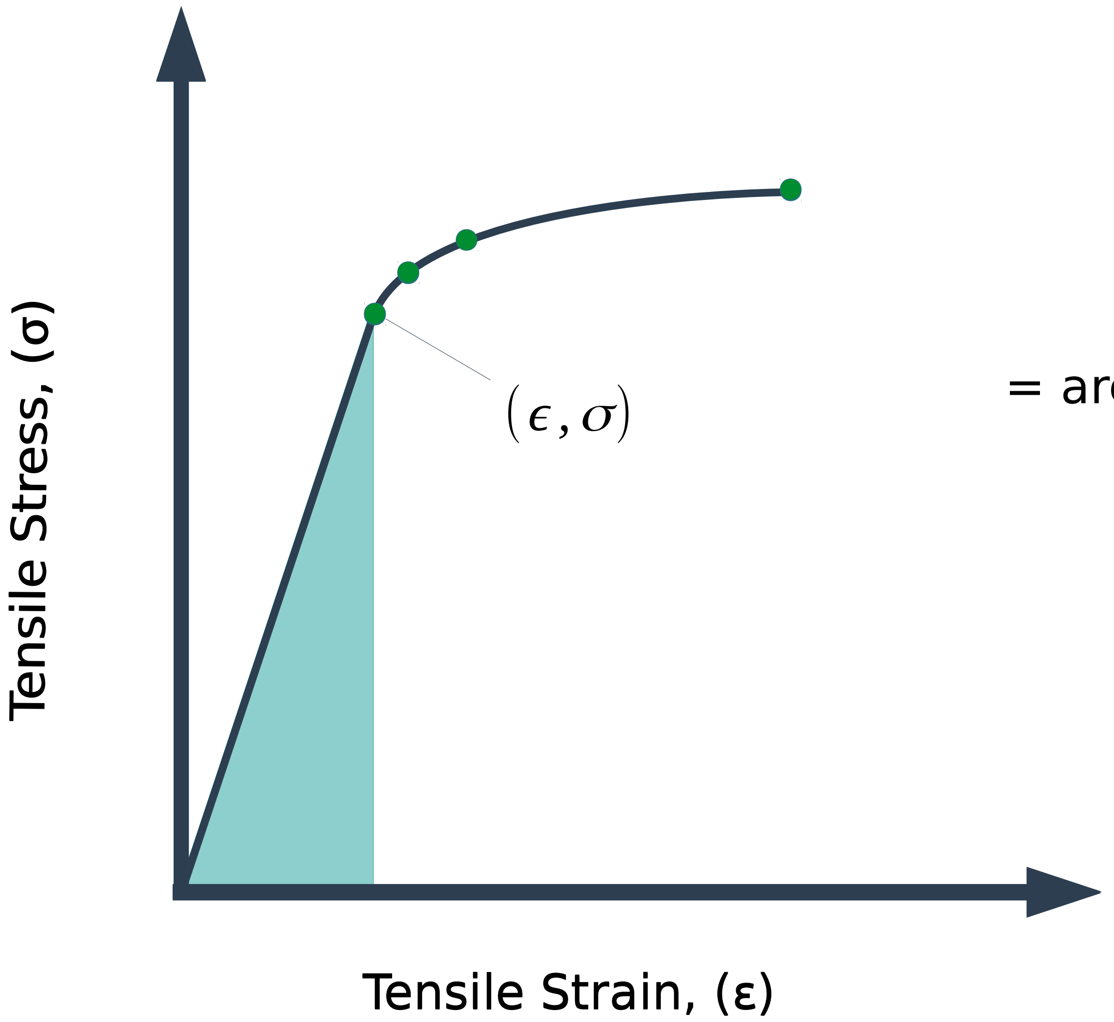
Critical Thinking Practice #1 Solution



What is the area of the shaded region?

$\epsilon\,\sigma$	$\frac{1}{2}\epsilon\,\sigma$
$(\epsilon\,\sigma)^2$	$\frac{1}{2}(\epsilon\,\sigma)^2$

Hooke's Law



$$\boxed{\text{Strain energy per unit volume} = \frac{1}{2}\sigma\epsilon}$$

= area under the modulus of elasticity curve

(before the limit of proportionality)

Review Problem #1

A $1\,m$ long wire with a cross sectional area of $0.5\,m^2$ stretches $\frac{2}{9.8}mm$ when a $1000\,kg$ load is attached. What is the work done in stretching the wire?

△ $W = 1\,N \cdot m$	▲ $W = 2\,N \cdot m$
○ $W = 4900\,N \cdot m$	● $W = 19600\,N \cdot m$

Review Problem #1 Solution

A $1\,m$ long wire with a cross sectional area of $0.5\,m^2$ stretches $\frac{2}{9.8}mm$ when a $1000\,kg$ load is attached. What is the work done in stretching the wire?

$W = 1\,N \cdot m$	$W = 2\,N \cdot m$
$W = 4900\,N \cdot m$	$W = 19600\,N \cdot m$

Review Problem #2 Solution

A $1\,m$ long wire with a cross sectional area of $0.5\,m^2$ stretches $\frac{2}{9.8}mm$ when a $1000\,kg$ load is attached. What is the energy per unit volume in the wire?

△ $1\frac{N}{m^2}$	▲ $2\frac{N}{m^2}$
○ $4900\frac{N}{m^2}$	● $19600\frac{N}{m^2}$

Review Problem #2

A $1\,m$ long wire with a cross sectional area of $0.5\,m^2$ stretches $\frac{2}{9.8}mm$ when a $1000\,kg$ load is attached. What is the energy per unit volume in the wire?

$1\frac{N}{m^2}$	$2\frac{N}{m^2}$
$4900\frac{N}{m^2}$	$19600\frac{N}{m^2}$

Review of Work

Property	Definition	Variable	Equation	SI Units
Work Done, Energy Stored, Strain Energy, Elastic P.E.	The amount of work required to stretch a wire a distance, x.	W	$W = \frac{1}{2} Fe$	$N \cdot m$
Energy per unit Volume, Strain energy per unit Volume	The amount of strain energy applied per unit volume.	none	$= \frac{W}{V}$ $= \frac{1}{2} \sigma \epsilon$	$\frac{N}{m^2}$

* also equal to area under modulus of elasticity plot

Class #3 Goals Reviewed

- **Review elasticity concepts from before break**
- **Review more complex tensile stress/strain examples**
- **Understand work done / energy in stretching a wire**

New Section!

Class #4 Goals

- **Check for understanding of last class**
- **Review more complex Young's Modulus problems**
- **Move on to Bulk Stress and Bulk Strain**

Quick Review Problem #1

$\frac{1}{2}Fe$ **represents which physical quantity?**

Strain Energy	Work done in stretching a wire
Energy Stored	All of these

Quick Review Problem #1 Solution

$\frac{1}{2}Fe$ **represents which physical quantity?**

Strain Energy	Work done in stretching a wire
Energy Stored	All of these

Quick Review Problem #2

$\frac{1}{2}ke^2$ **represents which physical quantity?**

△ Strain Energy	▲ Work done in stretching a wire
○ Energy Stored	● All of these

Quick Review Problem #2 Solution

$\frac{1}{2}ke^2$ **represents which physical quantity?**

Strain Energy	Work done in stretching a wire
Energy Stored	All of these

Complex Example

A uniform wire of density 7,800 kg/m³ is 1m long and weighs 10kg. When stretched by a force of 100N, the wire lengthens by 1mm. Calculate:

i. the value of the Young's modulus of the wire.

$$\boxed{E = 7.8 \times 10^{7} \frac{N}{m^{2}}}$$

ii. the energy per unit volume stored in the wire.

$$\boxed{\text{Energy per unit volume} = 39 \frac{N}{m^{2}}}$$

Word Problem #1

A steel metal wire has a Young's Modulus, (E) when a load of (m) is applied. If the load is decreased to (0.5m), what is the value of Young's Modulus?

Young's Modulus doubles

$$E_{new} = 2E$$

Young's Modulus does not change

$$E_{new} = E$$

Young's Modulus halves

$$E_{new} = \frac{1}{2}E$$

We cannot know with the data

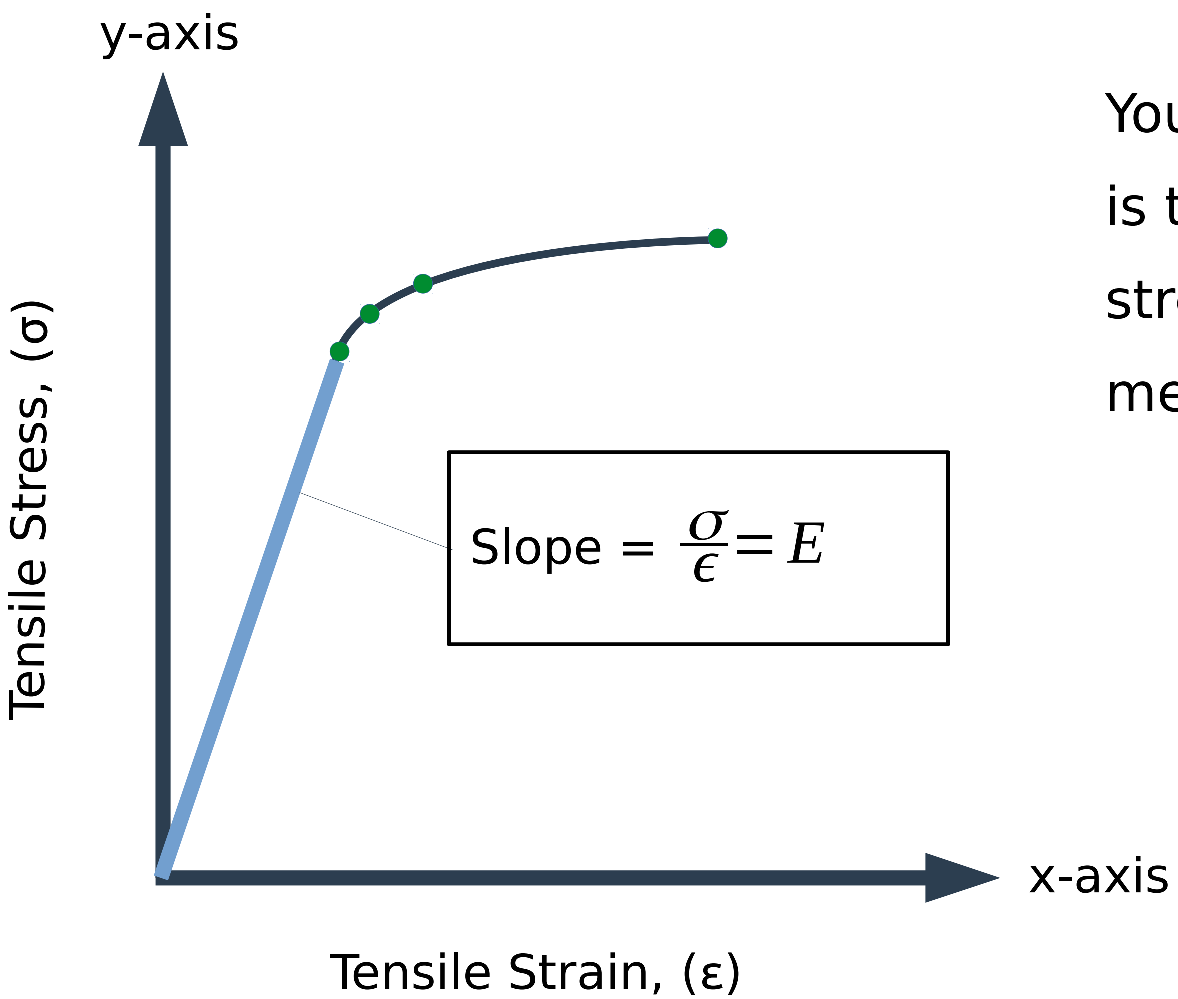
$$E_{new} = ?$$

Word Problem #1 Solution

A steel metal wire has a Young's Modulus, (E) when a load of (m) is applied. If the load is decreased to (0.5m), what is the value of Young's Modulus?

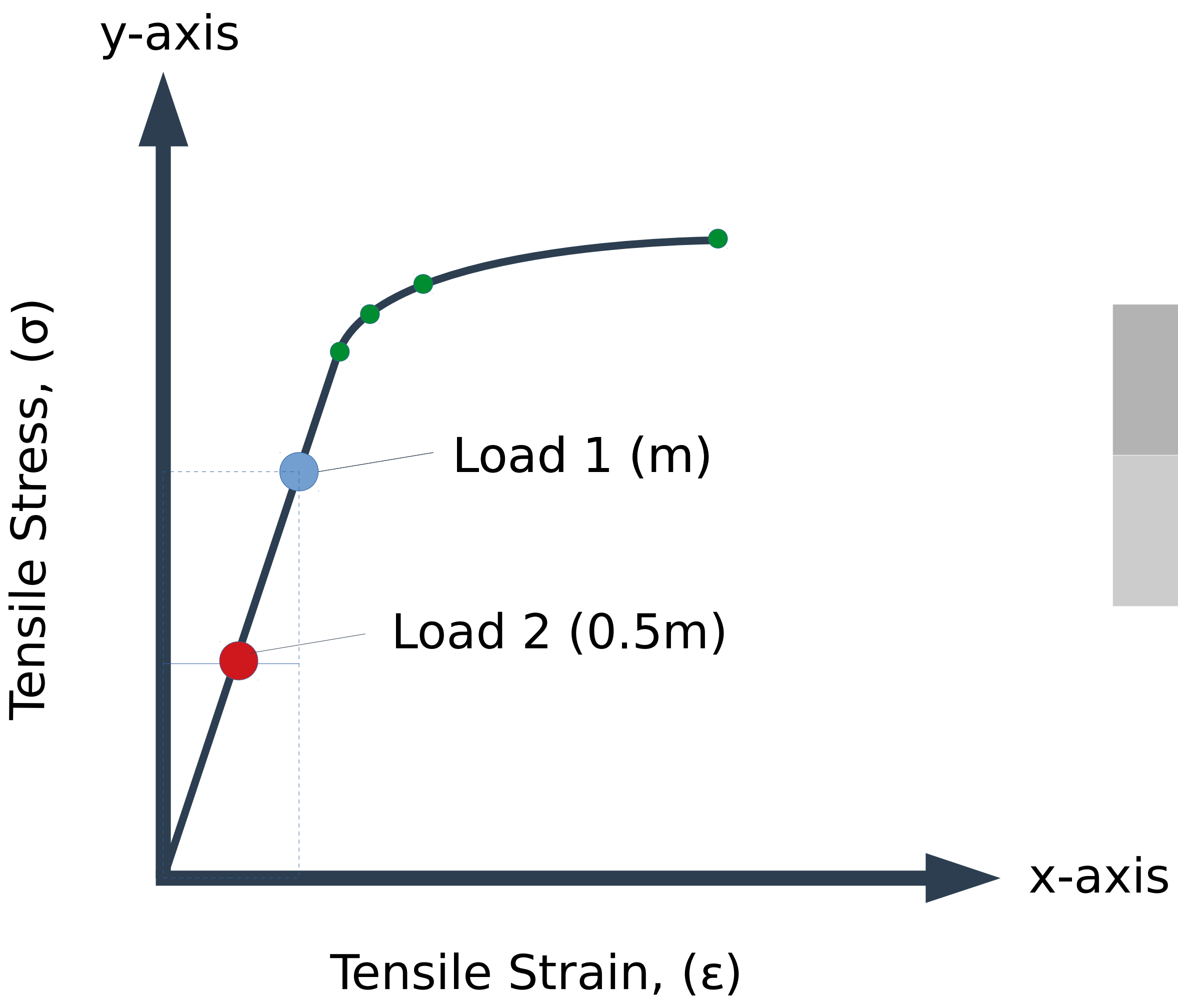
Young's Modulus doubles $E_{new} = 2E$	**Young's Modulus does not change** $E_{new} = E$
Young's Modulus halves $E_{new} = \frac{1}{2}E$	**We cannot know with the data** $E_{new} = ?$

Young's Modulus Review (again)



Young's Modulus of elasticity (E) is the ratio of the tensile stress to tensile strain measured in N/m^2.

Young’s Modulus Review (again)



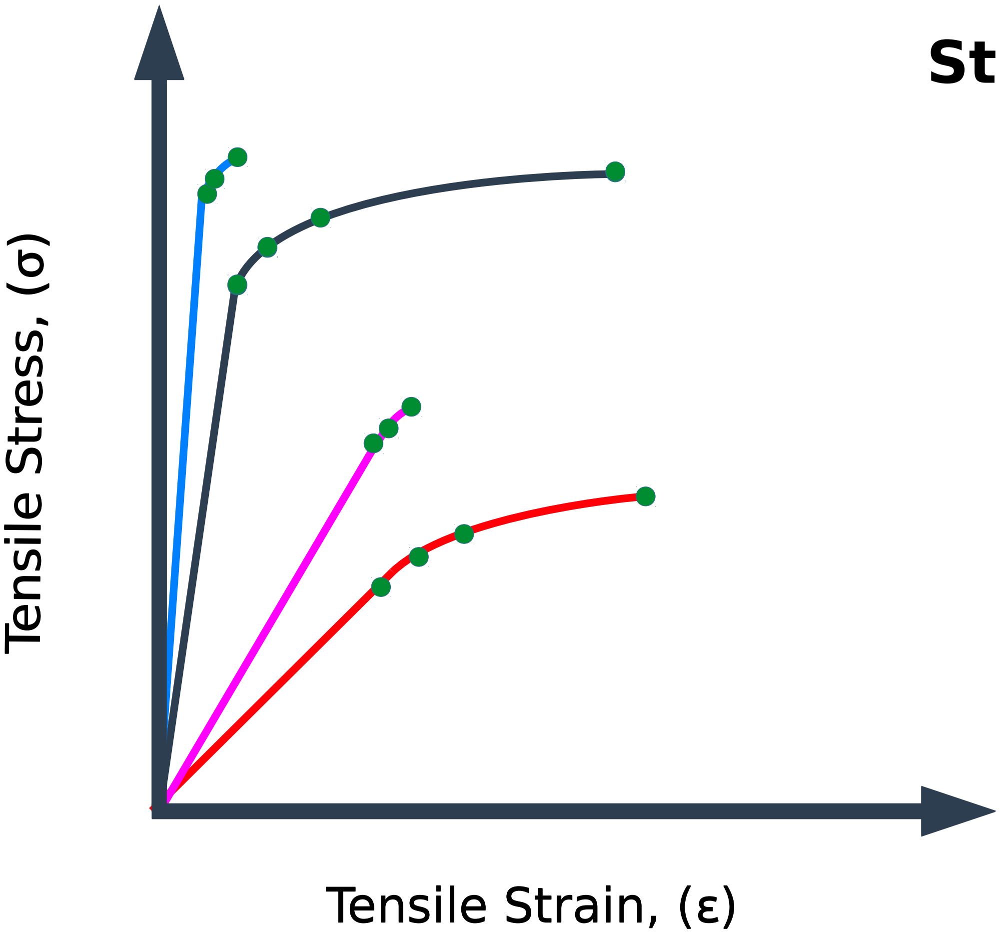
Mass →	Force →	Tensile Stress
m	mg	mg/A
0.5m	0.5mg	0.5(mg/A)

Same slope, same Young’s Modulus (E)

Word Problem #2

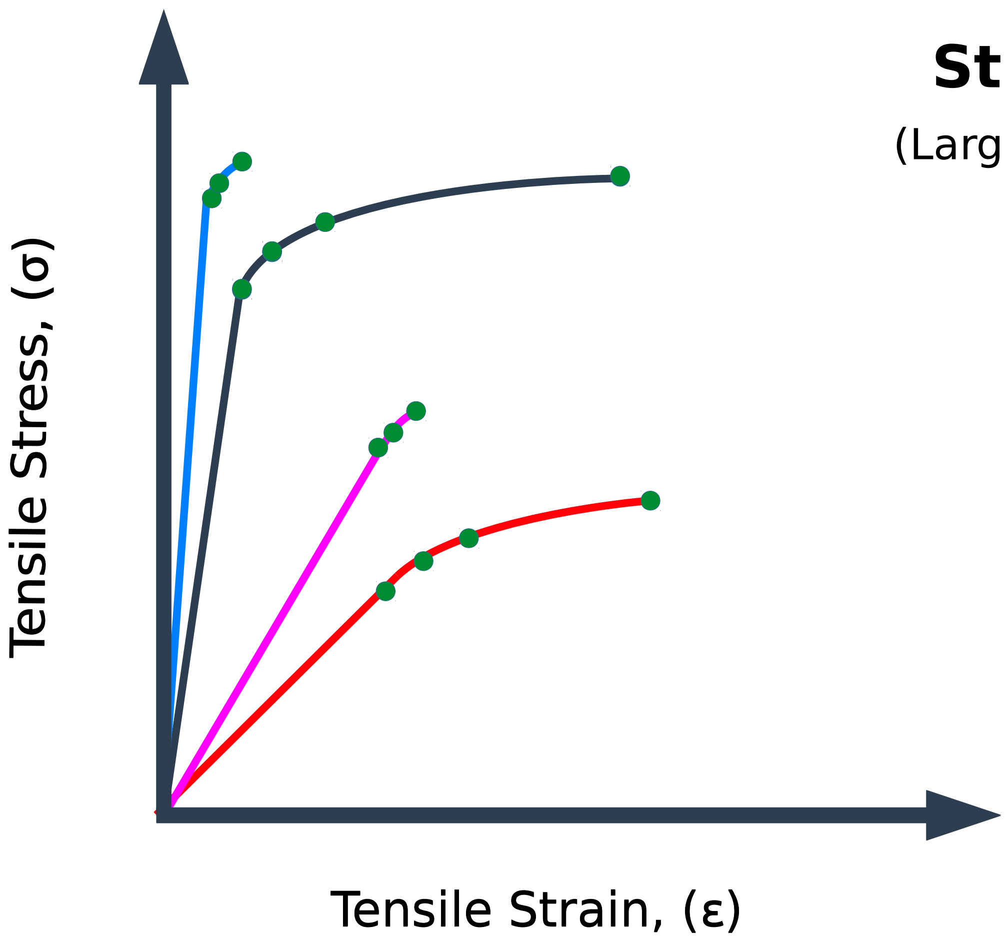
Which material can be described as:

Strong, Stiff, Ductile/Tough



Material Red	**Material Pink**
Material Black	**Material Blue**

Word Problem #2 Solution



Which material can be described as:

Strong, Stiff, Ductile/Tough

(Large UTS) (Large E) (Long plastic deformation)

△ **Material Red**	▲ **Material Pink**
○ **Material Black**	● **Material Blue**

Complex Review #1

A steel wire of length $l_1 = 1.0\,m$ and diameter $d_1 = 1\,cm$ stretches by the same amount as a copper wire of length $l_2 = 2.0\,m$ and diameter $d_2 = 2\,cm$ under a given load. What is the ratio of the Young's Modulus of steel to that copper? $\boxed{2}$

Important words:

"...stretches by the same amount..." → $e_1 = e_2$

"...under a given load." → $F_1 = F_2$

Complex Review #2

A vertical wire made of steel of length 1.0m and 2.0mm diameter has a load of 10.0kg applied to its lower end. What is the energy stored in the wire?

$$^{*}E_{steel} = 2 \times 10^{11} \frac{N}{m^2}$$

$$\boxed{W = 7.64 \times 10^{-3}\, N \cdot m}$$

Class #4 Goals Reviewed

- **Check for understanding of last class**
- **Review more complex Young’s Modulus problems**
- **Move on to Bulk Stress and Bulk Strain**

Next Section!

Class #5 Goals

- **<u>Very</u> quickly Review Bulk Stress / Bulk Strain**
- **Practice bulk stress/strain problems**
- **Move on to Shear Stress and Shear Strain**
- **Learn new vocabulary**

<u>Goal</u>: next class → ½ Elasticity, ½ Kinetic Theory of Gases

Too Bulky

Bulk Stress σ

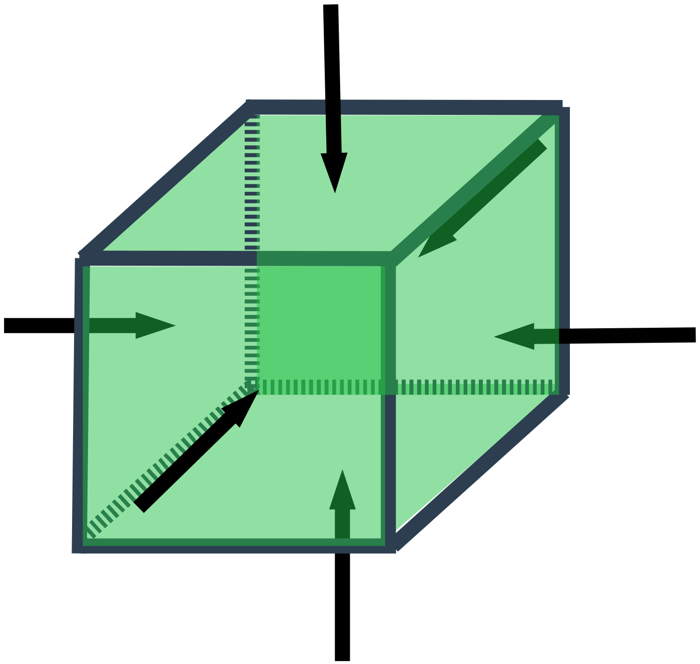
The difference in pressure

SI Unit: Pa

Bulk Strain ϵ

The fractional change in volume

SI Unit: none!

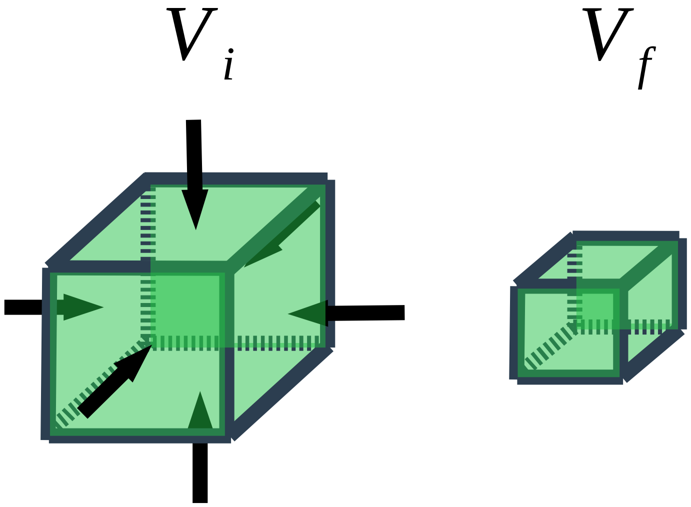


Under Pressure

Bulk Stress: $\Delta P = P_f - P_i$

Bulk Strain: $\dfrac{\Delta V}{V} = \dfrac{V_f - V_i}{V_i}$

i = initial   f = final



SI Unit: Pa

Bulk Modulus (K) – the bulk stress per bulk strain. $K = \dfrac{Bulk\ Stress}{Bulk\ Strain}$

Practice Problem #1

Given the information from the last slide, guess the equation for the bulk modulus?

$K = \frac{\Delta P}{\Delta V}$	$K = V \frac{\Delta P}{\Delta V}$
$K = \frac{\Delta P}{V}$	$K = \frac{\Delta P \Delta V}{V}$

Practice Problem #1

Given the information from the last slide, guess the equation for the bulk modulus?

$$K = \frac{\Delta P}{\Delta V}$$

$$K = V \frac{\Delta P}{\Delta V}$$

$$K = \frac{\Delta P}{V}$$

$$K = \frac{\Delta P \Delta V}{V}$$

Important note:

The Bulk Modulus is always positive

Consider an increase in pressure $P_f > P_i \rightarrow +\Delta P$

Results in a decrease in volume $V_f < V_i \rightarrow -\Delta V$

Before

$$\boxed{K = V \frac{\Delta P}{\Delta V}}$$

Therefore we need to introduce a negative sign to keep K positive!

Correct

$$\boxed{K = -V \frac{\Delta P}{\Delta V}}$$

Another expression for Bulk Modulus

$$K = -V \frac{\Delta P}{\Delta V} \Rightarrow K = -V_i \left(\frac{P_f - P_i}{V_f - V_i} \right)$$

Practice Problem #2

A volume has an initial volume of 2 m^3 at an initial pressure of 1 Pa. When the pressure is increased to 2 Pa, the volume shrinks to 1m^3. What is the bulk modulus?

△ $K = 0.5\,Pa$	▲ $K = 1\,Pa$
○ $K = 2\,Pa$	● $K = 4\,Pa$

Practice Problem #2

A volume has an initial volume of 2 m³ at an initial pressure of 1 Pa. When the pressure is increased to 2 Pa, the volume shrinks to 1m³. What is the bulk modulus?

△ $K = 0.5\,Pa$	▲ $K = 1\,Pa$
○ $K = 2\,Pa$	● $K = 4\,Pa$

Practice Problem

Water has a bulk modulus of $2.0 \times 10^{9}\,Pa$, and a density of $1000\,\frac{kg}{m^{3}}$ at atmospheric pressure (atm), $10^{5}\,Pa$.

a. What volume does 3kg of water occupy at atmospheric pressure?

$$\boxed{0.003\,m^{3}}$$

b. What is its change in volume when the external pressure is increased to 5 atm?

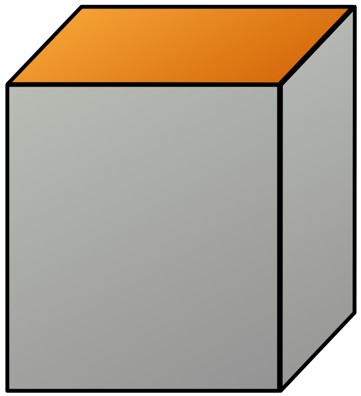
$$\boxed{-6 \times 10^{-7}\,m}$$

Shear Modulus

Shear Stress σ

The shear force per unit area with a turning or displacement effect due to a presence of a couple.

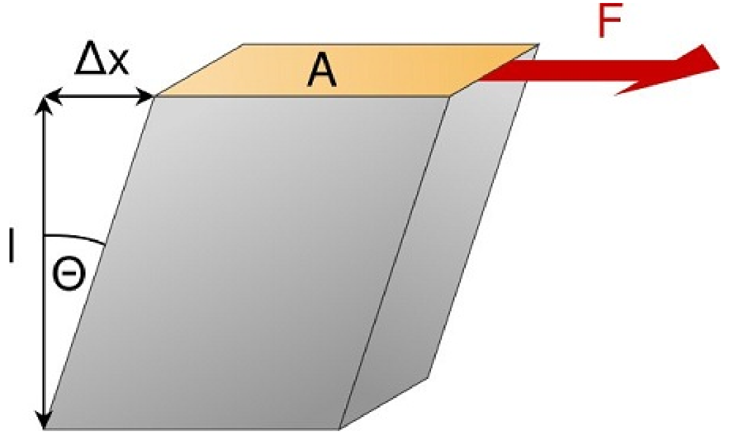
SI Unit: $\frac{N}{m^2}$



Shear Strain ϵ

The angle of the shear measured in radians.

SI Unit: None!



NECTA Vocabulary Question

Question: Differentiate between tensile stress and shear stress.

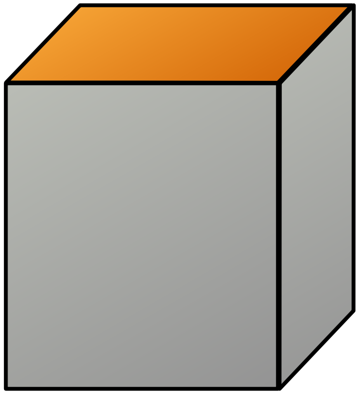
Answer: Tensile stress is the tensile force per unit area

while

Shear stress is defined as the shear force per unit area with a turning or displacement effect due to a presence of a couple.

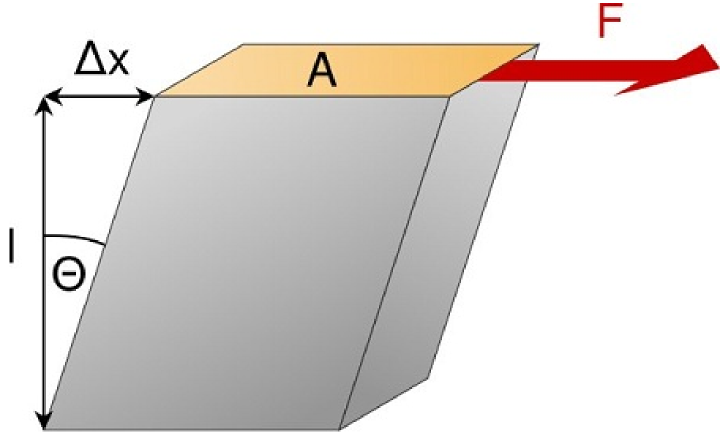
Shear Modulus

Shear Stress: $$\sigma_{shear} = \frac{F_{shearing}}{A_{being\ sheared}}$$



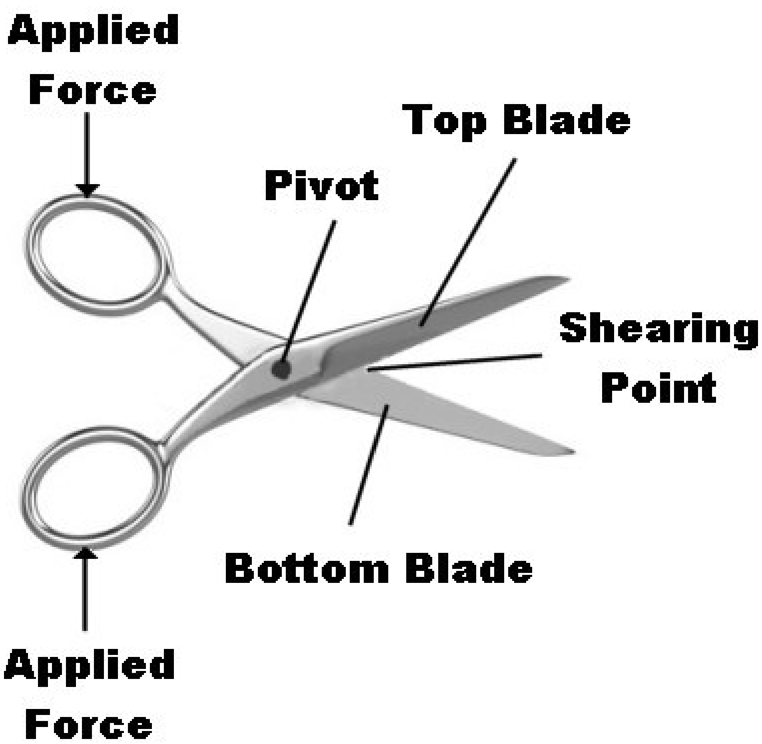
Shear Strain: $$\epsilon_{shear} = \theta$$ in radians

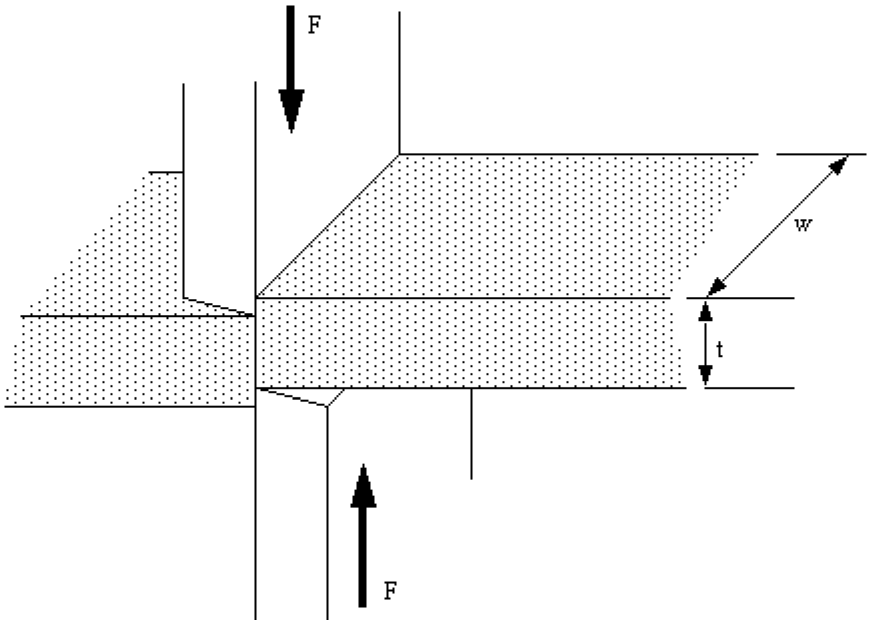
Shear Modulus: $$\mu = \frac{\sigma_{shear}}{\epsilon_{shear}}$$



A measure of how hard it is to slide one layer of a solid over another.

Shear Modulus Example





$F = 0.7tw\ UTS$

where,

F = force needed to shear

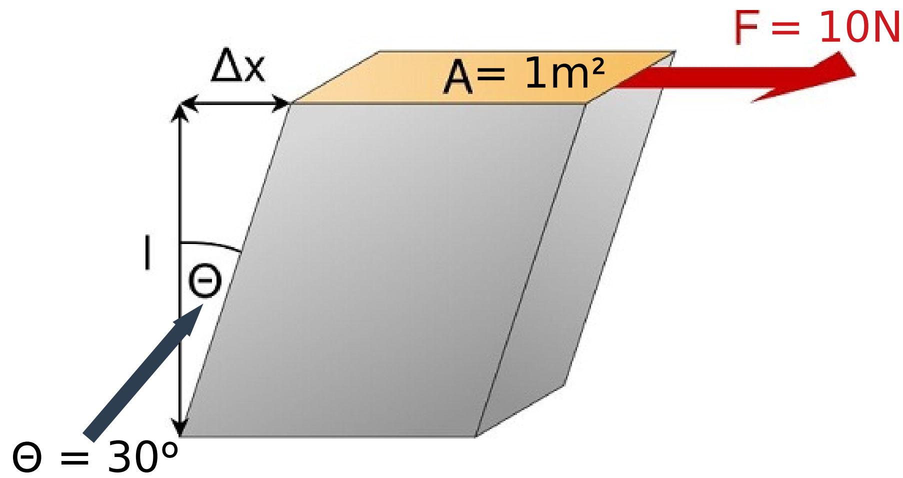
t = thickness of sheet

w = width of sheet

UTS = Ultimate Shear Strength of material

Practice Problem #3

What is the shear modulus, K given the diagram to the right?



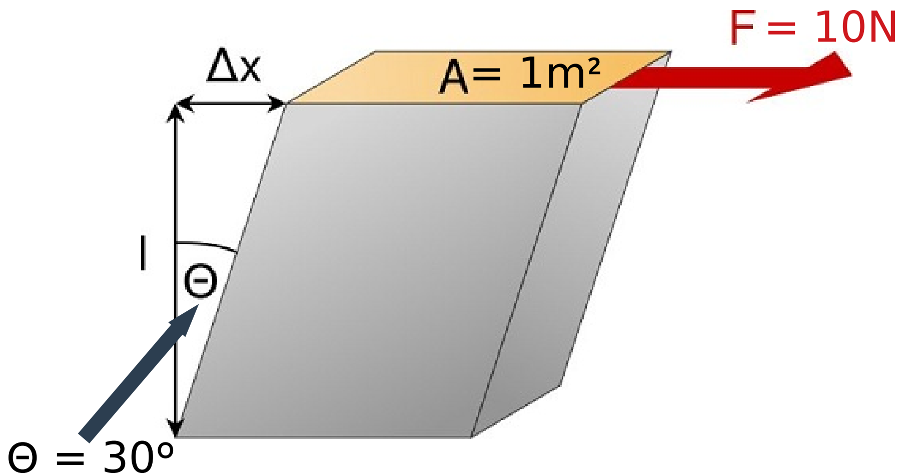
$\approx 0.\overline{33}\ \frac{N}{m^2}$

$\approx 19.10\ \frac{N}{m^2}$

$\approx 0.052\ \frac{N}{m^2}$

Practice Problem #3 Solution

What is the shear modulus, μ given the diagram to the right?

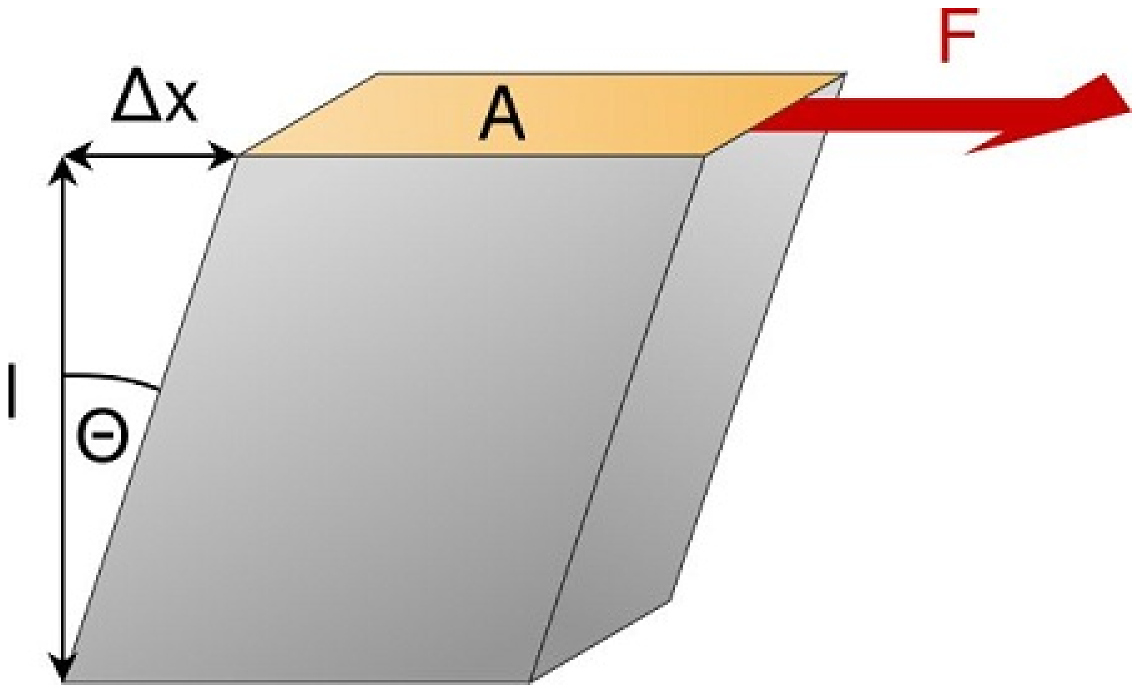


△ $\approx 0.\overline{33}\ \frac{N}{m^2}$	
○ $\approx 19.10\ \frac{N}{m^2}$	● $\approx 0.052\ \frac{N}{m^2}$

Shear Modulus

Note:

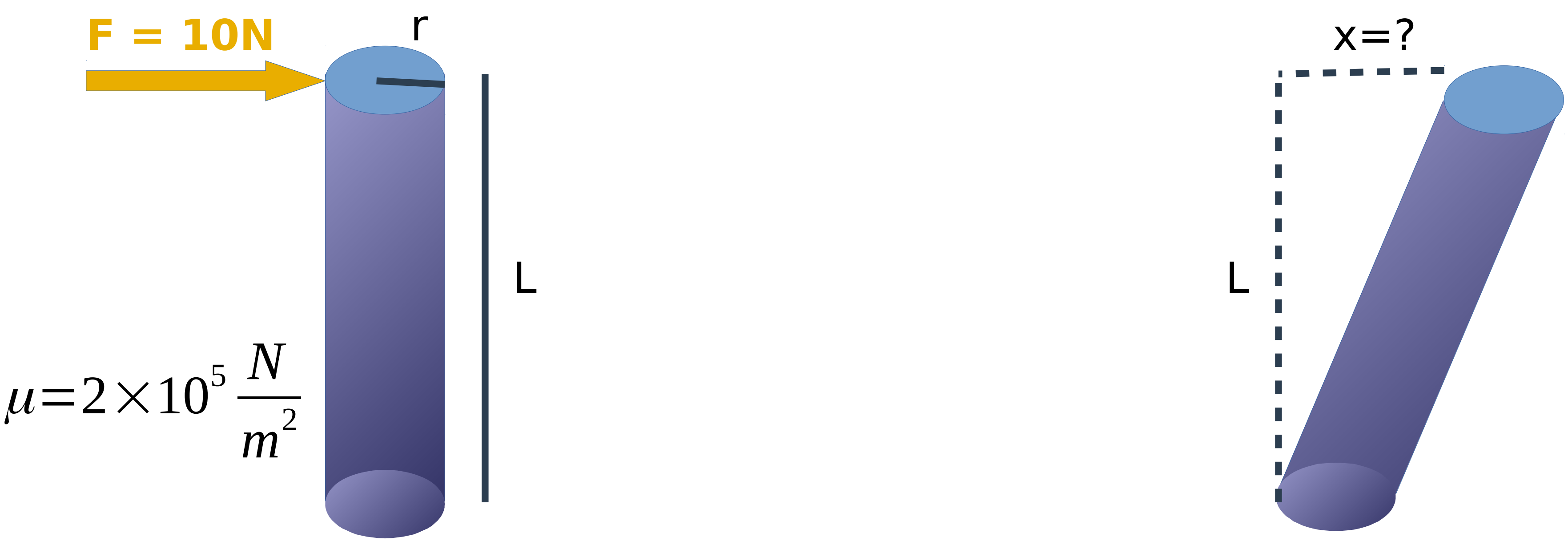
$$\frac{\Delta x}{l} = \tan(\Theta)$$



Word Problem - Shear

A cylinder of radius $r = 1\,cm$ and $L = 1\,m$ has a shear modulus of $\mu = 2 \times 10^5 \frac{N}{m^2}$. What is the displacement of the top of the cylinder when a tangential force of 10N is applied?

$\boxed{16.05\,cm}$



Fatigue

Fatigue is the weakening of a material caused by cyclic loading that results in progressive and localized structural damage and the growth of cracks.

Bridges – have largest loads during rush hours

(7:00am – 8:00am) before work

(4:00pm – 6:00pm) after work

Leads to material fatigue

Fatigue

Example:

SCOTT STERLING

Fatigue

Scott's Face

Impulse of ball

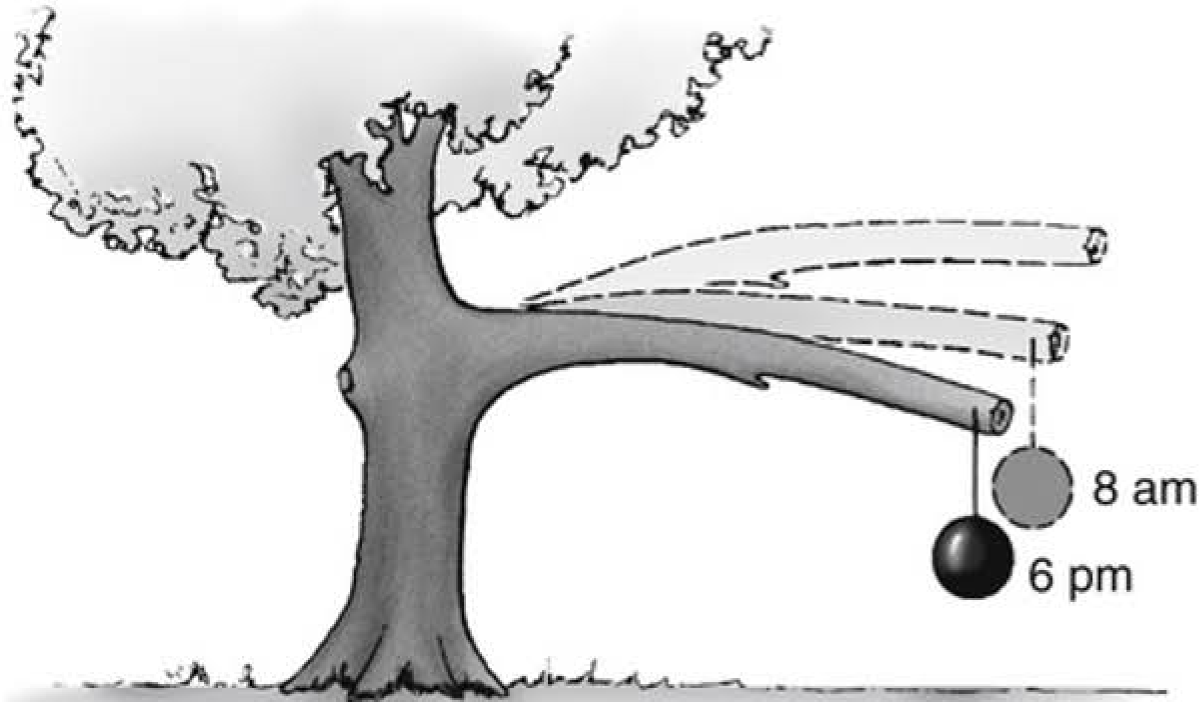
Fatigue is the weakening of a material caused by cyclic loading that results in progressive and localized structural damage and the growth of cracks.

Damaged face

Creep

Creep is the tendency of a solid material to move slowly or deform permanently under the influence of persistent mechanical stresses.

*Also why things at high temperatures more likely to break.



Wear

Wear is the damaging, gradual removal or deformation of material at solid surfaces.

Class #5 Goals Reviewed

- **<u>Very</u> quickly Review Bulk Stress / Bulk Strain**
- **Practice problems**
- **Move on to Shear Stress and Shear Strain**
- **Learn new vocabulary**

Goal: next class → ½ Elasticity, ½ Kinetic Theory of Gases

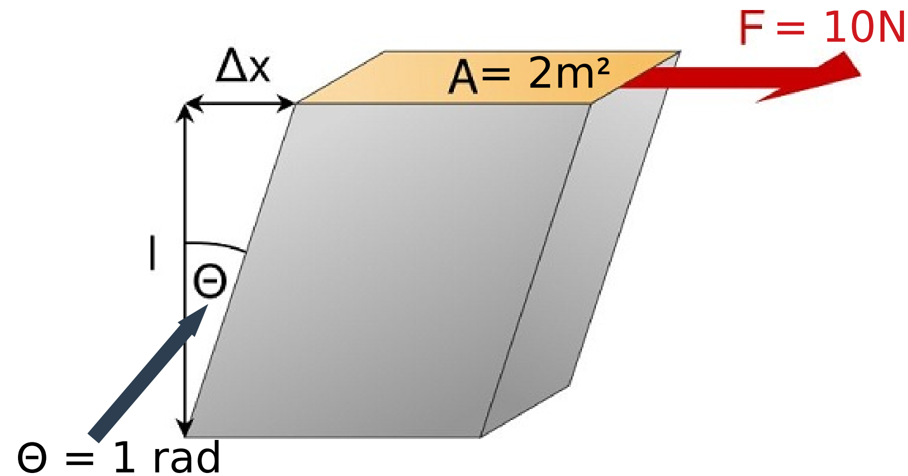
Final Part!

Class #6 Goals

- **Very quickly review vocab / bulk / shear**
- **Discuss rubber**
- **Discuss applications of Elasticity**

Review Problem #1

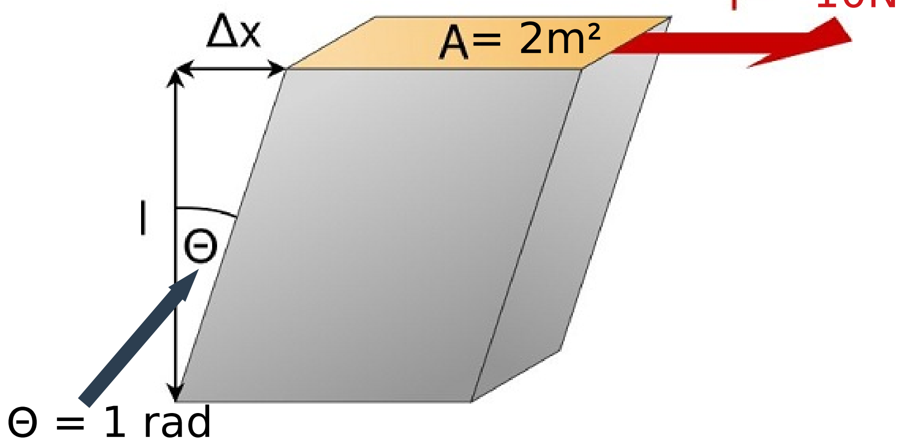
What is the shear modulus, μ given the diagram to the right?



△ $5 \ \frac{N}{m^2}$	
○ $0.05 \ \frac{N}{m^2}$	● $20 \ \frac{N}{m^2}$

Review Problem #1 Solution

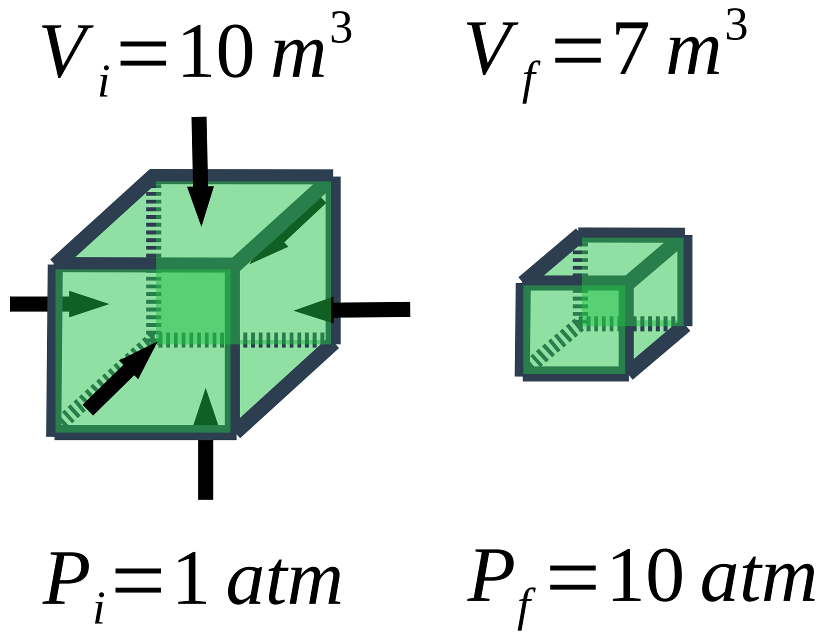
What is the shear modulus, μ given the diagram to the right?



$5 \ \frac{N}{m^2}$	
$0.05 \ \frac{N}{m^2}$	$20 \ \frac{N}{m^2}$

Review Problem #2

What is the bulk modulus, K given the diagram to the right?



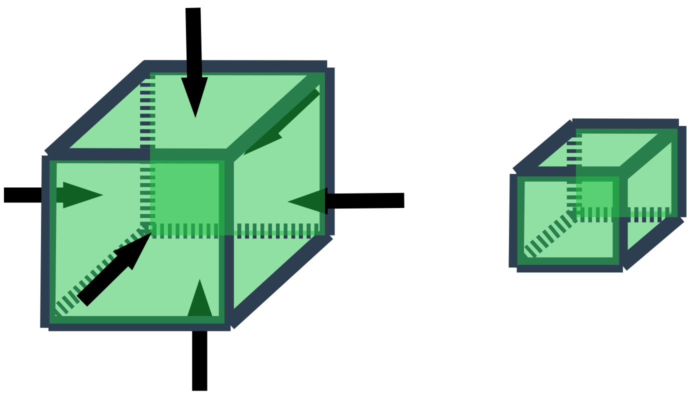
△ $3.33\ Pa$

○ $30\ Pa$

● $0.33\ Pa$

Review Problem #2 Solution

What is the bulk modulus, K given the diagram to the right?



△ $3.33\ Pa$	
○ $30\ Pa$	● $0.33\ Pa$

Review Problem #3

Scott Sterling received a cyclical load. This caused _______ on his face.

Creep

Wear

Fatigue

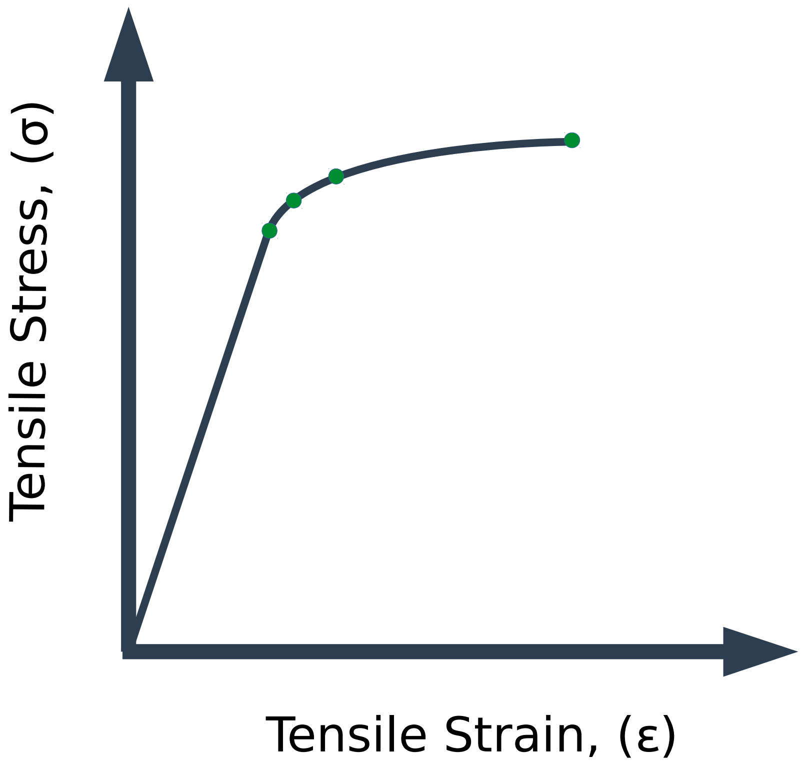
Review Problem #3 Solution

Scott Sterling received a cyclical load. This caused _______ on his face.

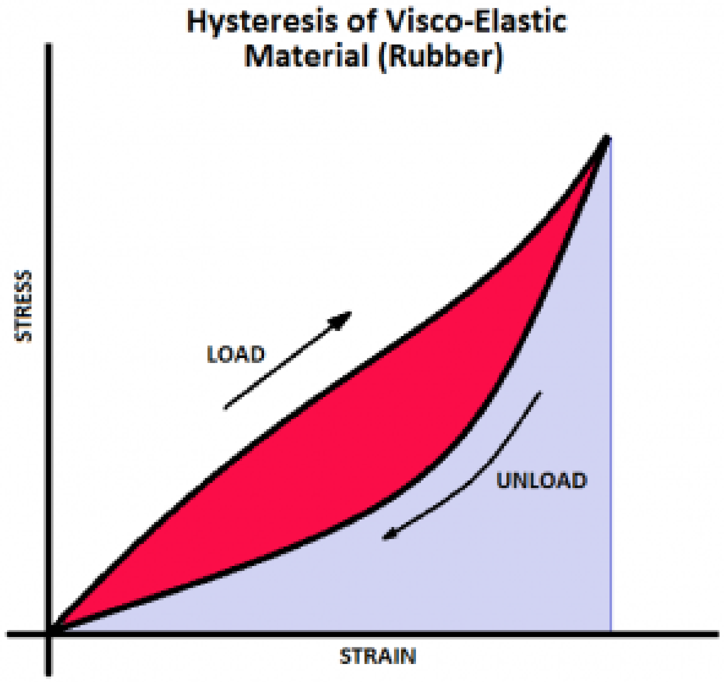
Creep	
Wear	Fatigue

Modulus of Elasticity Rubber

Modulus of Elasticity: Metals



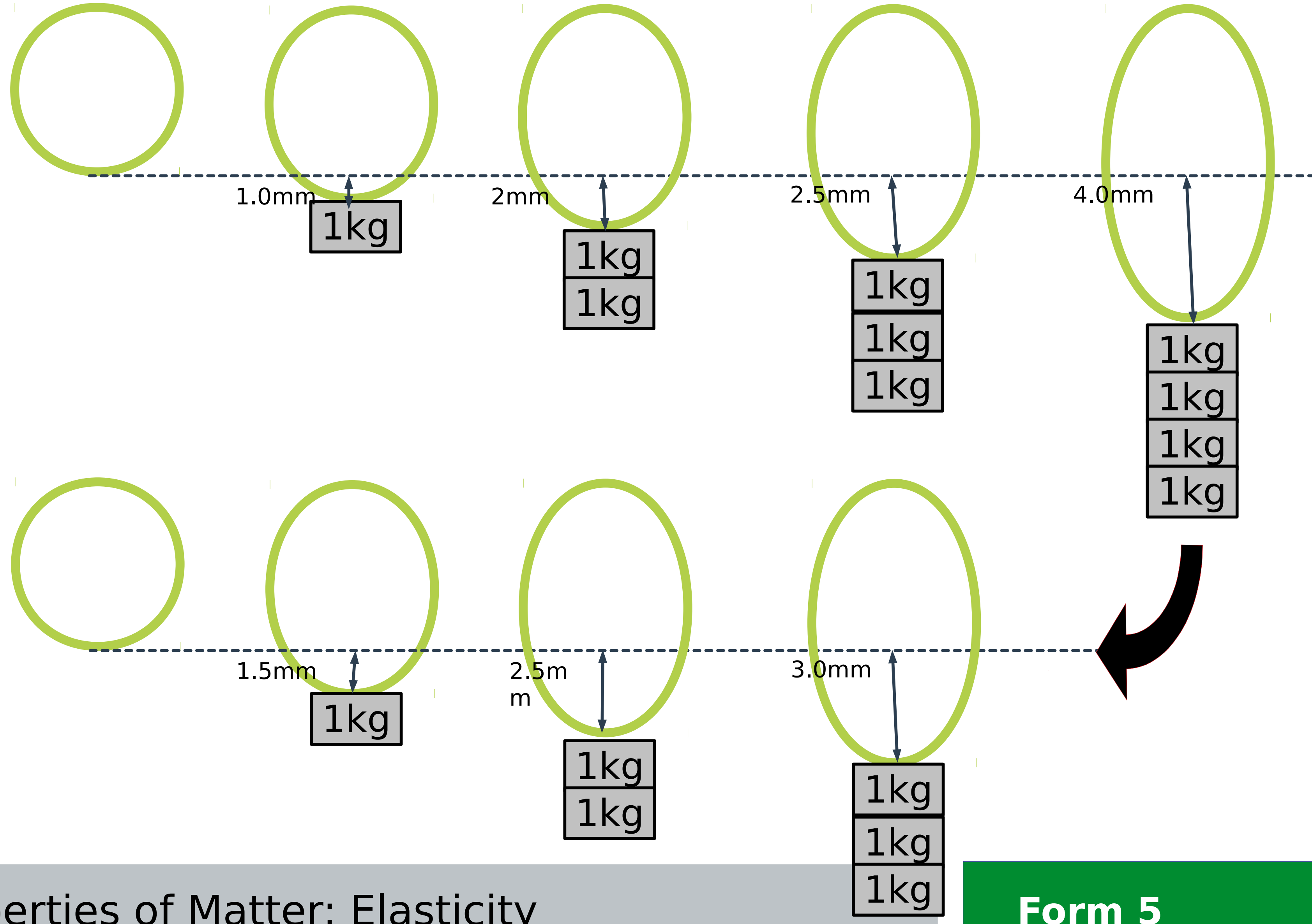
Modulus of Elasticity: Rubber



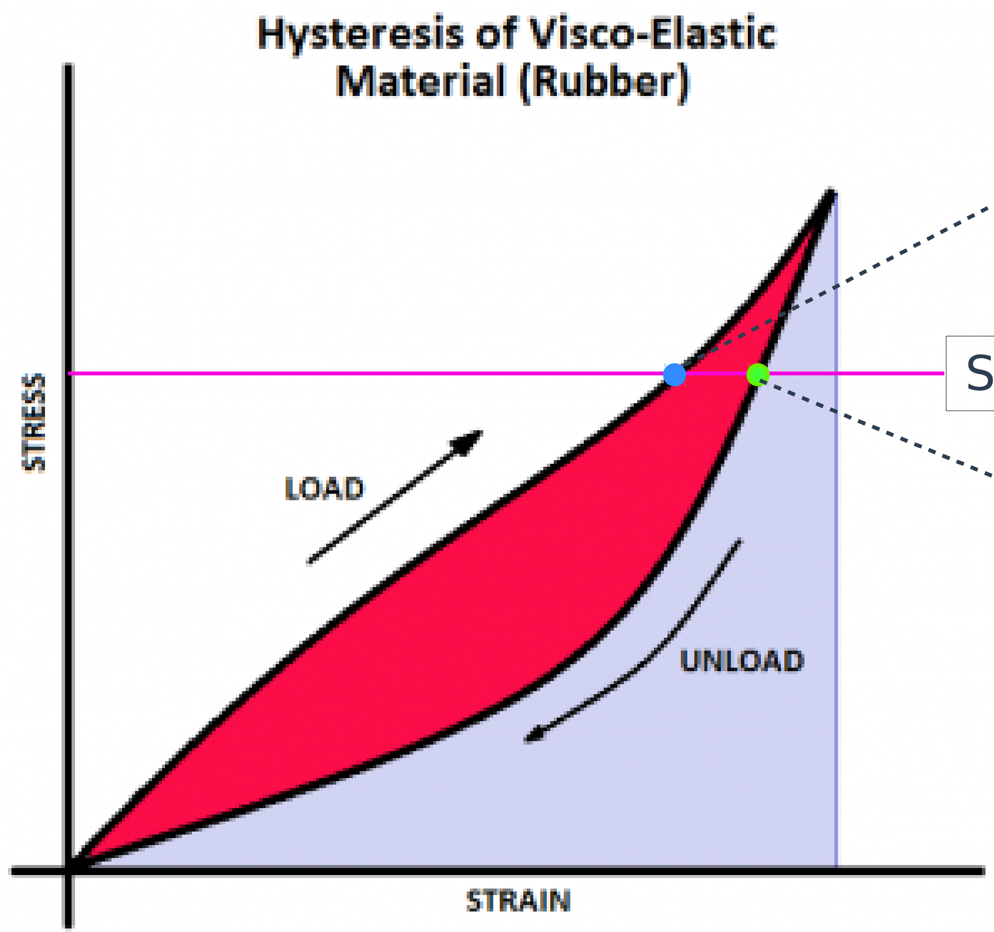
Hysteresis Loop (more details)

Adding mass: (loading)

Removing mass: (un-loading)



Hooke's Law



Same Stress, Different Strains

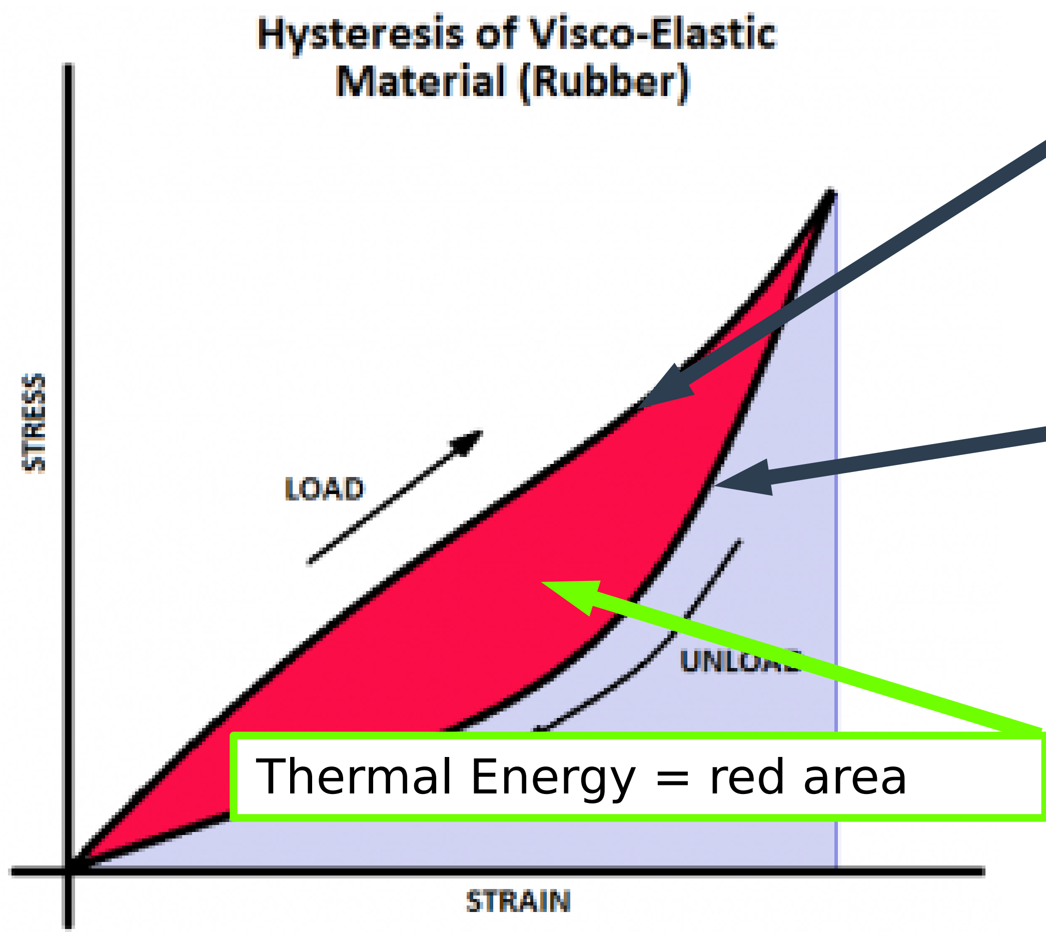
Loading

Lower Strain means less extension

Un-loading

Higher Strain means more extension

Hysteresis Loop



Total energy is all area below the <u>loading</u> curve

Energy done by stretching is area below <u>unloading</u> curve only

Thermal Energy =
Total Energy – Energy in stretching

Application 1 of Elasticity

The physics of elasticity is used to **pick materials to build with**.

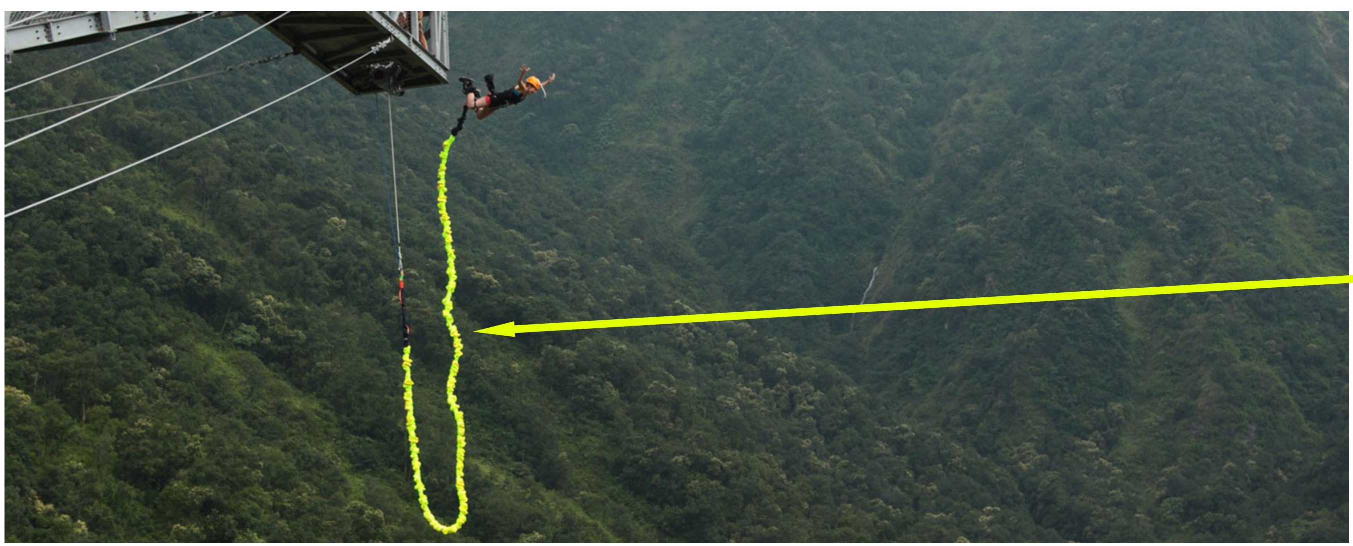
For example, building bridges → Most bridges are steel

$$UTS_{steel} = UTS_{concrete} \times 100$$

Application 2 of Elasticity

The physics of elasticity is used to make sure materials can support loads.

bungee jumping → If the bungee chord cannot support the load, the chord will break and someone will die.



Bungee chord

Application 3 of Elasticity

Clothes!

Different Elasticity → Different Purposes

<u>VS.</u>

The Jobs of Elasticity

- Materials Scientist
- Engineer (civil or mechanical)
- Construction manager

Quick Class Poll

Who would win in a football match?

Mwl. Gibson wearing **stretchy** clothes

Messi wearing **stiff** clothes

Mwl. Gibson

Lionel Messi

Quick Class Poll Solution

Who would win in a football match?

Mwl. Gibson wearing **stretchy** clothes

Messi wearing **stiff** clothes

Mwl. Gibson

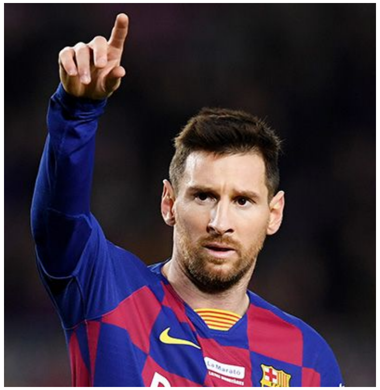
Lionel Messi

Quick Class Poll

Who would win in a fight?

Mwl. Gibson wearing **stiff** clothes

Messi wearing **stretchy** clothes



Mwl. Gibson

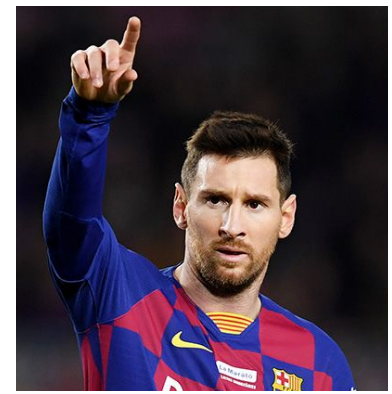
Lionel Messi

Quick Class Poll **Solution**

Who would win in a fight?

Mwl. Gibson wearing **stiff** clothes

Messi wearing **stretchy** clothes



Mwl. Gibson	Lionel Messi

Facts

Mwl. Gibson > Lionel Messi

Class #6 Goals Reviewed

- Very quickly review vocab / bulk / shear
- Discuss rubber
- Discuss applications of Elasticity

Done with Elasticity…

… on to the Kinetic Theory of Gases