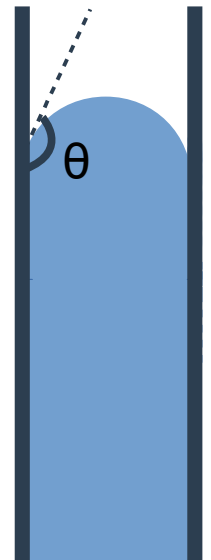
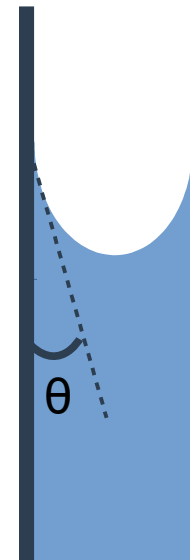
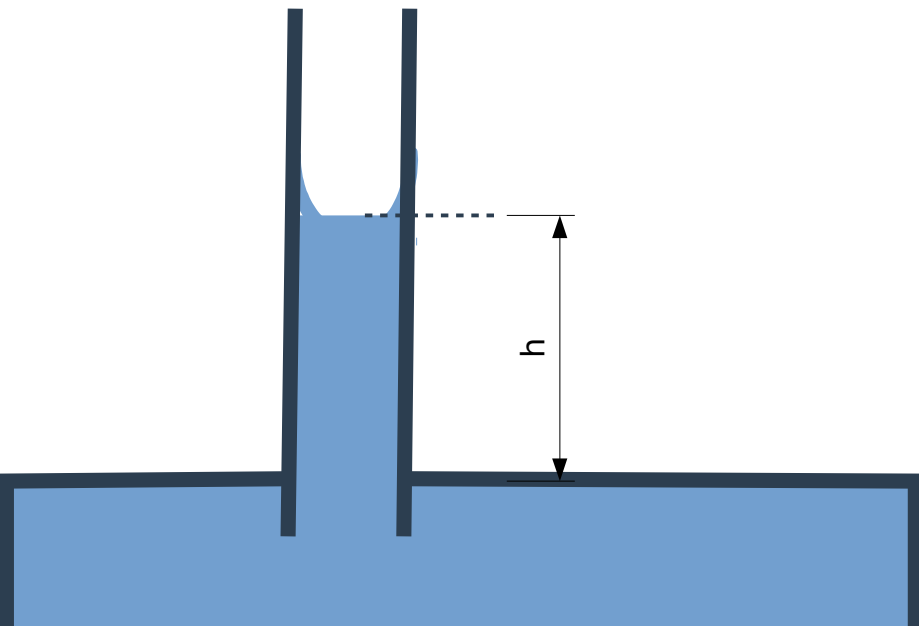


**Topic: Properties of Matter**  
**Sub-Topic: Surface Tension**



# Poll (PCB only)

We will have an after-lunch session (3:30pm-4:50pm) to replace the period that we missed on Tuesday. Which day works best?



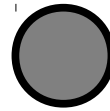
Monday



Tuesday



Wednesday



Thursday

# Surface Tension Class #1 Goals

- **Know the definition of surface tension and understand the related variables.**
- **Understand convex and concave meniscus.**
- **Know the direction of the force associated with surface tension.**

# Definition of Surface Tension



**Surface Tension** ( $\gamma$ )

Units:  $\frac{N}{m}$

- The force per unit length in the plane of a liquid surface acting in the surface and perpendicular to one side of an imaginary line drawn in the surface.
- In Terms of Molecular Interactions:  
The sum of all inter-molecular forces on the surface of a material.
- In Terms of Energy:  
The work done per unit area in increasing the surface area of a liquid at constant temperature.

# List of Related Variables

→ **Pressure** ( $P$ ) Units:  $\frac{N}{m^2}$

The force per unit area acting at right angles to the surface.

→ **Density** ( $\rho$ ) Units:  $\frac{kg}{m^3}$

The mass per unit volume of an object.

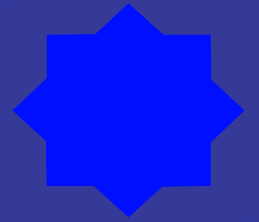
→ **Radius** ( $r$ ) Units:  $m$

Distance between the center of a circle and its perimeter

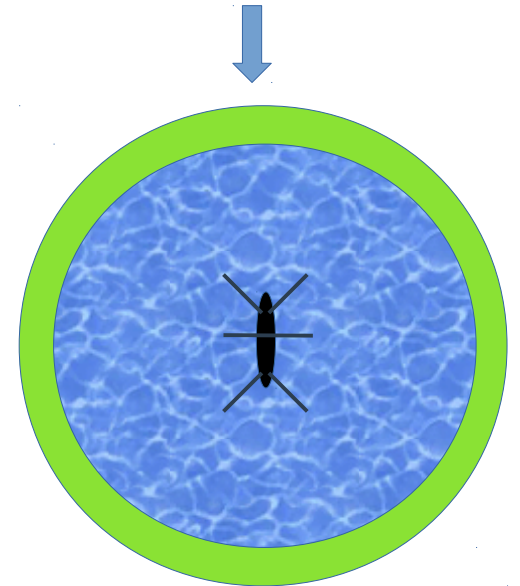
→ **Excess Pressure** ( $\Delta P$ ) Units:  $\frac{N}{m^2}$

The difference in pressure between two mediums.

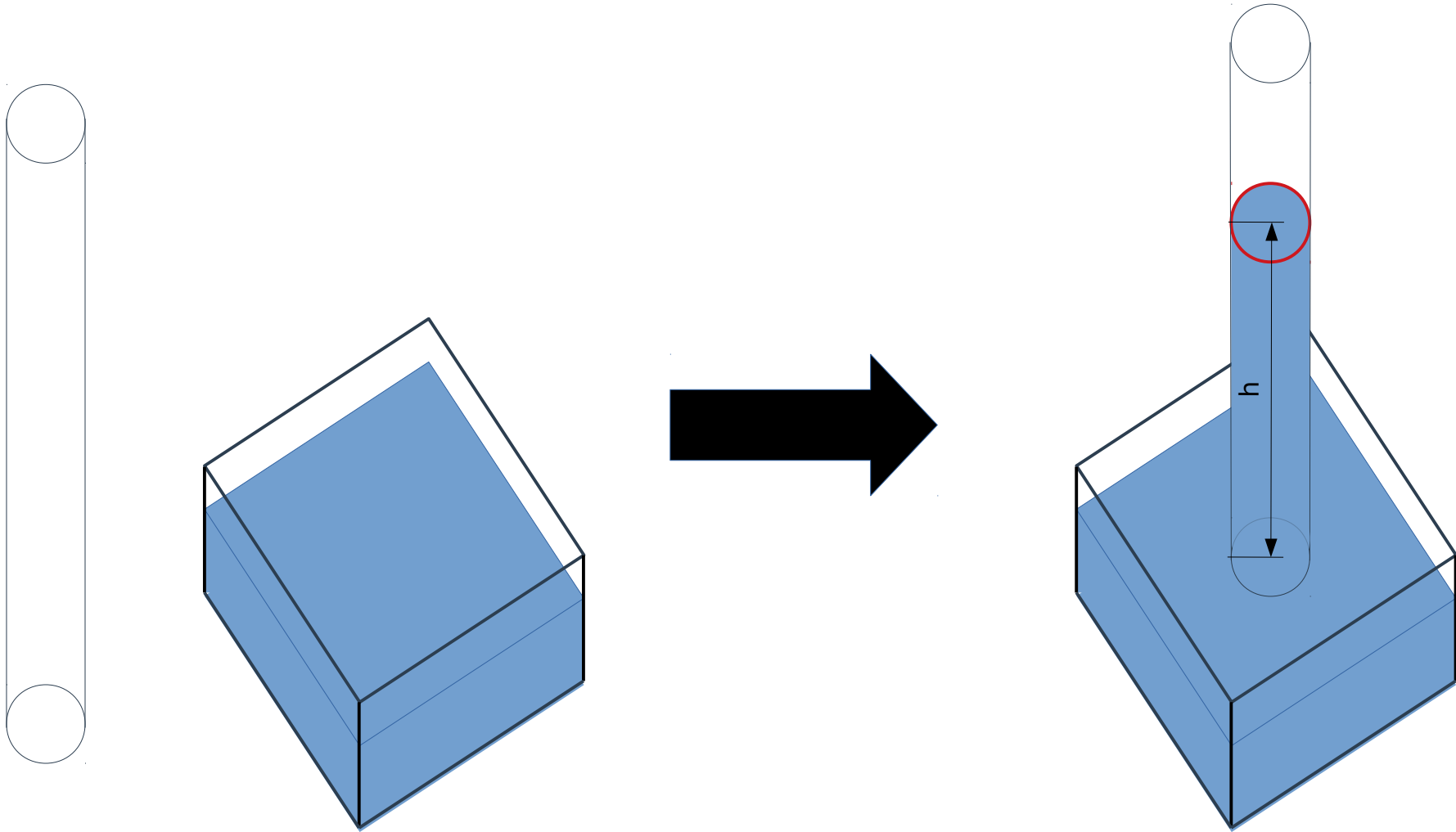
# Examples of Surface Tension



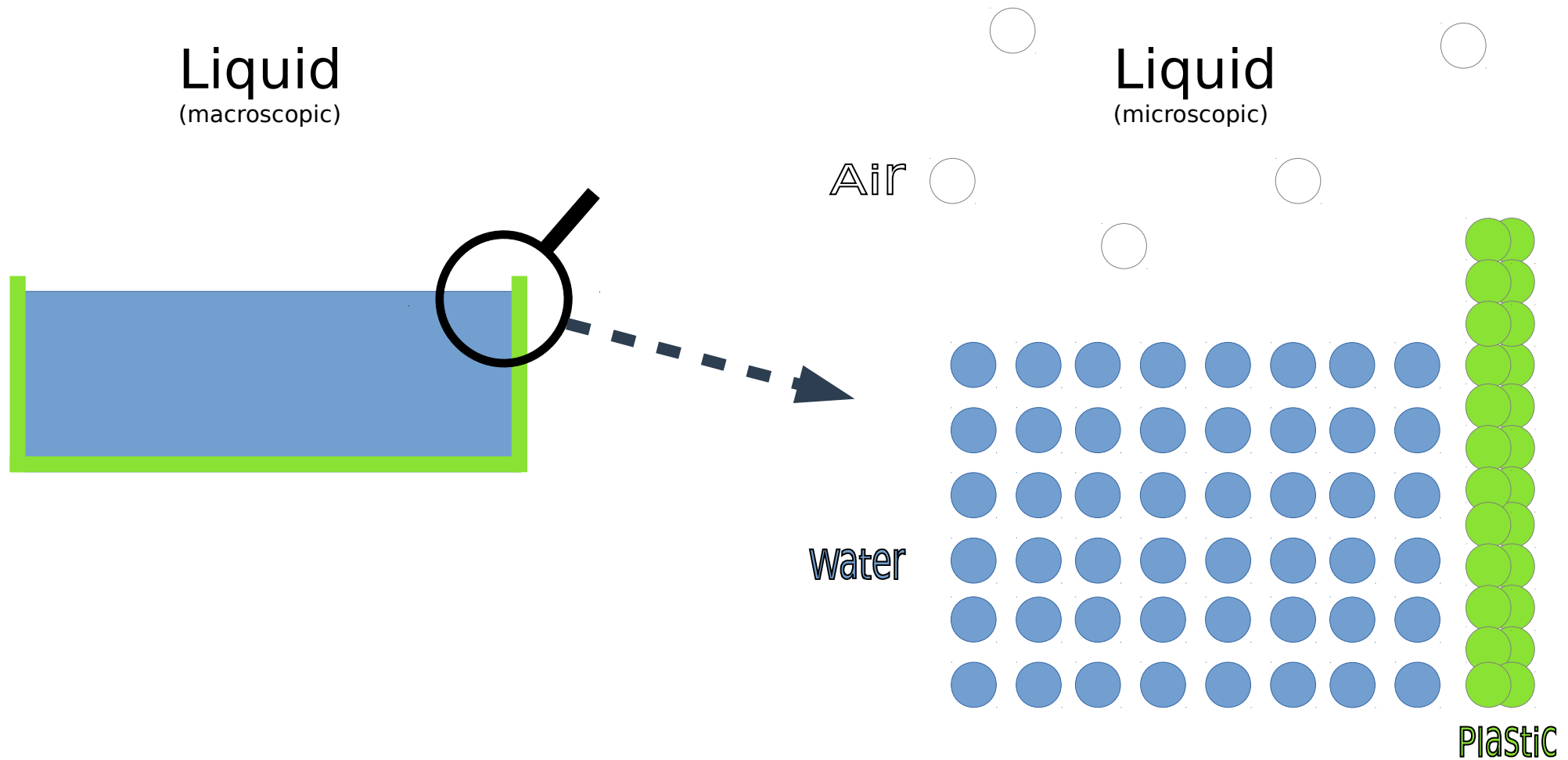
- Why liquid drops are spherical
- Why “pond skaters” can walk on water
- Why a needle floats in water



# Examples of Surface Tension



# Molecular Explanation





# Molecular Explanation



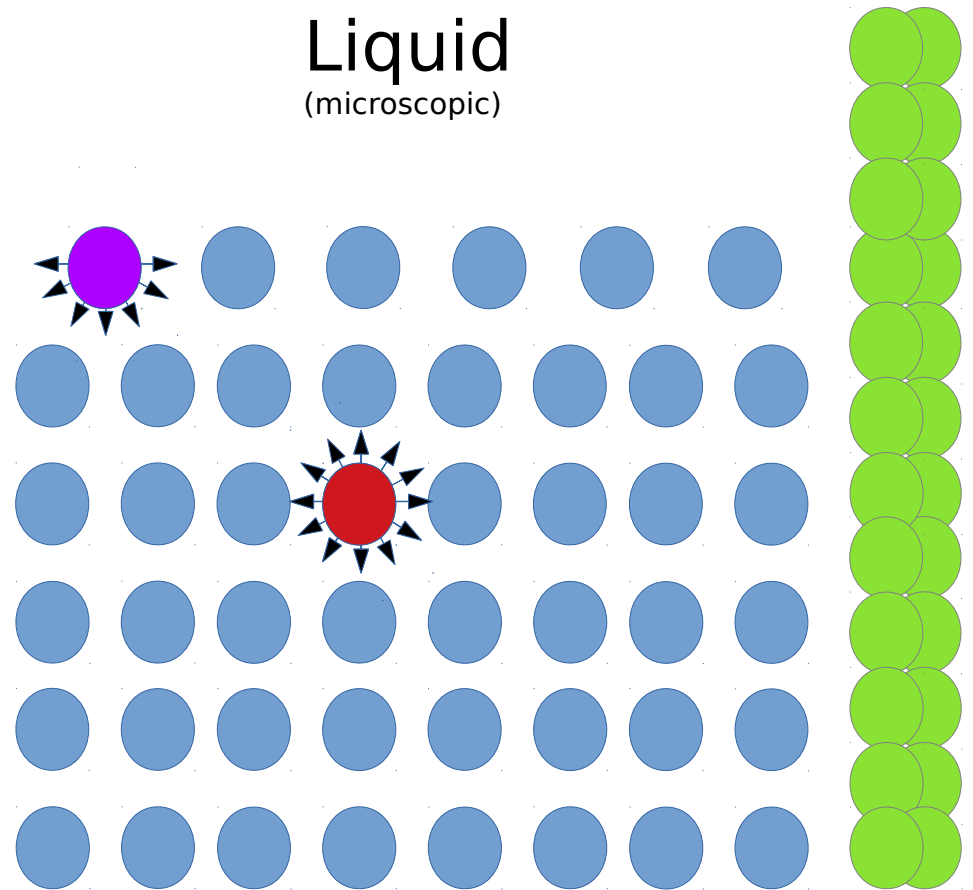
Molecule in the interior of liquid experiences inter-molecular forces from all directions



Molecule on the surface of liquid experiences half of the inter-molecular forces when compared to a molecule on the interior

\*Note that the inter-molecular in liquid forces are attractive, not repulsive

Liquid  
(microscopic)



# Molecular Explanation

PE = -1 eV

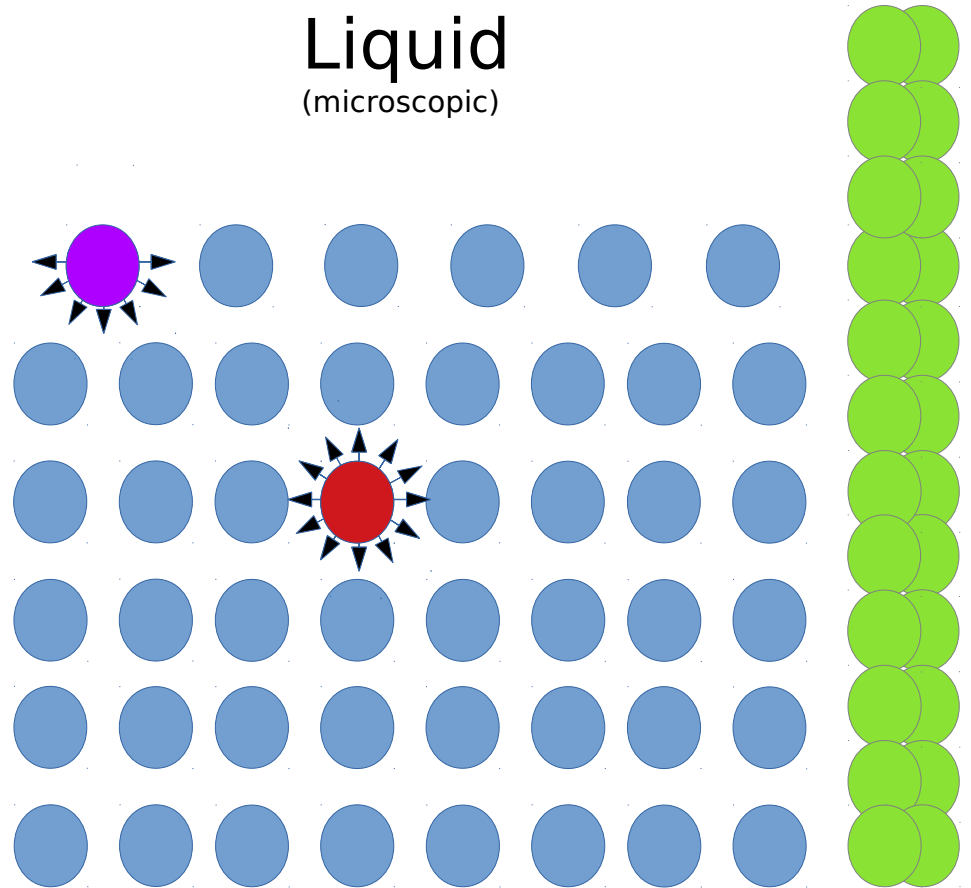


PE = -0.5 eV



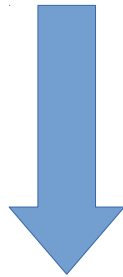
Therefore surface molecules have a **greater** potential energy than molecules in the interior.

Liquid  
(microscopic)



# Good thing to know ...

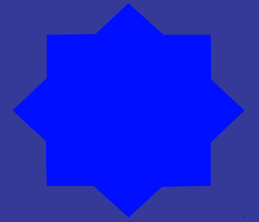
## Fact: All matter wants to minimize Potential Energy



If surface molecules in a liquid have a **greater** potential energy than molecules in the interior

- 1) In the presence of only surface tension forces, a liquid always assumes its lowest possible surface area (shape of a sphere) to minimize number of surface molecules.
- 2) To minimize the number of surface molecules, the spacing between surface molecules is greater than the spacing between molecules in the interior.

# Surface Tension vs. Free Surface Energy



**Surface Tension** ( $\gamma$ )

Units:  $\frac{N}{m}$

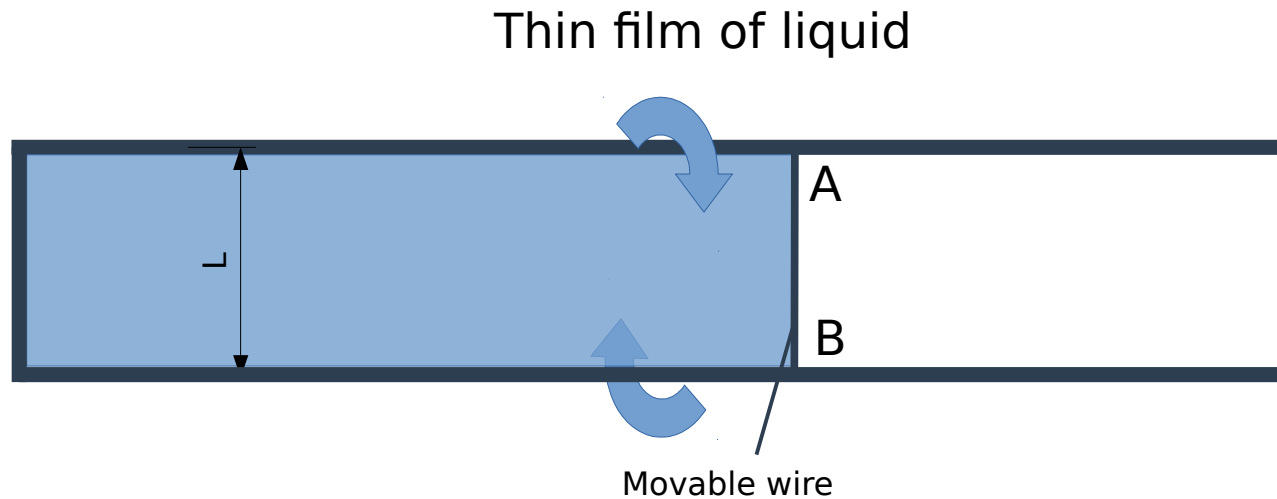
The sum of all inter-molecular forces on the surface of a material.

**Free Surface Energy** ( $\sigma$ )

Units:  $\frac{N}{m}$

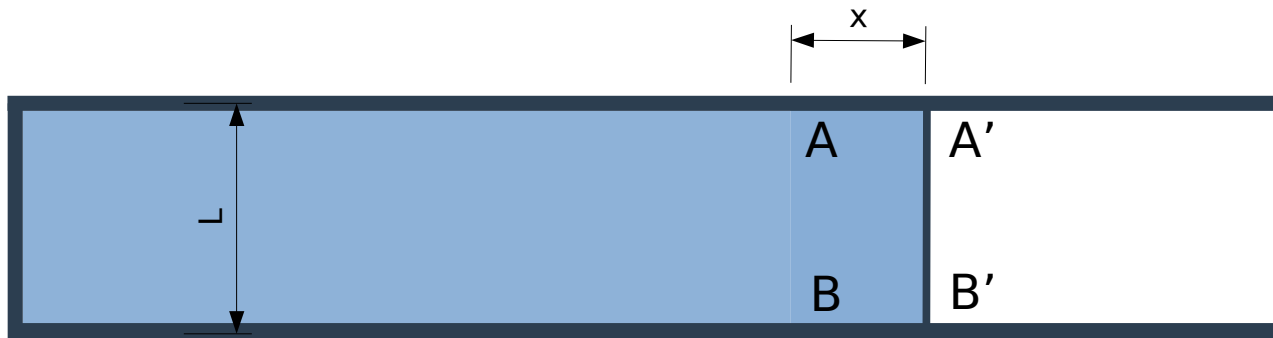
The work done per unit area in increasing the surface area of a liquid at constant temperature.

# Surface Tension vs. Free Surface Energy



$$F_{AB} = 2 \gamma L$$

# Surface Tension vs. Free Surface Energy



$$F_{AB} = 2 \gamma L$$

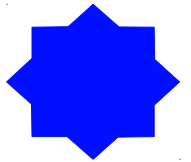
$$W = F \times \Delta x$$

$$W = 2 \gamma L x$$

$$\sigma = \frac{W}{A}$$

$$\sigma = \frac{2 \gamma L x}{2 L x}$$

$$\sigma = \gamma$$



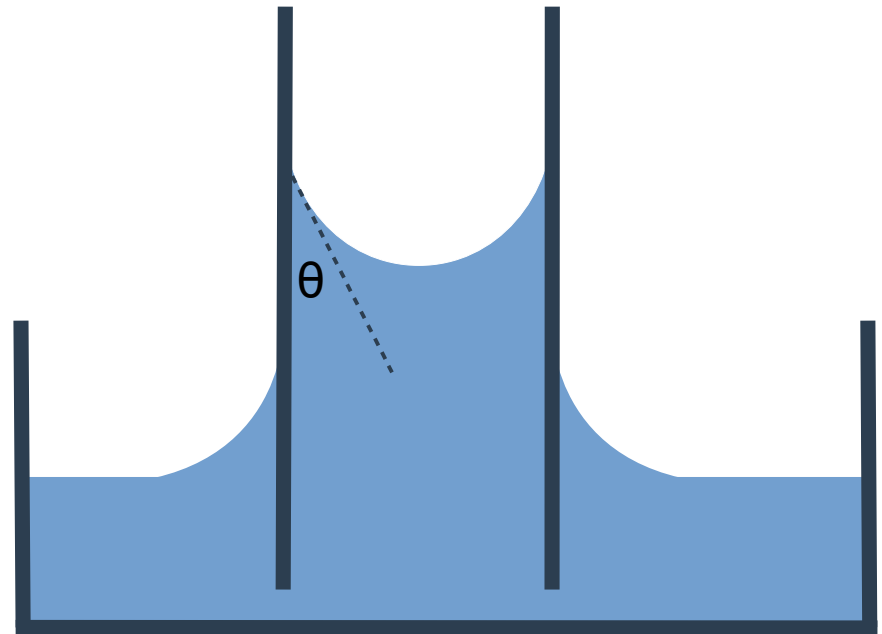
# Surface Tension: Angle of Contact

The surface of liquid is curved when it makes contact with a solid.

## The Angle of Contact

The angle between the solid surface and the tangent plane to the liquid surface at a point where it touches the solid.

\*the angle is measured through the liquid.



# Forces to consider

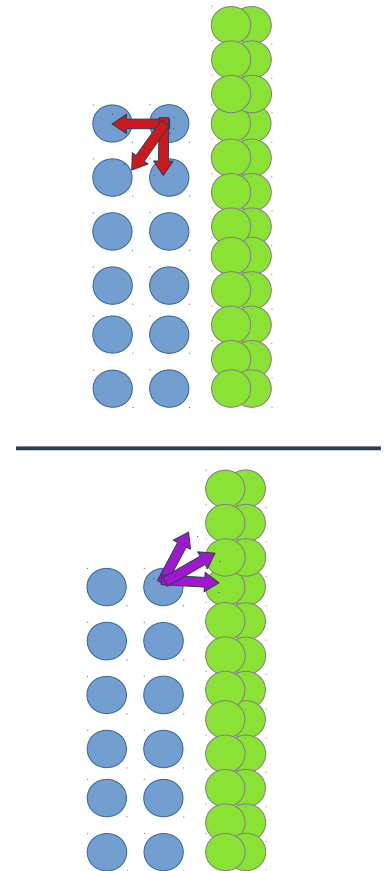
The angle of contact is determined by two forces:

## Cohesive Force

The attractive force exerted on a liquid molecule by other liquid molecules.

## Adhesive Force

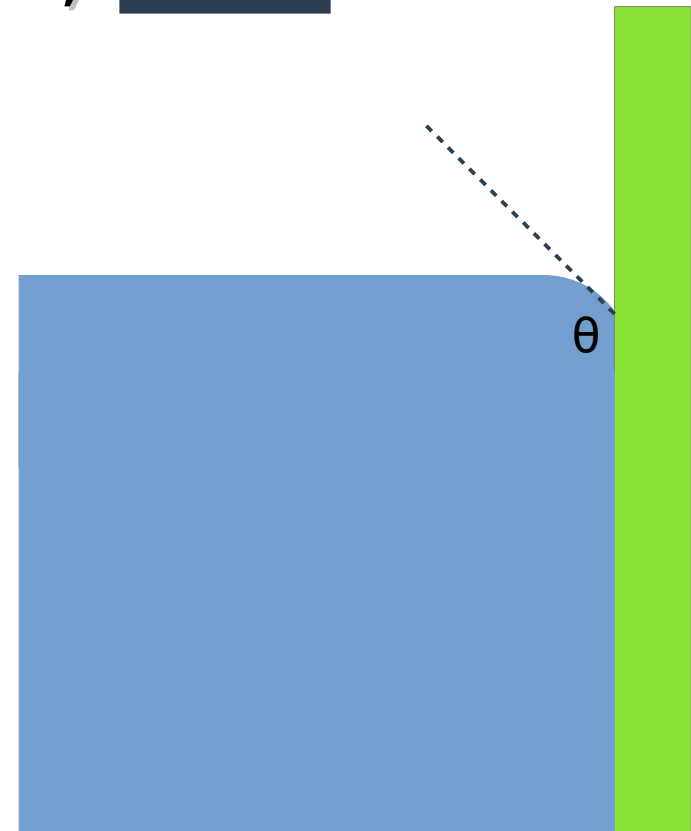
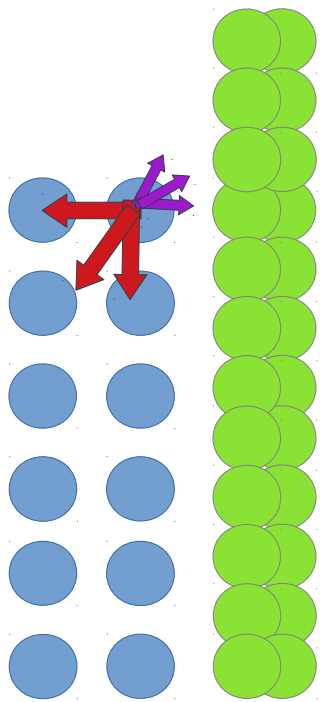
The attractive force exerted on a liquid molecule by molecules in the surface of a solid.





# Convex Meniscus

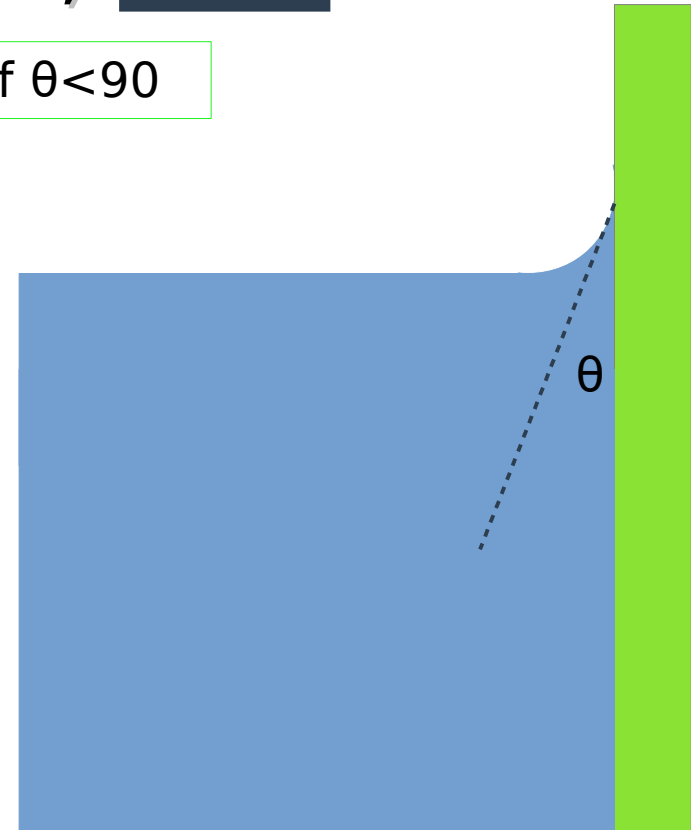
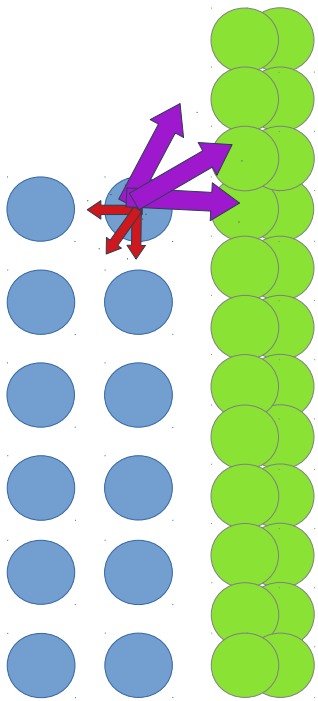
if **Cohesive Force** > **Adhesive Force**,  $\theta > 90^\circ$



# Concave Meniscus

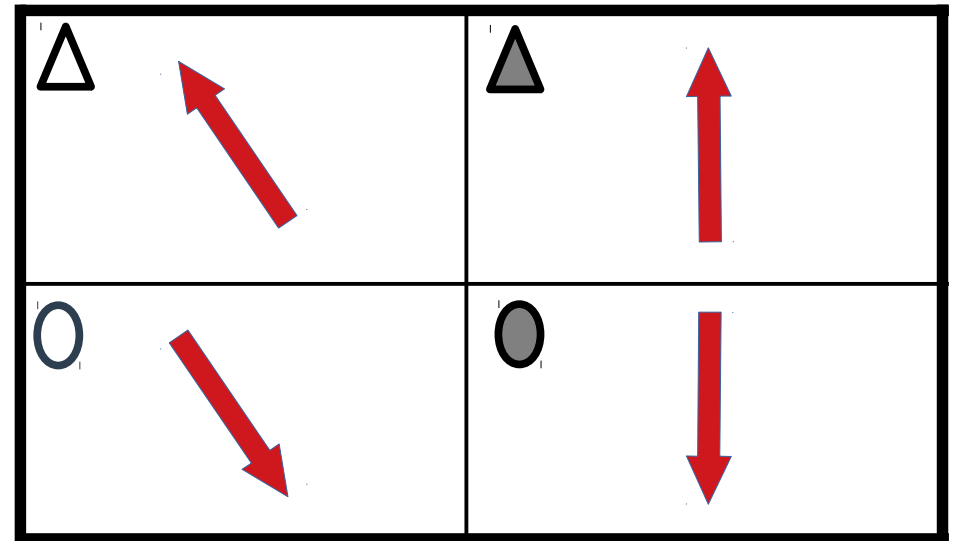
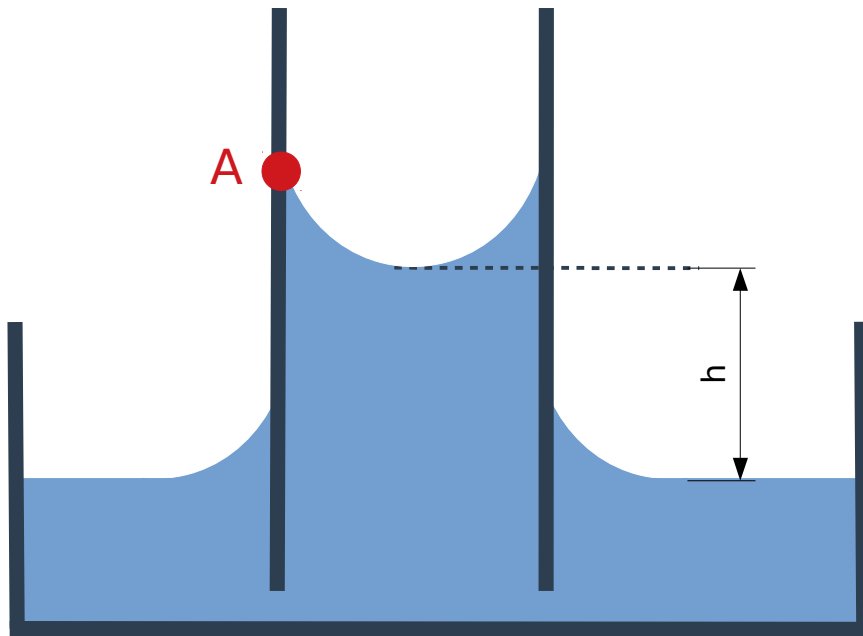
if **Cohesive Force** < **Adhesive Force** ,  $\theta < 90^\circ$

A liquid is said to 'wet' a surface if  $\theta < 90$



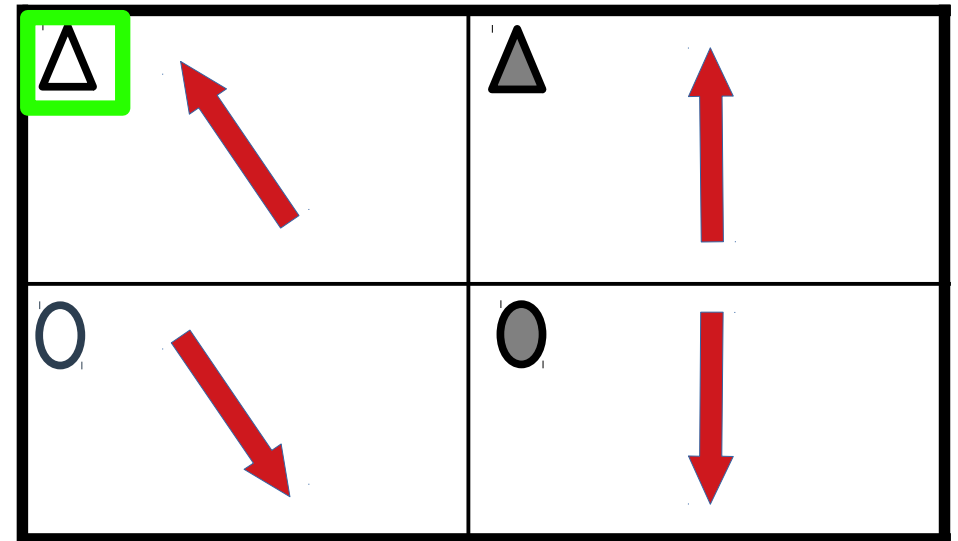
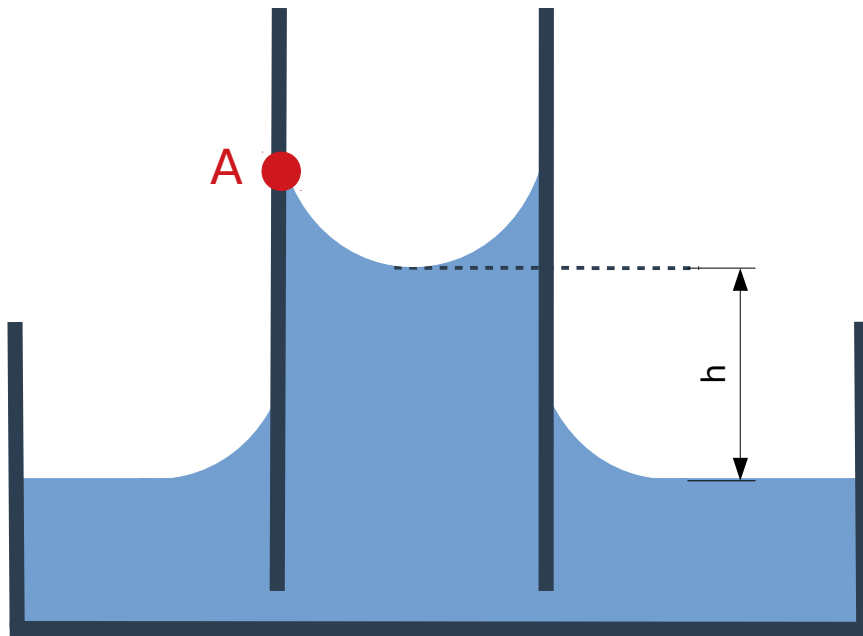
# Logic Question

A glass tube is placed in a liquid and the liquid rises up the tube to a height,  $h$ , above the water. Which direction is the force of surface tension acting at point A?



# Logic Question

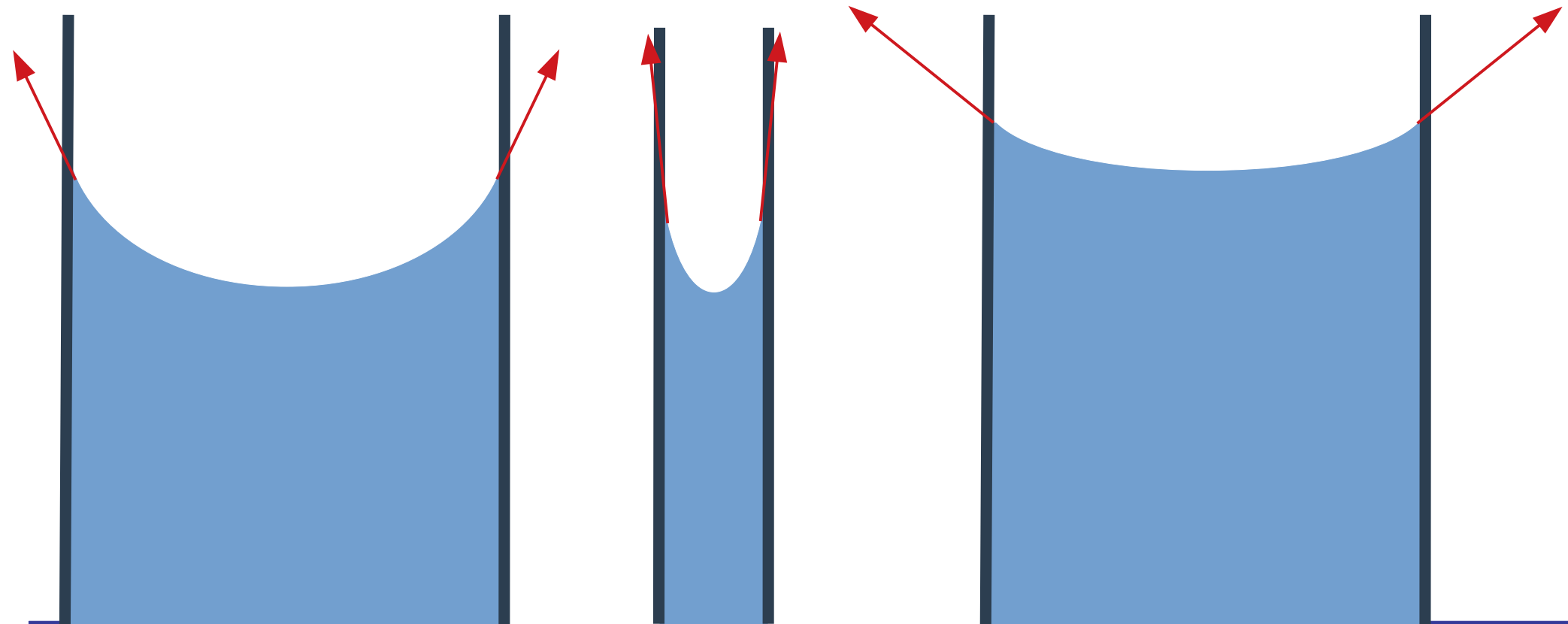
A glass tube is placed in a liquid and the liquid rises up the tube to a height,  $h$ , above the water. Which direction is the force of surface tension acting at point A?



# Direction of Surface Tension Force



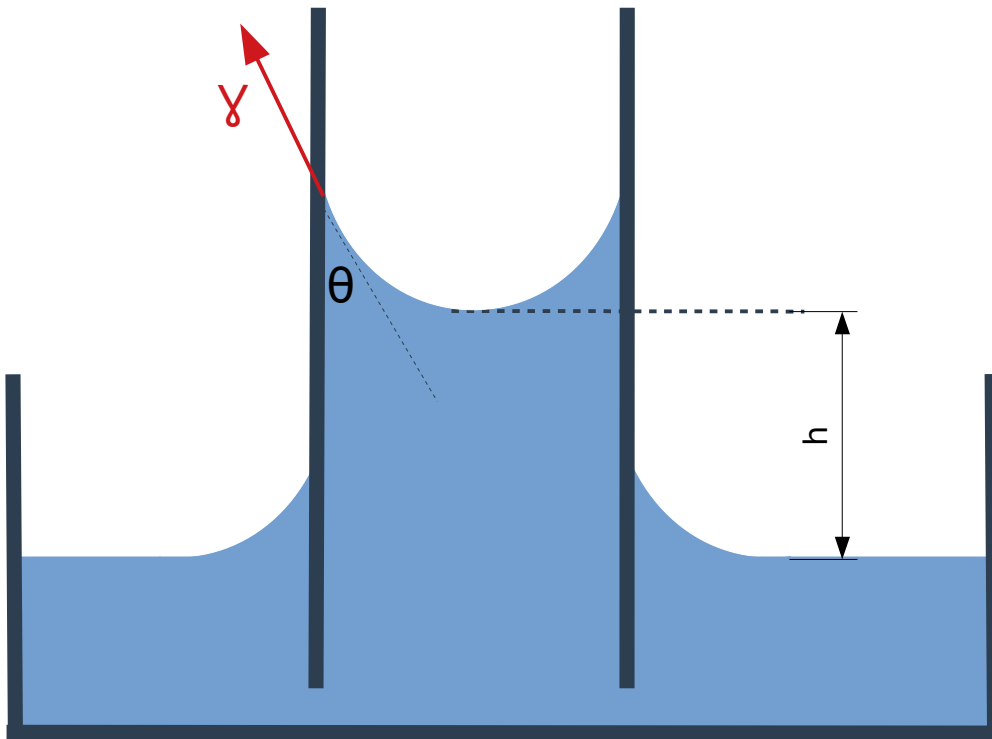
The surface tension force acts parallel to the surface of the liquid.



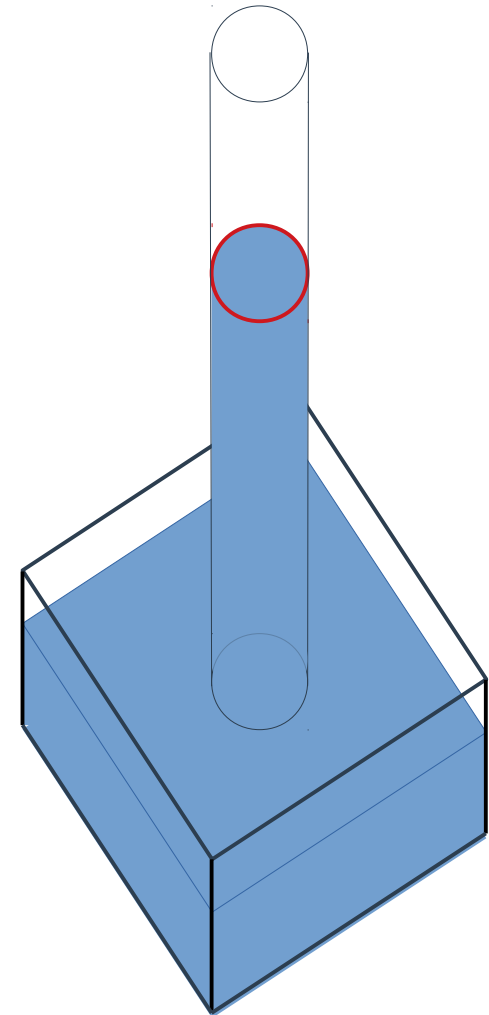
# Visualization



**2D**



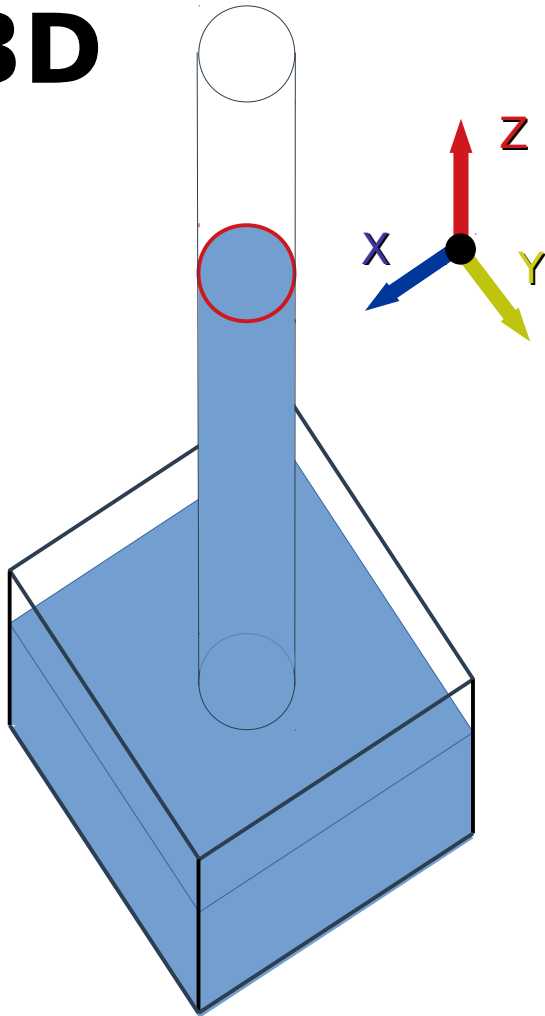
**3D**



# Guiding Questions



**3D**



1. With no surface tension forces, where would the water in the tube be?

The water would all be at the same height.  
The water would not rise in the tube.

2. Where is the surface tension force interacting with non-liquid molecules?

The surface tension force interacts with the perimeter of the tube (red circle).

3. In equilibrium, the surface tension forces balance with what other forces?

**Z**-direction: The surface tension forces balance with the force of gravity

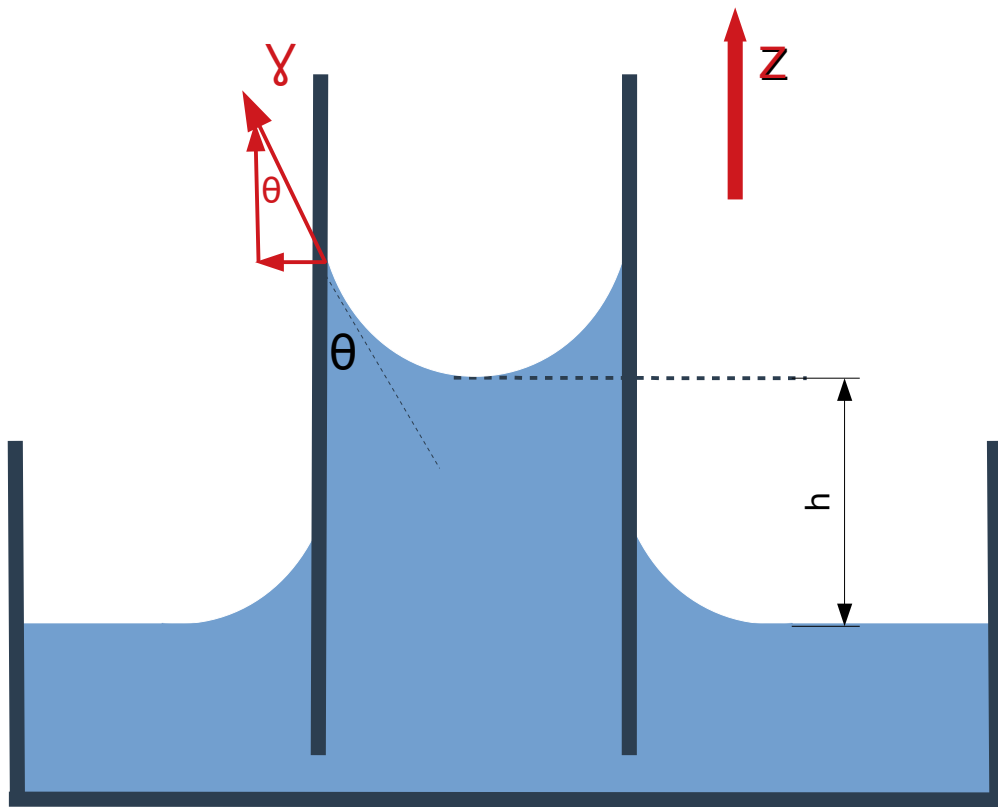
**XY**-direction: The surface tension forces balance with the normal force from the tube.

# Calculating Surface Tension Force

From Last Slide:

- The surface tension force interacts with the perimeter of the tube.
- (Z-direction) The surface tension forces balance with the force of gravity

$$\gamma \quad \text{Units: } \frac{N}{m}$$



Dimensional Analysis

$$\gamma \times C = F_{\text{surface tension}}$$

\*  $C$  = circumference

$$\frac{N}{m} \times m = N$$

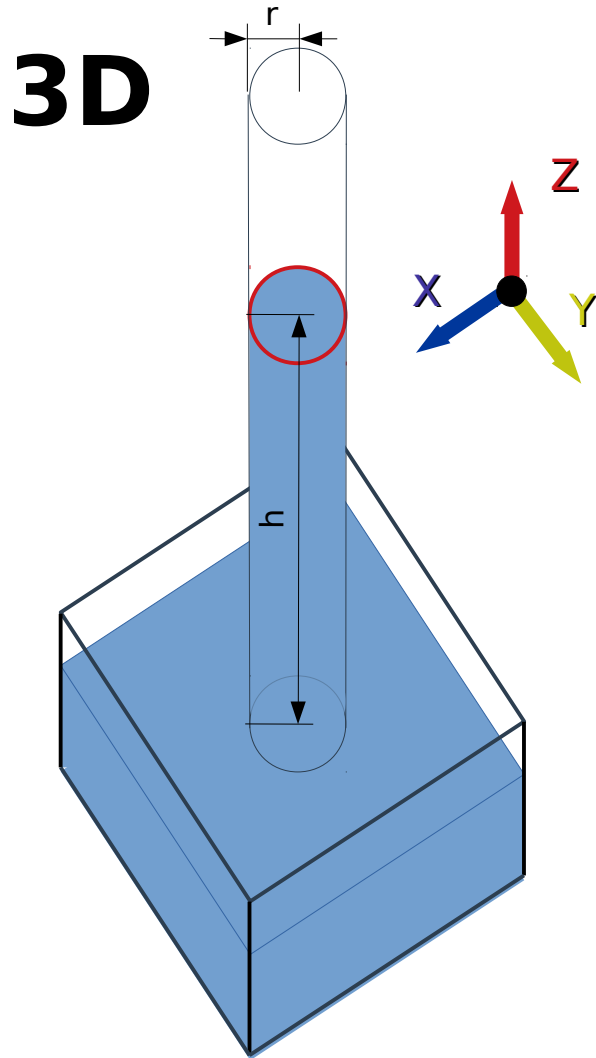


Force in Z-Direction

$$(\gamma \cos(\theta)) \times C = F_{\text{surface tension } Z} = mg$$



# Calculating Surface Tension Force



From last slide:  $\gamma \cos(\theta) \times C = mg$

$$C = 2\pi r$$

$$m = \rho V$$

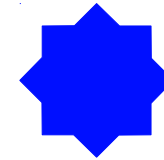
$$V_{\text{cylinder}} = \pi r^2 h$$

$$m = \rho \pi r^2 h$$

$$\gamma \cos(\theta) \times 2\cancel{\pi r} = (\rho \cancel{\pi r^2} h) g$$

$$\gamma \cos(\theta) \times 2 = \rho r h g$$

$$\gamma = \frac{\rho r h g}{2 \cos(\theta)}$$



# Surface Tension Class #1 Goals **Reviewed**

- **Know the definition of surface tension and understand the related variables.**
- **Understand convex and concave meniscus.**
- **Know the direction of the force associated with surface tension.**

**New Section!**

# Surface Tension Class #2 Goals

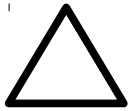


- **Understand how angle of contact is related to height in a surface tension experimental setup.**
- **Derive equations for excess pressure in an air bubble & a soap bubble.**
- **Complete several practice problems.**

# Review Question #1

A clean glass tube with a diameter of 2mm is placed into water at 20 degrees Celsius. How high does the water rise in the tube?

Note in this problem:  $\gamma_{\text{water}} = 0.0728 \frac{\text{N}}{\text{m}}$ ,  $g = 10 \frac{\text{m}}{\text{s}^2}$ ,  $\theta = 0$



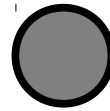
$$h = 7.28 \text{ mm}$$



$$h = 14.56 \text{ mm}$$



$$h = 7.28 \text{ cm}$$

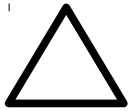


$$h = 14.56 \text{ cm}$$

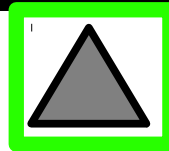
# Review Question #1 Solution

A clean glass tube with a diameter of 2mm is placed into water at 20 degrees Celsius. How high does the water rise in the tube?

Note in this problem:  $\gamma_{\text{water}} = 0.0728 \frac{\text{N}}{\text{m}}$ ,  $g = 10 \frac{\text{m}}{\text{s}^2}$



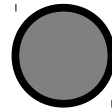
$$h = 7.28 \text{ mm}$$



$$h = 14.56 \text{ mm}$$



$$h = 7.28 \text{ cm}$$

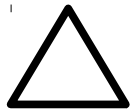


$$h = 14.56 \text{ cm}$$

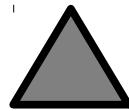
# Review Question #2



A clean glass capillary tube of internal diameter 0.5mm is held vertically with its lower end in water and with 1m of the tube above the surface. How high does the water rise in the tube?



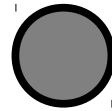
$$h = 29.71 \text{ mm}$$



$$h = 14.86 \text{ mm}$$

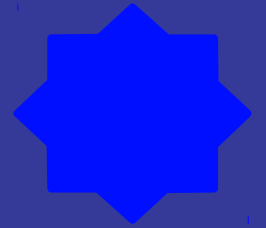


$$h = 59.43 \text{ mm}$$

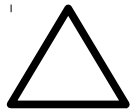


None of these

## Review Question #2 Solution



A clean glass capillary tube of internal diameter 0.5mm is held vertically with its lower end in water and with 1m of the tube above the surface. How high does the water rise in the tube?



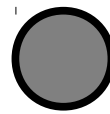
$$h = 29.71 \text{ mm}$$



$$h = 14.86 \text{ mm}$$



$$h = 59.43 \text{ mm}$$



None of these



# Critical Thinking Practice

A clean glass capillary tube of internal diameter 0.5mm is held vertically with its lower end in water and with 1m of the tube above the surface. How high does the water rise in the tube?

$$h = 59.43 \text{ mm}$$



If we now lower the clean glass capillary tube so that 20mm of the tube is above the surface, what happens?

We are forced to adopt a new and smaller height

What variables can we change in this equation?



$$\gamma = \frac{\rho r h g}{2 \cos(\theta)}$$

# Critical Thinking Practice



If we now lower the clean glass capillary tube so that 20mm of the tube is above the surface, what happens?

We are forced to adopt a new and smaller height

$$y = \frac{\rho r h g}{2 \cos(\theta)}$$

Does lowering a tube change the surface tension? **NO**

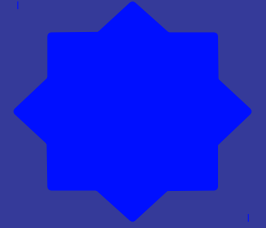
Does lowering a tube change liquid density? **NO**

Does lowering a tube change its radius? **NO**

Does lowering a tube change gravity? **NO**

Therefore the only thing that can change is the angle of contact!

# Critical Thinking Practice



A clean glass capillary tube of internal diameter 0.5mm is held vertically with its lower end in water and with 1m of the tube above the surface. How high does the water rise in the tube?

$$h = 59.43 \text{ mm}$$



If we now lower the clean glass capillary tube so that 20mm of the tube is above the surface, what happens?

**The angle of contact changes!**

# Critical Thinking



Surface Tension,  $\gamma$ , is always positive. A tube is placed in a liquid and the liquid forms a convex meniscus. What must be true?



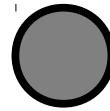
The liquid has 'wet' the surface.



The liquid has a negative density.



The liquid must be oil.



The height is negative.

# Critical Thinking **Solution**



Surface Tension,  $\gamma$ , is always positive. A tube is placed in a liquid and the liquid forms a convex meniscus. What must be true?



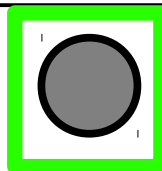
The liquid has 'wet' the surface.



The liquid has a negative density.



The liquid must be oil.



The height is negative.

# More about Convex Meniscus



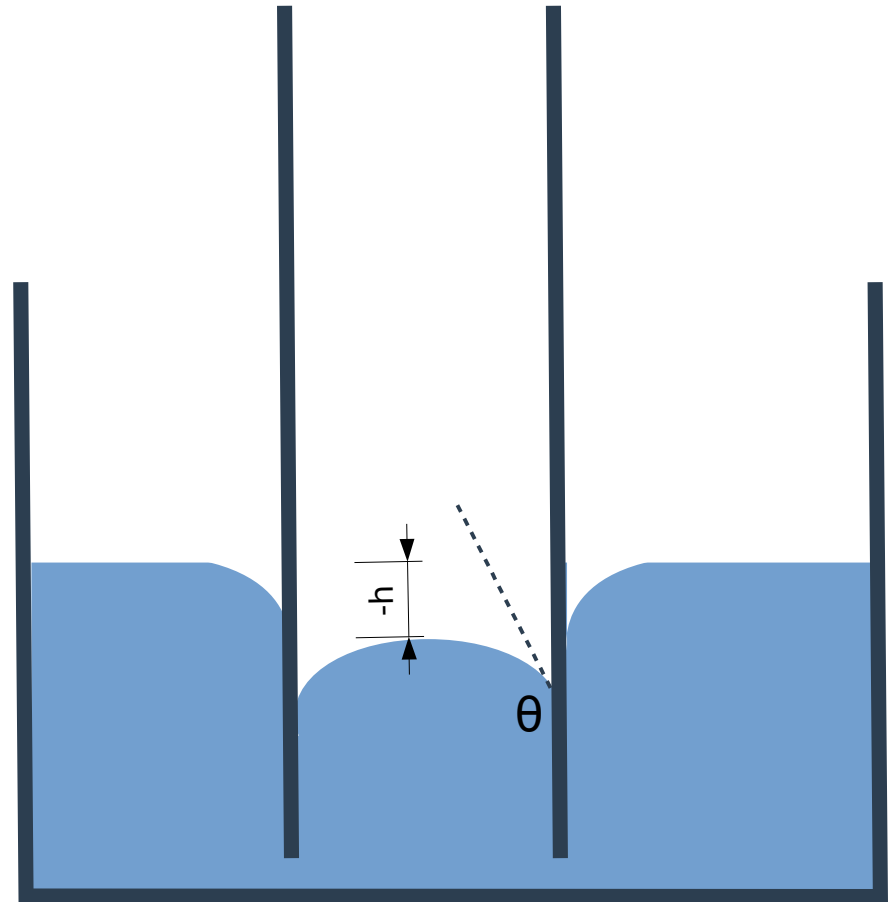
$$\gamma = \frac{\rho r h g}{2 \cos(\theta)}$$

$\gamma$  = always positive

Since  $\gamma$  is always positive:

if  $h$  is positive,  $\cos(\theta)$  is positive

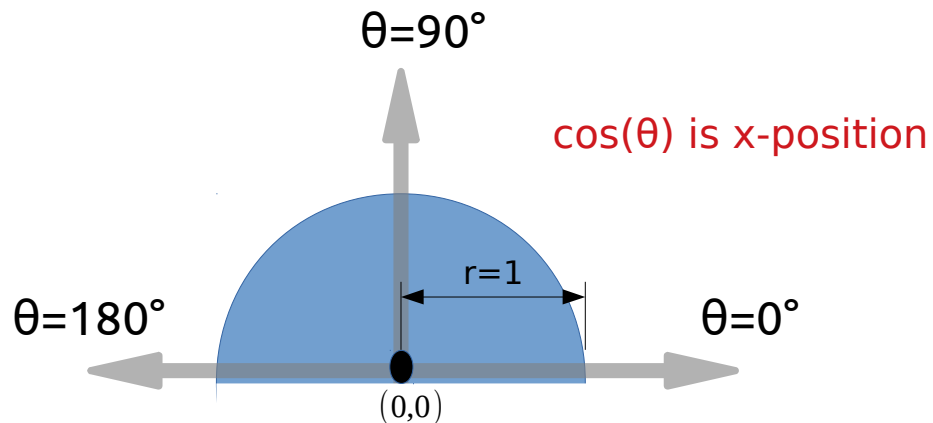
if  $h$  is negative,  $\cos(\theta)$  is negative



# More about Convex Meniscus

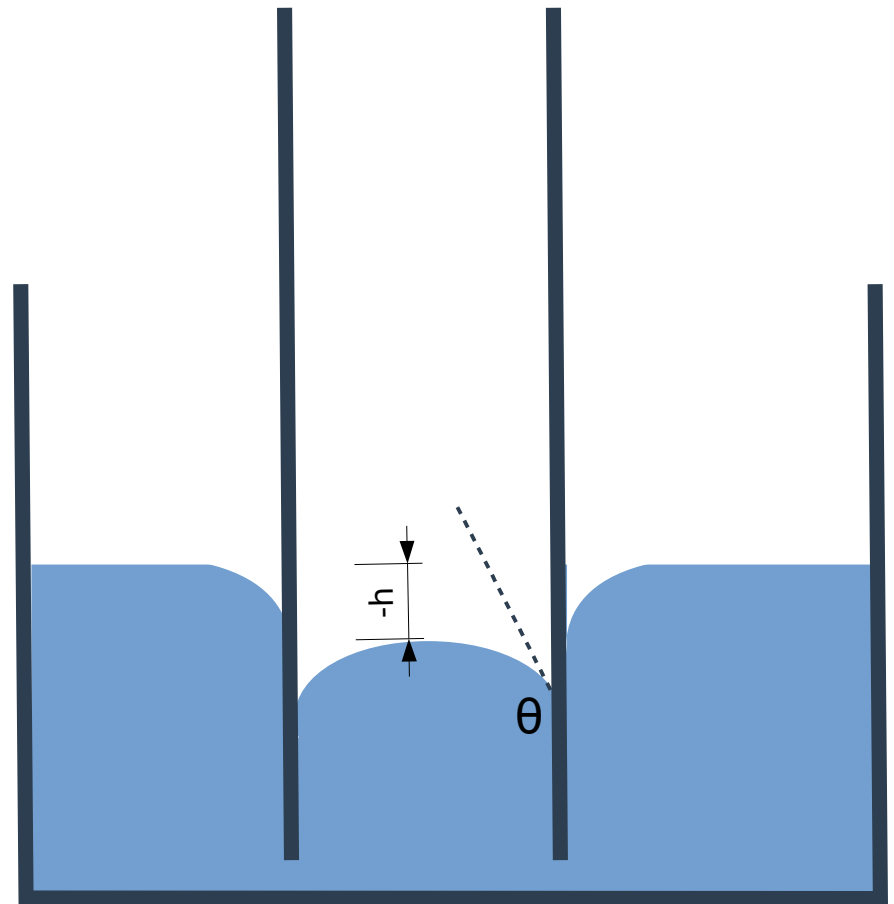


Where is  $\cos(\theta)$  negative?



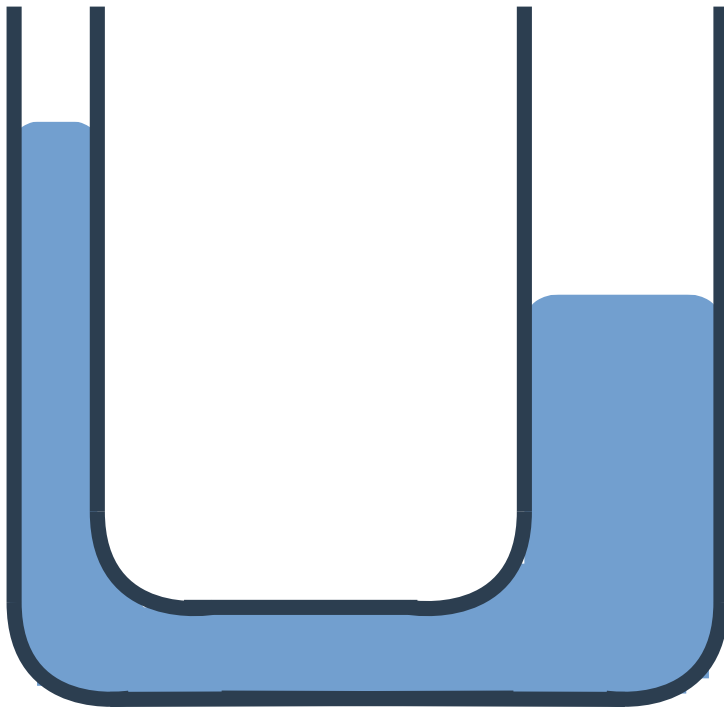
if  $0 < \theta < 90$  ,  $h$  is positive

if  $90 < \theta < 180$  ,  $h$  is negative

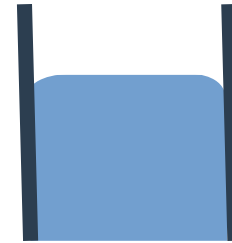


# Last thing about Convex Meniscus

## The U-Tube



In a U-Tube, even with a convex meniscus...



**...heights must be positive.**  
Remember this.



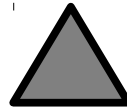
# Critical Thinking



Is it possible to have an angle of contact  $\theta = 90^\circ$  ?



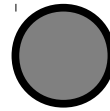
No, the surface tension would be equal to infinity.



No, the surface tension would be equal to zero.



No, the surface tension would be a negative number.



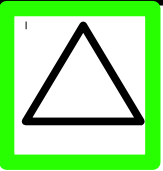
Yes

# Critical Thinking **Solution**

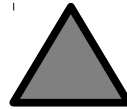


Is it possible to have an angle of contact  $\theta = 90^\circ$  ?

$$\gamma = \frac{\rho r h g}{2 \cos(\theta)}$$



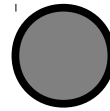
No, the surface tension would be equal to infinity.



No, the surface tension would be equal to zero.

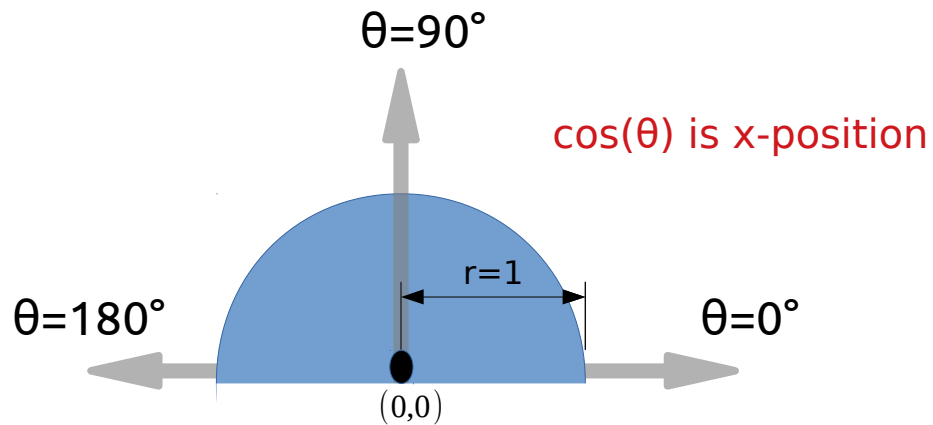


No, the surface tension would be a negative number.



Yes

# Visual explanation



$$\cos(90^\circ) = 0 \qquad \gamma = \frac{\rho r h g}{2 \cos(\theta)}$$

Arrows point from the above equations to the following:

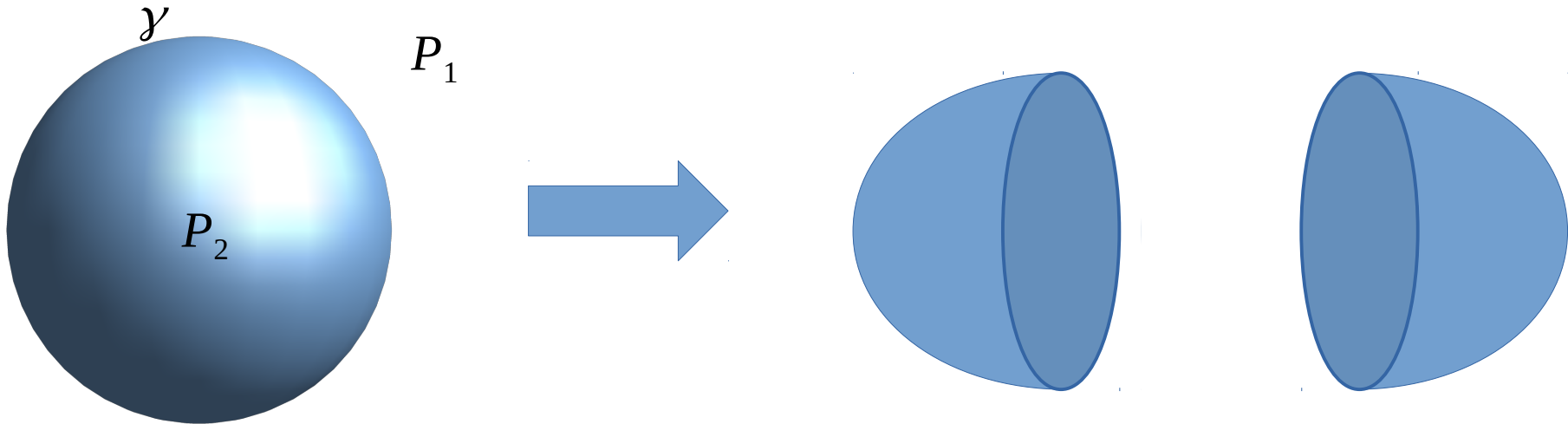
$$\gamma = \frac{\rho r h g}{2(0)}$$
$$\gamma = \frac{\rho r h g}{0} = \text{undefined}$$

# Excess Pressure in a Bubble

Consider a bubble in equilibrium. \*assume it has negligible ( $\approx 0$ ) thickness

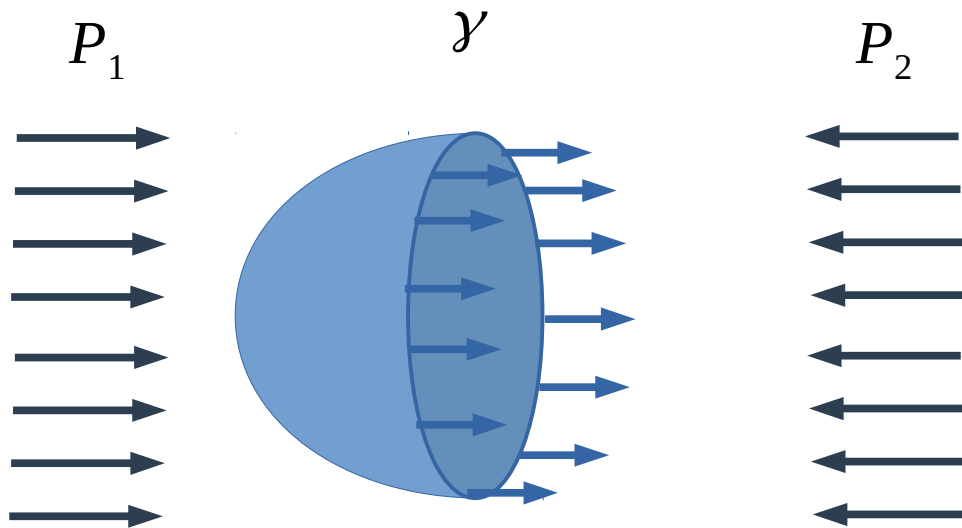
Outside pressure =  $P_1$   
Inside pressure =  $P_2$   
Surface tension =  $\gamma$

Calculate the excess pressure.



# Excess Pressure in a Bubble (continued)

At equilibrium, all forces are equal. We will look at forces in only one dimension.



$$P = \frac{F}{A} \quad \rightarrow \quad F = PA$$

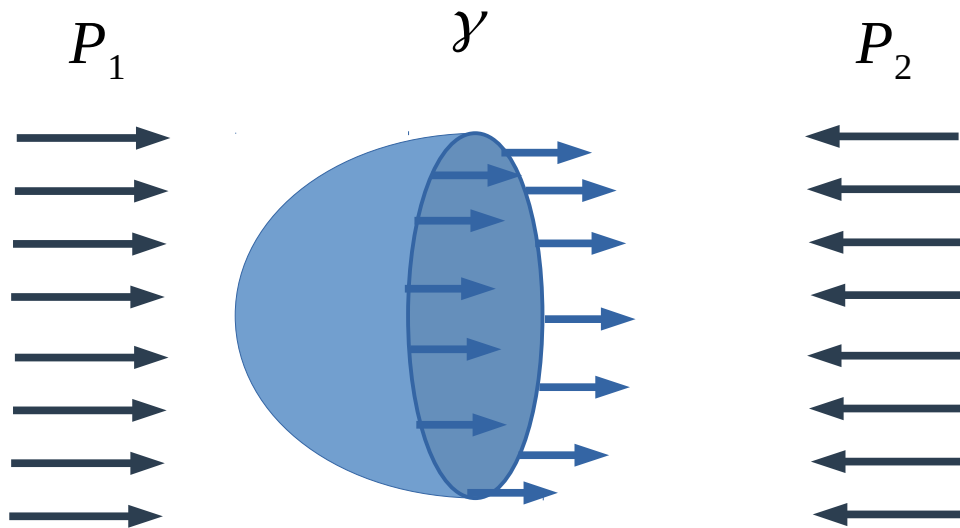
$$F_1 = P_1 A = P_1 \pi r^2$$

$$F_2 = P_2 A = P_2 \pi r^2$$

$$F_\gamma = \gamma C = \gamma(2\pi r)$$

# Excess Pressure in a Bubble (continued)

At equilibrium, all forces are equal. We will look at forces in only one direction



$$F_\gamma + F_1 = F_2$$

$$2\pi r \gamma + P_1 \pi r^2 = P_2 \pi r^2$$

$$2\cancel{\pi r} \gamma + P_1 \cancel{\pi r^2} = P_2 \cancel{\pi r^2}$$

$$2\gamma + P_1 r = P_2 r$$

$$2\gamma = r(P_2 - P_1)$$

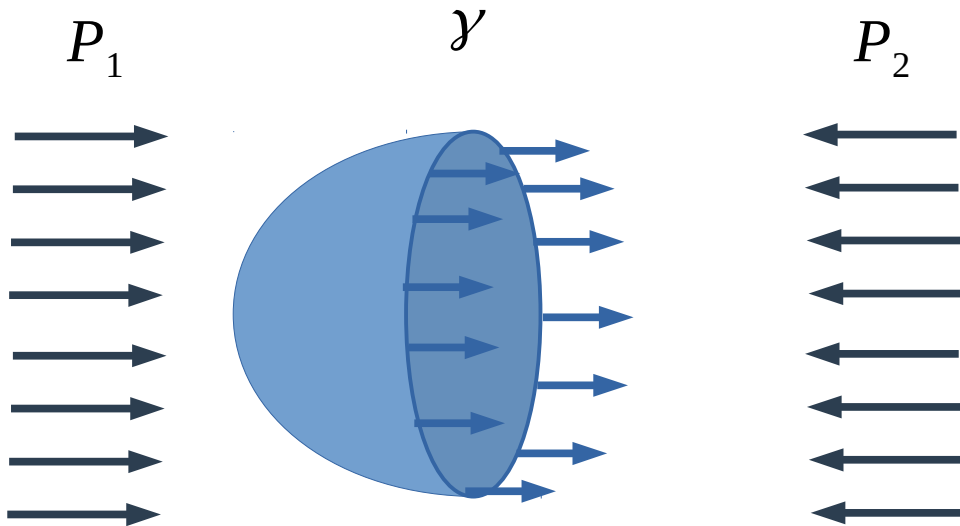
$$2\gamma = r(\Delta P)$$



$$\frac{2\gamma}{r} = \Delta P$$

# Clarification!

$$\Delta P = \frac{2\gamma}{r}$$

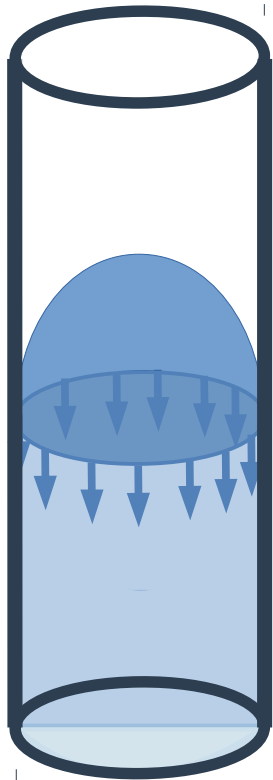


This equation works for bubbles with negligible thickness.

Examples include:

- Air Bubbles
- Spherical Drop

# Cool Fact!



By adding a  $\cos(\theta)$  term, this excess pressure formula can apply to liquids in a tube!

$$\Delta P = \frac{2\gamma \cos(\theta)}{r}$$

From Bernoulli's Principle

$$\Delta P = \rho g h$$

$$\frac{2\gamma \cos(\theta)}{r} = \rho g h$$

We already know this!

$$\gamma = \frac{\rho r h g}{2 \cos(\theta)}$$



# Critical Thinking Question

**Soap Bubbles** do not have a negligible thickness. This means two surfaces for surface tension approximately the same radius. What do you think the excess pressure of a soap bubble is?



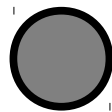
$$\Delta P = \frac{\gamma}{r}$$



$$\Delta P = \frac{2\gamma}{r}$$



$$\Delta P = \frac{4\gamma}{r}$$



$$\Delta P = \frac{8\gamma}{r}$$

# Critical Thinking Question **Solution**

**Soap Bubbles** do not have a negligible thickness. This means two surfaces for surface tension approximately the same radius. What do you think the excess pressure of a soap bubble is?



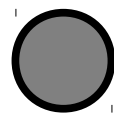
$$\Delta P = \frac{\gamma}{r}$$



$$\Delta P = \frac{2\gamma}{r}$$



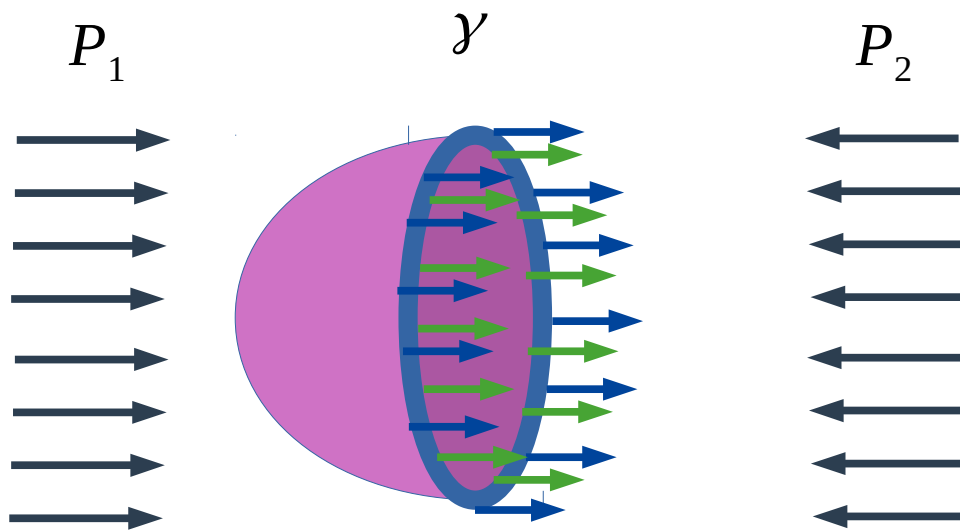
$$\Delta P = \frac{4\gamma}{r}$$



$$\Delta P = \frac{8\gamma}{r}$$

# Excess Pressure in a Soap Bubble

**Soap Bubbles do not have a negligible thickness. This means two surfaces for surface tension.**



$$2 F_{\gamma} + F_1 = F_2$$

$$2(2 \pi r \gamma) + P_1 \pi r^2 = P_2 \pi r^2$$

$$4 \cancel{\pi r} \gamma + P_1 \cancel{\pi r^2} = P_2 \cancel{\pi r^2}$$

$$4 \gamma + P_1 r = P_2 r$$

$$4 \gamma = r (P_2 - P_1)$$

$$4 \gamma = r (\Delta P)$$

$$\frac{4 \gamma}{r} = \Delta P$$

## **Surface Tension Class #2 Goals **Reviewed****

- **Understand how angle of contact is related to height in a surface tension experimental setup.**
- **Derive equations for excess pressure in a air bubble & a soap bubble.**
- **Complete several practice problems.**

**New Section!**

# Surface Tension Class #3 Goals



- **Solve a U-Tube problem**
- **Understand work related to surface tension**
- **Understand breaking/coalescing of bubbles**

# Review Question #1

What is the excess pressure of a soap bubble of radius  $r = 5\text{ cm}$  ?

$$^* \gamma_{\text{soap}} = 0.025 \frac{\text{N}}{\text{m}}$$



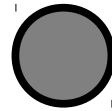
$$\Delta P = 0 \frac{\text{N}}{\text{m}^2}$$



$$\Delta P = 1 \frac{\text{N}}{\text{m}^2}$$



$$\Delta P = 2 \frac{\text{N}}{\text{m}^2}$$

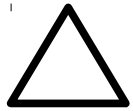


$$\Delta P = 4 \frac{\text{N}}{\text{m}^2}$$

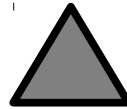
# Review Question #1 Solution

What is the excess pressure of a soap bubble of radius  $r = 5\text{ cm}$  ?

$$^* \gamma_{\text{soap}} = 0.025 \frac{\text{N}}{\text{m}}$$



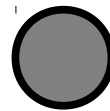
$$\Delta P = 0 \frac{\text{N}}{\text{m}^2}$$



$$\Delta P = 1 \frac{\text{N}}{\text{m}^2}$$



$$\Delta P = 2 \frac{\text{N}}{\text{m}^2}$$



$$\Delta P = 4 \frac{\text{N}}{\text{m}^2}$$



# Review Question #2

A hollow glass tube is placed into a liquid and forms a convex meniscus.  
The height is ①.

A liquid is poured into a U-tube with two limbs and forms 2 convex meniscus-es.  
The heights are ②.



① Negative

② Negative



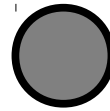
① Negative

② Positive



① Positive

② Positive



① Positive

② Negative

# Review Question #2 **Solution**

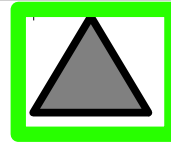
A hollow glass tube is placed into a liquid and forms a convex meniscus.  
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The heights are ②.



① Negative

② Negative



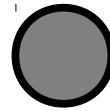
① Negative

② Positive



① Positive

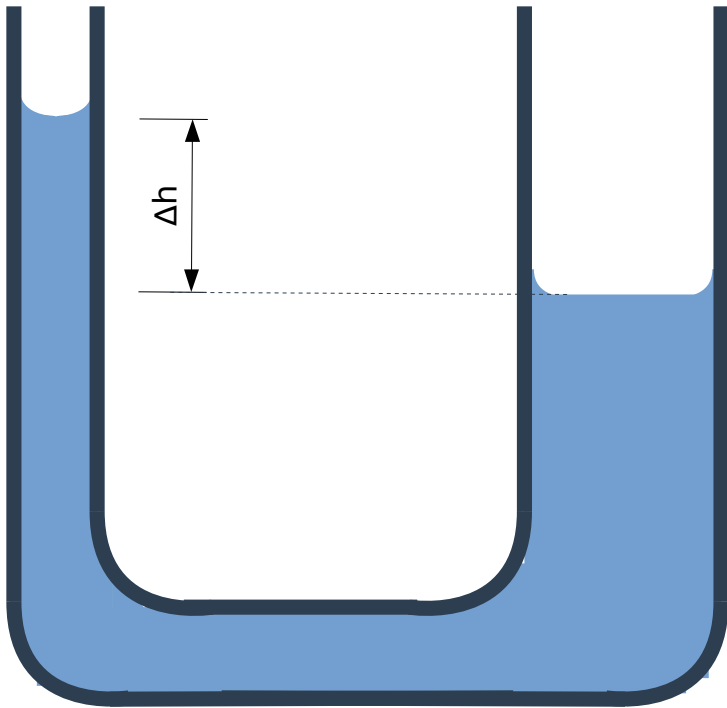
② Positive



① Positive

② Negative

# U-Tube Example



Water is poured into a U-Tube with limbs of radii  $r_1 = 1\text{ mm}$  and  $r_2 = 3\text{ mm}$  respectively.

What is responsible for the height difference?

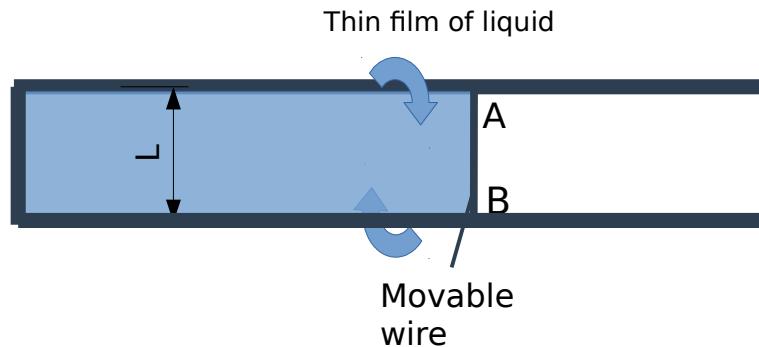
Capillarity – the phenomenon of water rising in a tube due to surface tension.

What is the height difference between the liquid in the two arms?

9.905 mm

# “Work work work work work” - Rhianna

## Recall from Day#1



$$F_{AB} = 2 \gamma L$$

$$W = F \times \Delta x$$

$$W = 2 \gamma L x \rightarrow W = \gamma (2 L x) \rightarrow$$

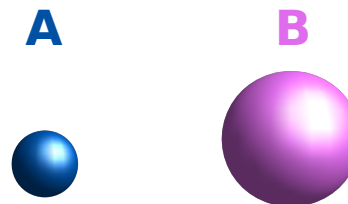
Work is in units of  
Newtons\*meters (N·m)

$$W = \gamma \Delta A$$

# Blowing Bubbles



Soap bubble A has a radius of 10 cm.  
Soap bubble B has a radius of 20 cm.



Compare the excess pressure inside these bubbles.

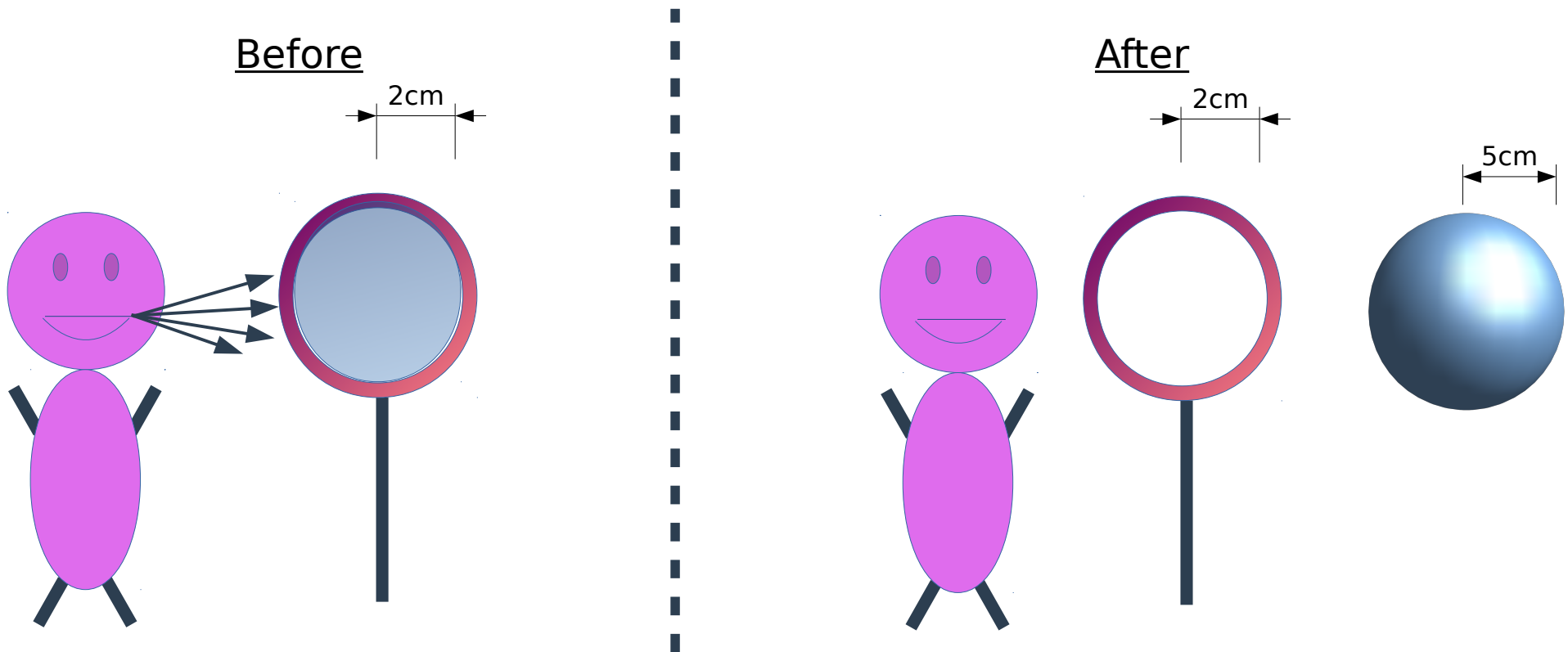
**The ratio of excess pressure is 2:1. The smaller bubble has a higher excess pressure.**

What is the ratio of the work required to create these bubbles?

**The ratio of work required is 4:1. It takes more work to create a larger bubble.**

# Blowing Bubbles pt. 2

A blowstick with an internal radius  $r=2\text{ cm}$  has a film of soap that is then blown into a bubble of radius  $R=5\text{ cm}$ . How much work was done?  $\gamma_{\text{soap}}=0.025\frac{\text{N}}{\text{m}}$



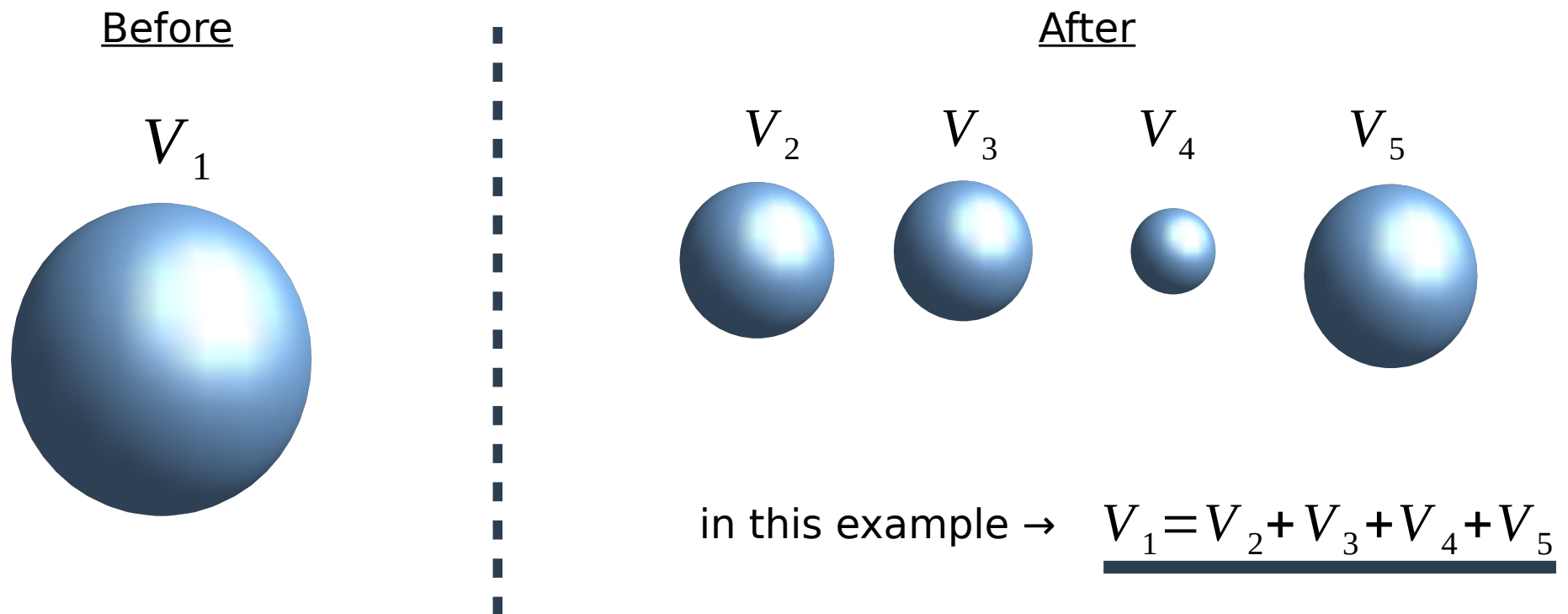
# Blowing Bubbles pt. 2

A blowstick with an internal radius  $r=2\text{ cm}$  has a film of soap that is then blown into a bubble of radius  $R=5\text{ cm}$ . How much work was done?  $\gamma_{\text{soap}}=0.025\frac{\text{N}}{\text{m}}$

$$W=0.00151\text{ N}\cdot\text{m}$$

# “Break it up, Break it up, Break it up, Break down” - Curtis Blow

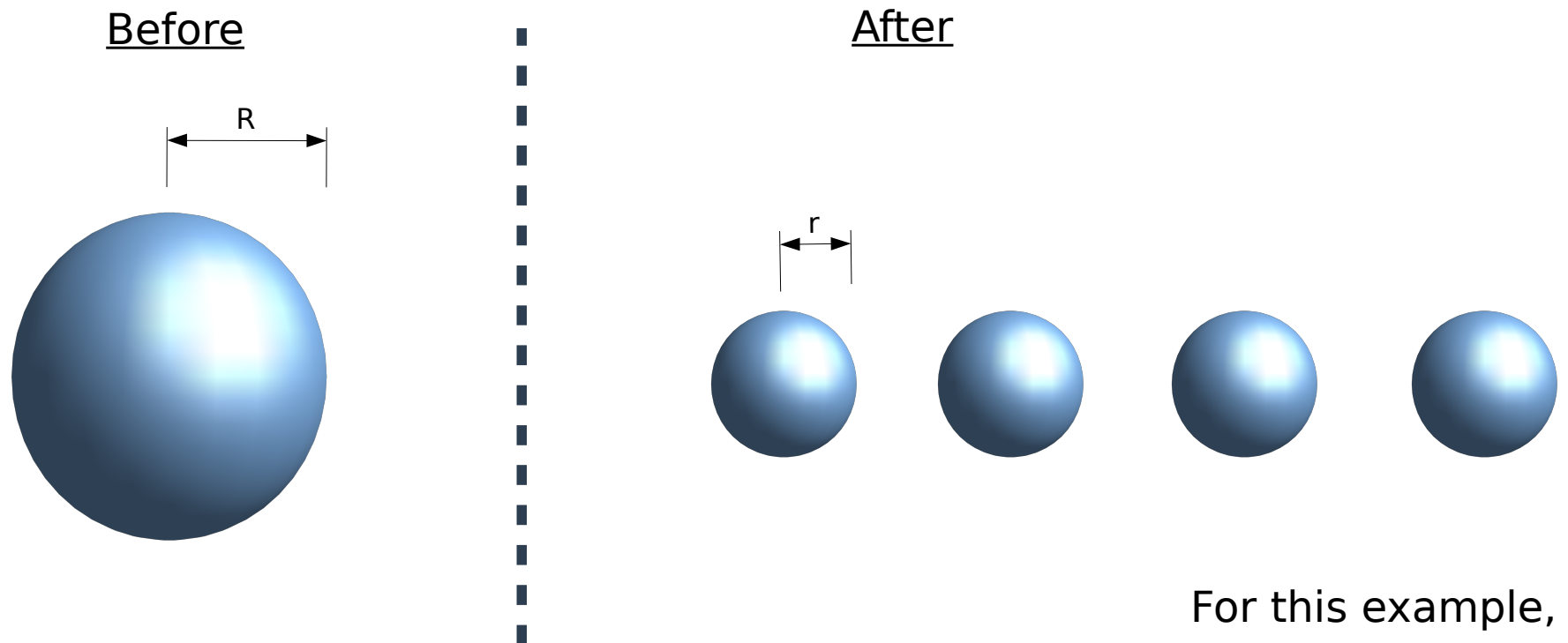
When “breaking up” a bubble into smaller bubbles, volume is conserved.





# Same Radius Break-up

If a bubble “breaks up” into  $n$  smaller identical bubbles, the equation is:  $R^3 = n r^3$



For this example,  $n=4$

# Practice Problem #1

**A soap bubble of diameter 66cm is broken into 27 identical bubbles.**

$$\gamma_{\text{soap}} = 0.025 \frac{\text{N}}{\text{m}}$$

What is the radius of the smaller bubbles?

$$r = 11 \text{ cm}$$

How much work has been spent?

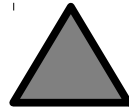
$$W = 0.1368 \text{ N}\cdot\text{m}$$

## Practice Problem #2

**How many 1cm radius bubbles can be created by a larger 1m radius bubble?**



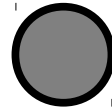
$n = 10^2$  bubbles



$n = 10^3$  bubbles



$n = 10^6$  bubbles



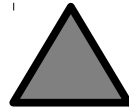
$n = 10^9$  bubbles

## Practice Problem #2 **Solution**

**How many 1cm radius bubbles can be created by a larger 1m radius bubble?**



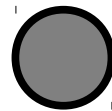
$n = 10^2$  bubbles



$n = 10^3$  bubbles



$n = 10^6$  bubbles



$n = 10^9$  bubbles

# **“Come Together” - The Beatles**

**Bubbles Coalescing is the same as:**

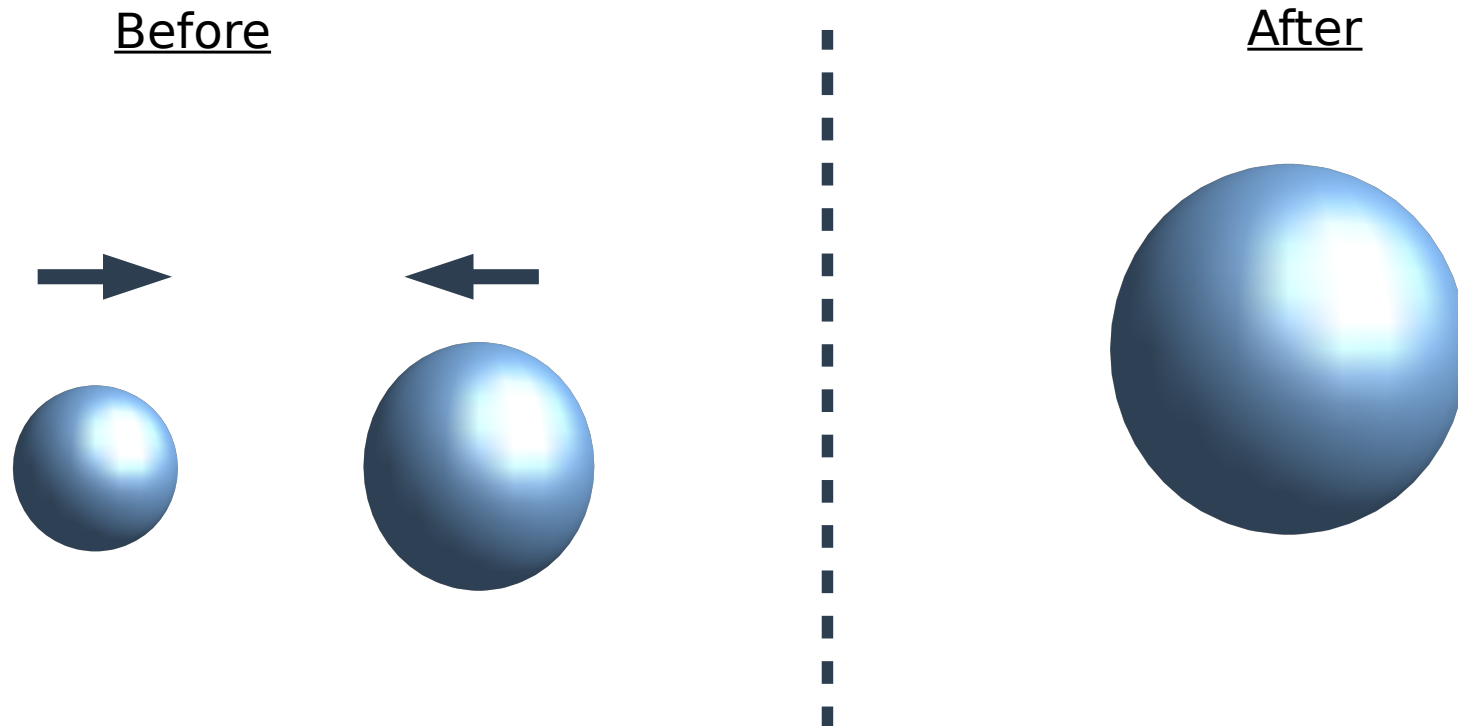
- Bubbles joining**
- Bubbles coming together**
- Bubbles meeting**

**There are 2 types of coalescing**

# Coalescing Type #1

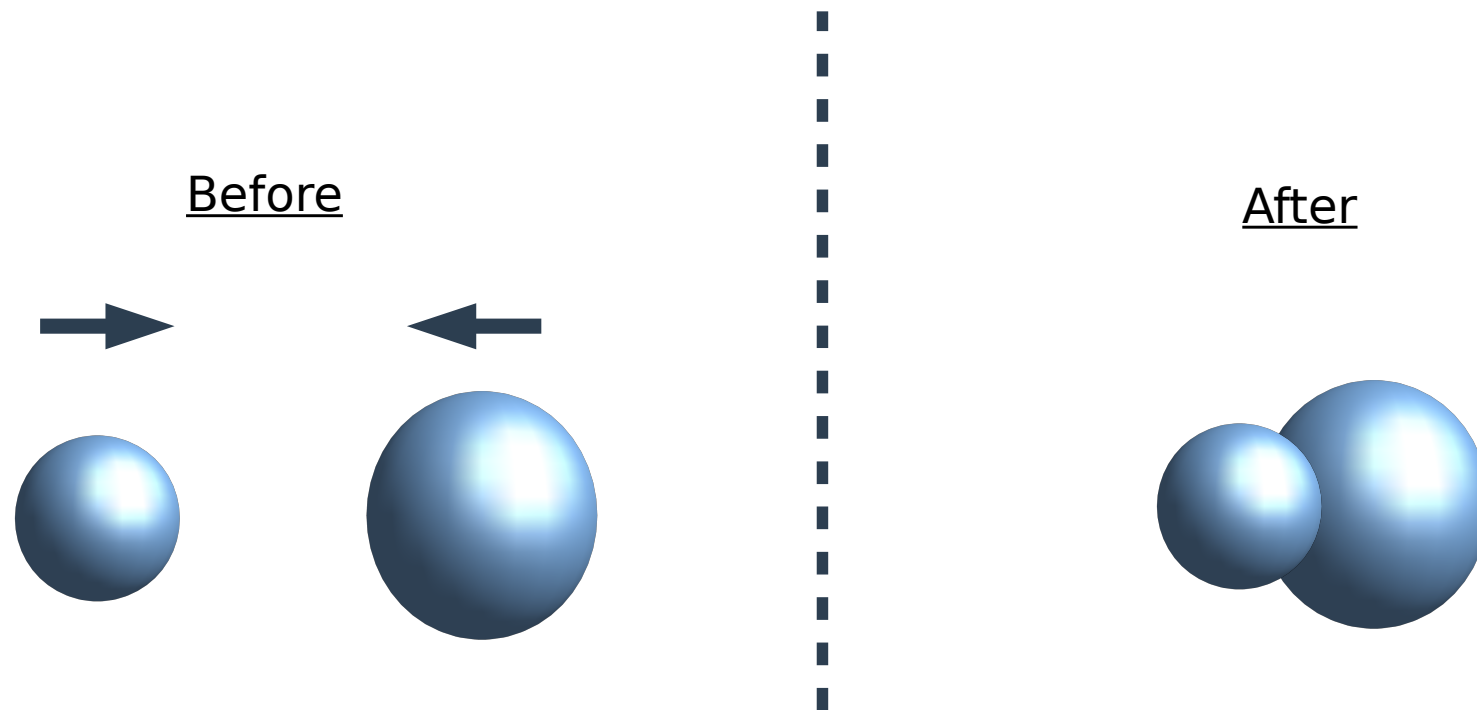
## 2 Bubbles Coalescing in a Vacuum

Surface Area is conserved, 1 bubble formed



# Coalescing Type #2

## 2 Bubbles Coalescing in Air ... a bit more complicated

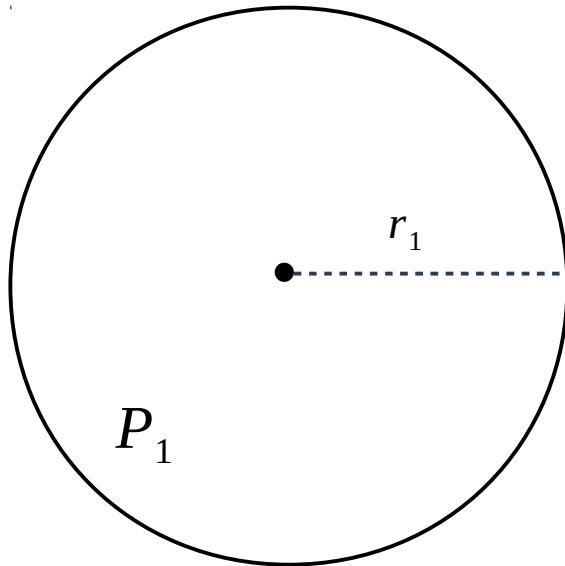


# Bubbles Coalescing in Air

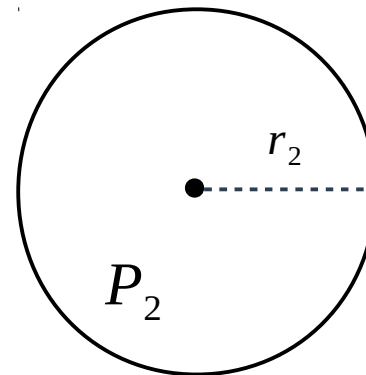


**Look at it in 2-Dimensions (example is soap bubbles)**

$$P_1 - P_{air} = \frac{4\gamma}{r_1}$$



$$P_2 - P_{air} = \frac{4\gamma}{r_2}$$



$P_{air}$



# Bubbles Coalescing in Air



## Derivation

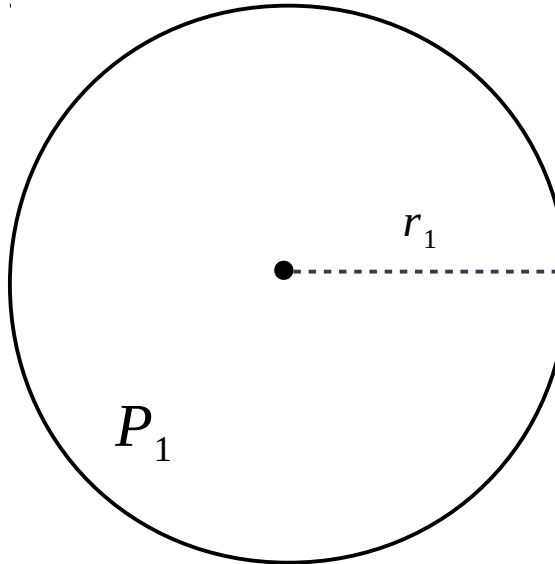
$$P_2 - P_1 = \frac{4\gamma}{r_2} - \frac{4\gamma}{r_1}$$

$$\Delta P_{2,1} = \frac{4\gamma}{r_2} - \frac{4\gamma}{r_1}$$

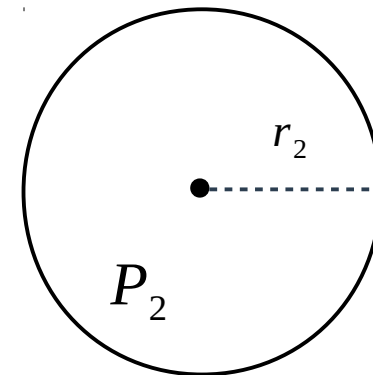
$$\frac{4\gamma}{r} = \frac{4\gamma}{r_2} - \frac{4\gamma}{r_1}$$

A new radius is formed!

$$P_1 - P_{air} = \frac{4\gamma}{r_1}$$

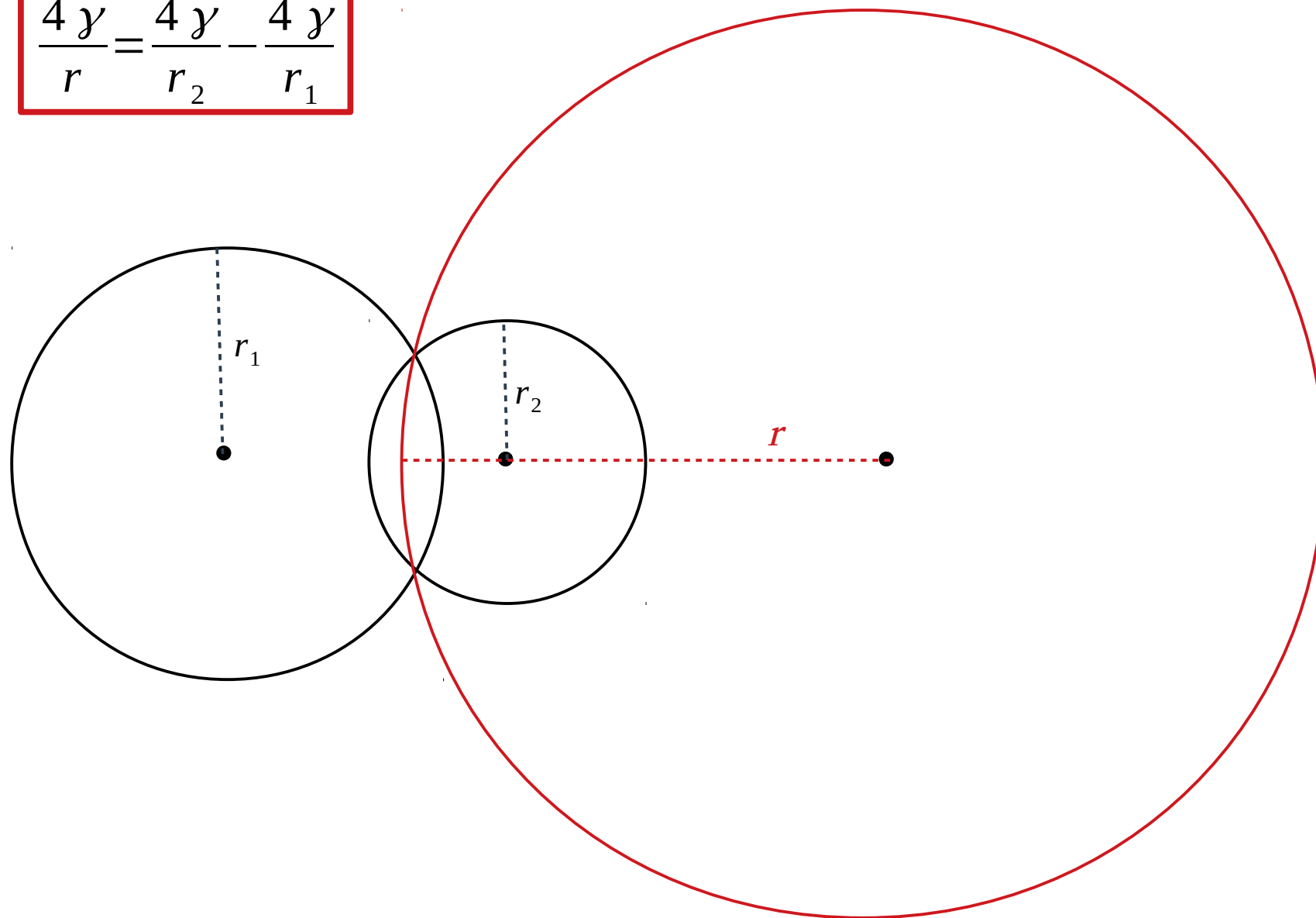


$$P_2 - P_{air} = \frac{4\gamma}{r_2}$$

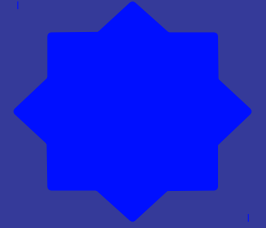


$P_{air}$

$$\frac{4\gamma}{r} = \frac{4\gamma}{r_2} - \frac{4\gamma}{r_1}$$

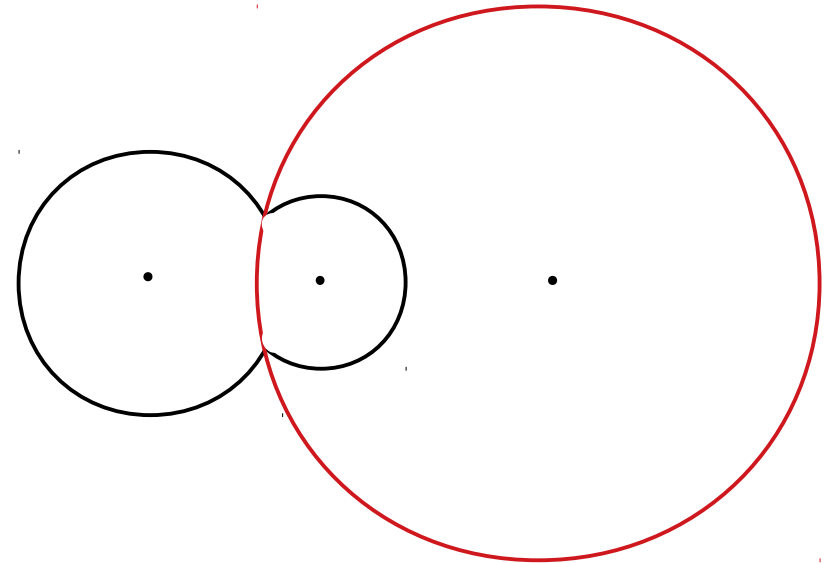


# Coalescing Type #2



## Bubbles Coalescing in Air

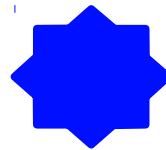
- New shared radius is formed
- Interface between bubbles is directed into the larger bubble **BECAUSE** smaller bubbles have higher excess pressure.



$$\frac{4\gamma}{r} = \frac{4\gamma}{r_2} + \frac{4\gamma}{r_1}$$



$$\frac{1}{r} = \frac{1}{r_2} + \frac{1}{r_1}$$



# Critical Thinking Practice #1

**What would the shared radius be if two identical soap bubbles of identical radii coalesce in air?**



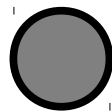
$$r_{new} = 0$$



$$r_{new} = \infty$$



$$r_{new} = r_1 + r_2$$



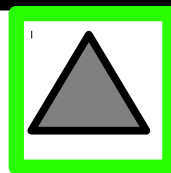
None of these

# Critical Thinking Practice #1 **Solution**

**What would the shared radius be if two identical soap bubbles of identical radii coalesce in air?**



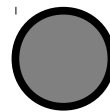
$$r_{new} = 0$$



$$r_{new} = \infty$$



$$r_{new} = r_1 + r_2$$



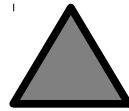
None of these

# Practice Problem #3

Two bubbles of radii 3cm and 4cm come together in a vacuum. What is the radius of the newly formed bubble?



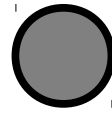
$$r_{new} = 3.12 \text{ cm}$$



$$r_{new} = 26.46 \text{ cm}$$



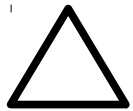
$$r_{new} = 10 \text{ cm}$$



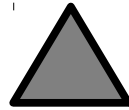
$$r_{new} = 5 \text{ cm}$$

## Practice Problem #3 **Solution**

Two bubbles of radii 3cm and 4cm come together in a vacuum. What is the radius of the newly formed bubble?



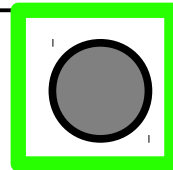
$$r_{new} = 3.12 \text{ cm}$$



$$r_{new} = 26.46 \text{ cm}$$



$$r_{new} = 10 \text{ cm}$$



$$r_{new} = 5 \text{ cm}$$

**NECTA Practice!**



# Class Poll

**We will have a night (8:00pm-9:15pm) review session of Surface Tension this week or next. What day?**



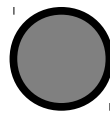
**Monday**



**Tuesday**



**Wednesday**



**Next Week**

# Surface Tension Class #4 Goals

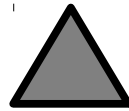
- Review all previous NECTA questions about surface tension
- Finish the unit of surface tension!

# Practice Problem #4

Two bubbles of radii 8cm and 7cm join together in air so that the common interface has a radius of  $r$ . What is  $r$ ?



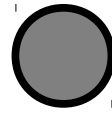
$$r_{new} = 113 \text{ cm}$$



$$r_{new} = 17 \text{ cm}$$



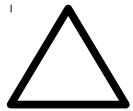
$$r_{new} = 28 \text{ cm}$$



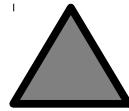
$$r_{new} = 56 \text{ cm}$$

## Practice Problem #4

Two bubbles of radii 8cm and 7cm join together in air so that the common interface has a radius of  $r$ . What is  $r$ ?



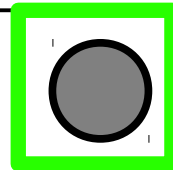
$$r_{new} = 113 \text{ cm}$$



$$r_{new} = 17 \text{ cm}$$



$$r_{new} = 28 \text{ cm}$$



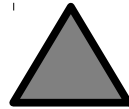
$$r_{new} = 56 \text{ cm}$$

# Critical Thinking Review #1

**What would the shared radius be if a VERY large bubble coalesced with a much smaller bubble in air?**



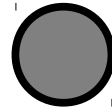
$$r_{new} = r_{small}$$



$$r_{new} = r_{large}$$



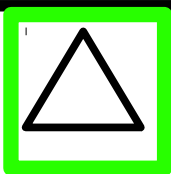
$$r_{new} = r_{large} - r_{small}$$



$$r_{new} = \infty$$

# Critical Thinking Review #1

What would the shared radius be if a **VERY** large bubble coalesced with a much smaller bubble in air?



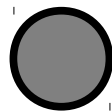
$$r_{new} = r_{small}$$



$$r_{new} = r_{large}$$



$$r_{new} = r_{large} - r_{small}$$



$$r_{new} = \infty$$

# Section 1 - Definitions



## **Define surface tension**

The force per unit length in the plane of a liquid surface acting in the surface and perpendicular to one side of an imaginary line drawn in the surface.

## **Define angle of contact**

The angle between the solid surface and the tangent plane to the liquid surface at a point where it touches the solid.

# Section 1 – Definitions (continued)



## **Define surface tension in terms of energy**

The work done per unit area in increasing the surface area of a liquid at constant temperature.

## **Explain the phenomenon of surface tension in terms of molecular theory**

Interior molecules in a liquid experience forces from all directions. Molecules on the surface only experience inter-molecular forces from below, resulting in an increase in potential energy. These surface molecules form a network such that they exist as a thin membrane, a scenario referred to as surface tension.



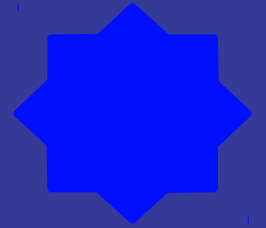
# Section 1 – Definitions (last part)



## **Differentiate surface tension from surface energy**

Surface tension is the elastic tendency of a fluid surface which makes it acquire the least surface area possible while surface energy is the energy required to form a new liquid surface.

## Section 2 – Surface Tension & Heights



**A clean glass capillary tube of internal diameter 0.7mm is held vertically with its lower end in water and with 1m of the tube above the surface.**

- i. How high does the water rise in the tube?
- ii. What happens if the tube is lowered until only 20mm of its length is above the surface?

## Section 2 – Surface Tension & Heights



**A clean open ended glass U-tube has vertical limbs one of which has a uniform internal diameter of 8.0mm and the other 40.0mm. Mercury is poured into the tube. You observe that the height of mercury is different in the two different limbs.**

Explain this observation.

Calculate the difference in heights.

$$\rho_{\text{mercury}} = 13600$$

$$\gamma_{\text{mercury}} = 0.46$$

$$\theta_{\text{mercury}} = 137$$

## Section 3b – Bubble Problems



**A soap bubble of radius 6.0cm and another soap bubble of radius 8.0cm are brought together so that the combined bubble has a common interface of radius  $r$ . Find the value of  $r$  and comment on the shape of the interface.**

$$* \gamma_{\text{soap}} = 0.025 \frac{\text{N}}{\text{m}}$$

## Section 3b – Bubble Problems



**Two spherical soap bubbles of radii 60mm and 20mm coalesce so that they have a common surface. If they are made from the same solution and the radii of the bubbles remain the same after they join together, calculate the radius of the curvature of their common surface.**

$$* \gamma_{\text{soap}} = 0.025 \frac{N}{m}$$

## Section 3b – Bubble Problems



**Two soap bubbles have radii in the ratio 4:9.**

1. Compare the excess pressure inside these bubbles.
2. Show that the ratio of work done in blowing these bubbles is 16:81

# Surface Tension Class #4 Goals **Reviewed**

- Review all previous NECTA questions about surface tension
- Finish the unit of surface tension!

**Done with  
Surface Tension...**

**... on to Elasticity**