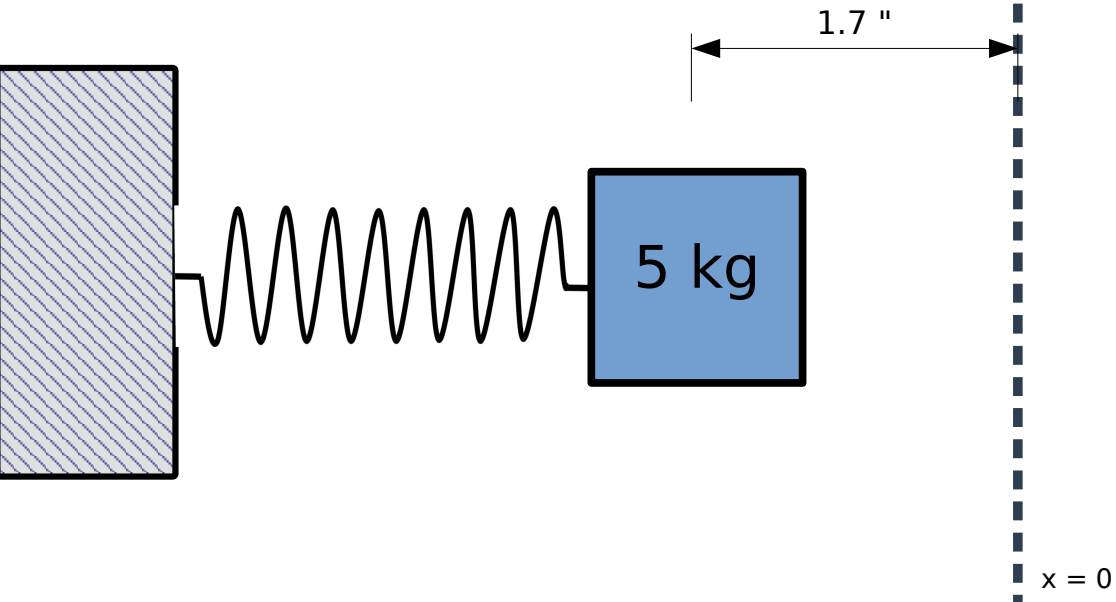


Topic:

Mechanics

Sub-Topic:

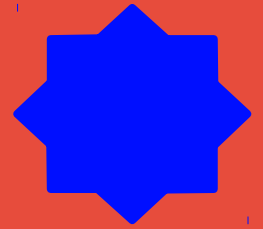
Simple Harmonic Motion



SHM Class #1 Goals

- To understand the definition of Simple Harmonic Motion and its characteristics.
- To derive the formulas surrounding Simple Harmonic Motion.
- To understand how velocity and acceleration change throughout Simple Harmonic Motion.

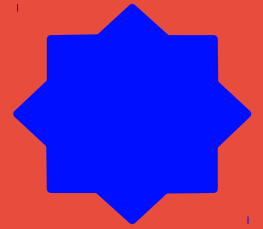
Definition of S.H.M.



Simple Harmonic Motion (S.H.M.)

- Periodic Motion
(repeats itself in equal intervals of time)
- The displacement is proportional to acceleration.
- Has a restoring force that always acts in the direction of equilibrium.

Definition of S.H.M. Terms



→ **Amplitude** (A)

The maximum displacement from equilibrium position.

→ **Period** (T)

The time it takes to complete one oscillation.

→ **Frequency** (f)

The number of complete oscillations in 1 second.

→ **Angular Frequency** (ω)

The number of radians completed in 1 second.

Examples of S.H.M.

- **Ideal spring systems (no friction)**
- **Ideal pendulum systems (no friction & small θ)**
- **Object moving up and down in water**

Simple Harmonic Motion - a circle

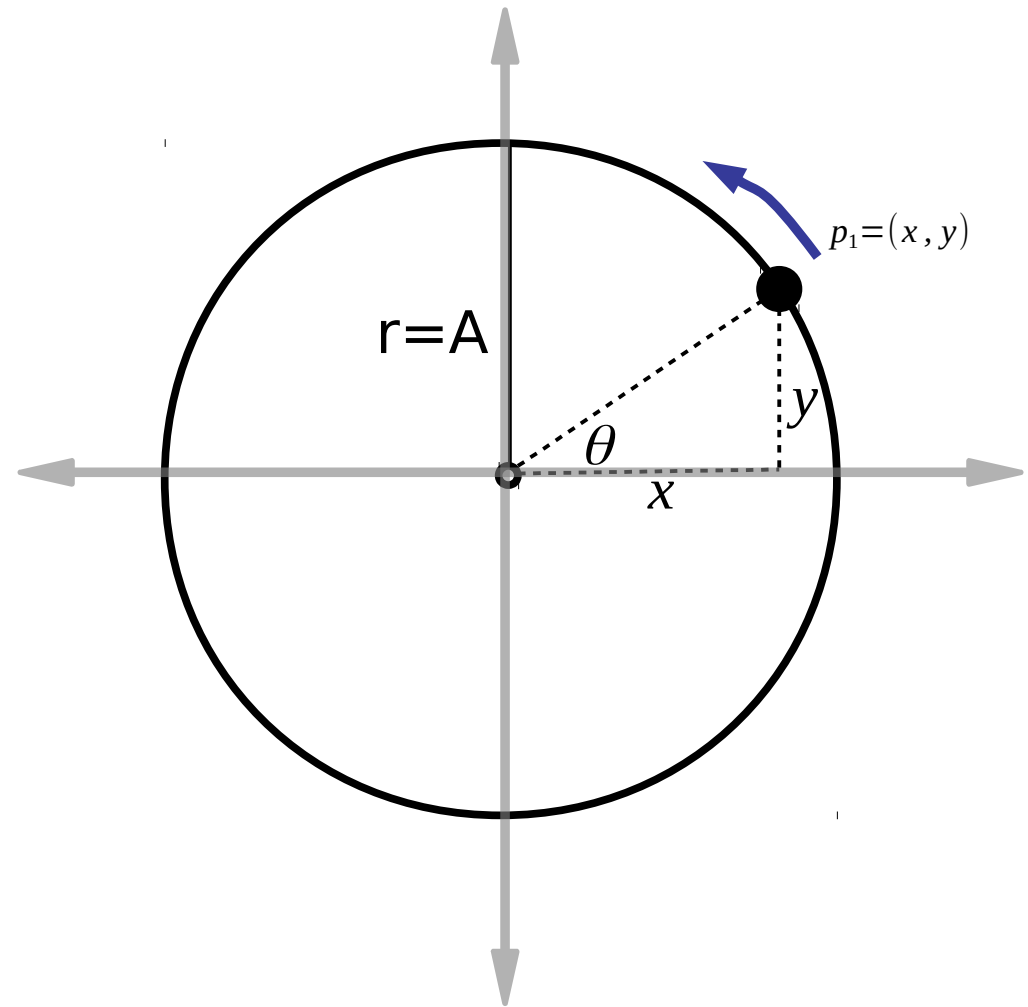


Consider a point moving around the circumference of a circle with radius A at a constant angular frequency ω

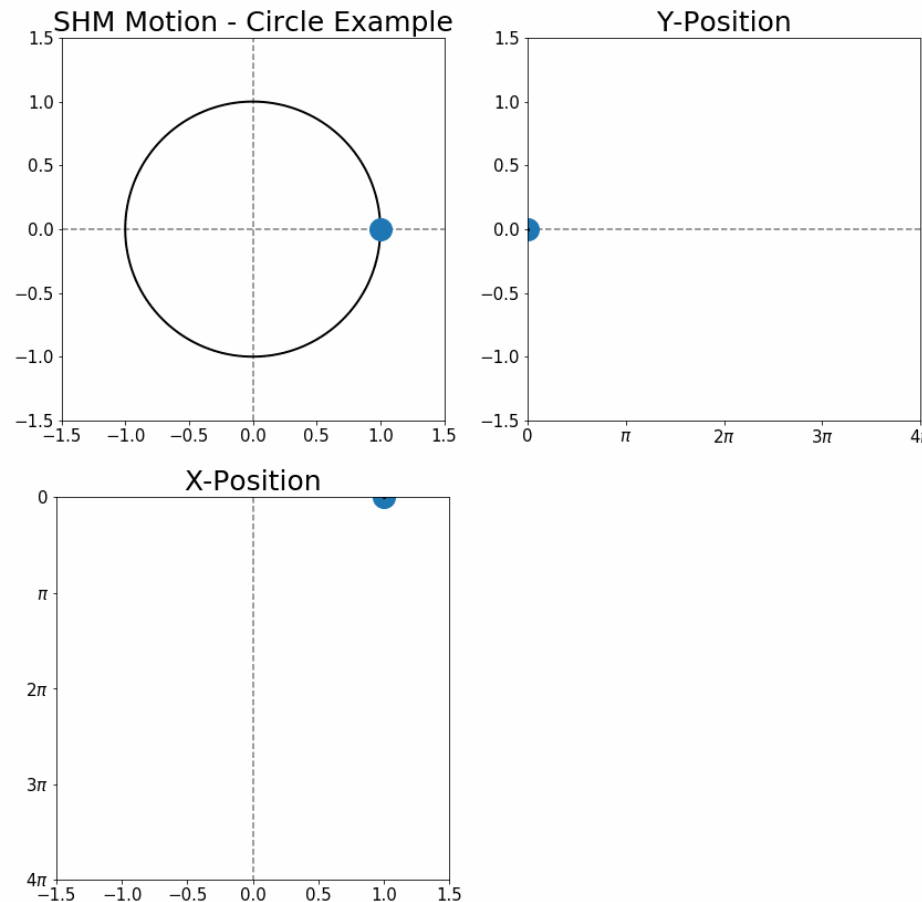
Is it simple harmonic...

...if we plot x-position against time?

...if we plot y-position against time?



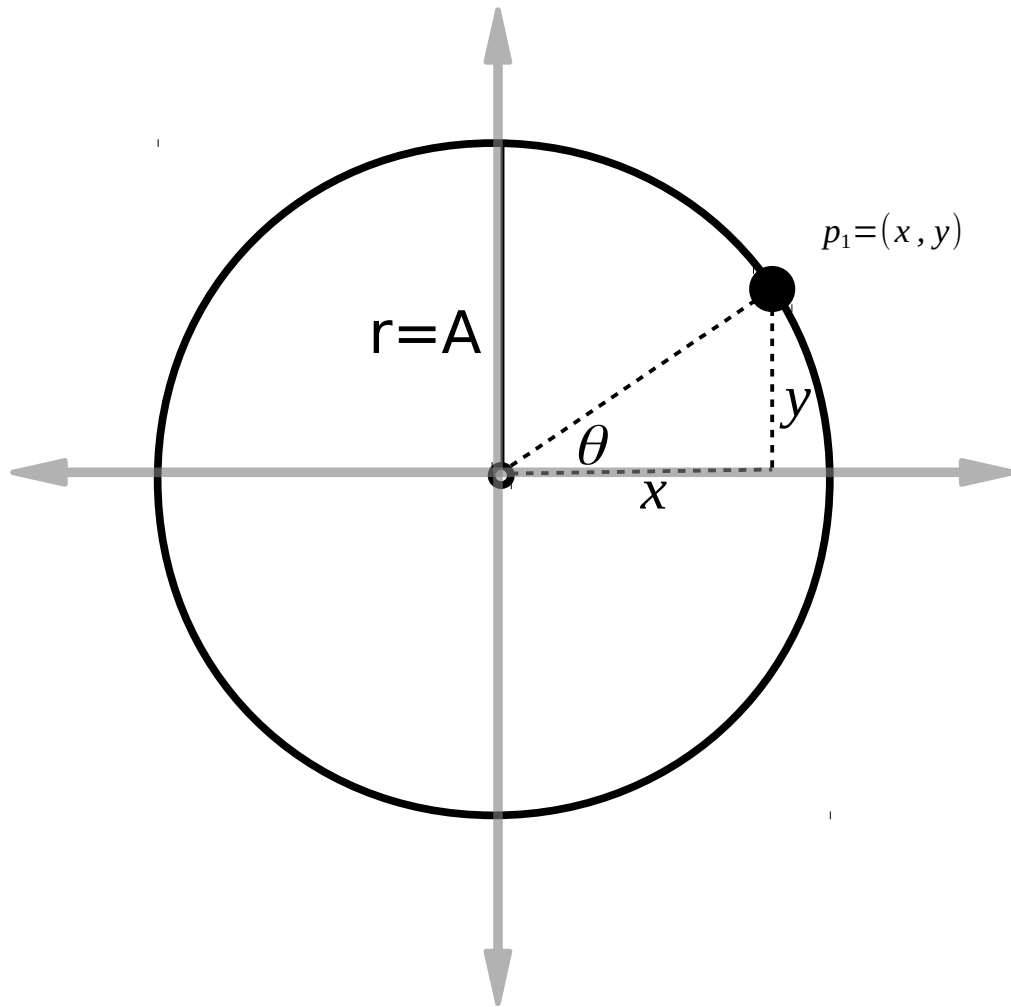
Circular Motion Equations (observed)



$$y = A \sin(\omega t)$$

$$x = A \cos(\omega t)$$

Circular Motion Equations (derived)



$\sin \theta = \frac{y}{A}$	$\cos \theta = \frac{x}{A}$
$\theta = \omega t$	

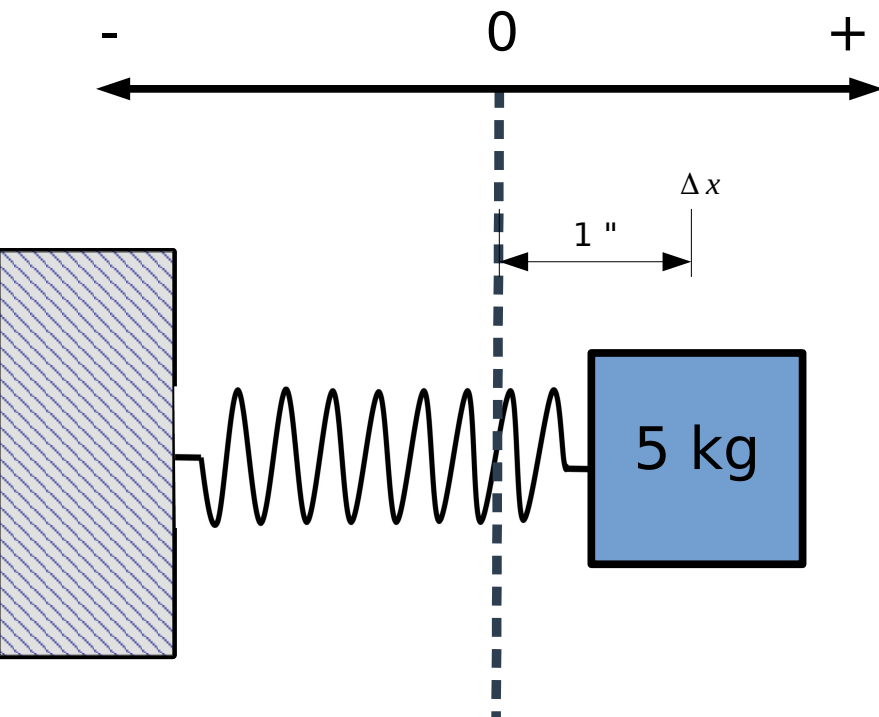
Using substitution

$$\begin{array}{c} \sin(\omega t) = \frac{y}{A} \\ \downarrow \\ y = A \sin(\omega t) \end{array} \quad \left| \quad \begin{array}{c} \cos(\omega t) = \frac{x}{A} \\ \downarrow \\ x = A \cos(\omega t) \end{array} \right.$$

Critical Thinking



Which equation, \triangle or \triangle , models the position of the 5 kg mass released from rest?



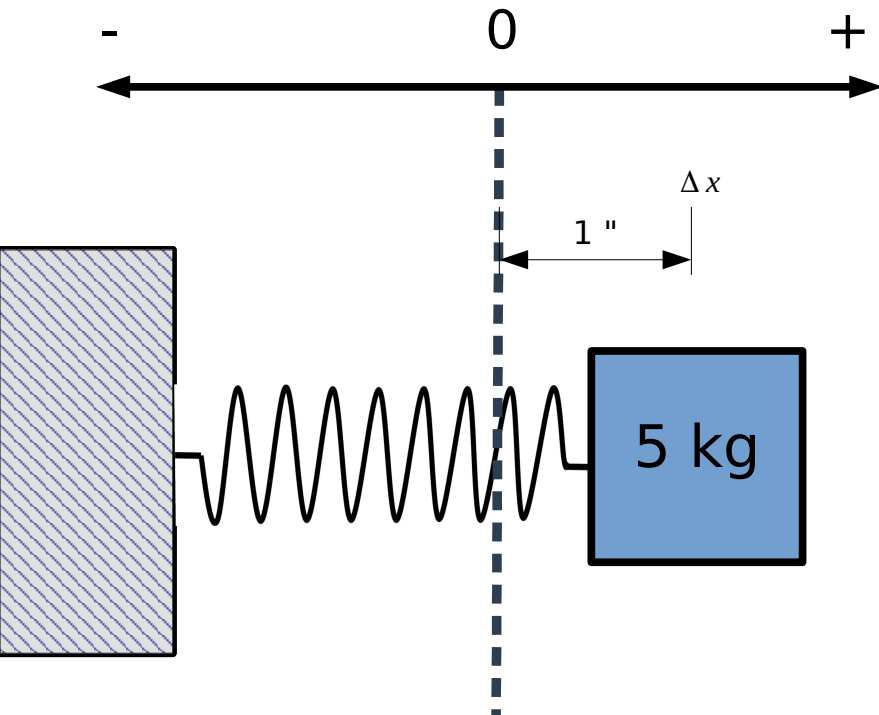
$$\triangle$$
$$A \sin(\omega t)$$

$$\triangle$$
$$A \cos(\omega t)$$

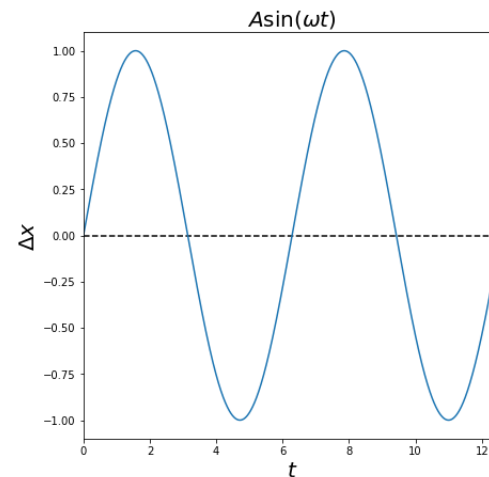
Critical Thinking Solution



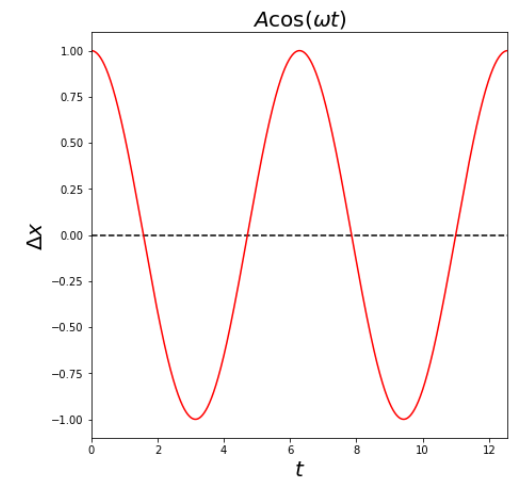
Which equation, \triangle or \blacktriangle , models the position of the 5kg mass released from rest?



\triangle
 $A \sin(\omega t)$

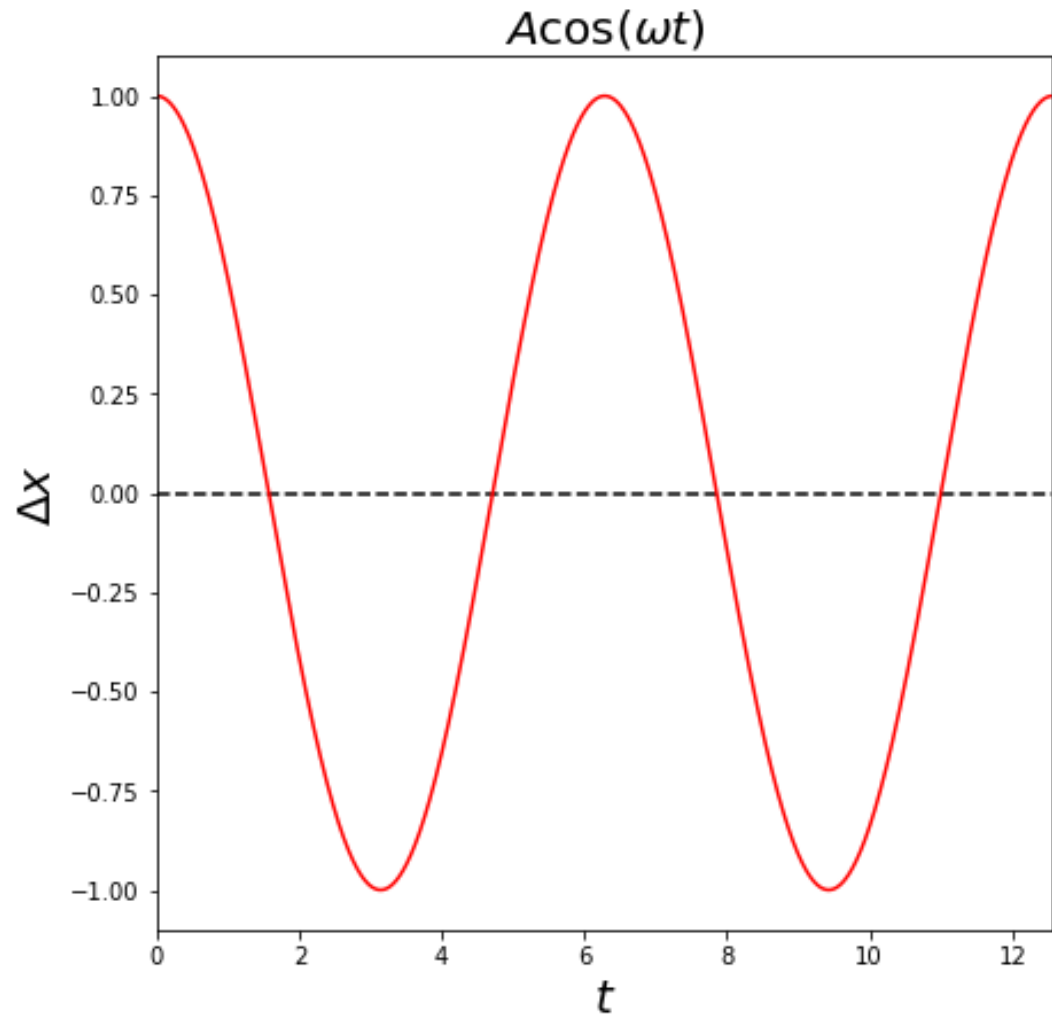


\blacktriangle
 $A \cos(\omega t)$



Discussion: how does this equation satisfy the requirements of SHM?

$$x = A \cos(\omega t)$$



Important Equations

Position

$$\star x = A \cos(\omega t)$$

Velocity

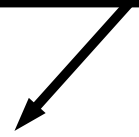
$$\star v = -A \omega \sin(\omega t)$$

Acceleration

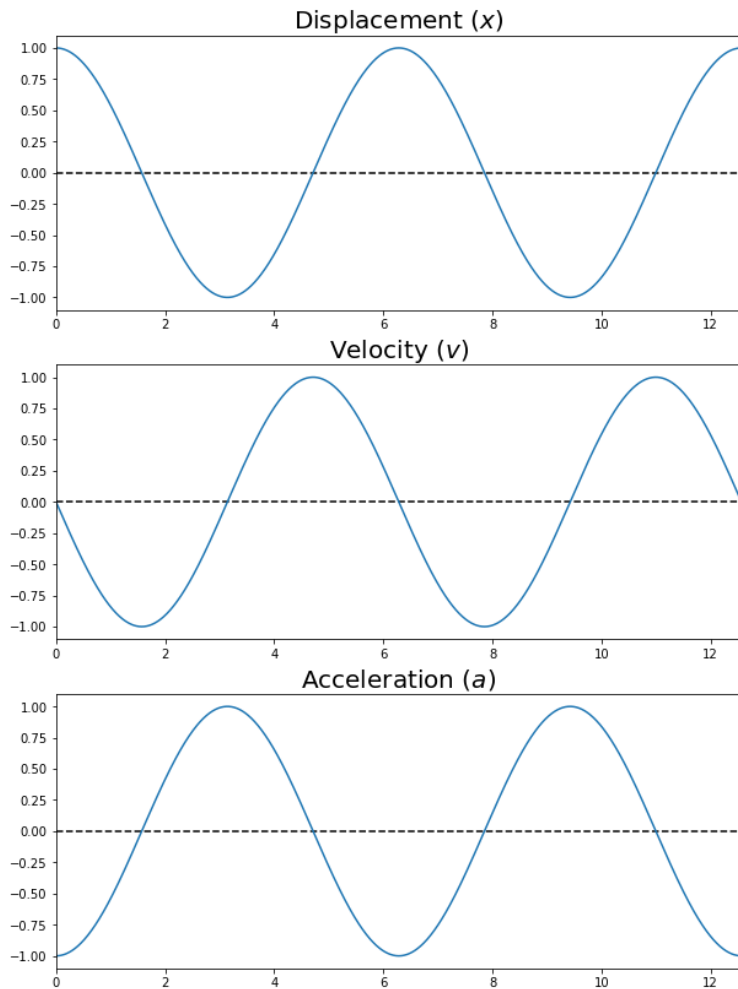
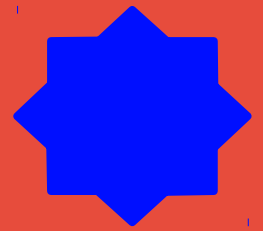
$$a = -A \omega^2 \cos(\omega t)$$



$$a = -\omega^2 (A \cos(\omega t))$$

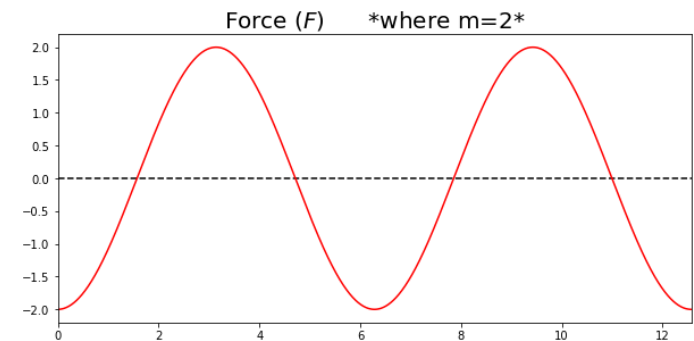
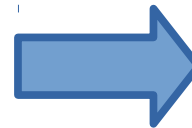

$$\star a = -\omega^2 x$$

Phases of Simple Harmonic Motion



Acceleration and Force are “in phase” because their graphs line up at any time.

$$F = m a$$



Applications – Mass on Spring

How is ω related to a spring system? Find the equation for ω .

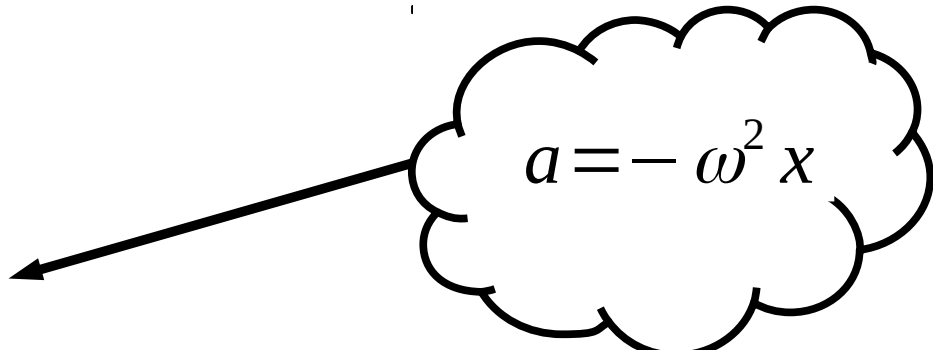
Recall

$$F_{\text{spring}} = -kx$$

Equation for force of a spring

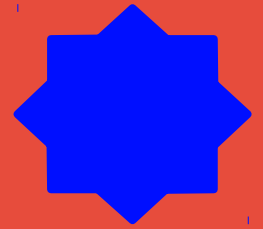
$$F = ma$$

Newton's Second Law


$$a = -\omega^2 x$$

Set $F_{\text{spring}} = F$ and solve!

Important Equations (part 2)



Position

$$x = A \cos(\omega t)$$

Velocity

$$v = -A \omega \sin(\omega t)$$

Acceleration

$$a = -\omega^2 x$$

Angular Frequency

$$\omega^2 = \frac{k}{m}$$

Frequency

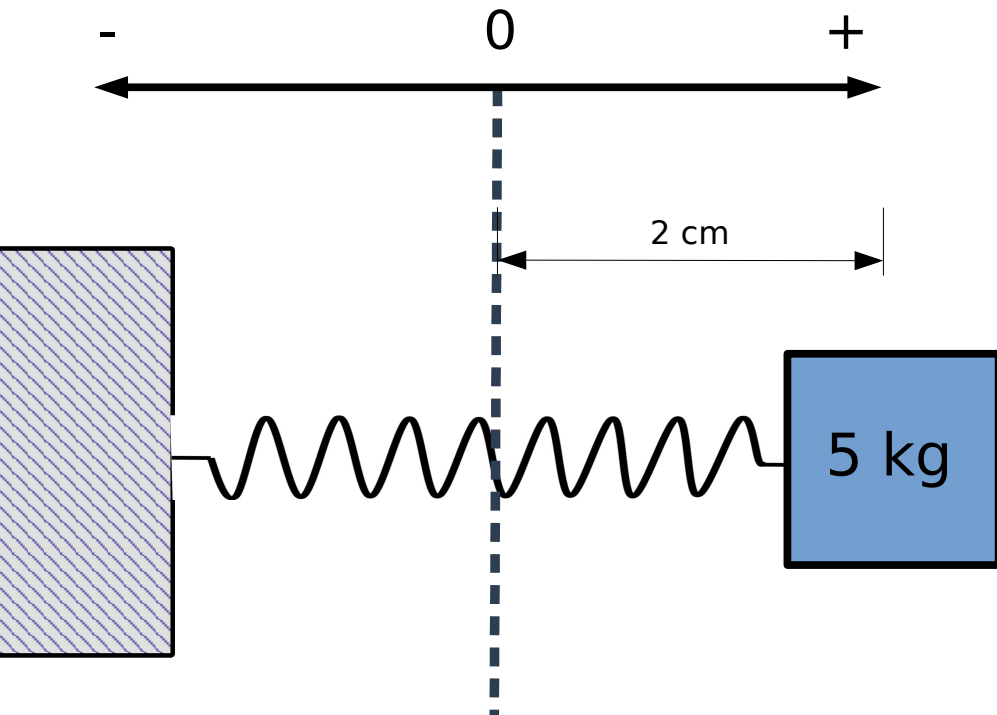
$$f = \frac{\omega}{(2\pi)}$$

Period

$$T = \frac{1}{f} = \frac{(2\pi)}{\omega}$$

Example Problem # 1

A spring with a mass of 5kg and a spring constant of 125 kg/s^2 is extended 2cm and released from rest.



- a) What is its position after π seconds?
- b) What is the period of oscillation?
- c) What is the acceleration after $\pi, 2\pi$ seconds?

Check for Understanding #1

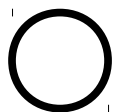
When does an object in Simple Harmonic Motion (SHM) experience maximum velocity? What is the position at this time? *** Assume A is positive ***



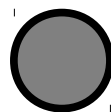
When: $\omega t = 0$
 $x = \Delta x_{max}$



When: $\omega t = 0$
 $x = 0$



When: $\omega t = \frac{\pi}{2}$
 $x = \Delta x_{max}$



When: $\omega t = \frac{\pi}{2}$
 $x = 0$

Check for Understanding #1 Solution

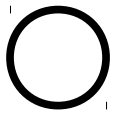
When does an object in Simple Harmonic Motion (SHM) experience maximum velocity? What is the position at this time? *** Assume A is positive ***



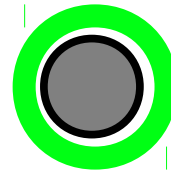
When: $\omega t = 0$
 $x = \Delta x_{max}$



When: $\omega t = 0$
 $x = 0$



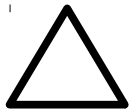
When: $\omega t = \frac{\pi}{2}$
 $x = \Delta x_{max}$



When: $\omega t = \frac{\pi}{2}$
 $x = 0$

Check for Understanding #2

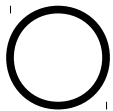
A spring in Simple Harmonic Motion (SHM) has a period of T with a mass m and spring constant k . We want a period of $2T$. Which option below provides this solution?



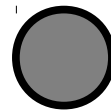
Keep k constant
Change $m \rightarrow 2m$



Change $k \rightarrow 4k$
Keep m constant



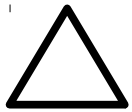
Change $k \rightarrow 0.5k$
Change $m \rightarrow 2m$



Change $k \rightarrow 2k$
Keep m constant

Check for Understanding #2 Solution

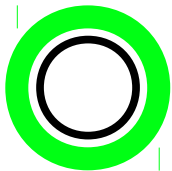
A spring in Simple Harmonic Motion (SHM) has a period of T with a mass m and spring constant k . We want a period of $2T$. Which option below provides this solution?



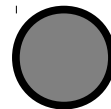
Keep k constant
Change $m \rightarrow 2m$



Change $k \rightarrow 4k$
Keep m constant



Keep k constant
Change $m \rightarrow 4m$



Change $k \rightarrow 2k$
Keep m constant

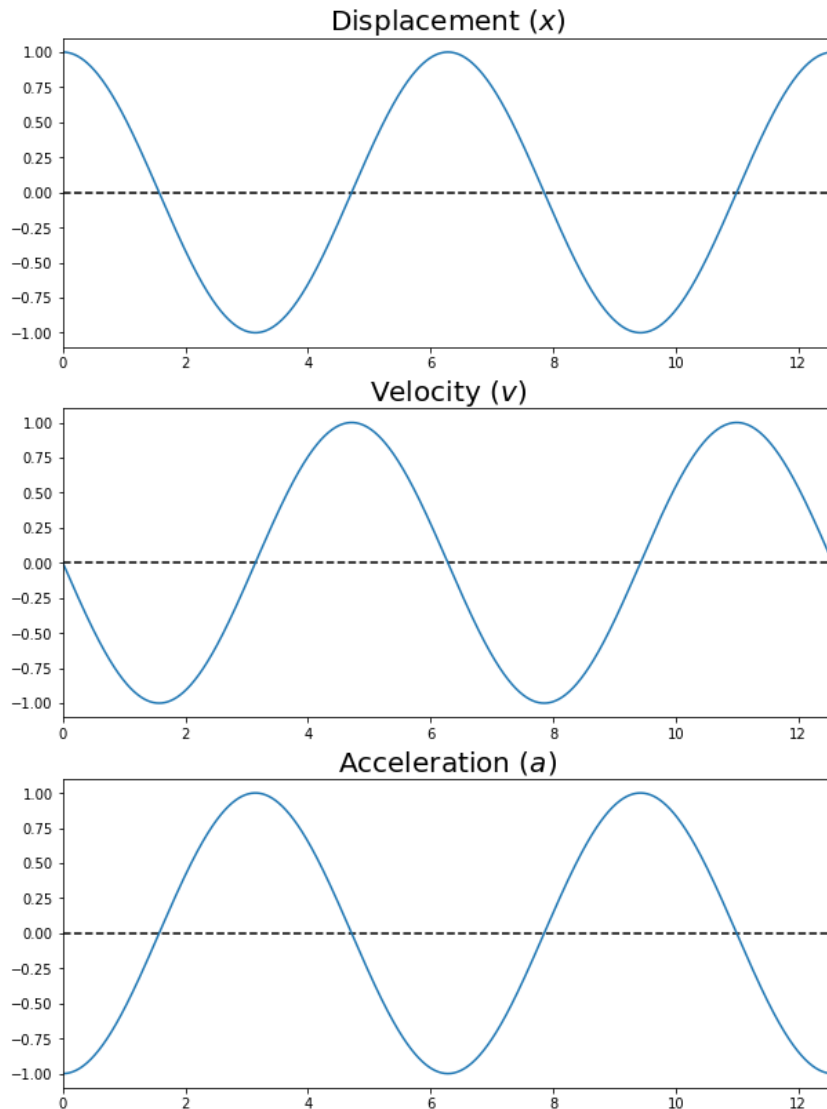
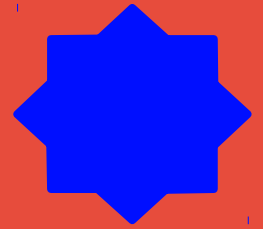
SHM Class #1 Goals **Review**

- To understand the definition of Simple Harmonic Motion and its characteristics.
- To derive the formulas surrounding Simple Harmonic Motion.
- To understand how velocity and acceleration change throughout Simple Harmonic Motion.

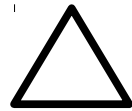
SHM Class #2 Goals

- To understand how SHM applies to pendulums.
- To derive formulas for a pendulum's SHM.
- To understand how gravity and pendulum length affect the period of a pendulum in SHM.

Review #1



When acceleration is greatest:
(positive direction)



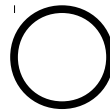
Velocity is greatest
(positive)

Displacement is zero



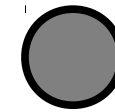
Velocity is zero

Displacement is
greatest (positive)



Velocity is greatest
(negative)

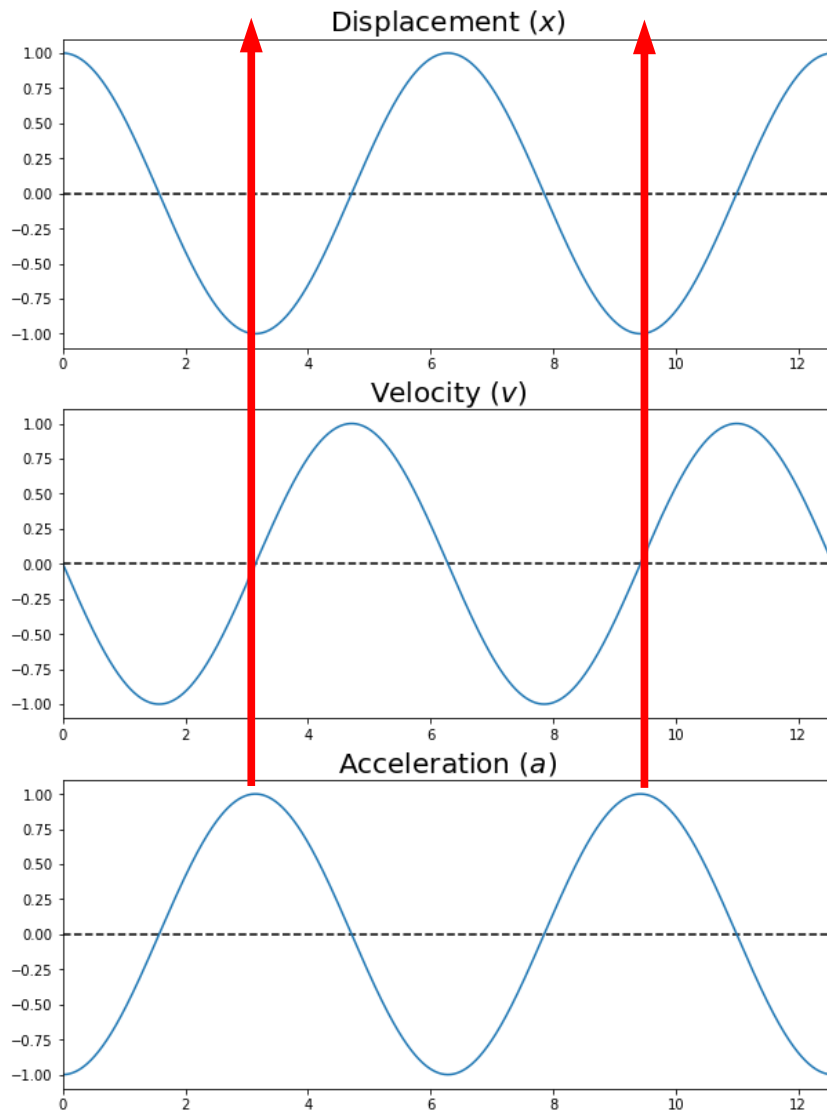
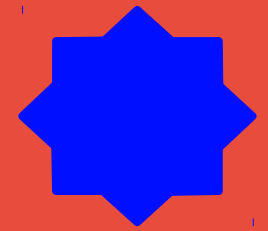
Displacement is zero



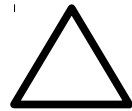
Velocity is zero

Displacement is
greatest (negative)

Review #1 Solution



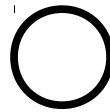
When acceleration is greatest:
(positive direction)



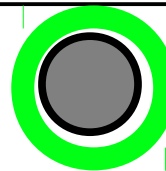
Velocity is greatest
(positive)
Displacement is zero



Velocity is zero
Displacement is greatest (positive)



Velocity is greatest
(negative)
Displacement is zero



Velocity is zero
Displacement is greatest (negative)

Guiding Question: Do Pendulums reflect SHM?

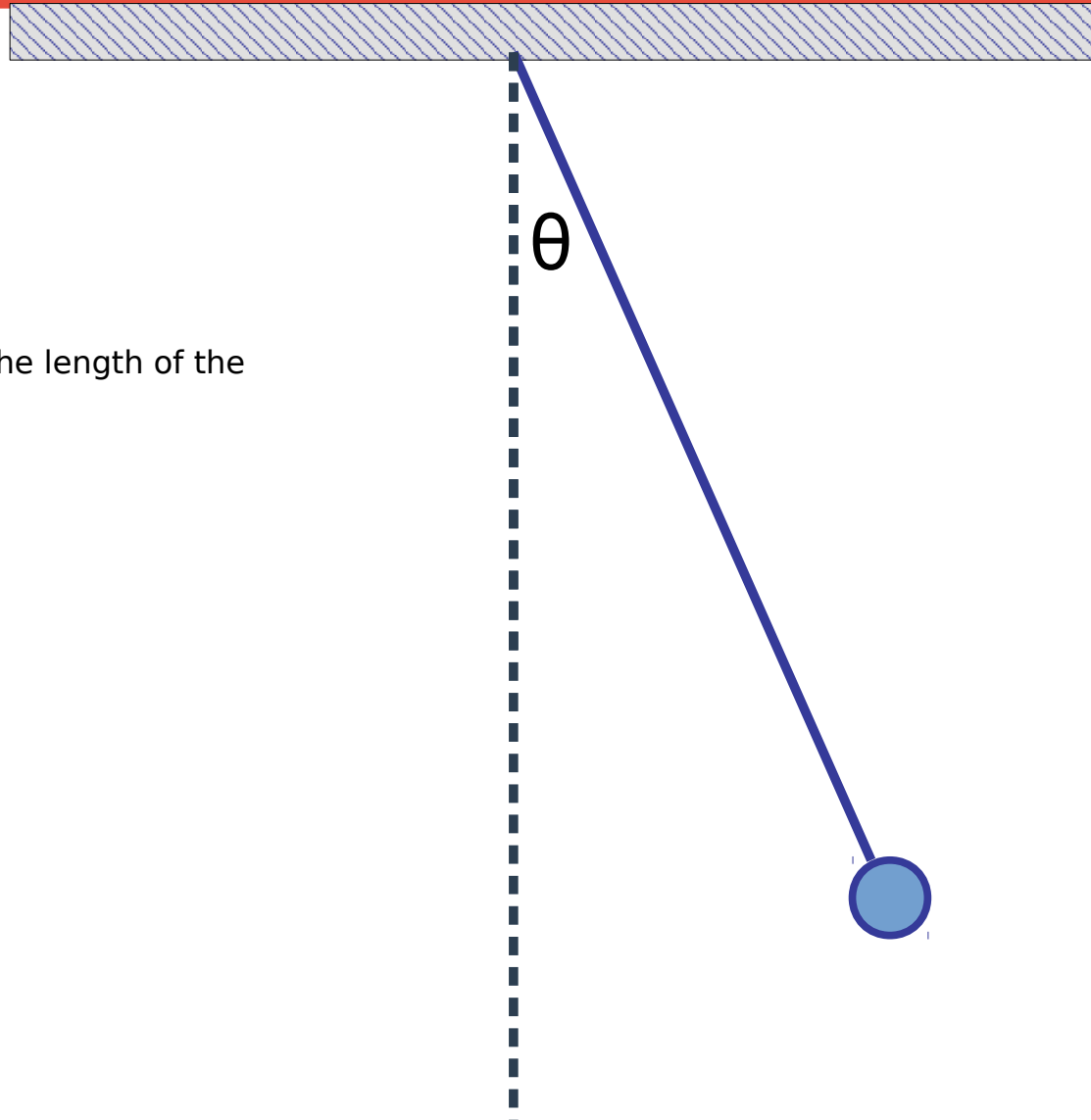


Linear Displacement

$$x = l \theta$$

Where l is the length of the pendulum

Linear Acceleration



Guiding Question: Do Pendulums reflect SHM?



Linear Displacement

$$x = l \theta$$

Where l is the length of the pendulum

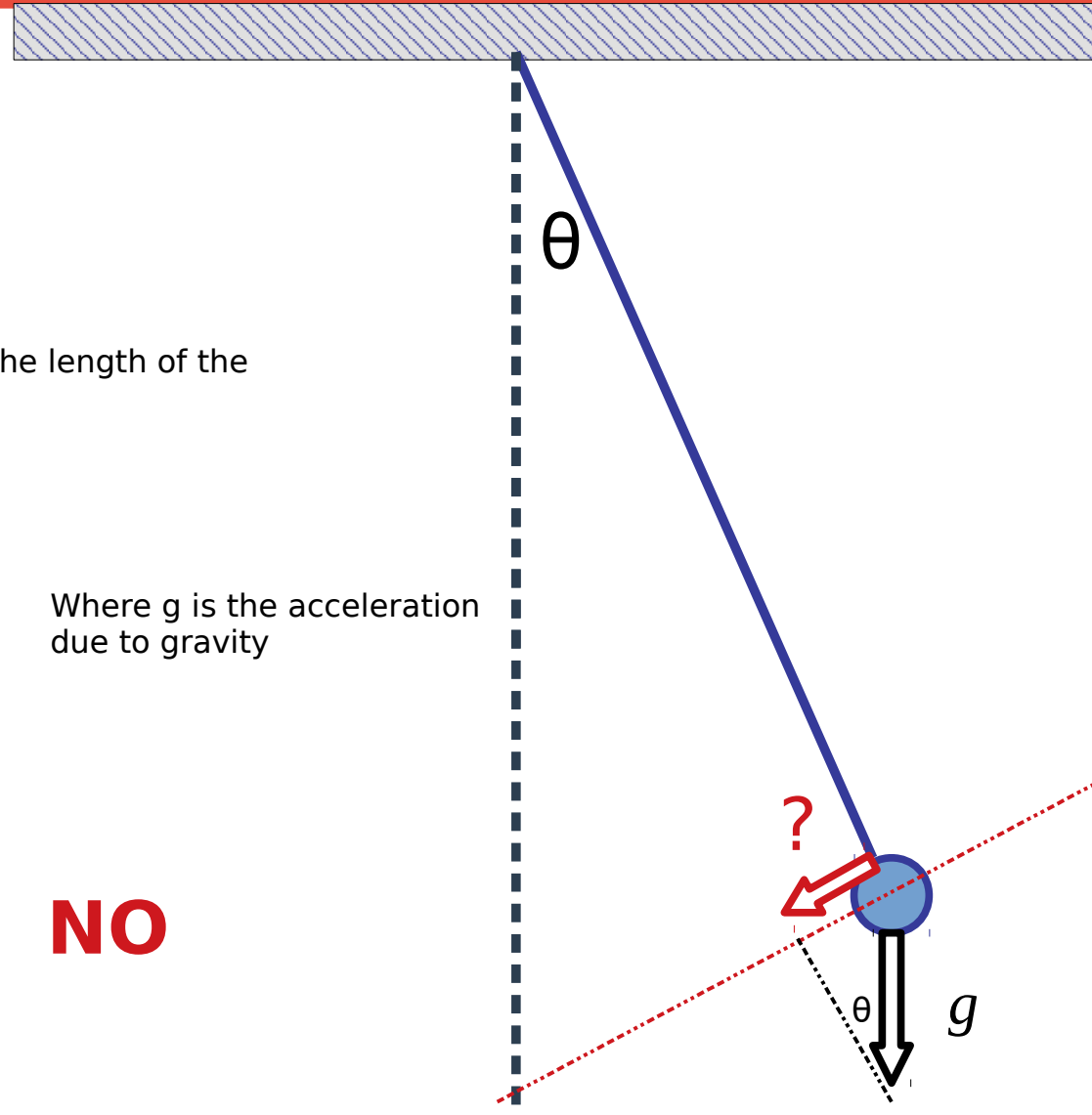
Linear Acceleration

$$a = -g \sin(\theta)$$

Where g is the acceleration due to gravity

Is displacement proportional to acceleration?

NO



Small Angle Approximation

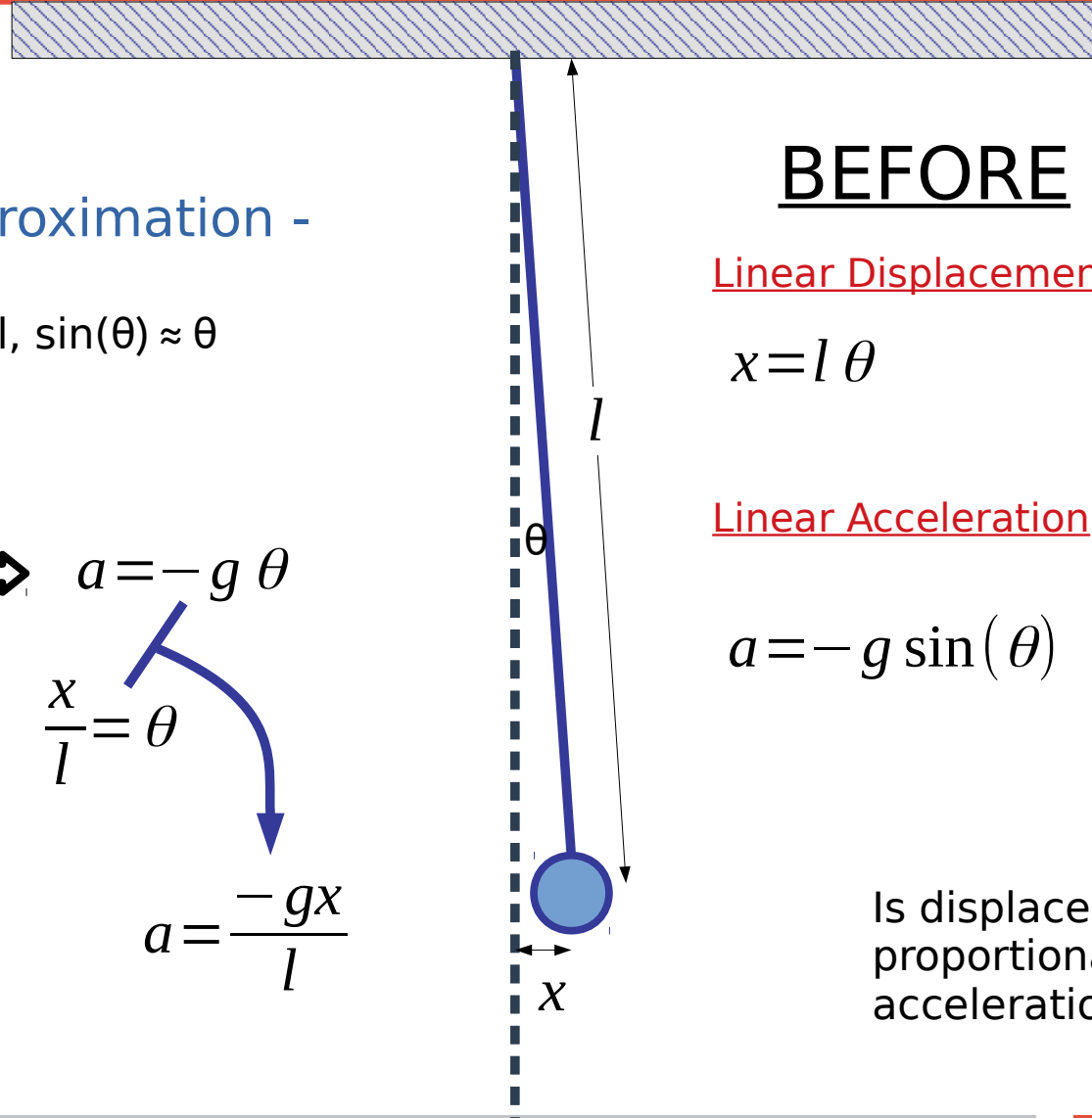
Small angle approximation -

When θ is very small, $\sin(\theta) \approx \theta$

$$a = -g \sin(\theta) \Rightarrow a = -g \theta$$

Since $x = l \theta$, $\frac{x}{l} = \theta$

$$a = \frac{-gx}{l}$$



BEFORE

Linear Displacement

$$x = l \theta$$

Linear Acceleration

$$a = -g \sin(\theta)$$

AFTER

Linear Displacement

$$x = l \theta$$

Linear Acceleration

$$a = \frac{-gx}{l}$$

Is displacement
proportional to
acceleration?

YES

Partner Derivation

With the person sitting next to you, use the equations:

$$a_{\text{pendulum}} = \frac{-gx}{l} = -\omega^2 x$$

$$f = \frac{\omega}{(2\pi)}$$

$$T = \frac{1}{f} = \frac{(2\pi)}{\omega}$$

to solve for $\rightarrow \omega$, f , and T for a pendulum.

Important Equations: Pendulum

Position

$$x_{\text{pendulum}} = l \theta$$

Acceleration

$$a_{\text{pendulum}} = \frac{-gx}{l}$$

Force

$$F_{\text{pendulum}} = \frac{-mgx}{l}$$

Angular Frequency

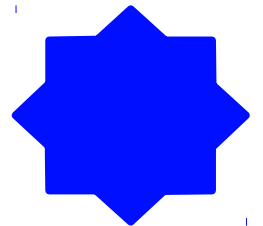
$$\omega_{\text{pendulum}} = \sqrt{\frac{g}{l}}$$

Frequency

$$f_{\text{pendulum}} = \frac{1}{2\pi} \sqrt{\frac{g}{l}}$$

Period

$$T_{\text{pendulum}} = 2\pi \sqrt{\frac{l}{g}}$$



Example Problem #1

1) You explore a planet and bring a 50cm long pendulum with you. You find that at small angles, it has a period of 1 second.

(a) What is the gravity on the planet?

(b) If you had a 4.5 m pendulum, what would its period be?

Example Problem #1 Solution

1) You explore a planet and bring a 50cm long pendulum with you. You find that at small angles, it has a period of 1 second.

(a) What is the gravity on the planet?

$$2\pi^2 \text{ m/s}^2$$

(b) If you had a 4.5 m pendulum, what would its period be?

3 seconds

Example Problem #2

2) You explore a different planet and bring the same 50cm long pendulum with you. Each day, it completes 10,800 complete oscillations.

(a) What is the gravity on the new planet?

Example Problem #2

2) You explore a different planet and bring the same 50cm long pendulum with you. Each day, it completes 10,800 complete oscillations.

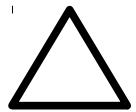
(a) What is the gravity on the new planet?

$$128 \pi^2 \text{ m/s}^2$$

Check for Understanding #1

A very slow clock with a small-angle pendulum with a period of 28π seconds is heard at sea level. What is its length?

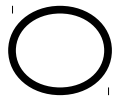
***Note** $g_{\text{sealevel}} = 9.8 \text{ m/s}^2$



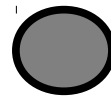
$1,920.8 \text{ m}$



192.1 m



137.2 m

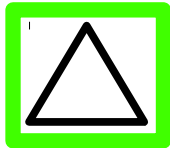


20 m

Check for Understanding Solution #1

A very slow clock with a small-angle pendulum with a period of 28π seconds is heard at sea level. What is its length?

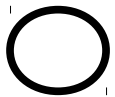
***Note** $g_{\text{sealevel}} = 9.8 \text{ m/s}^2$



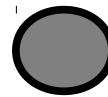
$1,920.8 \text{ m}$



192.1 m



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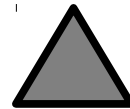
20 m

Check for Understanding #2

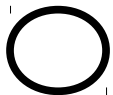
You want to double the period of a small angle pendulum. What do you do to frequency?



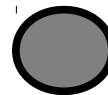
Multiply frequency by 2



Multiply frequency by 4



Divide frequency by 2



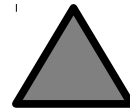
Divide frequency by 4

Check for Understanding Solution #2

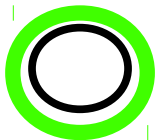
You want to double the period of a small angle pendulum. What do you do to frequency?



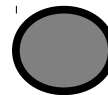
Multiply frequency by 2



Multiply frequency by 4



Divide frequency by 2



Divide frequency by 4

SHM Class #2 Goals **Review**

- To understand how SHM applies to pendulums.
- To derive formulas for a pendulum's SHM.
- To understand how gravity and pendulum length affect the period of a pendulum in SHM.

Poll



This week I will teach one afternoon review session.
Which day do you want?

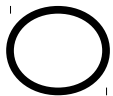
*if you do not want to go, do not participate



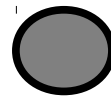
Monday (today)



Tuesday



Wednesday



Thursday

SHM Class #3 Goals

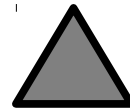
- To understand how kinetic and potential energy apply to SHM.
- To understand why kinetic and potential energy are related.
- To derive and apply a new formula for SHM velocity.

Review #1

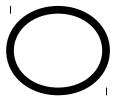
Do pendulums reflect/show Simple Harmonic Motion?



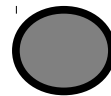
Yes, in all cases.



Yes, but only at small angles



No, never.



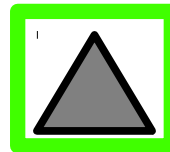
Yes, but only at large angles

Review #1 Solution

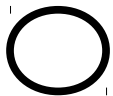
Do pendulums reflect/show Simple Harmonic Motion?



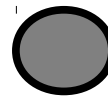
Yes, in all cases.



Yes, but only at small angles



No, never.



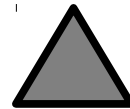
Yes, but only at large angles

Review #2

You want to double the period of a small angle pendulum. What do you do to frequency?



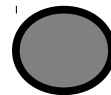
Multiply frequency by 2



Multiply frequency by 4



Divide frequency by 2



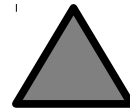
Divide frequency by 4

Review Solution #2

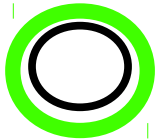
You want to double the period of a small angle pendulum. What do you do to frequency?



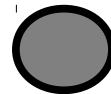
Multiply frequency by 2



Multiply frequency by 4



Divide frequency by 2



Divide frequency by 4

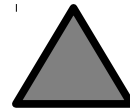
Review #3

A 5kg weight extends a spring 50cm. If you displace it further and release, what is the angular frequency, ω ?

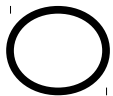
Assume $a_{\text{gravity}} = 10 \text{ m/s}^2$



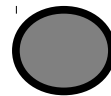
$$\omega = 20 \frac{\text{radian}}{\text{sec}}$$



$$\omega = 2\sqrt{5} \frac{\text{radian}}{\text{sec}}$$



$$\omega = 5 \frac{\text{radian}}{\text{sec}}$$



$$\omega = \sqrt{5} \frac{\text{radian}}{\text{sec}}$$

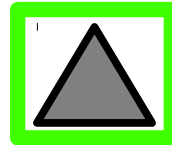
Review #3 Solution

A 5kg weight extends a spring 50cm. If you displace it further and release, what is the angular frequency, ω ?

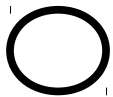
Assume $a_{\text{gravity}} = 10 \text{ m/s}^2$



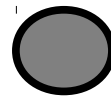
$$\omega = 20 \frac{\text{radian}}{\text{sec}}$$



$$\omega = 2\sqrt{5} \frac{\text{radian}}{\text{sec}}$$



$$\omega = 5 \frac{\text{radian}}{\text{sec}}$$



$$\omega = \sqrt{5} \frac{\text{radian}}{\text{sec}}$$

Energies in Simple Harmonic Motion

Kinetic Energy (KE):

Energy of movement at a given time.

Potential Energy (PE):

Energy stored in a system at a given time.

Total Energy (TE):

Total energy of a system at a given time.

Kinetic Energy of SHM



Recall

$$KE = \frac{1}{2} m v^2, \quad v = -A \omega \sin(\omega t), \quad \omega^2 = \frac{k}{m}$$

Substitution

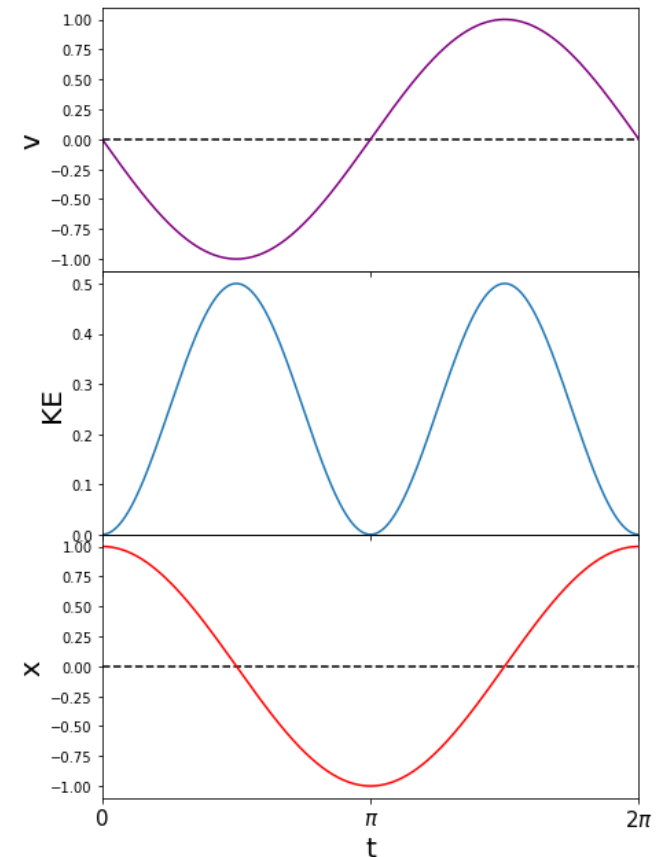
$$KE = \frac{1}{2} m (-A \omega \sin(\omega t))^2$$

$$KE = \frac{1}{2} m A^2 \omega^2 \sin^2(\omega t)$$

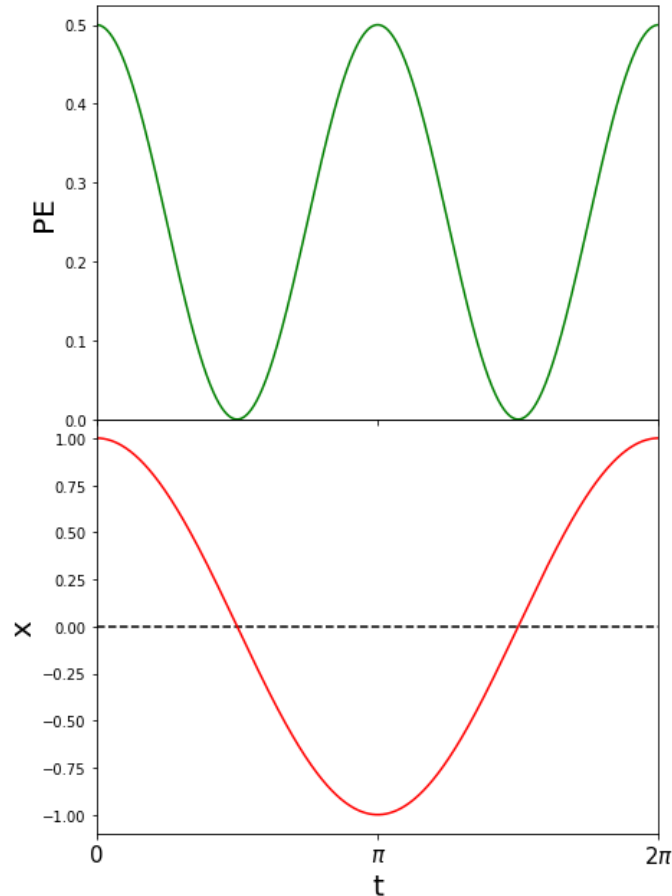
$$KE = \frac{1}{2} \cancel{m} A^2 \left(\frac{k}{\cancel{m}} \right) \sin^2(\omega t)$$

Substitution

$$\Rightarrow KE = \frac{1}{2} k A^2 \sin^2(\omega t)$$



Potential Energy of SHM



Recall

$$PE_{spring} = \frac{1}{2} k x^2, \quad x = A \cos(\omega t)$$

Substitution

$$PE_{spring} = \frac{1}{2} k (A \cos(\omega t))^2$$

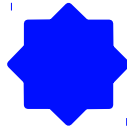
$$\Rightarrow PE_{spring} = \frac{1}{2} k A^2 \cos^2(\omega t)$$

Total Energy of SHM

$$KE = \frac{1}{2} k A^2 \sin^2(\omega t) \quad PE_{\text{spring}} = \frac{1}{2} k A^2 \cos^2(\omega t)$$

$$TE = KE + PE = \frac{1}{2} k A^2 (\underbrace{\sin^2(\omega t) + \cos^2(\omega t)}_1)$$

$$TE = \frac{1}{2} k A^2$$

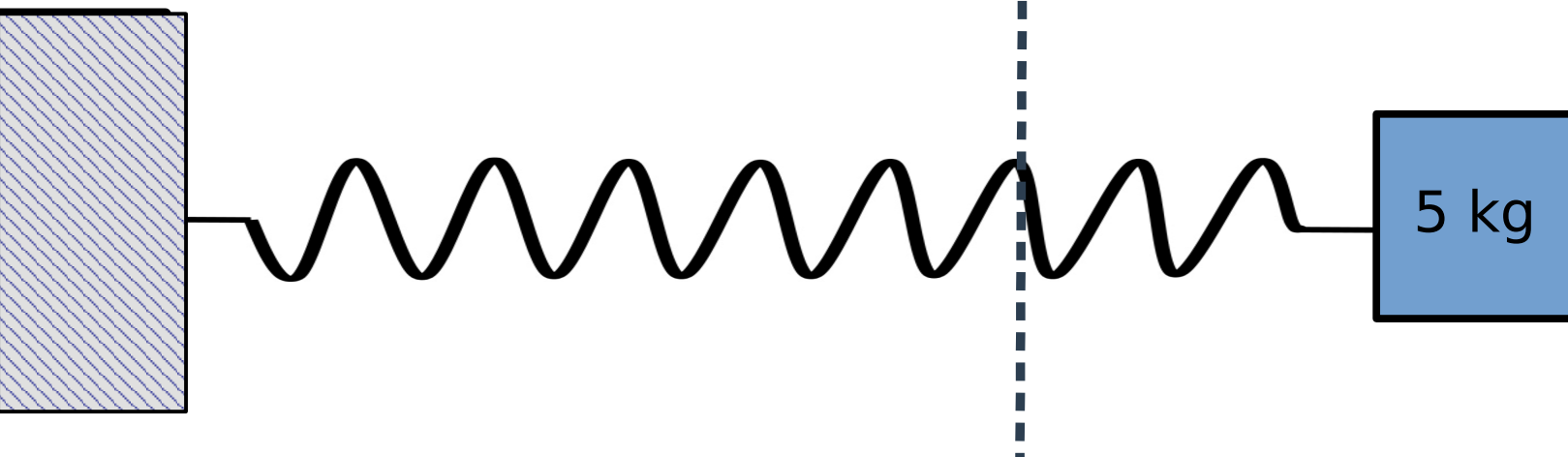
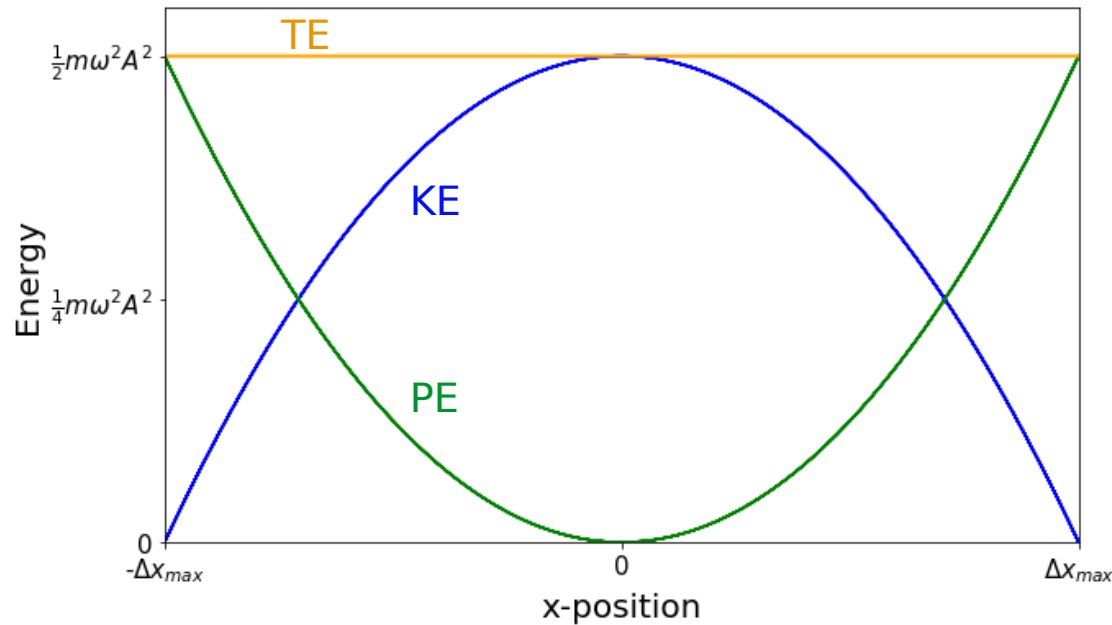


also written as

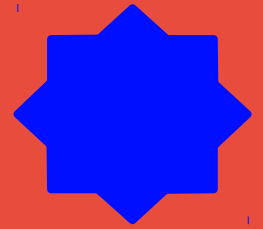
$$TE = \frac{1}{2} m \omega^2 A^2$$

because $\omega^2 = \frac{k}{m}$

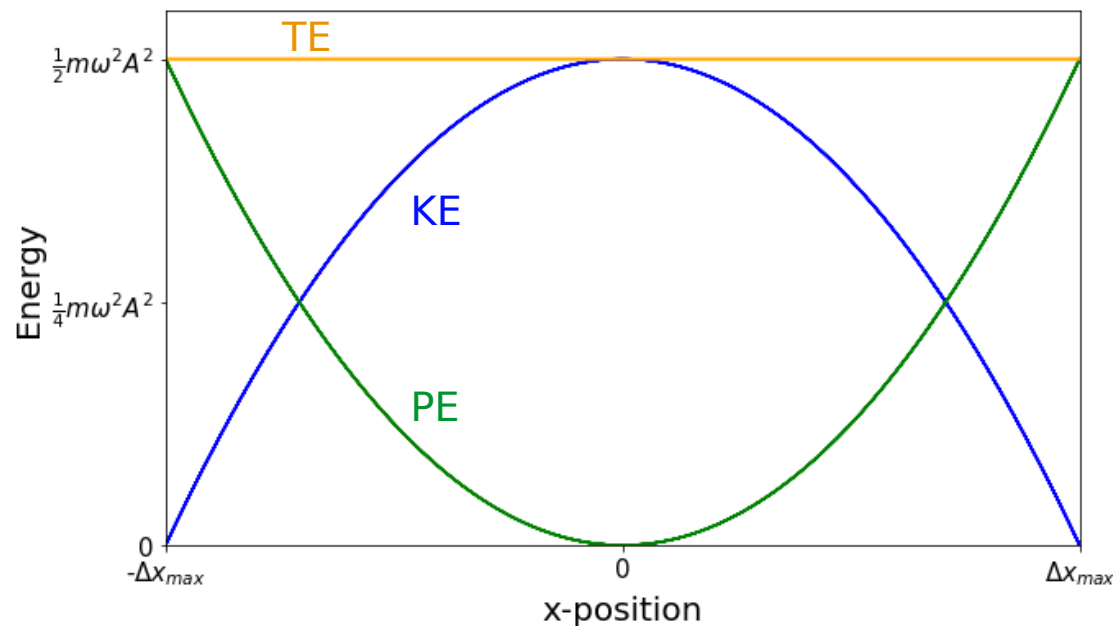
Conservation of Energy



Continual Interchange of Energy



Both PE and KE are dependent on displacement (see below). Displacement is proportional to acceleration. Therefore displacement is also proportional to the restoring force. The restoring force is responsible for the continual interchange of energy between PE and KE.



Another way to write Velocity

Recall: $TE = \frac{1}{2} k A^2$, $PE = \frac{1}{2} k x^2$, $KE = \frac{1}{2} m v^2$

$$TE = KE + PE$$

$$KE = TE - PE$$

$$KE = \frac{1}{2} k A^2 - \frac{1}{2} k x^2$$

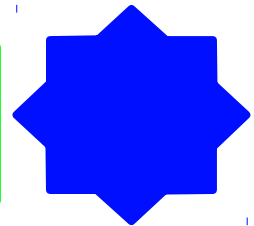
$$KE = \frac{1}{2} k (A^2 - x^2)$$

$$KE = \frac{1}{2} m \omega^2 (A^2 - x^2)$$

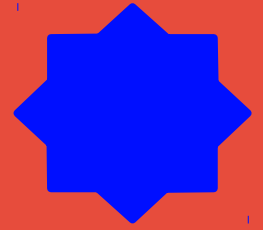
$$\frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 (A^2 - x^2)$$

$$v^2 = \omega^2 (A^2 - x^2)$$

$$v = \omega \sqrt{(A^2 - x^2)}$$



Important Equations



Kinetic Energy

$$KE = \frac{1}{2} m v^2$$

Total Energy

$$TE = \frac{1}{2} k A^2$$

$$TE = \frac{1}{2} m \omega^2 A^2$$

Potential Energy

$$PE = \frac{1}{2} k x^2$$

Velocity

$$v = \omega \sqrt{(A^2 - x^2)}$$

Example Problem #1

1) A 75g weight is attached to a spring with spring constant, $k=7.5$. It is extended 50 cm and then released.

(a) What is the velocity 40 cm from equilibrium?

(b) What is the velocity 30 cm from equilibrium?

(c) What is the maximum velocity?

Example Problem #1 solutions

1) A 75g weight is attached to a spring with spring constant, $k=7.5$. It is extended 50 cm and then released.

(a) What is the velocity 40 cm from equilibrium?

3 m/s

(b) What is the velocity 30 cm from equilibrium?

4 m/s

(c) What is the maximum velocity?

5 m/s

Example Problem #2

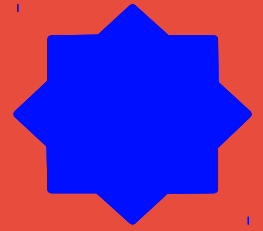


1) A very long spring in SHM has a velocity of 8 m/s at a distance of 6 m away from equilibrium. It has a velocity of 6 m/s at a distance of -8 m from equilibrium.

(a) What is the amplitude, A ?

(b) What is the angular frequency, ω ?

Example Problem #2 solutions



1) A very long spring in SHM has a velocity of 8 m/s at a distance of 6 m away from equilibrium. It has a velocity of 6 m/s at a distance of -8 m from equilibrium.

(a) What is the amplitude, A ?

10 m

(b) What is the angular frequency, ω ?

$1\frac{\text{radian}}{\text{sec}}$

Example Problem #3

1) A mass of 2 kg is attached to a spring with spring constant $k=36$. It is then set to oscillate at an amplitude of 1 m. At an unknown time, t , you observe a velocity of 2 m/s. At this time:

(a) What is the kinetic energy, KE?

(b) What is total energy, TE?

(c) What is the Potential Energy, PE?

Example Problem #3 solutions

1) A mass of 2 kg is attached to a spring with spring constant $k=36$. It is then set to oscillate at an amplitude of 1 m. At an unknown time, t , you observe a velocity of 2 m/s. At this time:

(a) What is the kinetic energy, KE?

4 J

(b) What is total energy, TE?

18 J

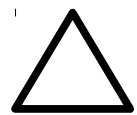
(c) What is the Potential Energy, PE?

14 J

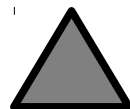
Check for Understanding #1

In the last problem we found: $KE=4$, $PE=14$, $TE=18$

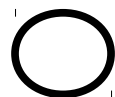
What point in the oscillation is this object at this time?



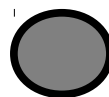
x is close to its negative maximum amplitude



x is close to its positive maximum amplitude



Either option above

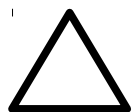


x is close to equilibrium

Check for Understanding #1 solution

In the last problem we found: $KE=4$, $PE=14$, $TE=18$

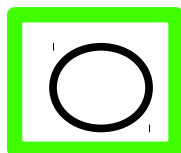
What point in the oscillation is this object at this time?



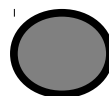
x is close to its negative maximum amplitude



x is close to its positive maximum amplitude



Either option above



x is close to equilibrium

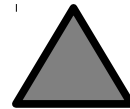
Check for Understanding #1

What is an objects PE when $v = v_{max}$? What is the maximum velocity?



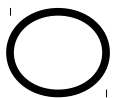
$$PE = 0$$

$$v_{max} = \omega A$$



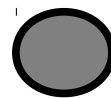
$$PE = \frac{1}{2} m \omega^2 A^2$$

$$v_{max} = \omega A$$



$$PE = 0$$

$$v_{max} = \frac{1}{2} \omega A$$

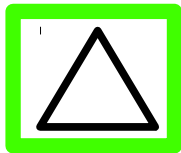


$$PE = \frac{1}{2} m \omega^2 A^2$$

$$v_{max} = \frac{1}{2} \omega A$$

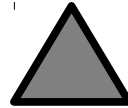
Check for Understanding #1 Solution

What is an objects PE at $v = v_{max}$. What is the maximum velocity?



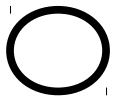
$$PE = 0$$

$$v_{max} = \omega A$$



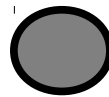
$$PE = \frac{1}{2} m \omega^2 A^2$$

$$v_{max} = \omega A$$



$$PE = 0$$

$$v_{max} = \frac{1}{2} \omega A$$



$$PE = \frac{1}{2} m \omega^2 A^2$$

$$v_{max} = \frac{1}{2} \omega A$$

SHM Class #3 Goals Review

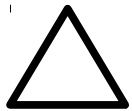
- To understand how kinetic and potential energy apply to SHM.
- To understand why kinetic and potential energy are related.
- To derive and apply a new formula for SHM velocity.

Poll

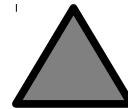


What style of evening/night session do you want?

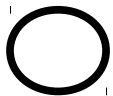
*if you do not want to go, do not participate



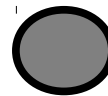
Review questions that I
solve and you take notes



Group work



Review questions that you
work on individually



Review all PowerPoint slides

Finishing up Energies in SHM

Here we go!

Review Problem #1

1) A mass of 2 kg is attached to a spring with spring constant $k=36$. It is then set to oscillate at an amplitude of 1 m. At an unknown time, t , you observe a velocity of 2 m/s. At this time:

(a) What is the kinetic energy, KE?

(b) What is total energy, TE?

(c) What is the Potential Energy, PE?

Review Problem #1 solutions

1) A mass of 2 kg is attached to a spring with spring constant $k=36$. It is then set to oscillate at an amplitude of 1 m. At an unknown time, t , you observe a velocity of 2 m/s. At this time:

(a) What is the kinetic energy, KE?

$4 J$

(b) What is total energy, TE?

$18 J$

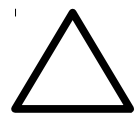
(c) What is the Potential Energy, PE?

$14 J$

Check for Understanding #1

In the last problem we found: $KE=4$, $PE=14$, $TE=18$

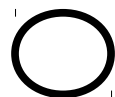
What point in the oscillation is this object at this time?



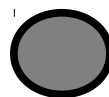
x is close to its negative maximum amplitude



x is close to its positive maximum amplitude



Either option above

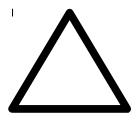


x is close to equilibrium

Check for Understanding #1 solution

In the last problem we found: $KE=4$, $PE=14$, $TE=18$

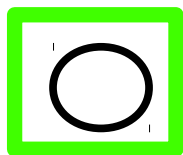
What point in the oscillation is this object at this time?



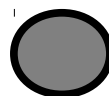
x is close to its negative maximum amplitude



x is close to its positive maximum amplitude



Either option above



x is close to equilibrium

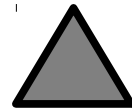
Check for Understanding #2

What is an objects PE when $v = v_{max}$? What is the maximum velocity?



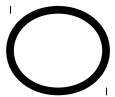
$$PE = 0$$

$$v_{max} = \omega A$$



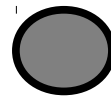
$$PE = \frac{1}{2} m \omega^2 A^2$$

$$v_{max} = \omega A$$



$$PE = 0$$

$$v_{max} = \frac{1}{2} \omega A$$

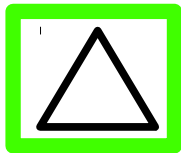


$$PE = \frac{1}{2} m \omega^2 A^2$$

$$v_{max} = \frac{1}{2} \omega A$$

Check for Understanding #2 Solution

What is an objects PE at $v = v_{max}$. What is the maximum velocity?



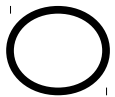
$$PE = 0$$

$$v_{max} = \omega A$$



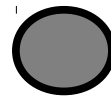
$$PE = \frac{1}{2} m \omega^2 A^2$$

$$v_{max} = \omega A$$



$$PE = 0$$

$$v_{max} = \frac{1}{2} \omega A$$

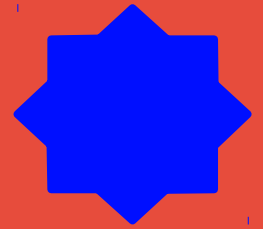


$$PE = \frac{1}{2} m \omega^2 A^2$$

$$v_{max} = \frac{1}{2} \omega A$$

SHM Class #4 Goals

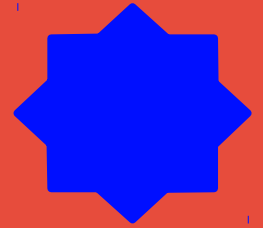
- To know how to solve all of the previous NECTA problems about SHM.
- To be confident about knowing SHM.



■ Define Simple Harmonic Motion in terms of amplitude period and frequency as applied to SHM.

SHM is a periodic motion in which acceleration is directly proportional to displacement from equilibrium and in a direction towards that fixed point.

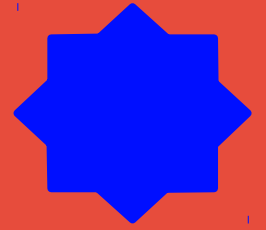
- **Amplitude** is the maximum displacement from the equilibrium position.
- **Period** is the time that it takes to make one complete oscillation.
- **Frequency** is the number of complete oscillations made per second.



■ **Explain what is responsible for the continual interchange of Potential Energy (PE) and Kinetic Energy (KE) in a mechanical oscillation. At what points in SHM is the magnitude of the acceleration greatest? Where is it least?**

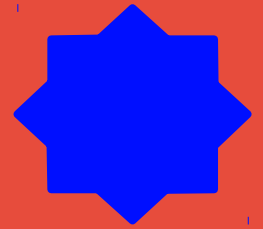
Both PE and KE are dependent on displacement. In a SHM system, displacement is proportional to acceleration. Acceleration is proportional to the restoring force. Therefore displacement is proportional to the restoring force. PE and KE are therefore dependent on the restoring force, which is responsible for their continual interchange of energy.

- The magnitude of the acceleration is greatest when the magnitude of the displacement is greatest and velocity is equal to zero.
- The magnitude of the acceleration is least when the magnitude of the displacement is 0 and the magnitude of velocity is at a maximum.



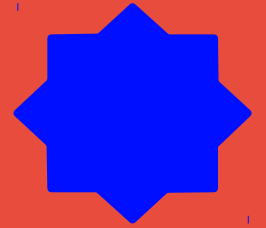
■ A small mass of 325g is attached to one end of a helical spring and produces an extension of 35mm. The mass is now set into oscillation of amplitude 50cm. Calculate the:

- i. Period of oscillation
- ii. Velocity as it passes the equilibrium point
- iii. Maximum kinetic energy of the system
- iv. Potential energy when the mass is 46cm from equilibrium



■ A small mass of 325g is attached to one end of a helical spring and produces an extension of 35mm. The mass is now set into oscillation of amplitude 50cm. Calculate the:

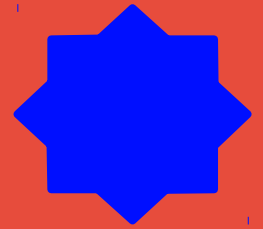
- i. Period of oscillation $\frac{\pi}{\sqrt{70}} \text{ sec}$
- ii. Velocity as it passes the equilibrium point $\sqrt{70} \text{ m/s}$
- iii. Maximum kinetic energy of the system 11.375 J
- iv. Potential energy when the mass is 48cm from equilibrium 10.4832 J



▪ What is the criterion for an object to execute simple harmonic motion (SHM)?

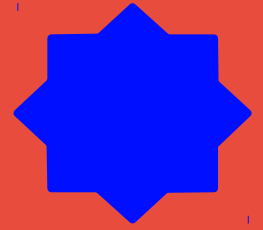
An object in SHM must satisfy the following criteria:

- 1) The motion is periodic/cyclical. It systematically repeats in periods of time.
- 2) The acceleration is proportional to the displacement.
- 3) A restoring force is present that acts in a direction towards equilibrium.

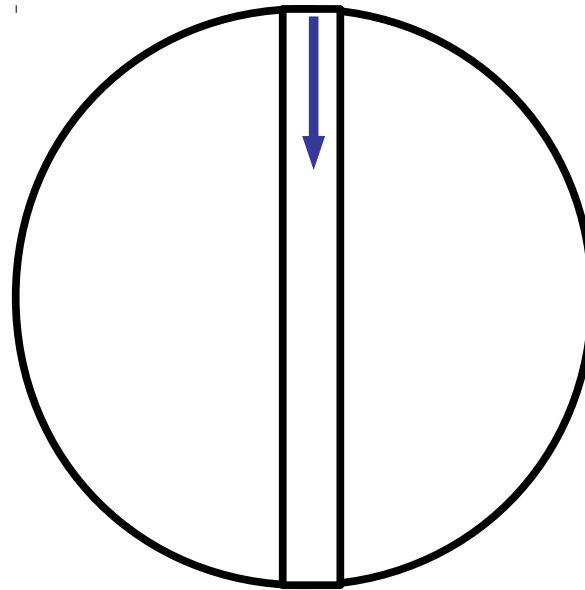


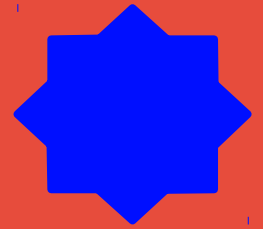
- **A body executing simple harmonic motion (SHM) is associated with the restoring force acting on it, its displacement, its velocity, and its acceleration. Which of these four physical quantities are in phase?**

The acceleration and the restoring force are in phase. This is because force is proportional to acceleration by a value of mass. Since mass is constant, acceleration and the restoring force are in phase.



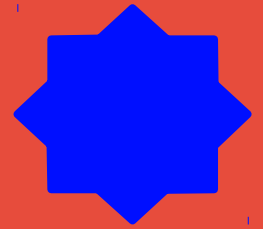
- Suppose a tunnel is dug through the earth from one side to the other along a diameter. Show that the motion of a particle dropped into the tunnel is simple harmonic. You may assume the density, ρ , of earth to be uniform.





▪ A particle moving with SHM has velocities of 9 cm s^{-1} and 12 cm s^{-1} at distances of 4 cm and 3 cm respectively from its equilibrium position. Find the:

- i. Amplitude of Oscillation
- ii. Period
- iii. Velocity of the particle as it passes equilibrium position

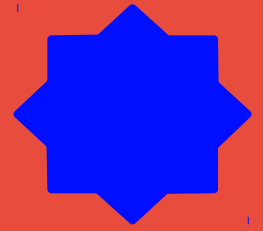


- A particle moving with SHM has velocities of 9 cm s^{-1} and 12 cm s^{-1} at distances of 4 cm and 3 cm respectively from its equilibrium position. Find the:

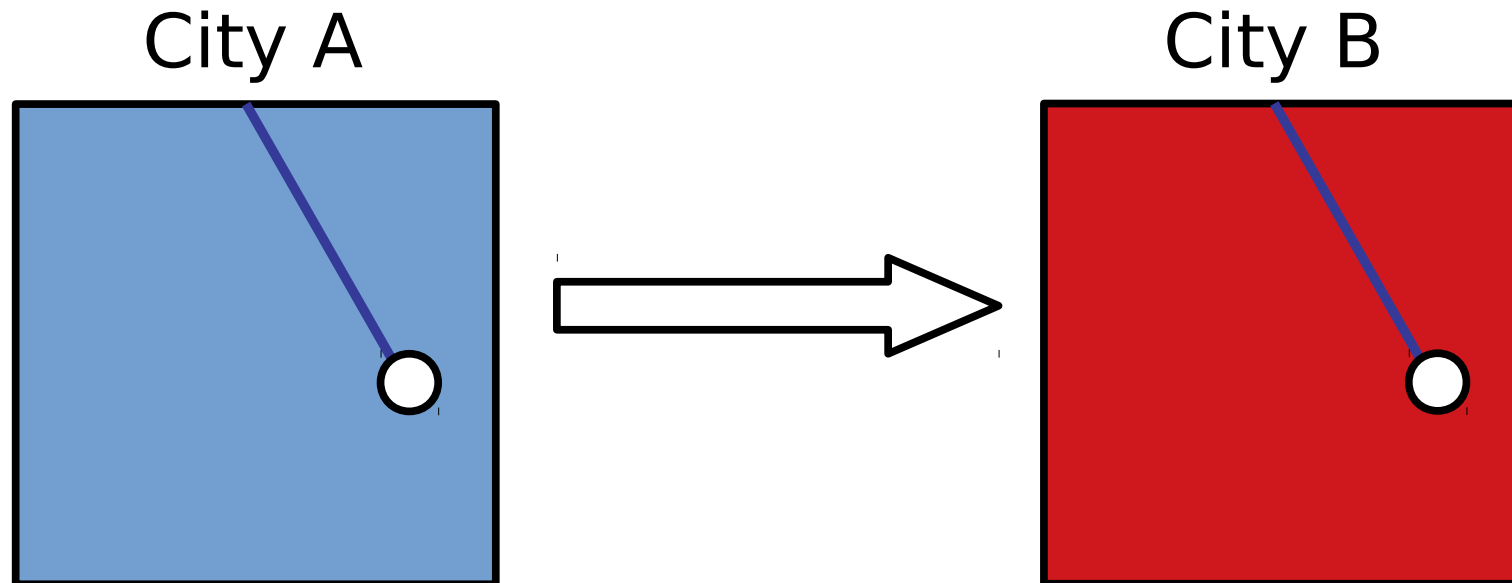
i. Amplitude of Oscillation 0.05 m

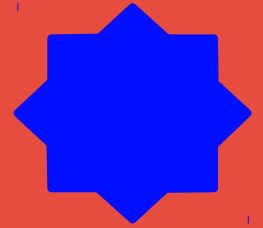
ii. Period $\frac{2\pi}{3} \text{ sec}$

iii. Velocity of the particle as it passes equilibrium position 0.15 m/s

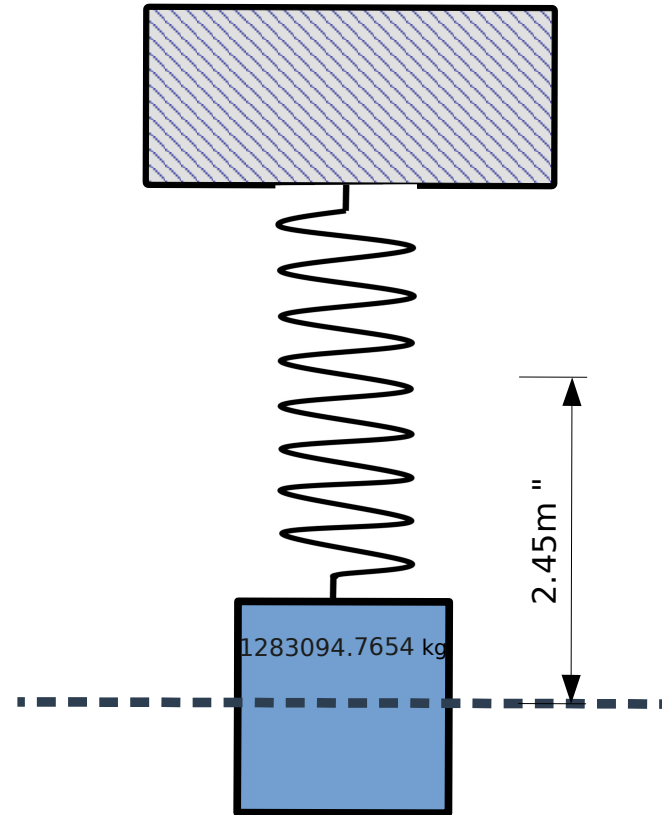


- A simple pendulum has a period of 1 second in city A, where the acceleration due to gravity is 9.45 ms^{-1} . It is taken to city B where it is found to gain 2 minutes per day. Calculate the value of the acceleration due to gravity in city B.





- A light spiral spring is loaded with a mass of 1283094.7654 kg and it extends by 2.45 m . Calculate the period of small vertical oscillation.



SHM Class #4 Goals **Reviewed**

- To know how to solve all of the previous NECTA problems about SHM.
- To be confident about knowing SHM.

Done with SHM!!!!

... now on to surface tension