Computational Systems Biology 11/10/19

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Stoechiometrics

$$S + E \underset{R_2}{\overset{R_1}{\rightleftharpoons}} C \xrightarrow{R_3} P + E$$

$$\Rightarrow \frac{d}{dt}\begin{pmatrix} S \\ E \\ C \\ P \end{pmatrix} = \begin{array}{c} \begin{array}{ccc} R_1 & R_2 & R_3 \end{array} \\ \begin{bmatrix} -1 & 1 & 0 \\ -1 & 1 & 1 \\ 1 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \\ N \end{array} \begin{array}{c} \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} \\ R \end{array}$$

ODEs

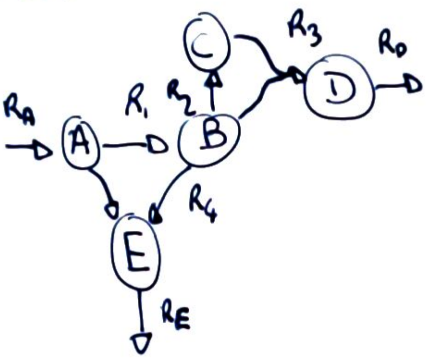
$$\frac{dS}{dt} = -R_1 + R_2$$

$$\frac{dE}{dt} = -R_1 + R_2 + R_3$$

$$\frac{dC}{dt} = R_1 - R_2 - R_3$$

$$\frac{dP}{dt} = R_3$$

Exercise 1



1) Derive stoechiometric matrix

$$N = \begin{array}{c} \begin{array}{ccccccc} R_A & R_D & R_E & R_1 & R_2 & R_3 & R_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & +1 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array}$$

Background - theory concepts

1) A set of vectors $\{v_i\}_{i=1}^{n}$ is called linearly independent iff

$$\sum_{i=1}^{n} a_i v_i = 0 \iff a_i = 0, \ \forall i \in 1, \ldots, n.$$

ex $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ are linearly independent

$$a_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} + a_2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0 \iff a_1 = a_2 = 0$$

2) The nullspace of a matrix $N^{n \times a}$, is the set

$$Null(N) = \{ R \mid N \cdot R = 0 \}$$

3) The rank of N is the number of independent columns or rows of N.

4) For a matrix $N^{n \times a}$ the following hold:

A] $\text{Rank}(N) + \text{Nullity}(N) = a$

B] $0 \leq \text{Rank}(N) \leq a$

C] $\text{Rank}(N) = \text{Rank}(N^T)$

Exercise 2

Find the rank of

$$N = \begin{bmatrix} 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

<u>Row echelon</u>

e.g. $A_1 = \begin{bmatrix} 0 & 2 & 1 & -1 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Reduced row echelon

$$A_2 = \begin{bmatrix} 1 & 4 & 0 & 7 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$N^T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 1 \end{bmatrix} \xrightarrow[\substack{R_4' = R_3 \\ R_5' = R_4}]{R_3' = R_5} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 1 \end{bmatrix}$$

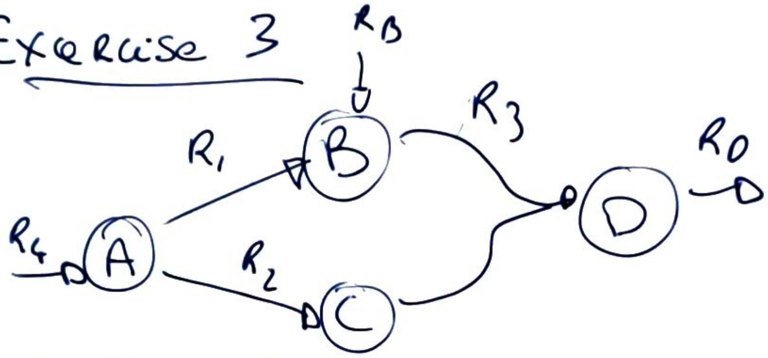
$$\overset{R_4 = -R_4}{=} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 1 \end{bmatrix} \overset{R_3' = R_3 + R_1}{=} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 1 \end{bmatrix}$$

$$R_5' = R_5 + R_1 - R_2$$
$$R_6' = R_6 + R_2 + R_3 - R_4$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

4) $\text{Rank}(N^T) = 4 \Leftrightarrow \text{Rank}(N) = 4$

Exercise 3



a) Find the stoechiometric matrix N.

b) Find the matrix equation that characterizes the steady state fluxes

$$\frac{dx}{dt} = 0$$

Ex 3

$$N = \begin{bmatrix} 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

$\text{Rank}(N) = 4$

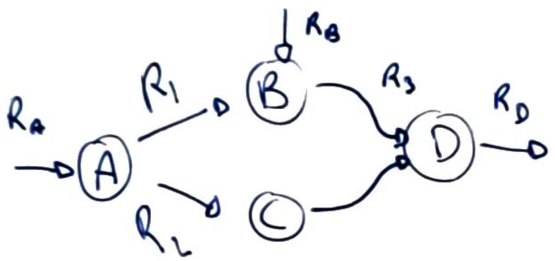
c ~~d~~) Compute dimensionality of the nullspace of N.

$$\text{Rank}(N) = \text{nullity}(N) = 9$$

$$\Leftrightarrow \quad \text{nullity}(N) = 9 - \text{Rank}(N)$$

$$= 6 - 4 = 2$$

d)



Given that $R_D = 1$, find a functional relationship between R_A and R_B

For A: $\quad R_1 + R_2 = R_A \quad ①$

For B: $\quad R_1 + R_B = R_3 \quad ②$

For C: $\quad R_2 = R_3$

For D: $\quad R_3 = R_D$

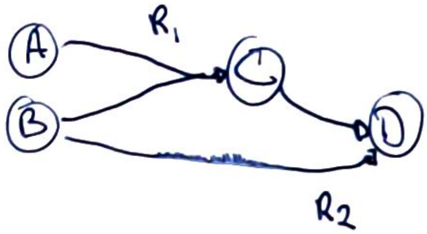
$$\left.\begin{matrix} R_2 = R_3 \\ R_3 = R_D \end{matrix}\right\} R_2 = R_3 = R_D = 1$$

$$① - ② \rightarrow R_1 + R_2 - R_1 - R_B = R_A - R_3$$

$$\Leftrightarrow \quad R_2 - R_B = \cancel{R_2,} R_A - R_3$$

$$\Leftrightarrow \quad R_A + R_B = 2$$

Exercice 4



1) Derive Stoechiometric matrix

$$\begin{array}{c} \begin{array}{cc} R_1 & R_2 \end{array} \\ \begin{bmatrix} -1 & 0 \\ -1 & -1 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} \\ A \\ B \\ C \\ D \end{array}$$

2] Rank of the stoechiometric matrix.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad Rank(N) = 2$$