

# Course 2: Supervised Learning



**IMT Atlantique**  
Bretagne-Pays de la Loire  
École Mines-Télécom

## Last session

- 1 What is not AI ?
- 2 AI definition
- 3 Applications
- 4 Open issues

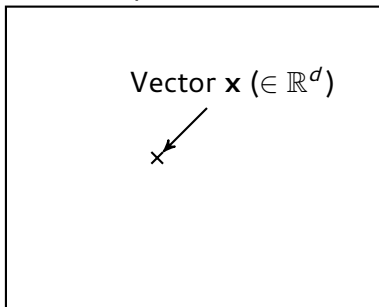
## Today's session

- Learning from labeled examples
- Challenges of supervised learning

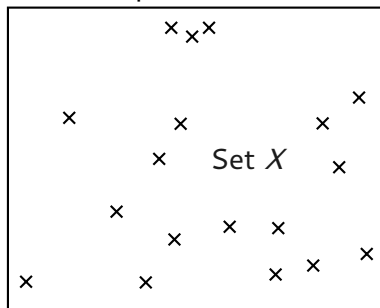
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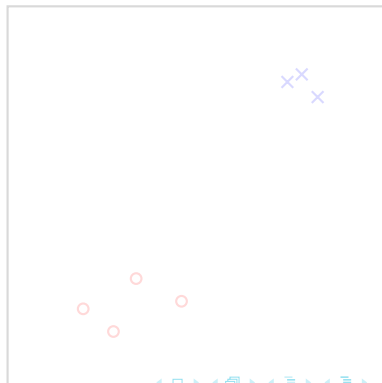
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Supervised learning methods use **labels**  $y$  associated to examples  $x \in X$  to learn a function  $f$  such as  $y = f(X)$ , with the aim of **generalizing** ( $\neq$  memorizing) to unlabeled examples.

## Examples

- Regression ( $y$  is scalar)
- Classification ( $y$  is categorical)
- Tons of applications:
  - Pattern recognition,
  - Prediction...



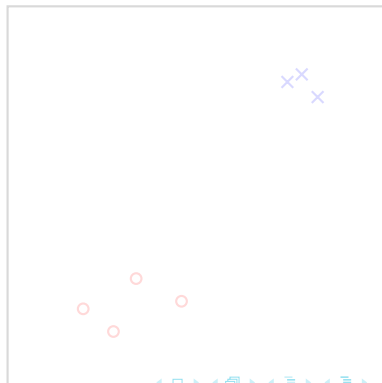
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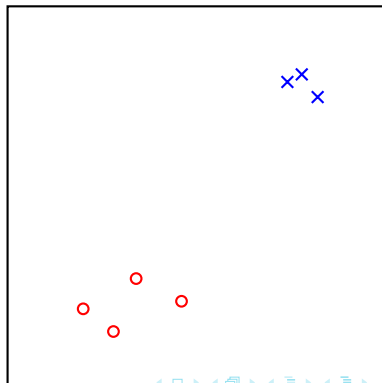
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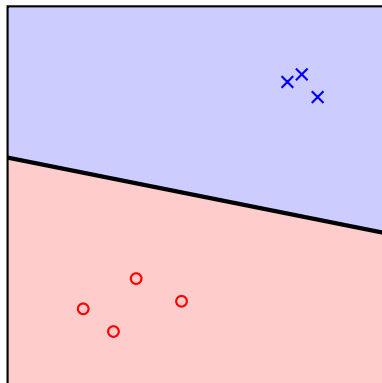
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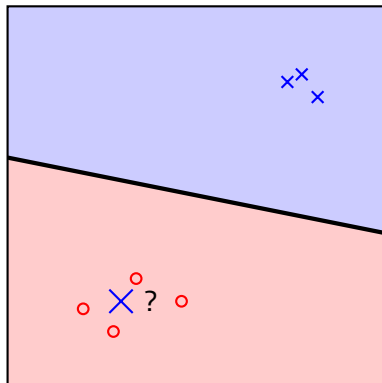
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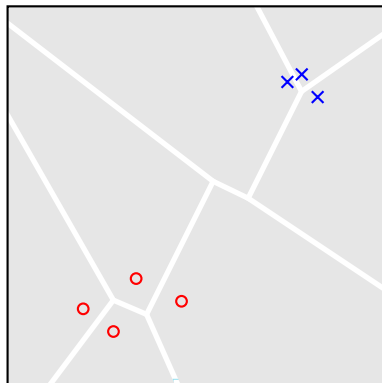
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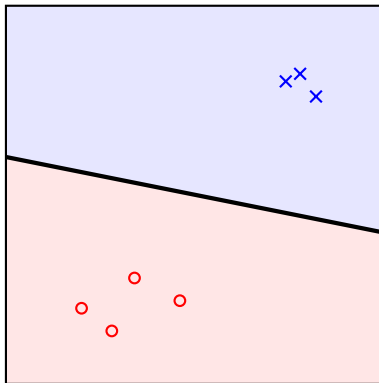
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# Challenges of supervised learning (1/5)

## An ill-defined problem

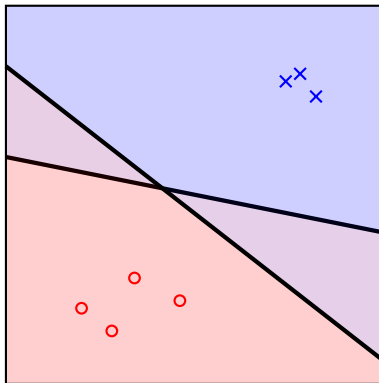
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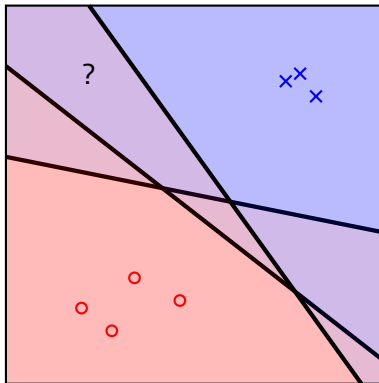
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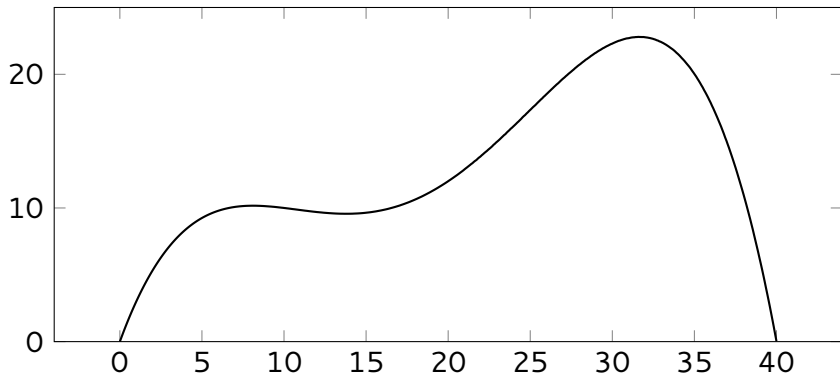
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## Bias/variance trade-off

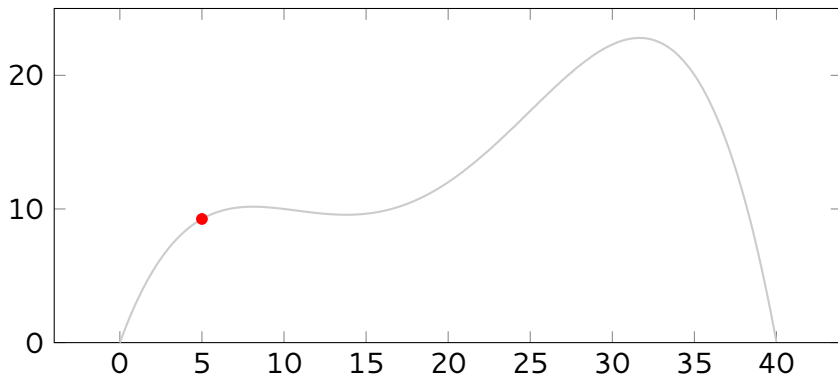
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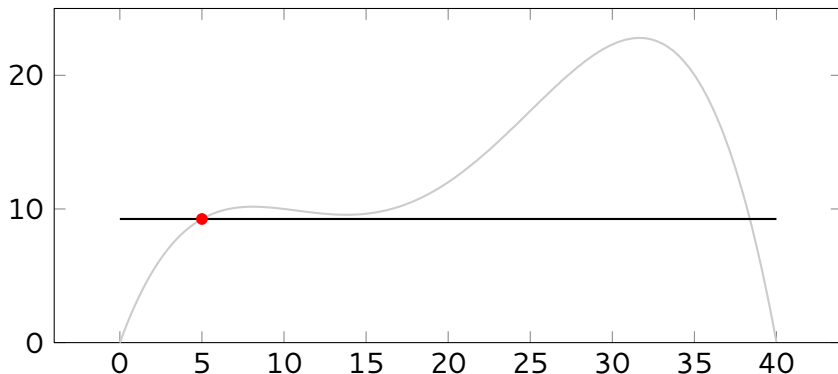




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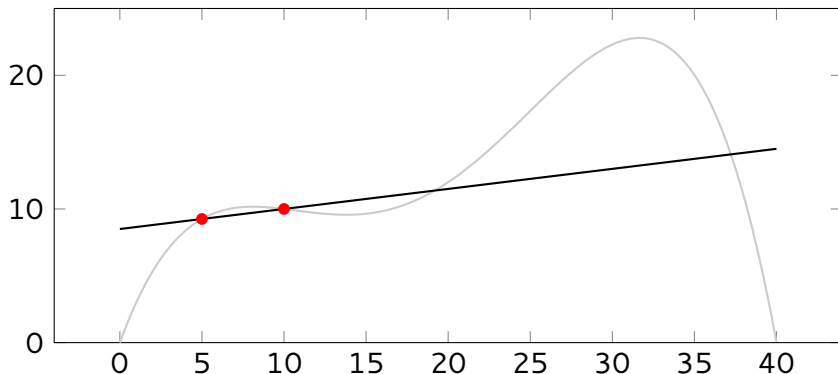
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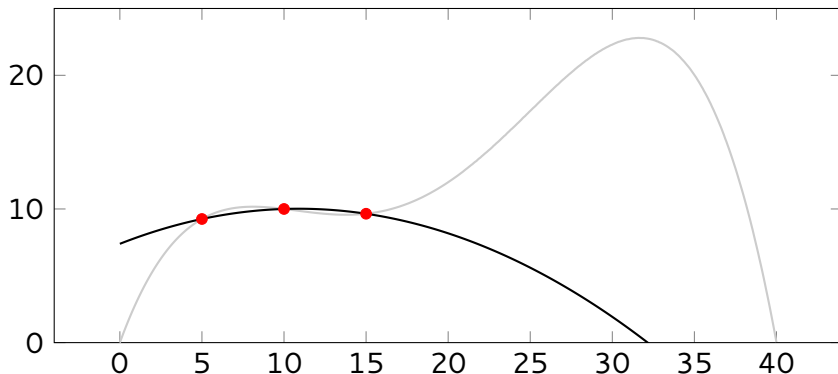
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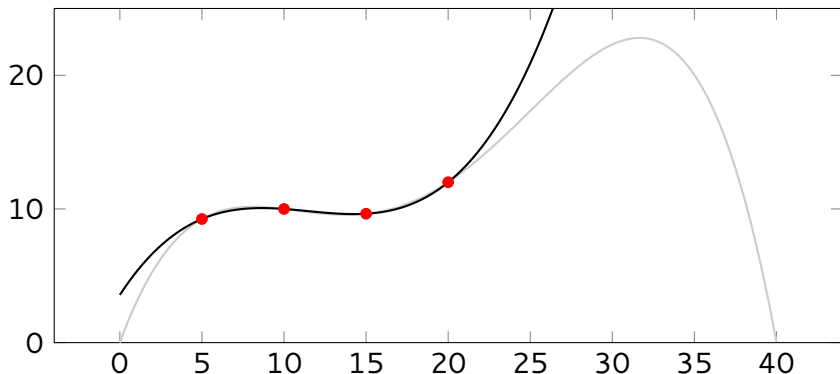
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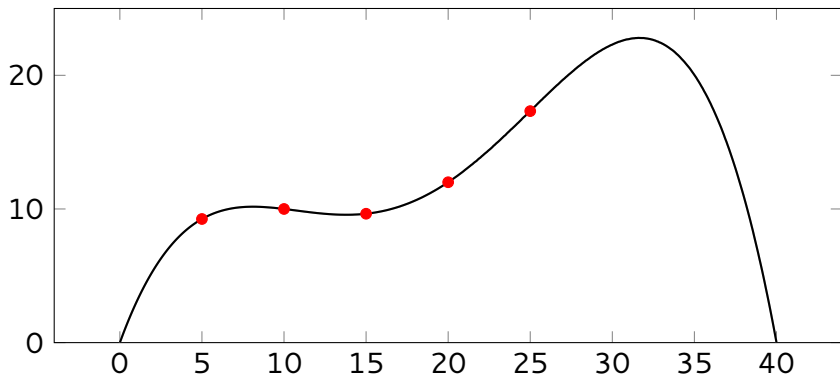
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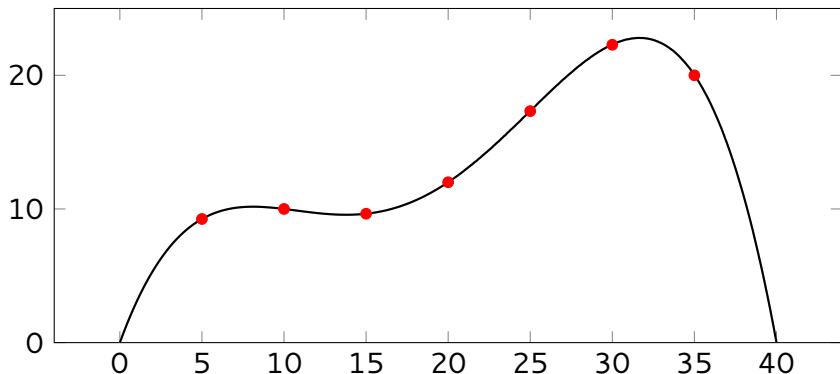
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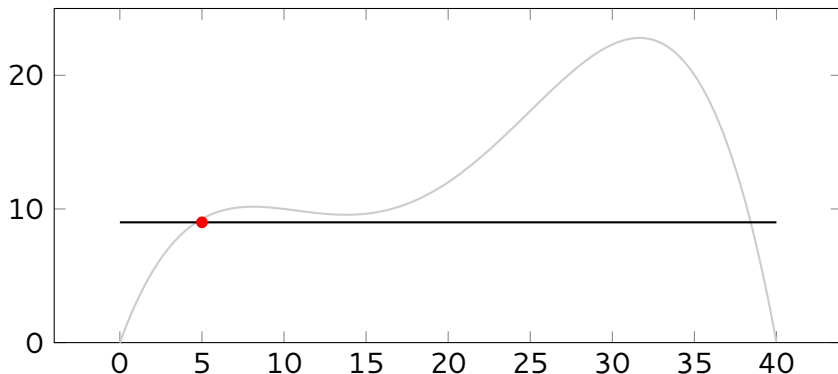
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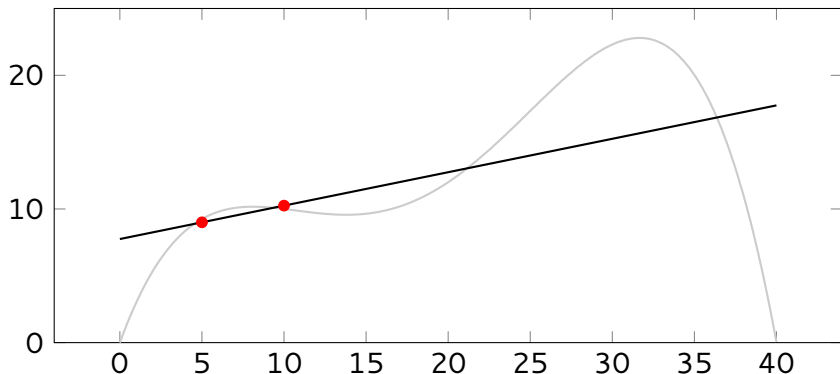
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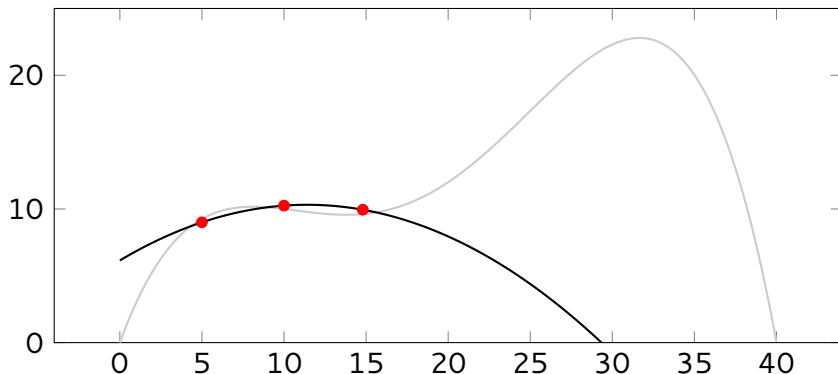




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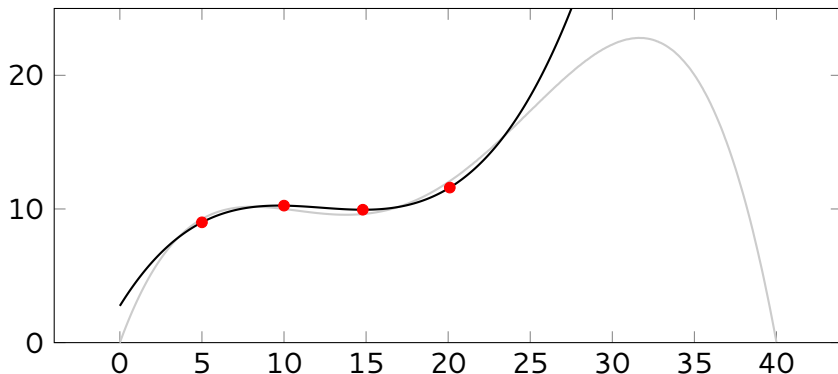
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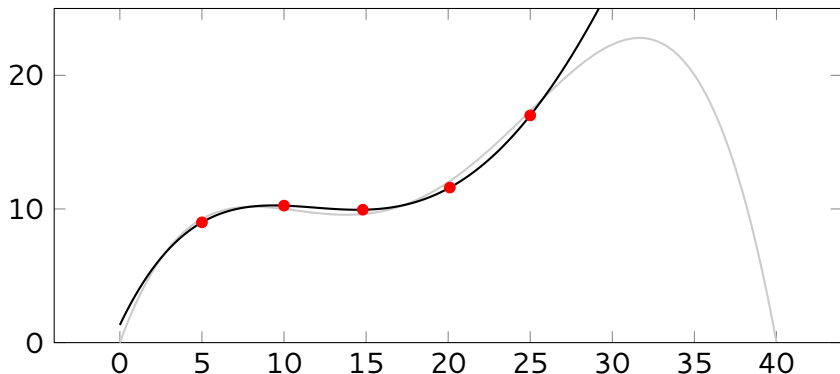
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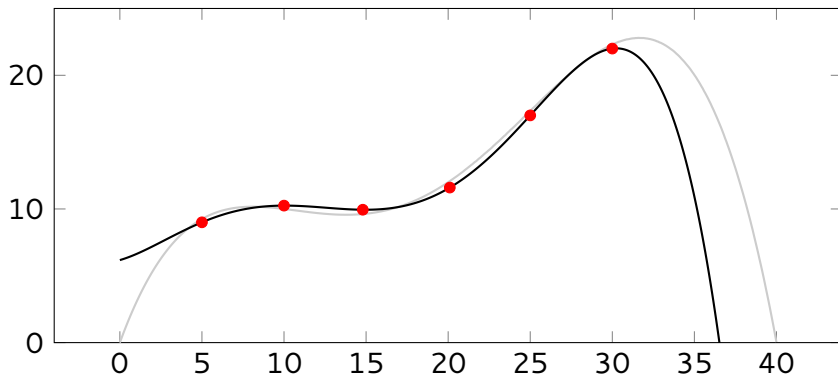
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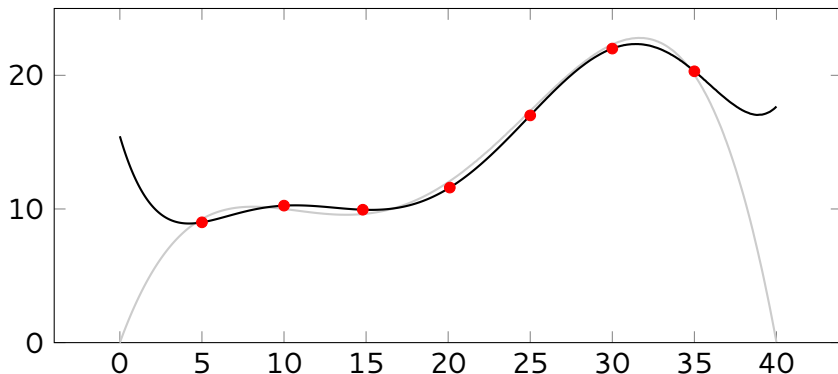
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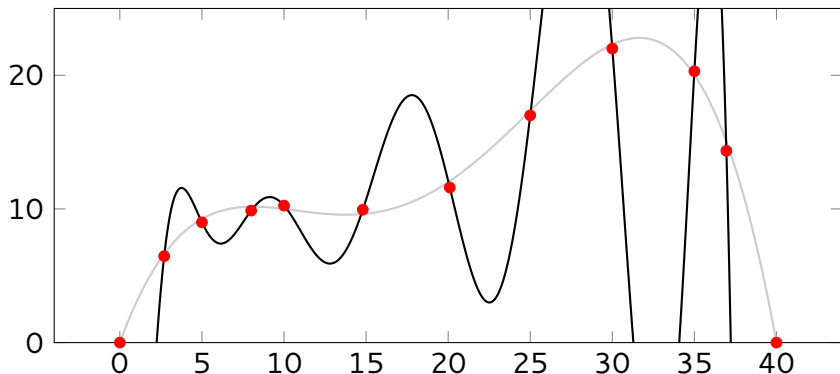
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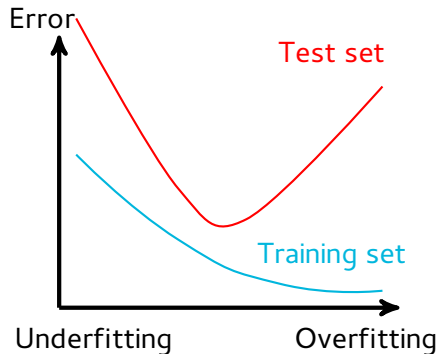
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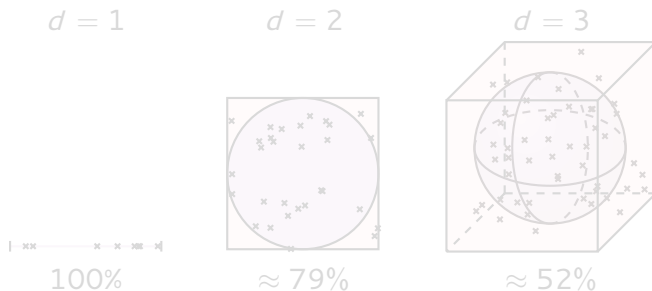
## Crossvalidation

- To quantify overfitting, split training dataset in two parts:
  - 1 A first part is used to train,
  - 2 A second part is used to validate,

# Challenges of supervised learning (3/5)

## Curse of dimensionality

- Geometry is not intuitive in high dimension,
- Efficient methods in 2D are not necessarily still valid.



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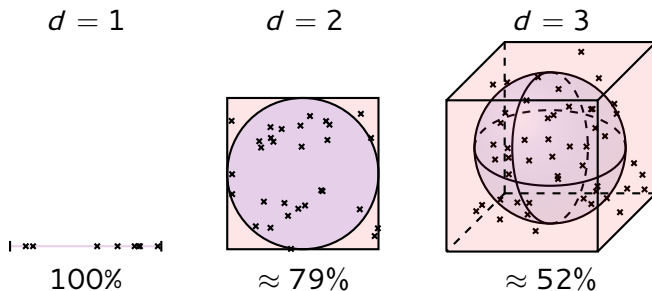
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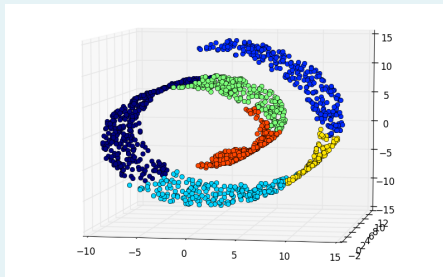


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## Riemannian manifolds

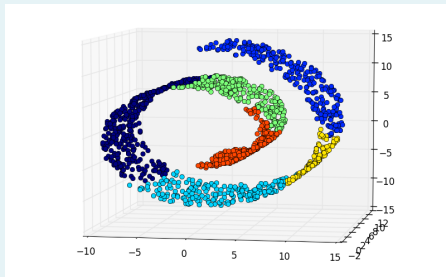


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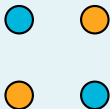


# Challenges of supervised learning (4/5)

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# Challenges of supervised learning (5/5)

## Computation time

Example on ImageNet, simply going through all images:

- $n = 10.000.000$ ,  $d \approx 1.000.000$ ,
- $\approx 10^{13}$  elementary operations,
- $\approx 2\text{h}45$  on a modern processor.

## Scalability

- Finding the best solution to a problem would be feasible with unlimited computation time,
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- Let us fix  $d$ ,
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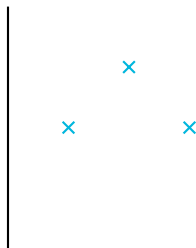


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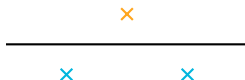


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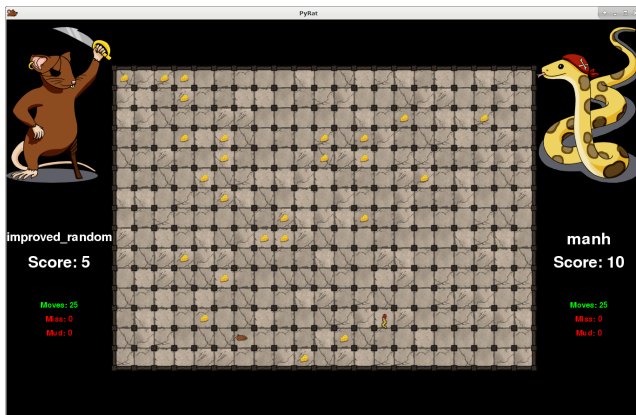
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VC is 3.

# Non-symmetric PyRat without walls / mud



Both players follow a deterministic greedy algorithm.

Supervised learning - Two tasks

- Predict the outcome of a game from the start configuration.
- Learn the next move using a dataset of winners

# Lab Session 2 and assignments for Session 3

## TP Supervised Learning (TP1)

- Basics of machine learning using sklearn (including new definitions / concepts)
- Tests on PyRat datasets using the two tasks (predicting winner and predicting moves to play)

## Project 1 (P1)

You will be choose a supervised learning method. You have to prepare a Jupyter Notebook on this method, including:

- A brief description of the theory behind the method,
- Basic tests on simulated data to show the influence of parameters and hyperparameters
- Tests on PyRat Datasets on at least ONE of the two tasks (predicting winner or playing)

During Session 3 you will have 7 minutes to present your notebook.