Peer Instruction

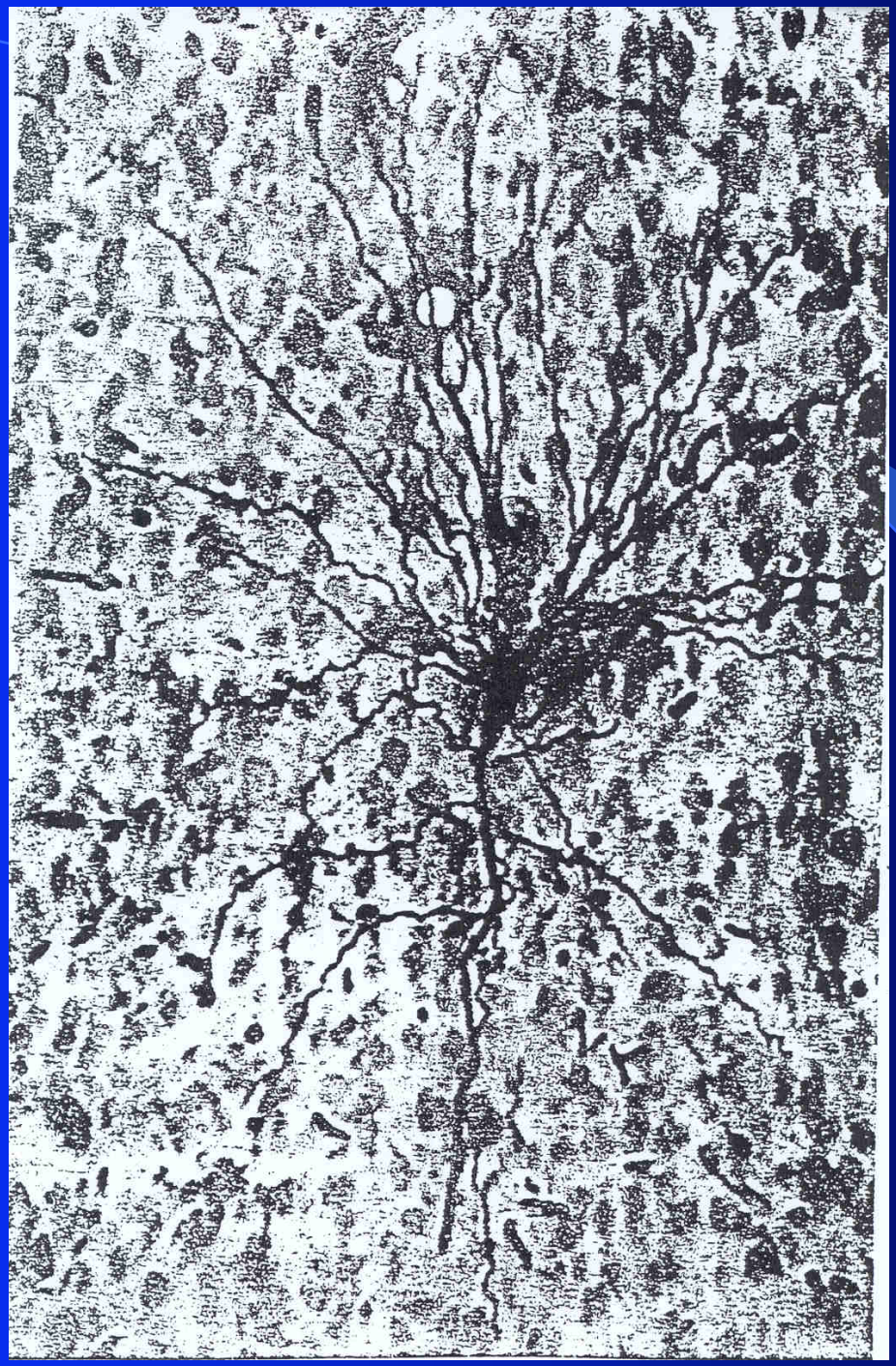
- I pose a *challenge question* (usually multiple choice), which will help solidify understanding of topics we have studied
  - Might not just be one correct answer
- You each get some time (1-2 minutes) to come up with your answer and vote – use Mentimeter
- Then you get some time to convince your group (neighbors) why you think you are right (2-3 minutes)
  - Learn from and teach each other!
- You vote again. May change your vote if you want.
- We discuss together the different responses, show the votes, give you opportunity to justify your thinking, and give you further insights

Peer Instruction (PI) Why

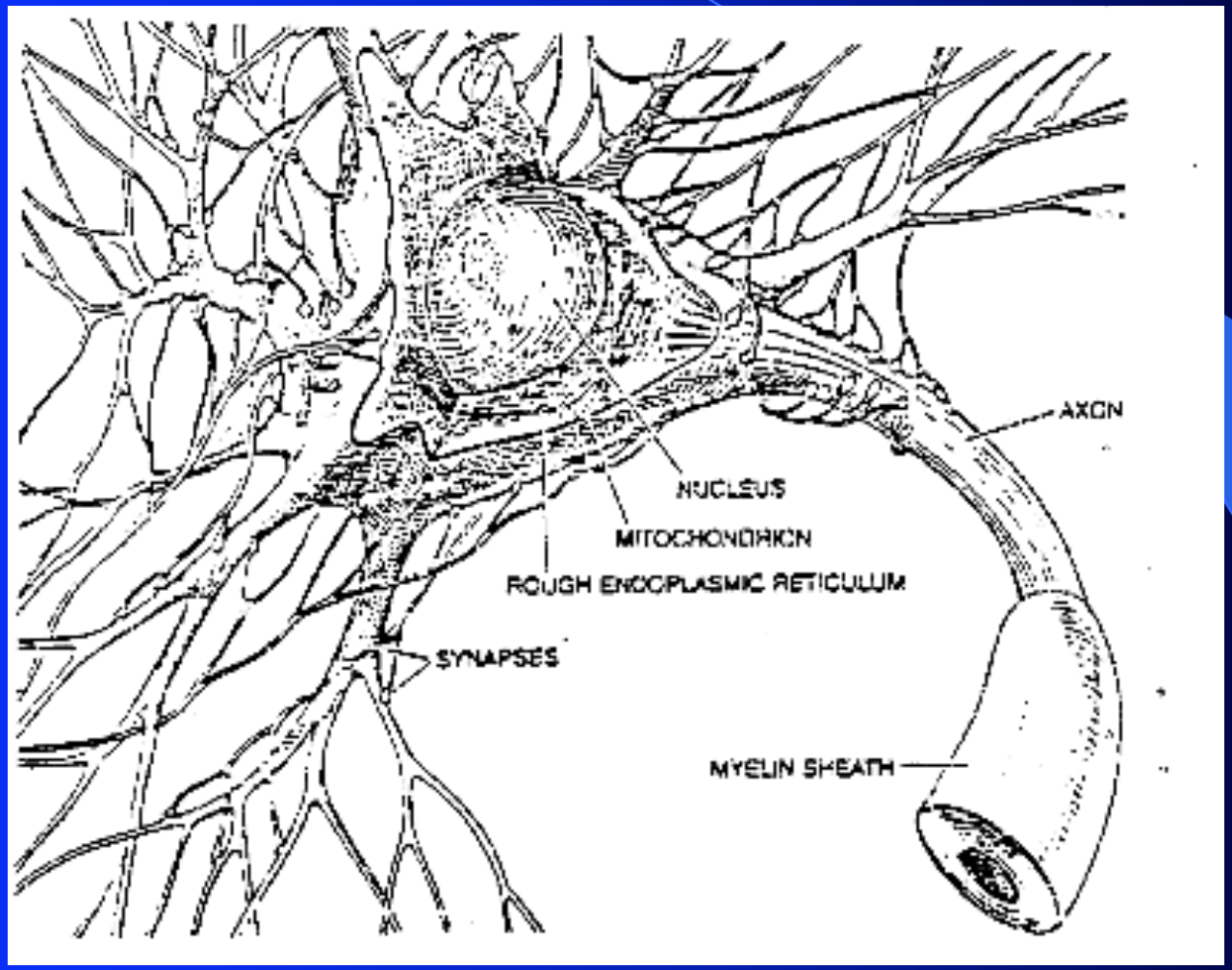
- Studies show this approach improves learning
- Learn by doing, discussing, and teaching each other
  - Curse of knowledge/expert blind-spot
  - Compared to talking with a peer who just figured it out and who can explain it in your own jargon
  - You never really know something until you can teach it to someone else – More improved learning!
- Just as important to understand why wrong answers are wrong, or how they could be changed to be right.
  - Learn to reason about your thinking and answers
- More enjoyable - You are involved and active in the learning

How Groups Interact

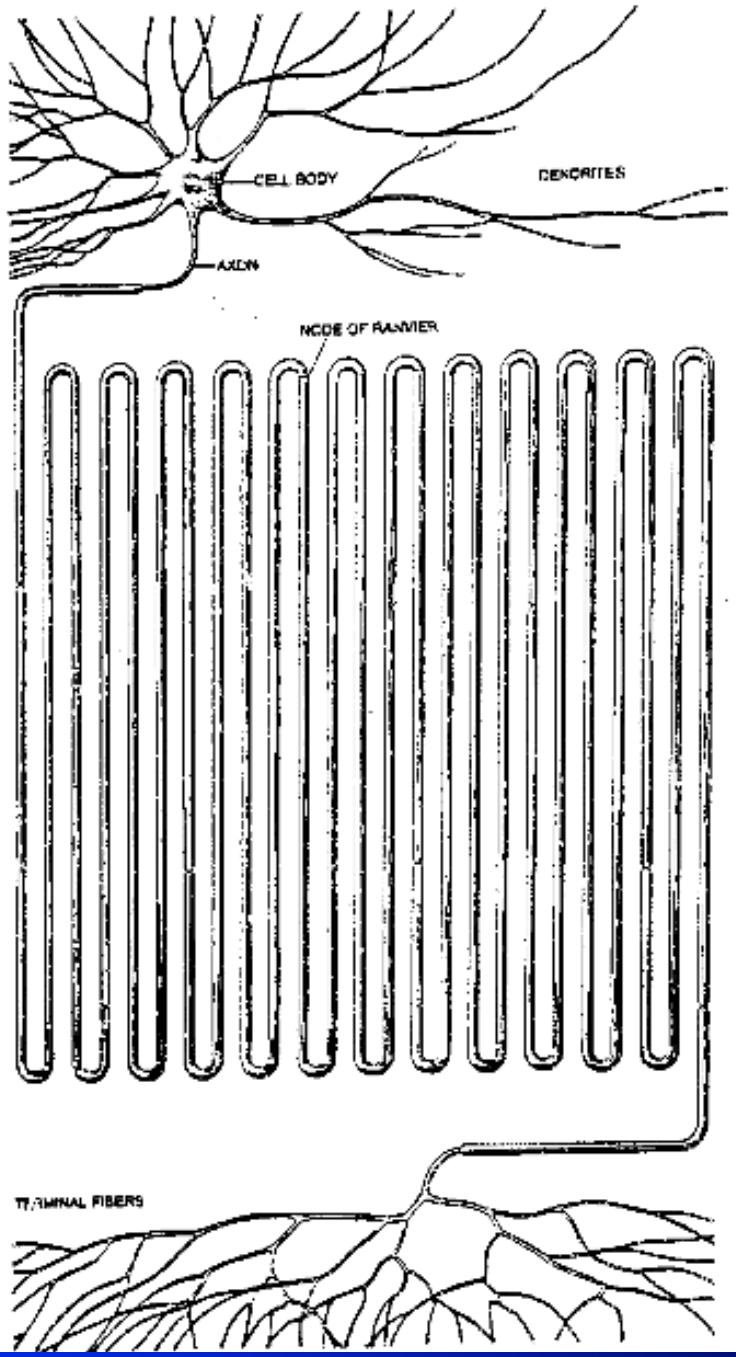
- Best if group members have different initial answers
- 3 is the “magic” group number
  - You can self-organize "on-the-fly" or sit together specifically to be a group
  - Can go 2-4 on a given day to make sure everyone is involved
- Teach and learn from each other: Discuss, reason, articulate
- If you know the answer, listen to where colleagues are coming from first, then be a great humble teacher, you will also learn by doing that, and you’ll be on the other side in the future
  - I can’t do that as well because every small group has different misunderstandings and you get to focus on your particular questions
- Be ready to justify to the class your vote and justifications!



Basic Neuron



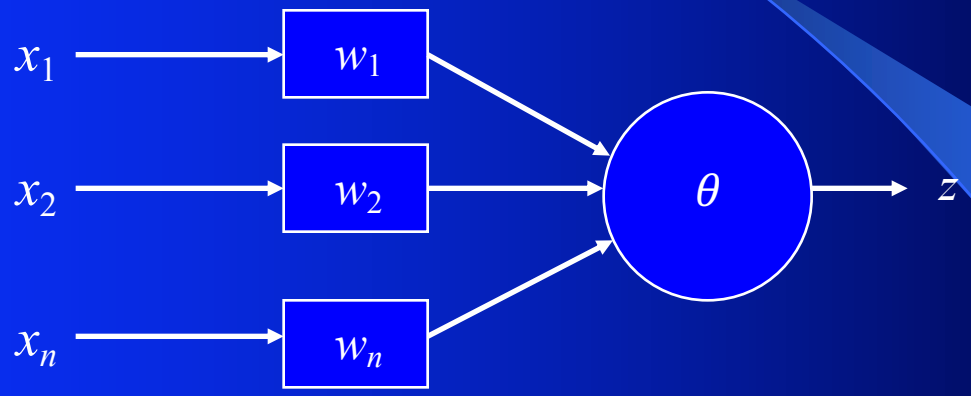
Generic Neuron and Neurites



Perceptron Learning Algorithm

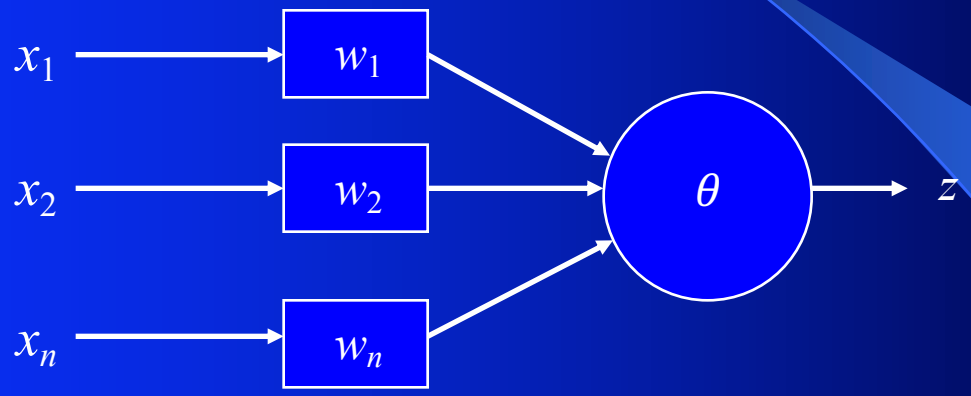
- First neural network learning model in the 1960's
- Simple and limited (single layer models)
- Basic concepts are similar for multi-layer models so this is a good learning tool
- Still used in many current applications (modems, etc.)

Perceptron Node – Threshold Logic Unit



$$z = \begin{cases} 1 & \text{if } \sum_{i=1}^{n} x_i w_i \geq \theta \\ 0 & \text{if } \sum_{i=1}^{n} x_i w_i < \theta \end{cases}$$

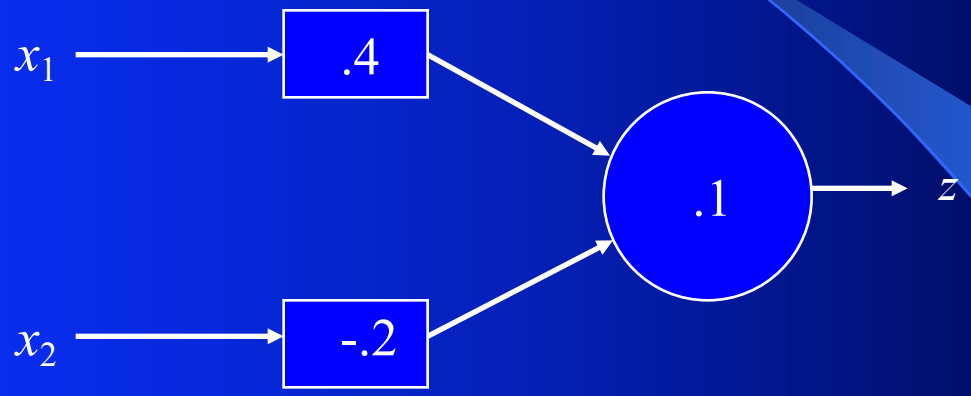
Perceptron Node – Threshold Logic Unit



- Learn weights such that an objective function is maximized.
- What objective function should we use?
- What learning algorithm should we use?

$$z = \begin{cases} 1 & \text{if } \sum_{i=1}^{n} x_i w_i \geq \theta \\ 0 & \text{if } \sum_{i=1}^{n} x_i w_i < \theta \end{cases}$$

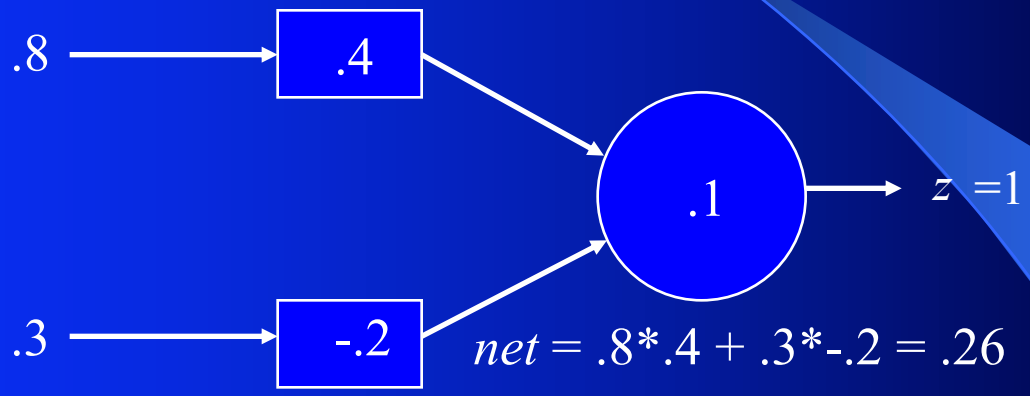
Perceptron Learning Algorithm



x_1	x_2	t
.8	.3	1
.4	.1	0

$$z = \begin{cases} 1 & \text{if } \sum_{i=1}^{n} x_i w_i \geq \theta \\ 0 & \text{if } \sum_{i=1}^{n} x_i w_i < \theta \end{cases}$$

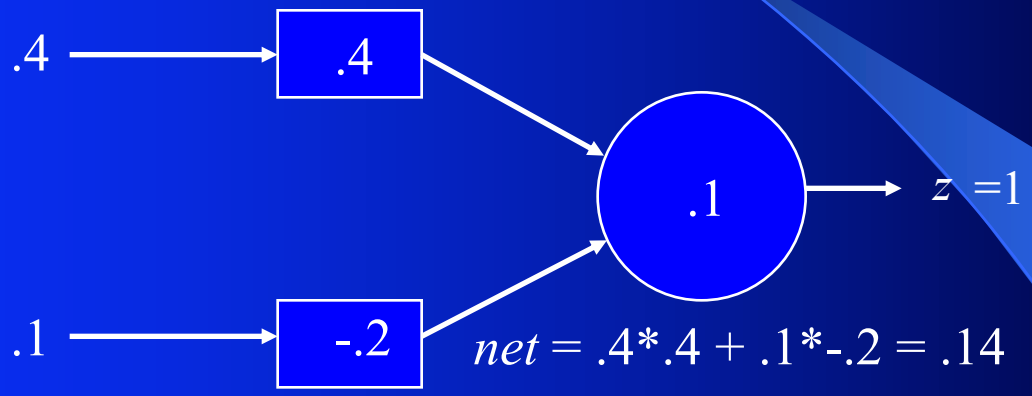
First Training Instance



x_1	x_2	t
.8	.3	1
.4	.1	0

$$z = \begin{cases} 1 & \text{if } \sum_{i=1}^{n} x_i w_i \geq \theta \\ 0 & \text{if } \sum_{i=1}^{n} x_i w_i < \theta \end{cases}$$

Second Training Instance



x_1	x_2	t
.8	.3	1
.4	.1	0

$$z = \begin{cases} 1 & \text{if } \sum_{i=1}^{n} x_i w_i \geq \theta \\ 0 & \text{if } \sum_{i=1}^{n} x_i w_i < \theta \end{cases}$$

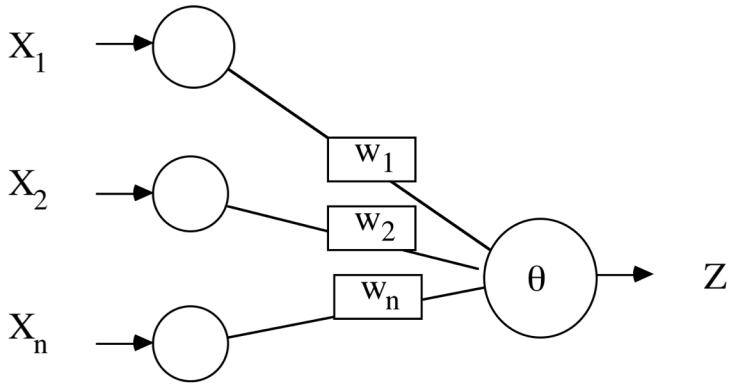
$$\Delta w_i = (t - z) * c * x_i$$

Perceptron Rule Learning

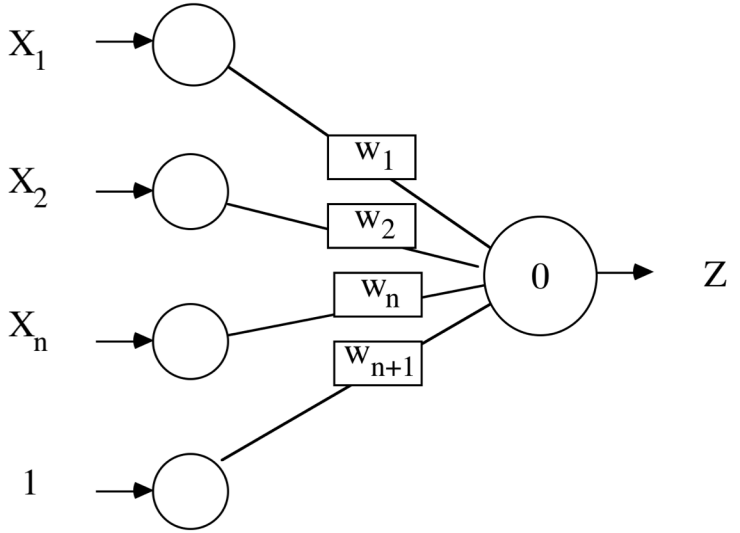
$$\Delta w_i = c(t - z)\, x_i$$

- Where w_i is the weight from input i to perceptron node, c is the learning rate, t is the target for the current instance, z is the current output, and x_i is i^{th} input
- Least perturbation principle
  - Only change weights if there is an error
  - small c rather than changing weights sufficient to make current pattern correct
  - Scale by x_i
- Create a perceptron node with n inputs
- Iteratively apply a pattern from the training set and apply the perceptron rule
- Each iteration through the training set is an *epoch*
- Continue training until total training set error ceases to improve
- Perceptron Convergence Theorem: Guaranteed to find a solution in finite time if a solution exists

Weight Versus Threshold



Do you need to adjust Theta? Yes, in most cases



where $w_{n+1} = -\theta$

Augmented Pattern Vectors

1 0 1 -> 0

1 0 0 -> 1

Augmented Version

1 0 1 1 -> 0

1 0 0 1 -> 1

- Treat threshold like any other weight. No special case. Call it a *bias* since it biases the output up or down.
- Since we start with random weights anyways, can ignore the -θ notion, and just think of the bias as an extra available weight. (note the author uses a -1 input)
- Always use a bias weight

Perceptron Rule Example

- Assume a 3 input perceptron plus bias (it outputs 1 if net > 0, else 0)
- Assume a learning rate c of 1 and initial weights all 0: $\Delta w_i = c(t - z)\, x_i$
- Training set 0 0 1 -> 0
  1 1 1 -> 1
  1 0 1 -> 1
  0 1 1 -> 0

Pattern	Target	Weight Vector	Net	Output	ΔW
0 0 1 1	0	0 0 0 0			

Example

- Assume a 3 input perceptron plus bias (it outputs 1 if net > 0, else 0)
- Assume a learning rate c of 1 and initial weights all 0: $\Delta w_i = c(t - z)\, x_i$
- Training set 0 0 1 -> 0
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Pattern	Target	Weight Vector	Net	Output	ΔW
0 0 1 1	0	0 0 0 0	0	0	0 0 0 0
1 1 1 1	1	0 0 0 0			

Example

- Assume a 3 input perceptron plus bias (it outputs 1 if net > 0, else 0)
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Pattern	Target	Weight Vector	Net	Output	ΔW
0 0 1 1	0	0 0 0 0	0	0	0 0 0 0
1 1 1 1	1	0 0 0 0	0	0	1 1 1 1
1 0 1 1	1	1 1 1 1			

Challenge Question - Perceptron

- Assume a 3 input perceptron plus bias (it outputs 1 if net > 0, else 0)
- Assume a learning rate c of 1 and initial weights all 0: $\Delta w_i = c(t - z)\, x_i$
- Training set
  0 0 1 -> 0
  1 1 1 -> 1
  1 0 1 -> 1
  0 1 1 -> 0

Pattern	Target	Weight Vector	Net	Output	ΔW
0 0 1 1	0	0 0 0 0	0	0	0 0 0 0
1 1 1 1	1	0 0 0 0	0	0	1 1 1 1
1 0 1 1	1	1 1 1 1			

- Once this converges the final weight vector will be
  A. 1 1 1 1
  B. -1 0 1 0
  C. 0 0 0 0
  D. 1 0 0 0
  E. None of the above

Example

- Assume a 3 input perceptron plus bias (it outputs 1 if net > 0, else 0)
- Assume a learning rate c of 1 and initial weights all 0: $\Delta w_i = c(t - z)\, x_i$
- Training set 0 0 1 -> 0
  1 1 1 -> 1
  1 0 1 -> 1
  0 1 1 -> 0

Pattern	Target	Weight Vector	Net	Output	ΔW
0 0 1 1	0	0 0 0 0	0	0	0 0 0 0
1 1 1 1	1	0 0 0 0	0	0	1 1 1 1
1 0 1 1	1	1 1 1 1	3	1	0 0 0 0
0 1 1 1	0	1 1 1 1			

Example

- Assume a 3 input perceptron plus bias (it outputs 1 if net > 0, else 0)
- Assume a learning rate c of 1 and initial weights all 0: $\Delta w_i = c(t - z)\, x_i$
- Training set 0 0 1 -> 0
  1 1 1 -> 1
  1 0 1 -> 1
  0 1 1 -> 0

Pattern	Target	Weight Vector	Net	Output	ΔW
0 0 1 1	0	0 0 0 0	0	0	0 0 0 0
1 1 1 1	1	0 0 0 0	0	0	1 1 1 1
1 0 1 1	1	1 1 1 1	3	1	0 0 0 0
0 1 1 1	0	1 1 1 1	3	1	0 -1 -1 -1
0 0 1 1	0	1 0 0 0			

Example

- Assume a 3 input perceptron plus bias (it outputs 1 if net > 0, else 0)
- Assume a learning rate c of 1 and initial weights all 0: $\Delta w_i = c(t - z)\, x_i$
- Training set 0 0 1 -> 0
  1 1 1 -> 1
  1 0 1 -> 1
  0 1 1 -> 0

Pattern	Target	Weight Vector	Net	Output	ΔW
0 0 1 1	0	0 0 0 0	0	0	0 0 0 0
1 1 1 1	1	0 0 0 0	0	0	1 1 1 1
1 0 1 1	1	1 1 1 1	3	1	0 0 0 0
0 1 1 1	0	1 1 1 1	3	1	0 -1 -1 -1
0 0 1 1	0	1 0 0 0	0	0	0 0 0 0
1 1 1 1	1	1 0 0 0	1	1	0 0 0 0
1 0 1 1	1	1 0 0 0	1	1	0 0 0 0
0 1 1 1	0	1 0 0 0	0	0	0 0 0 0

Perceptron Homework

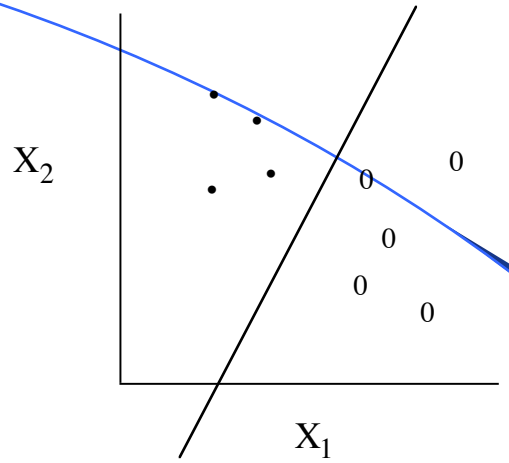
- Assume a 3 input perceptron plus bias (it outputs 1 if net > 0, else 0)
- Assume a learning rate c of 1 and initial weights all 1: $\Delta w_i = c(t - z)\, x_i$
- Show weights after each pattern for just one epoch
- Training set 1 0 1 -> 0
  1 1 0 -> 0
  1 0 1 -> 1
  0 1 1 -> 1

Pattern	Target	Weight Vector	Net	Output	ΔW
		1 1 1 1			

Training Sets and Noise

- Assume a Probability of Error at each bit
- 0 0 1 0 1 1 0 0 1 1 0 -> 0 1 1 0
- i.e. P(error) = .05
- Or a probability that the algorithm is applied wrong (opposite) occasionally
- Averages out over learning

Linear Separability



2-d case (two inputs)

$W_1X_1 + W_2X_2 > \theta \ (Z=1)$

$W_1X_1 + W_2X_2 < \theta \ (Z=0)$

So, what is decision boundary?

$W_1X_1 + W_2X_2 = \theta$

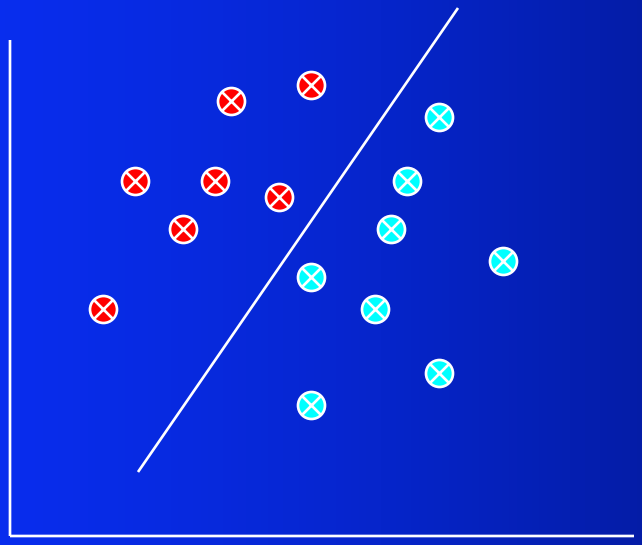
$X_2 + W_1X_1/W_2 = \theta/W_2$

$X_2 = (-W_1/W_2)X_1 + \theta/W_2$

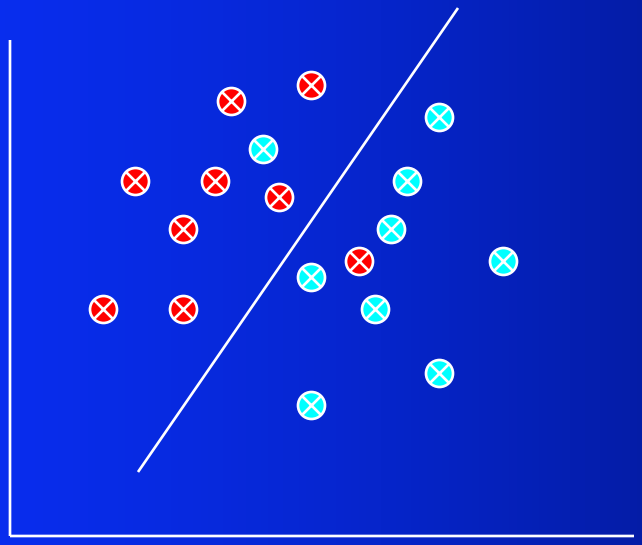
$Y = MX + B$

If no bias weight then the hyperplane must go through the origin

Linear Separability

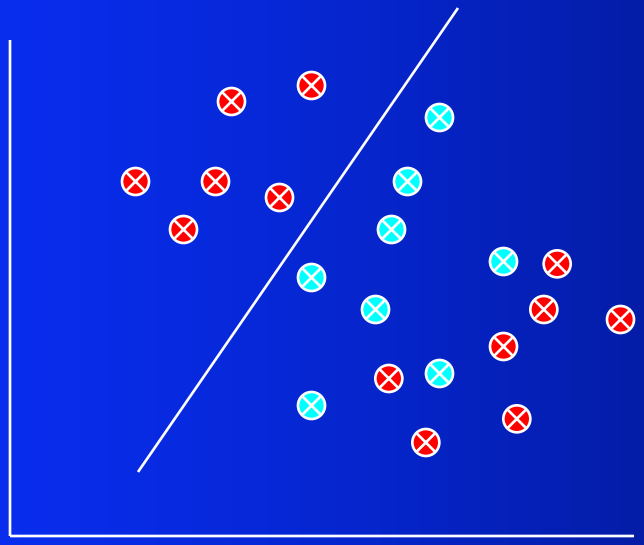


Linear Separability and Generalization



When is data noise vs. a legitimate exception

Limited Functionality of Hyperplane

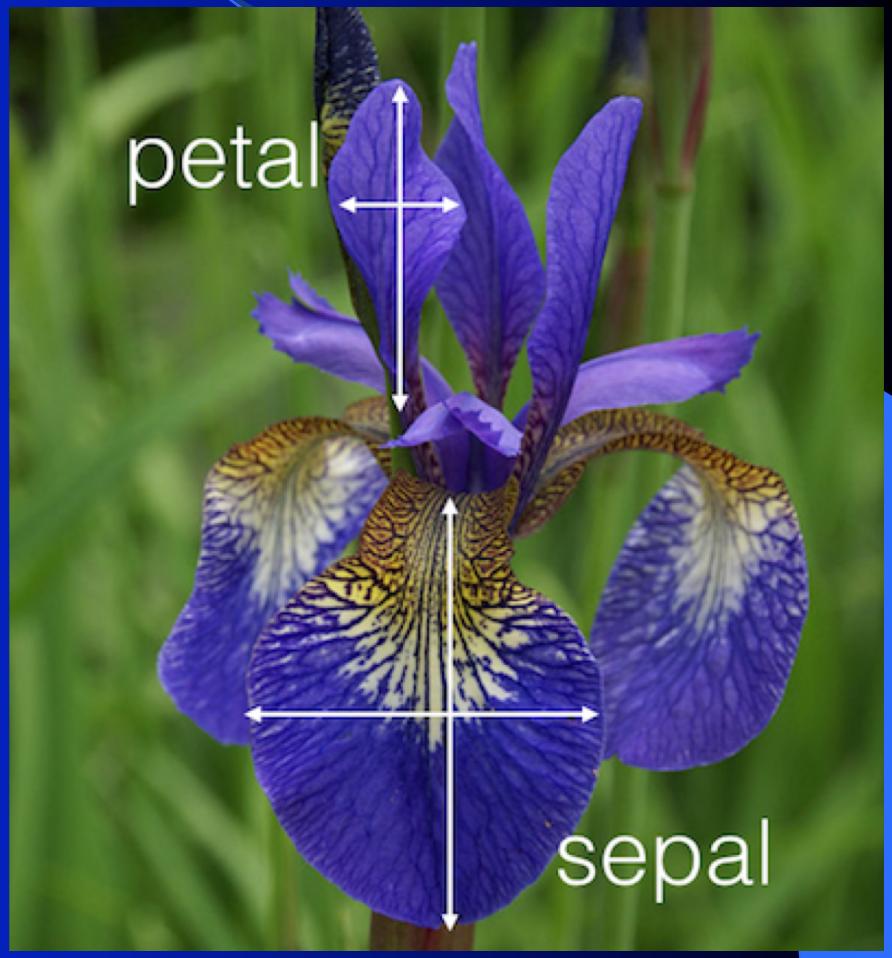


How to Handle Multi-Class Output

- This is an issue with any learning model which only supports binary classification (perceptron, SVM, etc.)
- Create 1 perceptron for each output class, where the training set considers all other classes to be negative examples
  - Run all perceptrons on novel data and set the output to the class of the perceptron which outputs high
  - If there is a tie, choose the perceptron with the highest net value
- Create 1 perceptron for each pair of output classes, where the training set only contains examples from the 2 classes
  - Run all perceptrons on novel data and set the output to be the class with the most wins (votes) from the perceptrons
  - In case of a tie, use the net values to decide
  - Number of models grows by the square of the output classes

UC Irvine Machine Learning Data Base
Iris Data Set

4.8,3.0,1.4,0.3,	Iris-setosa
5.1,3.8,1.6,0.2,	Iris-setosa
4.6,3.2,1.4,0.2,	Iris-setosa
5.3,3.7,1.5,0.2,	Iris-setosa
5.0,3.3,1.4,0.2,	Iris-setosa
7.0,3.2,4.7,1.4,	Iris-versicolor
6.4,3.2,4.5,1.5,	Iris-versicolor
6.9,3.1,4.9,1.5,	Iris-versicolor
5.5,2.3,4.0,1.3,	Iris-versicolor
6.5,2.8,4.6,1.5,	Iris-versicolor
6.0,2.2,5.0,1.5,	Iris-viginica
6.9,3.2,5.7,2.3,	Iris-viginica
5.6,2.8,4.9,2.0,	Iris-viginica
7.7,2.8,6.7,2.0,	Iris-viginica
6.3,2.7,4.9,1.8,	Iris-viginica



Objective Functions: Accuracy/Error

- How do we judge the quality of a particular model (e.g. Perceptron with a particular setting of weights)
- Consider how accurate the model is on the data set
  - *Classification accuracy* = # Correct/Total instances
  - *Classification error* = # Misclassified/Total instances (= 1 – acc)
- Usually minimize a Loss function (aka cost, error)
- For real valued outputs and/or targets
  - Pattern error = Target – output: Errors could cancel each other
    - $\Sigma|t_i - z_i|$ (L1 loss)
    - Common approach is *Squared Error* = $\Sigma(t_i - z_i)^2$ (L2 loss)
  - Total sum squared error = Σ pattern sqaured errors = $\Sigma\ \Sigma\ (t_i - z_i)^2$
- For nominal data, pattern error is typically 1 for a mismatch and 0 for a match
  - For nominal (including binary) output and targets, SSE and classification error are equivalent

Mean Squared Error

- Mean Squared Error (MSE) – SSE/n where n is the number of instances in the data set
  - This can be nice because it normalizes the error for data sets of different sizes
  - MSE is the average squared error per pattern
- Root Mean Squared Error (RMSE) – is the square root of the MSE
  - This puts the error value back into the same units as the features and can thus be more intuitive
    - Since we squared the error on the SSE
  - RMSE is the average distance (error) of targets from the outputs in the same scale as the features

Challenge Question - Error

- Given the following data set, what is the L1 ($\Sigma|t_i - z_i|$), SSE/L2 ($\Sigma(t_i - z_i)^2$), MSE, and RMSE error for the entire data set?

x	y	Output	Target	Data Set
2	-3	1	1	
0	1	0	1	
.5	.6	.8	.2	
L1				x
SSE				x
MSE				x
RMSE				x

A. .4 1 1 1
B. 1.6 2.36 1 1
C. .4 .64 .21 0.46
D. 1.6 1.36 .68 .82
E. None of the above

Challenge Question - Error

- Given the following data set, what is the L1 ($\Sigma|t_i - z_i|$), SSE/L2 ($\Sigma(t_i - z_i)^2$), MSE, and RMSE error for the entire data set?

x	y	Output	Target	Data Set
2	-3	1	1	
0	1	0	1	
.5	.6	.8	.2	
L1				1.6
SSE				1.36
MSE				1.36/3 = .45
RMSE				.45^.5 = .67

A. .4 1 1 1
B. 1.6 2.36 1 1
C. .4 .64 .21 0.46
D. 1.6 1.36 .68 .82
E. None of the above

SSE Homework

- Given the following data set, what is the L1, SSE (L2), MSE, and RMSE error of Output1, Output2, and the entire data set? Fill in cells that have an x.

x	y	Output1	Target1	Output2	Target 2	Data Set
-1	-1	0	1	.6	1.0	
-1	1	1	1	-.3	0	
1	-1	1	0	1.2	.5	
1	1	0	0	0	-.2	
L1		x		x		x
SSE		x		x		x
MSE		x		x		x
RMSE		x		x		x

Gradient Descent Learning: Minimize (Maximze) the Objective Function

Deriving a Gradient Descent Learning Algorithm

- Goal is to decrease overall error (or other objective function) each time a weight is changed
- Total Sum Squared error one possible objective function E:

$$\Sigma (t_i - z_i)^2$$

- Seek a weight changing algorithm such that $\frac{\partial E}{\partial w_{ij}}$ is negative
- If a formula can be found then we have a gradient descent learning algorithm
- Delta rule is a variant of the perceptron rule which gives a gradient descent learning algorithm with perceptron nodes

Delta rule algorithm

- Delta rule uses (target - net) before the net value goes through the threshold in the learning rule to decide weight update

$$\Delta w_i = c(t - net)x_i$$

- Weights are updated even when the output would be correct
- Because this model is single layer and because of the SSE objective function, the error surface is guaranteed to be parabolic with only one minima
- Learning rate
  - If learning rate is too large can jump around global minimum
  - If too small, will work, but will take a longer time
  - Can decrease learning rate over time to give higher speed and still attain the global minimum (although exact minimum is still just for training set and thus…)

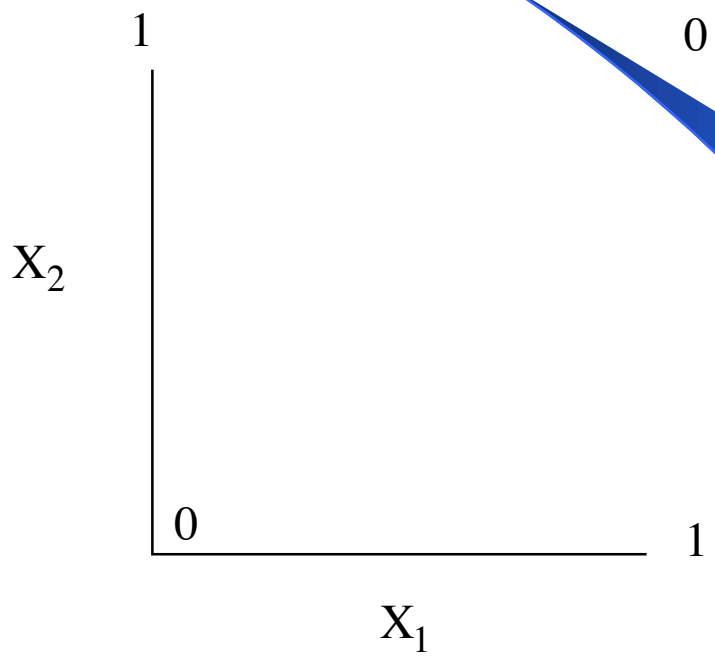
Batch vs Stochastic Update

- To get the true gradient with the delta rule, we need to sum errors over the entire training set and only update weights at the end of each epoch
- Batch (gradient) vs stochastic (on-line, incremental)
  - With the stochastic delta rule algorithm, you update after every pattern, just like with the perceptron algorithm (even though that means each change may not be exactly along the true gradient)
  - Stochastic is more efficient and best to use in almost all cases, though not all have figured it out yet
    - We'll take about this a little bit when we get to Backpropagation

Perceptron rule vs Delta rule

- Perceptron rule (target - thresholded output) guaranteed to converge to a separating hyperplane if the problem is linearly separable. Otherwise may not converge – could get in cycle
- Singe layer Delta rule guaranteed to have only one global minimum. Thus it will converge to the best SSE solution whether the problem is linearly separable or not.
  - Could have a higher misclassification rate than with the perceptron rule and a less intuitive decision surface – we will discuss this later with regression
- Stopping Criteria – For these models stop when no longer making progress
  - When you have gone a few epochs with no significant improvement/change between epochs (including oscillations)

Exclusive Or



Is there a dividing hyperplane?

Linearly Separable Boolean Functions

- d = # of dimensions

Linearly Separable Boolean Functions

- d = # of dimensions
- $P = 2^d$ = # of Patterns

Linearly Separable Boolean Functions

- d = # of dimensions
- $P = 2^d$ = # of Patterns
- $2^P = 2^{2^d}$ = # of Functions

n	Total Functions	Linearly Separable Functions
0	2	2
1	4	4
2	16	14

Linearly Separable Boolean Functions

- d = # of dimensions
- $P = 2^d$ = # of Patterns
- $2^P = 2^{2^d}$ = # of Functions

n	Total Functions	Linearly Separable Functions
0	2	2
1	4	4
2	16	14
3	256	104
4	65536	1882
5	4.3×10^9	94572
6	1.8×10^{19}	1.5×10^7
7	3.4×10^{38}	8.4×10^9

Linearly Separable Functions

$$LS(P,d) = 2\sum_{i=0}^{d}\frac{(P-1)!}{(P-1-i)!i!} \quad for\ P > d$$

$$= 2^P \quad for\ P \leq d$$

(All patterns for d=P)

i.e. all 8 ways of dividing 3 vertices of a cube for d=P=3

Where P is the # of patterns for training and d is the # of inputs

$$\lim_{d \to \infty} (\#\ of\ LS\ functions) = \infty$$

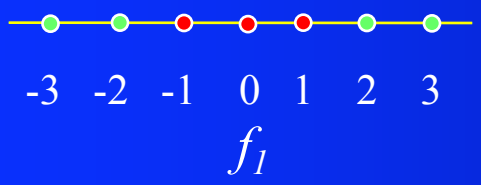
Linear Models which are Non-Linear in the Input Space

- So far we have used $$f(\mathbf{x},\mathbf{w}) = sign(\sum_{1=1}^{n} w_i x_i)$$
- We could preprocess the inputs in a non-linear way and do $$f(\mathbf{x},\mathbf{w}) = sign(\sum_{1=1}^{m} w_i \phi_i(\mathbf{x}))$$
- To the perceptron algorithm it looks just the same and can use the same learning algorithm, it just has different inputs - SVM
- For example, for a problem with two inputs x and y (plus the bias), we could also add the inputs x^2, y^2, and $x \cdot y$
- The perceptron would just think it is a 5 dimensional task, and it is linear in those 5 dimensions
  - But what kind of decision surfaces would it allow for the original 2-d input space?

Quadric Machine

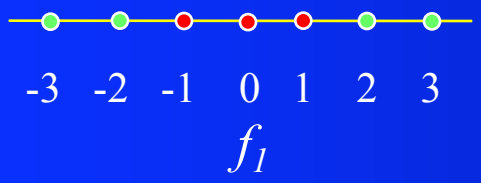
- All quadratic surfaces (2nd order)
  - ellipsoid
  - parabola
  - etc.
- That significantly increases the number of problems that can be solved
- But still many problem which are not quadrically separable
- Could go to 3rd and higher order features, but number of possible features grows exponentially
- Multi-layer neural networks will allow us to discover high-order features automatically from the input space

Simple Quadric Example



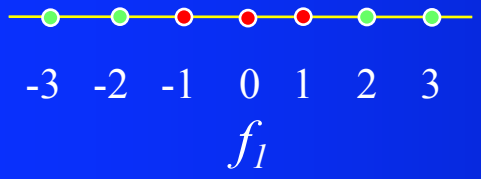
- Perceptron with just feature f_1 cannot separate the data
- Could we add a transformed feature to our perceptron?

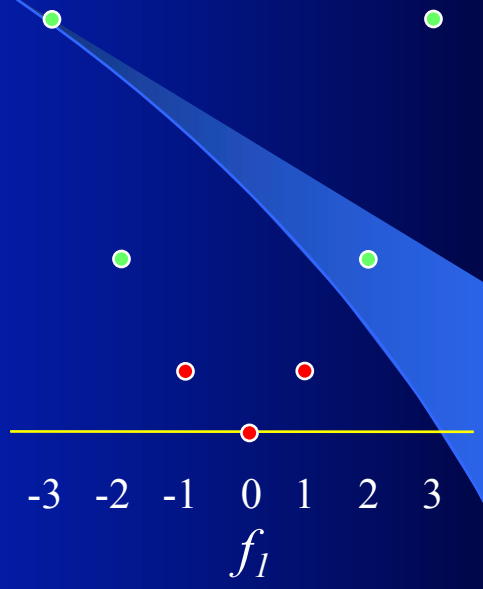
Simple Quadric Example



- Perceptron with just feature f_1 cannot separate the data
- Could we add a transformed feature to our perceptron?
- $f_2 = f_1^2$

Simple Quadric Example





- Perceptron with just feature f_1 cannot separate the data
- Could we add another feature to our perceptron $f_2 = f_1^2$
- Note could also think of this as just using feature f_1 but now allowing a quadric surface to separate the data

Quadric Machine Homework

- Assume a 2 input perceptron expanded to be a quadric perceptron (it outputs 1 if net > 0, else 0). Note that with binary inputs of -1, 1, that x^2 and y^2 would always be 1 and thus do not add info and are not needed (they would just act like two more bias weights)
- Assume a learning rate *c* of .4 and initial weights all 0: $\Delta w_i = c(t - z)\, x_i$
- Show weights after each pattern for one epoch with the following non-linearly separable training set (XOR).
- Has it learned to solve the problem after just one epoch?
- Which of the quadric features are actually needed to solve this training set?

x	y	Target
-1	-1	0
-1	1	1
1	-1	1
1	1	0