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Convex Hull report

Sec 001

1. Source code

```
#Tangent line class with index of left point in left array and index of right point in right array
class Tan:
	def __init__(self, indexL, indexR, slope):
		self.indexL = indexL
		self.indexR = indexR
		self.slope = slope

#Gets slope of two points
def slope(self, p1, p2):
	rise = p1.y() - p2.y()
	run = p1.x() - p2.x()
	return rise/run

#Returns the hull and the index of right most point
def clockwiseOrder(self, points):
	if len(points) == 1:
		return points, 0
	elif len(points) == 2:
		return points, 1
	else:
		clockwisePoints = [points[0]]
		slopeTo1 = self.slope(points[0], points[1])
		slopeTo2 = self.slope(points[0], points[2])
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        #In the rare case the slopes are the same we see which point is closer x-wise
        if slopeTo1 == slopeTo2:
            if points[1].x() < points[2].x():
                clockwisePoints.append(points[1])
                clockwisePoints.append(points[2])
                return clockwisePoints, 2
            else:
                clockwisePoints.append(points[2])
                clockwisePoints.append(points[1])
                return clockwisePoints, 1

        #If the slopes aren't equal, the point with the highest slope is next in clockwise ordering
        if slopeTo1 > slopeTo2:
            clockwisePoints.append(points[1])
            clockwisePoints.append(points[2])
        else:
            clockwisePoints.append(points[2])
            clockwisePoints.append(points[1])
        #Which of those points is right most?
        if clockwisePoints[1].x() > clockwisePoints[2].x():
            return clockwisePoints, 1
        return clockwisePoints,2

#Combines the two hulls
def combine(self, left, rmiL, right, rmiR):
    #This is the upper tan to start with, right most point of left hull and leftmost point of right hull
    #I'm using indexes to keep track of points in order to go counter clockwise or clockwise
    upperTan = Tan(rmiL, 0, self.slope(left[rmiL], right[0]))
    #self.showTan(left[upperTan.indexL],right[upperTan.indexR])
    moved1 = True
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        moved2 = True
        #While either side of the tangent has moved, we will continue to search for the upper tangent
        while moved1 or moved2:
            #We move the left point counterclockwise, if the new slope is less than the old slope, 
update the slope
            if self.slope(left[(upperTan.indexL - 1) % len(left)],right[upperTan.indexR]) < 
upperTan.slope:
                #self.eraseTan(left[upperTan.indexL],right[upperTan.indexR])
                upperTan.indexL = (upperTan.indexL - 1) % len(left)
                upperTan.slope = self.slope(left[upperTan.indexL],right[upperTan.indexR])
                #self.showTan(left[upperTan.indexL],right[upperTan.indexR])
                moved1 = True
            #If the new slope is greater than or equal to the old slope, do nothing
            else:
                moved1 = False

            #We move the right point clockwise, if the new slope is greater than the old slope, 
update the slope
            if self.slope(left[upperTan.indexL],right[(upperTan.indexR+1) % len(right)]) > 
upperTan.slope:
                #self.eraseTan(left[upperTan.indexL],right[upperTan.indexR])
                upperTan.indexR     = ((upperTan.indexR + 1) % len(right))
                upperTan.slope = self.slope(left[upperTan.indexL],right[upperTan.indexR])
                #self.showTan(left[upperTan.indexL],right[upperTan.indexR])
                moved2 = True
            #New slope was less than or equal than old slope, do nothing
            else:
                moved2 = False

        #Compute the lower tangent
        lowerTan = Tan(rmiL, 0, self.slope(left[rmiL], right[0]))
        #self.showTan(left[lowerTan.indexL],right[lowerTan.indexR])
        moved1 = True
        moved2 = True

        #We keep trying to find a better lower tangent if either of the points of lowerTan has changed
        while moved1 or moved2:
            #We move the left point of lowerTan clockwise, if the new slope is greater than the old,
we update the slope.
            if self.slope(left[(lowerTan.indexL+1) % len(left)],right[lowerTan.indexR]) >
lowerTan.slope:
                #self.eraseTan(left[lowerTan.indexL],right[lowerTan.indexR])
                lowerTan.indexL = (lowerTan.indexL+1) % len(left)
                lowerTan.slope = self.slope(left[lowerTan.indexL], right[lowerTan.indexR])
                #self.showTan(left[lowerTan.indexL],right[lowerTan.indexR])
                moved1 = True
            else:
                moved1 = False

            #We move the right point of lower tan counter clockwise, if the new slope is less than
old, update
            if self.slope(left[lowerTan.indexL], right[(lowerTan.indexR-1) % len(right)]) <
lowerTan.slope:
                #self.eraseTan(left[lowerTan.indexL],right[lowerTan.indexR])
                lowerTan.indexR = (lowerTan.indexR-1) % len(right)
                lowerTan.slope = self.slope(left[lowerTan.indexL], right[lowerTan.indexR])
                #self.showTan(left[lowerTan.indexL],right[lowerTan.indexR])
                moved2 = True
            else:
                moved2 = False

        #combine hulls with the upperTan and lowerTan
        newHull = []
        #gets leftmost of left to first point of uppertan
        newHull.extend(left[0:upperTan.indexL+1])

        #special cases when combining the rest of the hull
        if lowerTan.indexR < upperTan.indexR:
            newHull.extend(right[upperTan.indexR:])
            newHull.extend(right[0:(lowerTan.indexR+1) % len(right)])
        else:
            newHull.extend(right[upperTan.indexR:lowerTan.indexR + 1])
        if lowerTan.indexL != 0:
            newHull.extend(left[lowerTan.indexL:])



        #find new right most index
        newRmi = 0
        for i in range(len(newHull)):
            if newHull[i].x() > newHull[newRmi].x():
                newRmi = i

        #erase tans
        #self.eraseTan(left[upperTan.indexL], right[upperTan.indexR])
        #self.eraseTan(left[lowerTan.indexL], right[lowerTan.indexR])

        #erase hulls
        #self.eraseHull(left)
        #self.eraseHull(right)

        return newHull, newRmi


    #This is the divide and conquer function
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def getHull(self, points):
    #Base case: if the hull is less than 4 points we create a base hull
    if len(points) < 4:
        return self.clockwiseOrder(points)
    #We split the hull in 2 and give the left most half of points to getHull
    left, rmiL = self.getHull(points[0:math.ceil(len(points)/2)])
    #self.showHull(left)

    #We take the right half of the hull and give it to getHull
    right, rmiR = self.getHull(points[math.ceil(len(points)/2):])
    #self.showHull(right)

    #We then take both halves of hull and combine them
    return self.combine(left, rmiL, right, rmiR)

#End my functions
```

2. Time and Space Complexity

Time

The overall time complexity of the algorithm is O(n * log(n)) where n is the number of points.

The divide-and-conquer function “getHull” divides the number of points in two parts, each one half the size of the original array, and the combine method is puts the hulls together in linear time. Using the Master’s Theorem, a = 2, b = 2 and c = 1. Since log2(2) == 1, logb(a) = c, so the time complexity is O(n * log(n))

The “combine” method is O(n) because worst case, we iterate through the total number of points (number of points in the left hull + number of points in the right hull) to find the right most point of the combined hull.

The “clockwiseOrder” method is done in O(n) time in the worst case were we have 3 points and decide which point is next in the clockwise ordering.

The “slope” function is done in constant time.

Space

The overall space complexity of the algorithm is O(n) where n is the number of points. When we divide, we keep passing the original points to new calls of “getHull”, and split that in half until we have less than 3. This means there are never more than n points existing simultaneously.

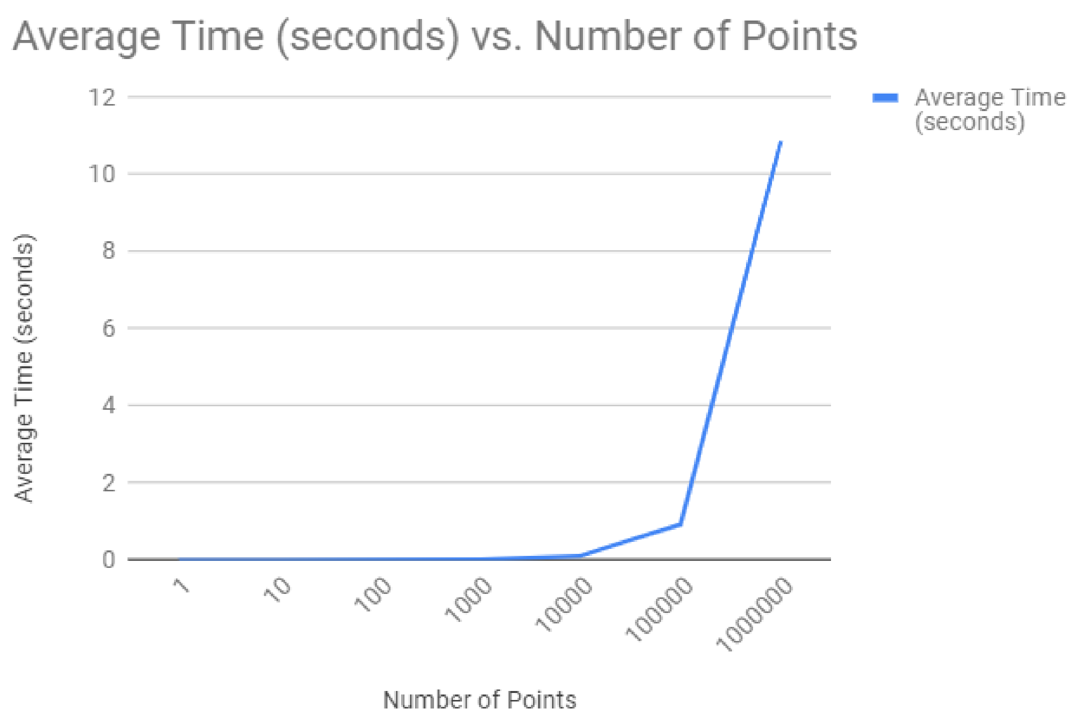
The divide-and-conquer method “getHull” splits the total number of points in half until there are only three, so the most points that exist are n.

The “combine” method only stores 4 extra points worst case which is when the hulls contain all the points of their halves and the tangents also contain two points from each.

The “clockwiseOrder” only uses the number of points it is given so its space complexity is O(n)

The slope function space complexity is O(n) where n is 2.

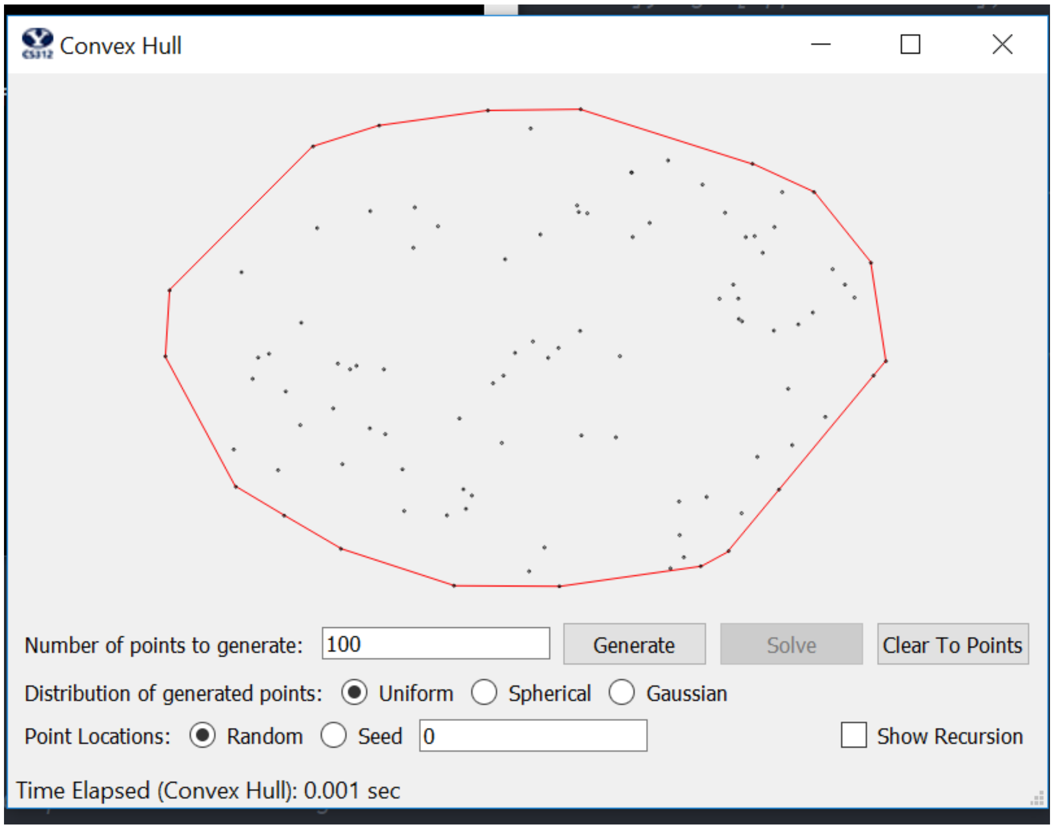
3. Raw experimental outcome



After taking the average of ten tests for each respective number of points, I found the trend above.

The pattern of the plot most closely follows n * log(n)

4. Screenshots


Convex Hull

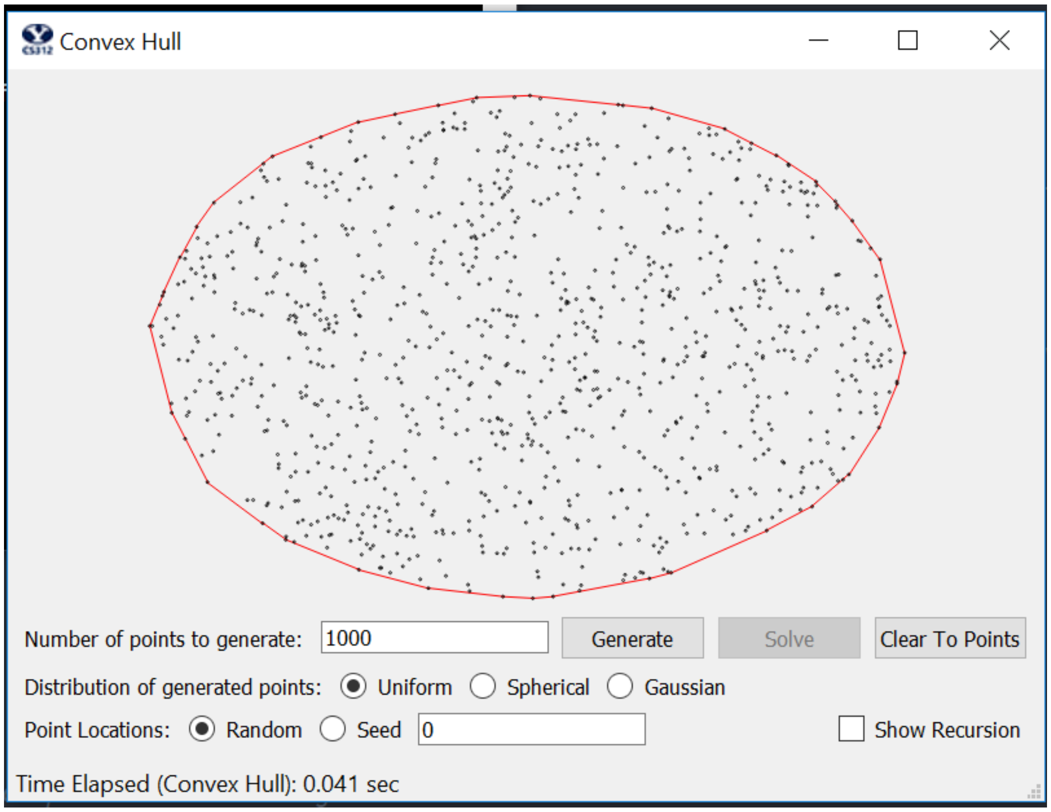
Number of points to generate: 100

Distribution of generated points: [✓ Uniform] Spherical Gaussian

Point Locations: [✓ Random] Seed 0

Show Recursion

Time Elapsed (Convex Hull): 0.001 sec


Convex Hull

Number of points to generate: 1000

Distribution of generated points: [✓ Uniform] Spherical Gaussian

Point Locations: [✓ Random] Seed 0

Show Recursion

Time Elapsed (Convex Hull): 0.041 sec