The Turducken Cooking Challenge

ME-408 Final Project
December 11, 2017

Pranav Joneja,
Mingyang Lee
Yueyue Li,
Jacob Maarek

Table of Contents

Introduction

The objective of this project is to determine a valid cooking process of a Thanksgiving turducken using a conventional oven and two heating skewers. The dish consists of three parts: The outer layer of turkey, an intermediate layer of ducken and some stuffing on the inside. The goal is to fully cook the meat as fast as possible while minimizing the volume that is burnt. Finite Element Analysis software is used to perform a transient thermal analysis to determine the cooking process necessary to achieve this goal. The parameters that define the cooking process are related to the two boundary conditions: the oven and the skewers.

Design & Model Assumptions

1. All quantities are expressed in U.S. Imperial units (inches, pounds force, Fahrenheit).
2. The whole bird is initially fully refrigerated, which means the temperature at the start of the cooking process is 42°F.
3. The bird is defined as fully cooked when no part has a temperature less than 185°F.
4. At the end of the cooking process, any part of the bird with a temperature above 265°F is considered burned. These parts should be counted as part of the burned volume.
5. The oven is considered to be a typical domestic convection oven that operates uniformly with a single convection coefficient. The temperature of the oven can be changed exactly once by the customer and such a change should be modeled as a step change (i.e. the oven itself has no thermal mass and does not need to be simulated directly).
6. The two electric resistance skewers are identical to each other in every way and are assumed to provide a constant heat flux through their outside surface during operation. The operating state of the skewers can be changed exactly once (from on to off or vice versa) as long as the time when they are switched is specified.
7. All food materials are homogeneous mixtures. The material properties of the mixtures are assumed to be an average of the components that constitute them.
8. The food materials that make up the turducken have non-linear material properties due to
    a. Variations in water content as the food dries during cooking, and
    b. Variations in physical property when the meat burns.
9. A symmetry plane cuts through the turducken so only one half is analyzed for computational efficiency.

Altair HyperMesh and ANSYS Workbench were the Finite Element Analysis software packages chosen for this design project. HyperMesh was used to discretize the turducken geometry into a mesh. ANSYS Workbench was used to analyze the cooking process as a transient thermal simulation involving non-linear material properties, varying boundary conditions and adaptive time-step solving.

Approach

Oven Cooking Approach

At the start of the cooking process, the fully refrigerated turducken is very cold (42°F). In order to achieve the fastest cooking time, the oven should initially be set its maximum temperature setting to quickly bring up the temperature of the bird. This is because convection from the oven on the turkey is directly related to the difference between the temperature of the oven and temperature of the turkey skin. By setting the oven to its maximum setting of 500°F, the turkey will initially heat up rapidly, and therefore minimize overall cook time.

Then, after a period of time when the skin of the turkey begins to burn, the oven should be set at its lowest temperature setting (350°F). This will minimize the total burnt volume in the long term and allow the temperature to distribute more evenly for the duration of the cooking process.

The appropriate amount of cooking time at the high and low oven temperature settings can be determined after running the transient thermal analysis a few times.

Skewer Design Concept

The skewer is intended to help cook the inside of the turducken by providing a direct heat flux that conducts through the interior of the bird. This should help make the overall temperature gradient more uniform during the cooking process and thus help speed up cooking time.

Without any other influence, the convection from the oven will cook the outside (turkey) before cooking the interior (ducken, stuffing). In addition, looking at material properties, the stuffing as a whole should cook faster than the ducken. A hand calculation of the cooking process (described in detail later) verifies this result. Through this process of elimination, the assumed approach is to design the skewer to cook the ducken directly. This approach means the skewer should pierce the turkey and ducken but only provide a heat flux to the ducken directly. The skewer should be switched off when too much of the ducken has burn. The exact placement of the skewer is described in the 'geometry' section.

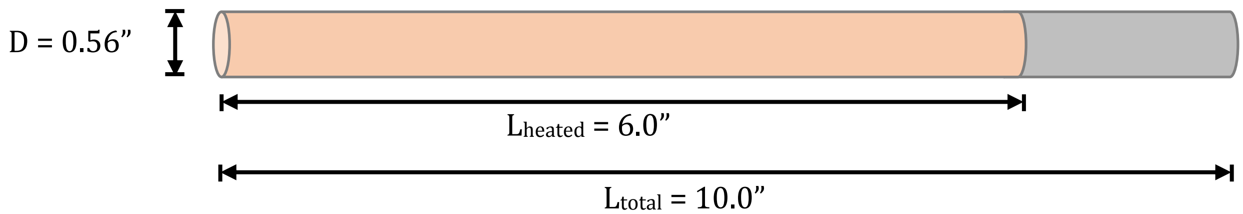
The chosen material for the skewer is silver because it has a very high thermal conductivity, thus maximizing the heat flux through its surface into the turkey. In addition, silver is food-safe. The appropriate magnitude of heat flux can be determined after running the transient thermal analysis a few times.

	Silver
Thermal Conductivity *[BTU/hr.ft.°F]*	247.87
Density *[lbm/in^3]*	0.379
Specific Heat Capacity *[BTU/lb.°F]*	0.057
Melting Point *[°F]*	*1760*

Figure 1: Silver material properties for skewer design from Engineer's Edge [2]

Geometry

Skewer Geometry



	Silver
Cross Sectional Area *[in²]*	0.25
Diameter *[in]*	0.282
Total Length *[in]*	10.0
Heated Length *[in]*	6.0
Area of Heated Curved Surface *[in²]*	10.63

The skewer is designed to be a simple cylindrical rod shape with constant cross-sectional area of 0.25”. A circular cross-section is chosen because it maximizes the area of the curved surface that will provide a heat flux to the ducken. In the finite element model the heat flux is only applied to portion of the ducken in direct contact with this outside this curved surface of the skewer. Selecting this portion means the heated length of the skewer is 6.0” The circular area at the tip is negligible and is not included.

Skewer Placement

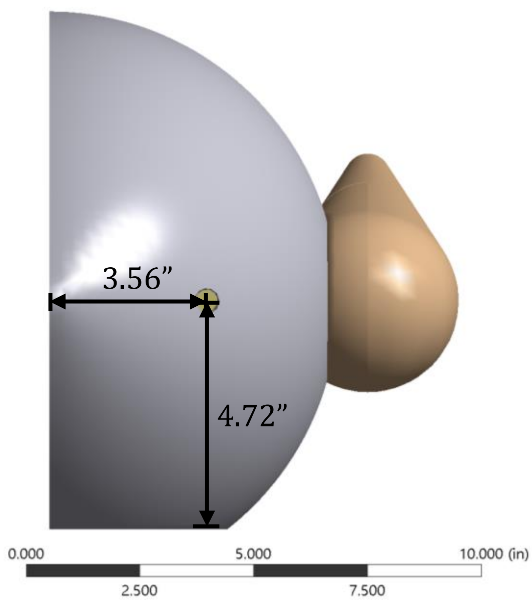


Figure 2: Front view of geometry showing skewer insertion point

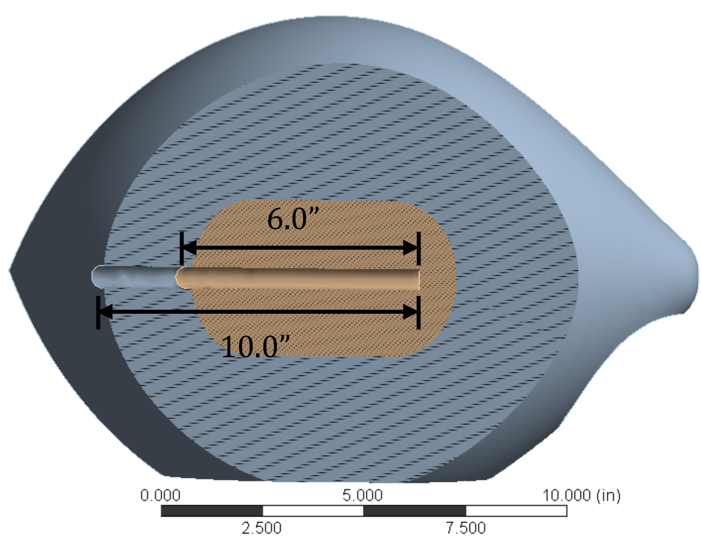


Figure 3: Section view showing depth of skewer insertion

For the purposes of this description, the 'back' of the turducken refers to the region where there is an opening for the stuffing void. As such, the 'front' of the turkey refers to the region diametrically opposite to the back.

From the front of the turducken, the skewer is inserted 3.56" from the plane of symmetry and 4.72" above the plane of the base and parallel to the base, as shown in Figure 2. The skewer is inserted a total depth of 10.63" such that the heated length of 6.0" is contact with the ducken only, as shown in Figure 3. Once the skewer is properly inserted, it should not be touching the stuffing at all (see Figure 4).

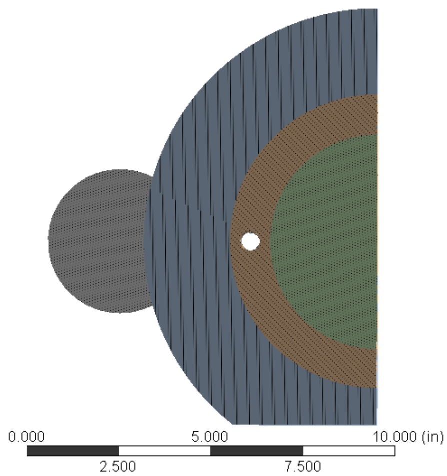


Figure 4: Section view showing internal location of skewer

Turducken Geometry

The geometry of the turducken was provided and is considered fixed. It is approximately 18'' in length and width, and approximately 10'' in height. The volume of the turkey is about 1100 in^3.

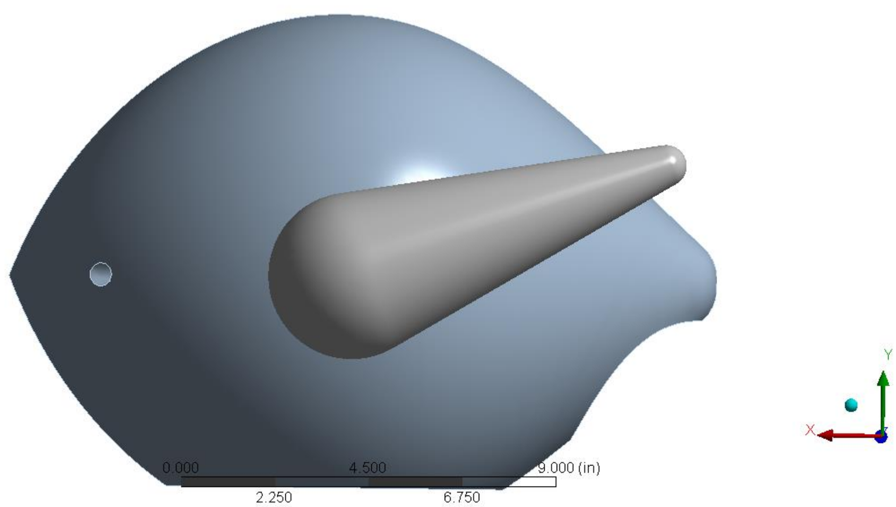


Figure 5: Outside geometry of the turkey and wing

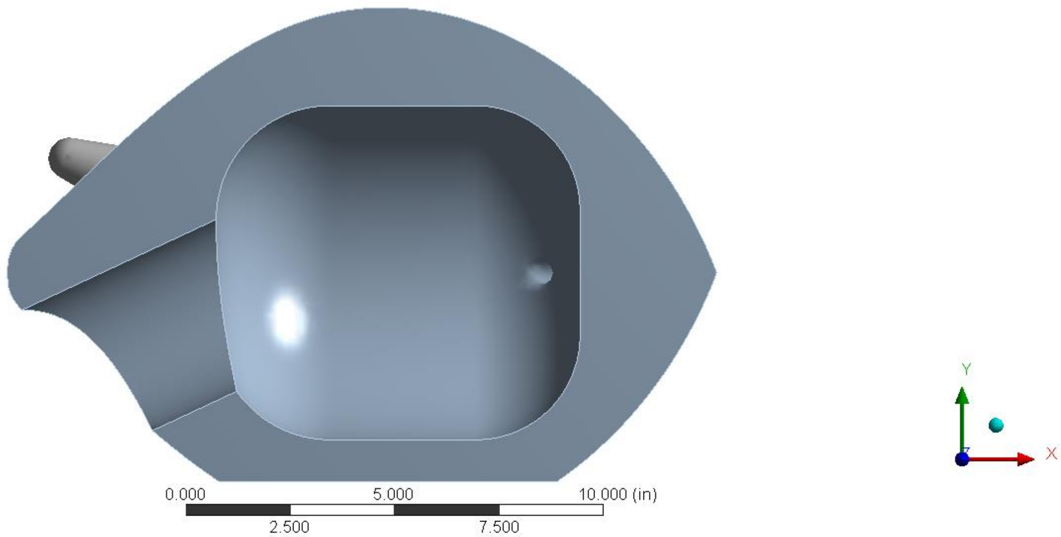


Figure 6: Symmetry plane view of turkey with inside void for ducken and stuffing

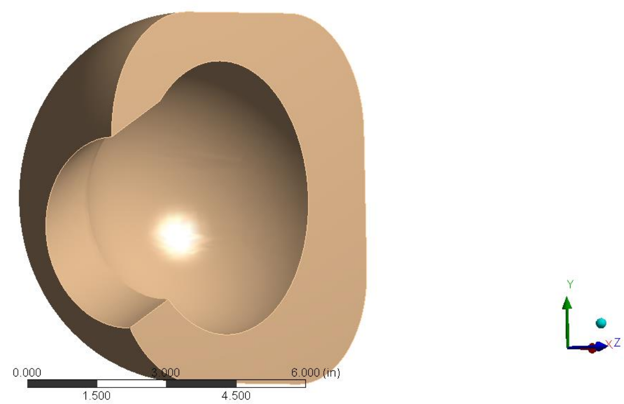


Figure 7: Three-quarters view of the ducken with the void for stuffing

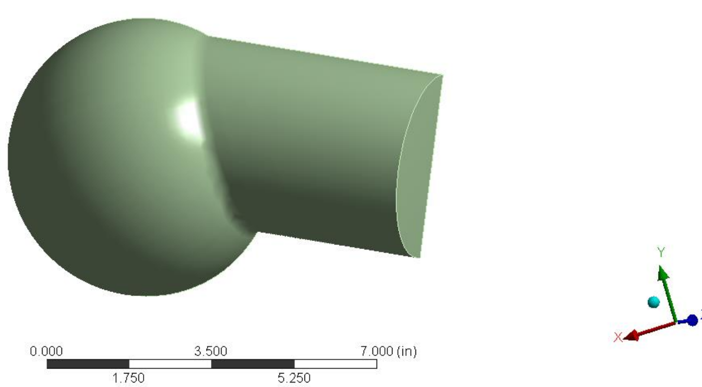


Figure 8: Opposite side view of the stuffing

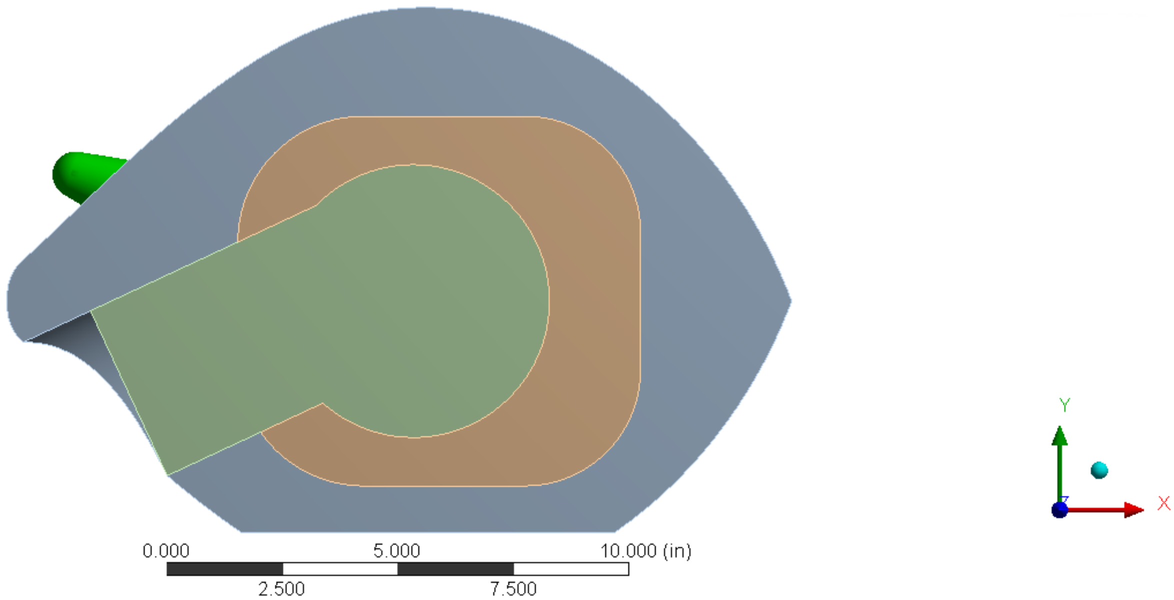


Figure 9: Symmetry plane view of the turducken as a complete assembly

Materials

Properties of Interest

For a transient thermal analysis, the pertinent material properties are thermal conductivity, density and specific heat capacity for each of the food materials in the turducken.

All of these material properties are dependent the proportions of food components: water, protein, fat, carbohydrates, fiber and ash. These material properties also change as temperature increases due to chemical and physical transformations that take place within the material.

Food Components

Figure 10 shows the composition of three turducken materials at 42°F.

	TURKEY	***DUCKEN***		***STUFFING***	
% Composition	*100%*	*80%*	*20%*	*50%*	*50%*
Food Components	**-**	***Duck***	***Chicken***	***Peas***	***Potatoes***
Water	0.7	0.49	0.66	0.79	0.79
Protein	0.2	0.11	0.19	0.054	0.021
Fat	0.08	0.39	0.15	0.004	0.0001
Carbohydrates	0	0	0	0.14	0.18
Fiber	0	0	0	0.051	0.016
Ash	0.0088	0.0068	0.0079	0.0087	0.0089

Figure 10: Nutrient composition of the three turducken materials. Source: 2006 ASHRAE Handbook [1]

Sample Calculation

A sample calculation of the thermal conductivity (k), density (ρ), and specific heat capacity (c) of turkey at 42°F is carried out below.

First, calculate the k_i, ρ_i, c_i values at 42°F for water, which is one component of turkey.

$$k_w = 3.1064 \times 10^{-1} + 6.4226 \times 10^{-4} t - 1.1955 \times 10^{-6} t^2$$

$$\rho_w = 6.2174 \times 10^{1} + 4.7425 \times 10^{-3} t - 7.2397 \times 10^{-8} t^2$$

$$c_w = 9.9827 \times 10^{-1} - 3.7879 \times 10^{-5} t + 4.0347 \times 10^{-7} t^2$$

Similar equations for other food components (protein, fat, …) can be found in the same chapter of the 2006 ASHRAE Handbook [1].

The overall k, ρ, c for turkey are found by computing the weighted average as follows:

$$k = \sum x_i k_i = x_w k_w + x_{protein} k_{protein} + x_{fat} k_{fat} + x_{carb} k_{carb} + x_{fiber} k_{fiber} + x_{ash} k_{ash}$$

Where x is the fraction of the food component in the material and k is the conductivity of that component only. The density and specific heat capacity is computed in a similar way and shown in Figure 11.

	Nutrient Composition	k (BTU/h.ft².F)	ρ (lb/ft³)	c (BTU/lb.°F)
Water	0.7	0.34	62.17	1.00
Protein	0.2	0.11	76.05	0.48
Fat	0.08	0.10	58.2	0.37
Carbohydrates	0	0	0	0
Fiber	0	0	0	0
Ash	0.0088	0.0026	151.6	0.26
	Weighted Average	**0.26**	**64.72**	**0.82**

Figure 11: Material properties for Turkey at 42°F

Temperature choices for tabular material properties

As the temperature increases, chemical and physical transformations take place within the material. It is important to recalculate material properties at these particular temperatures.

Temperature (°F)	Selection Reason
42	Initial temperature
212	Water boiling point
213	All liquid water vaporized, thus no water content
265	Last temperature point before the material turns to ash
266	Assume everything besides water turns to ash
2000	Upper limit, assume same properties from T = 266

Figure 12: Temperature points of interest for tabular material property data

Putting it all together, the tabular material data that were input into ANSYS are presented in Figure 13 below.

	Temp. *(°F)*	***Thermal Conductivity k*** *(BTU/h.ft²*.F)*	***Density ρ*** *(lb/ft³)*	***Specific Heat Capacity c*** *(BTU/lb.°F)*
TURKEY	42	0.265	64.721	0.824
	212	0.314	58.609	0.814
	213	0.038	15.051	0.134
	500	0.040	13.181	0.135
	4000	0.040	13.181	0.135
DUCKEN	42	0.224	63.282	0.710
	212	0.255	59.336	0.708
	213	0.050	26.984	0.203
	265	0.050	25.776	0.204
	266	0.005	72.771	0.156
	4000	0.005	72.771	0.156
STUFFING	42	0.224	63.282	0.710
	212	0.316	52.218	0.789
	213	0.006	3.150	0.022
	265	0.006	2.800	0.022
	266	0.006	1.169	0.022
	4000	0.006	1.169	0.022

Figure 13: Non-linear material data table input into ANSYS

Boundary Conditions

The boundary constraints can be thought of as the methods through which the turducken is cooked. The boundary conditions are therefore chosen in order to meet the stated goals of minimizing total cooking time while also minimizing burnt volume at the end of the cooking process. These boundary condition choices are limited by the real-world capabilities of the cooking tools used. The intent and thought process in selecting boundary conditions are presented in the sections below.

Convection from Oven

The idealized oven in this project has a uniform convection film coefficient (h). To determine a realistic value for h for this analysis, a research paper on "Measurements of heat transfer coefficients within convection ovens" was consulted [3]. The authors of this paper back-calculated from transient temperature data collected empirically to determine a range of convection coefficients that the selected ovens could operate at. The ovens in their study were a typical domestic fan oven and a commercial batch oven. From their results, they found that the typical range of h values was between 15 and 40 W m^{-2} K^{-1} [3]. The idealized oven in this turducken project calls for a single h value, so a number in the middle of this range was chosen: 30 W m^{-2} K^{-1} (1.0192×10^{-5} BTU/s-in^2-°F)

The convection boundary condition is applied only on the surfaces of the turducken that are exposed to the air inside the oven directly. These surfaces are highlighted in Figure 14.

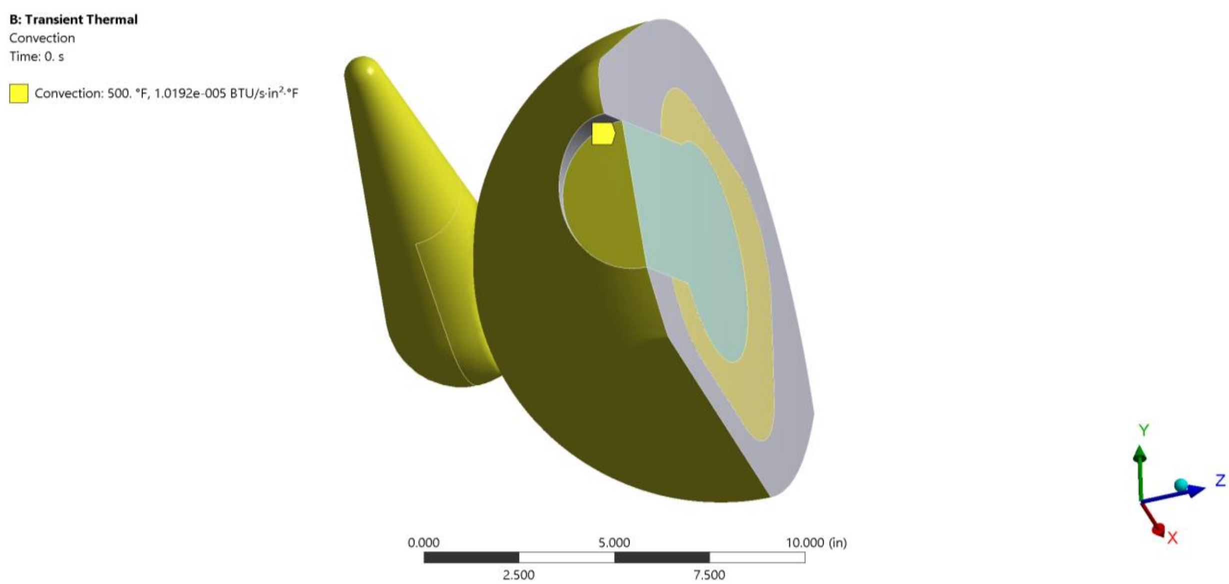


Figure 14: Convection Boundary Condition applied to exterior surfaces of the turducken

Note the gap in the boundary condition applied near the contact between the turkey and the stuffing. This gap is the result of the fact that this surface is continuous into the interior of the turkey (see Figure 5). Selecting this surface to apply the boundary condition means creating an inaccurate solution where the convection heats up the interior of the turkey. Leaving it unselected is the best choice, however it still does affect the results of the FEA.

The remaining choice is deciding what temperature to set the oven to. As outlined previously in the 'approach' section of this paper, the best choice is to start the cooking process at the highest temperature setting to quickly heat up the refrigerated meat and thus minimize the overall cook time. When the 'skin' of the turkey starts to burn, the oven temperature should be set to its lowest temperature setting to prevent further burning.

After running the transient thermal analysis a number of times, through trial and error, a decision was made:

The optimal convection boundary condition is the oven temperature set to 500°F for the first 30 minutes followed by the oven temperature set to 350°F for the remainder of the cook time. The film coefficient should be 1.0192×10^{-5} BTU/s-in^2-°F throughout.

Heat Flux from Skewer

The purpose of the skewer is to heat up the turducken from the inside, thus helping to keep the temperature gradient more uniform and speed up total cooking time. A microwave is an existing kitchen appliance that heats food with the same idea. Even though a microwave works primarily through radiation (and some convection), the total energy supplied by a microwave can be used as a comparable metric for designing how much energy the skewer should provide through heat flux.

A typical microwave can provide a maximum of 1200 Watts. When cooking refrigerated meat in a microwave, it's a good idea to use the 'defrost' setting [citation needed]. The defrost setting pulses the microwave radiation so that the total power delivered to the food is a fraction of the maximum. The defrost setting for meat can be as low as 10% of the maximum, effectively providing 120 Watts. When defrosting turkey in a microwave, the U.S. Dept of Health and Human Services suggests 6 minutes per pound of turkey[4]. The previously reported approximate volume of the turkey is 1100in^3 and the density at 42°F was computed to be approximately 64.7 lb/ft^3. A Fermi estimation of the total time required to defrost a turkey in a microwave follows:

$$1200\,Watts \times 10\% = 120\,Watts$$

$$6\frac{min}{lb} \times 64.7\frac{lb}{ft^3} \times \left(\frac{1ft}{12\,in}\right)^3 \times 1100in^3 = 247.11\,min \approx 240\,min\ or\ (2\,hours)$$

A skewer can be designed to provide an equivalent amount of power through heat flux for an equivalent duration (hence, the same amount of energy). The previously reported heated "active" area of the skewer is 10.63 in^2. Thus, the required power is:

$$\frac{120\,Watts \times \left(3.412\frac{BTU}{hr}\right) \times \left(\frac{1\,hour}{3600sec}\right)}{10.63in^2} = 1.07 \times 10^{-2}\frac{BTU}{s\,in^2}$$

For the same duration of 2 hours.

The skewer design concept calls for the heat flux to only be applied to the surface in contact with the ducken directly, as shown in Figure 15.

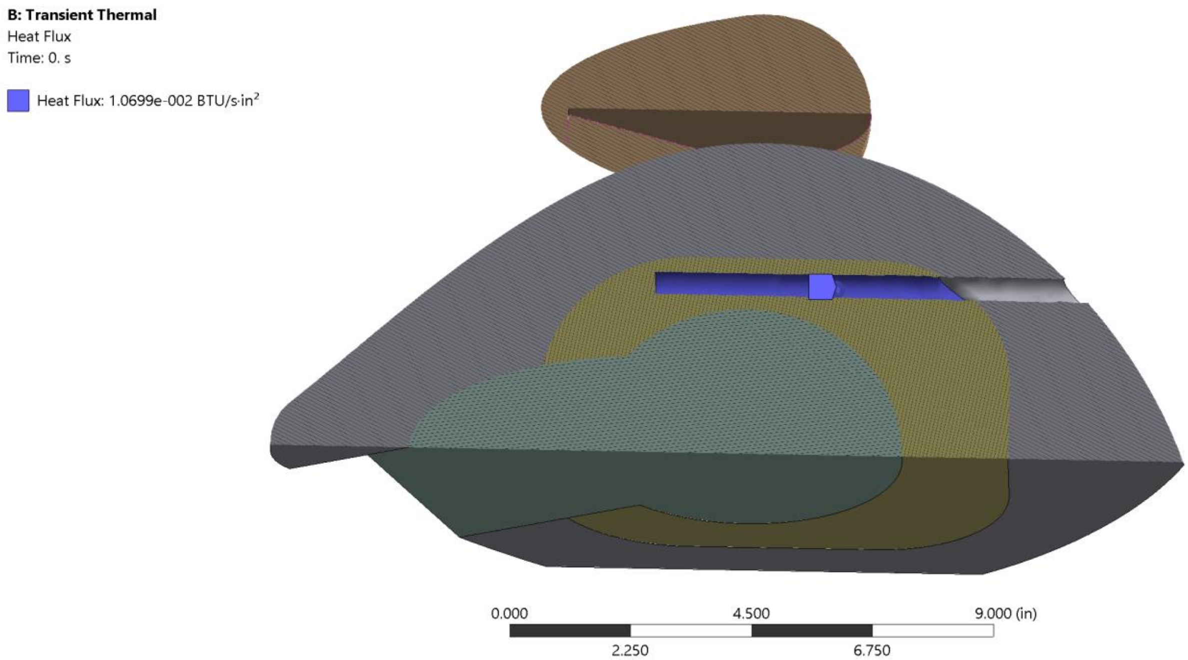


Figure 15: Oblique top section view showing the surface of the ducken where the skewer heat flux BC is applied.

Other Boundary Conditions considered

Radiation

If the turducken is considered as a black body, the total power radiated is proportional to $\sigma(T_{turducken}^4 - T_{oven}^4)$, where σ is the Stefan-Boltzmann constant and is extremely small. Thus radiation is negligible and does not need to be accounted for in the finite element model.

Furthermore, the research paper on convection in ovens suggests that the range of the convection coefficients actually already includes the effects of radiation inside the oven [3]. The h value effectively includes the total net effects of both radiation and convection together.

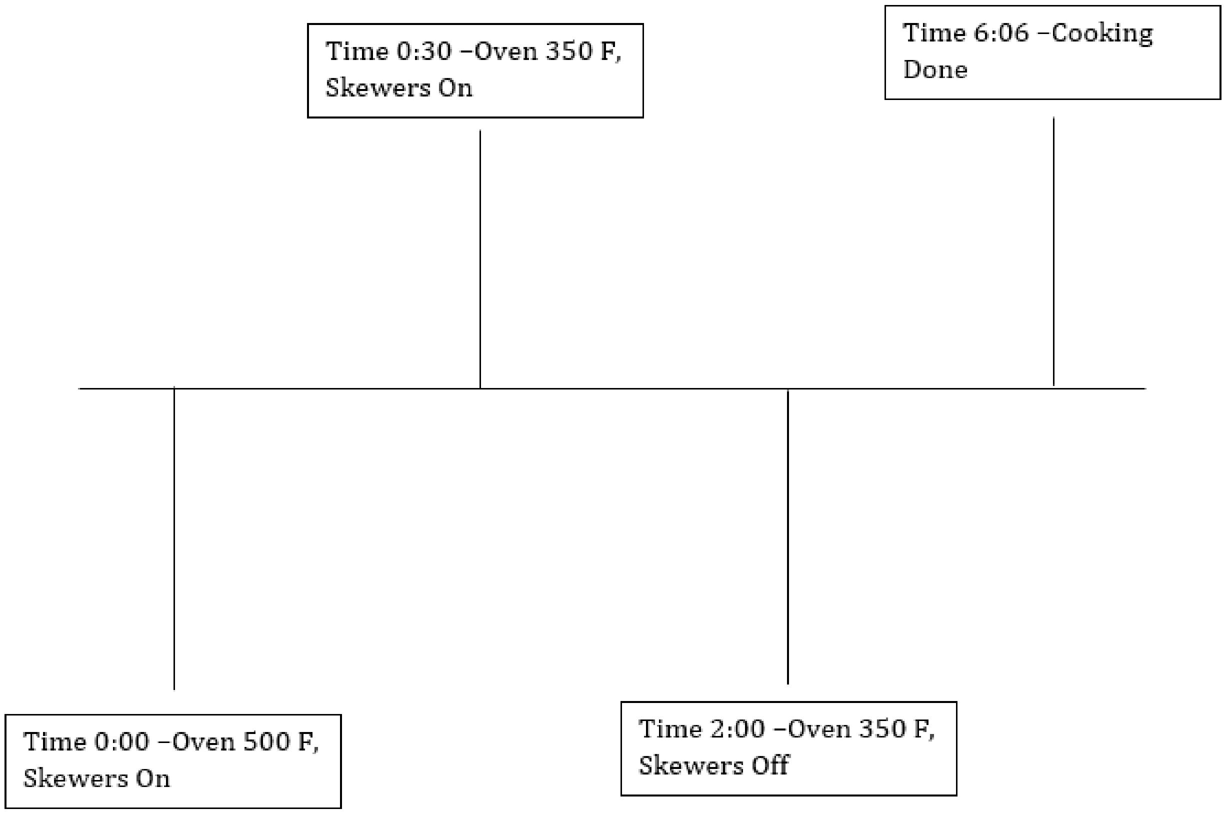
Conduction through oven rack

In real life, the turducken would be placed on a metal rack in the oven. However, the surface area of the turducken that would be in contact with the metal would be relatively small. Furthermore, if the rack is made of stainless steel, then the limited thermal conductivity of the steel means that very little heat conducts through the rack. In effect, it is not necessary to model this boundary condition in this thermal analysis.

Heat Flow

No heat flows through the symmetry plane of the turducken.

Summary


Time 0:30 –Oven 350 F, Skewers On
Time 6:06 –Cooking Done
Time 0:00 –Oven 500 F, Skewers On
Time 2:00 –Oven 350 F, Skewers Off

Meshing

Since the transient analysis will be run at many, many time steps, it is important to reduce the total element count so that the simulation is computationally efficient. In terms of meshing, the goal should be to create as many mapped regions as possible since mapped volumes have up to four times fewer elements than tetrahedral meshed ones.

To ensure an adequate element density, it was decided that there needs to be at least 20 elements through the diameter of the stuffing opening. Since this diameter is 0.1 units, the chosen element size is 0.005 units. For consistency in meshing, this element size is held constant throughout the turducken mesh.

To create mappable volumes, the turducken geometry was sliced using the solid edit tool in HyperMesh. The goal was to great prism-like volumes that can be mapped one after another. The yellow, translucent volumes in Figure 16 show that the after careful solid editing, it is possible to make the vast majority of the turducken volume mappable.

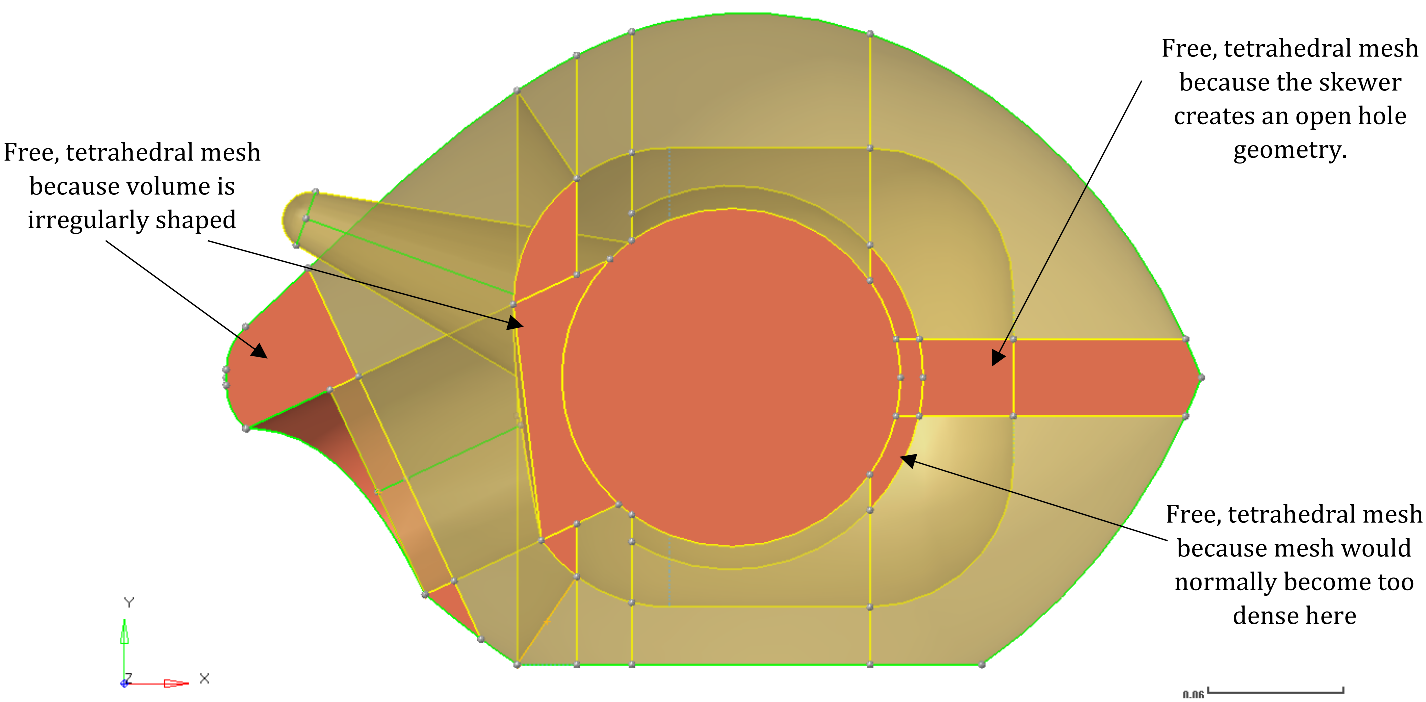


Figure 16: Mappable volumes shown in HyperMesh after solid editing. Translucent yellow means mappable.

The meshing approach was to start at the outside (turkey layer) and work towards the center. When the mesh gets inevitably gets too dense, the idea was to switch to a free tetrahedral mesh for a small, thin volume to return to an adequate element size (0.005).

Lastly, the hemisphere at the center of the stuffing was meshed using hexacore because it is over-constrained by the mapped meshes surrounding it and it’s too large to only use tetrahedral mesh. Instead, the hexacore method allows some mesh efficiency to be regained.

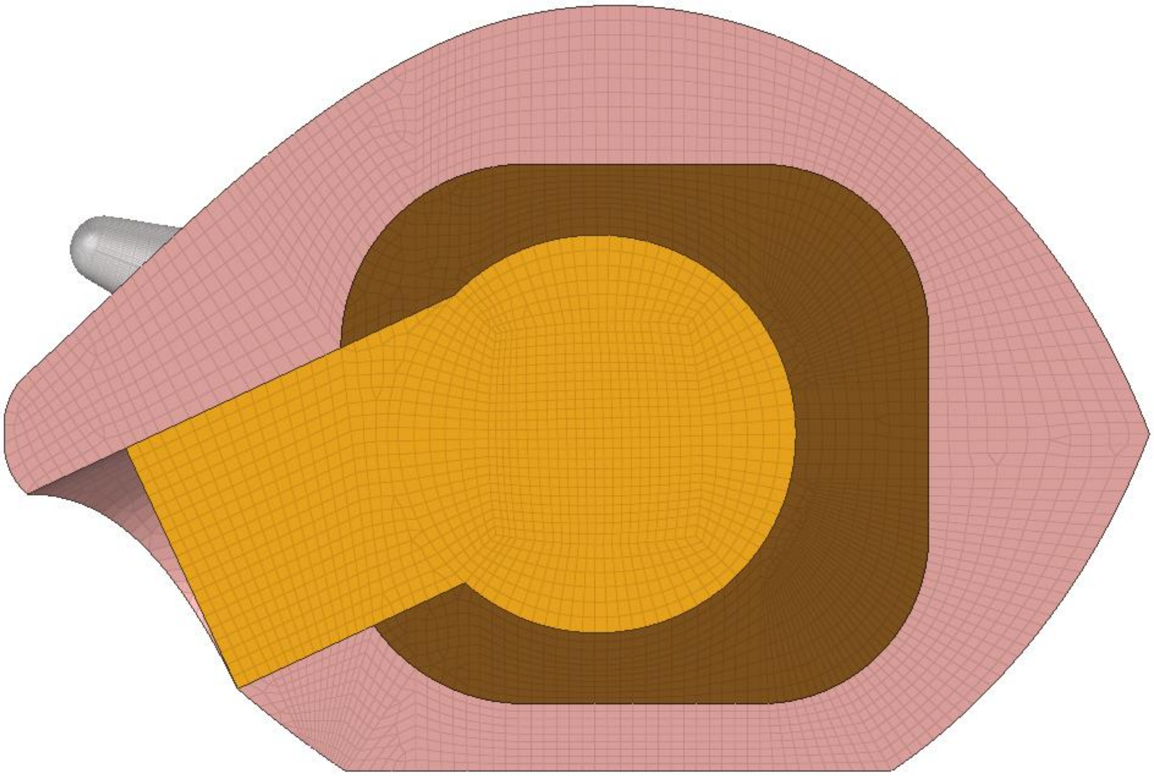


Figure 17: Symmetry plane view of meshed turducken

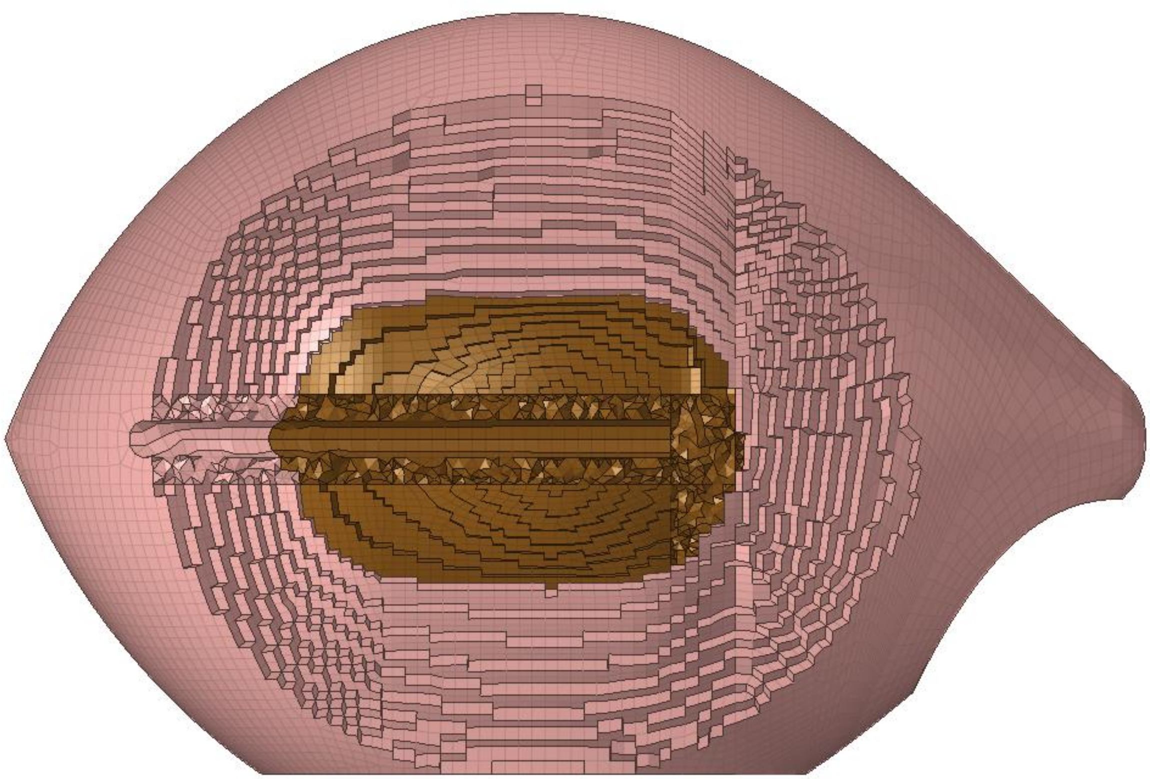


Figure 18: Opposite to symmetry plane view with cutaway to show mesh around the skewer

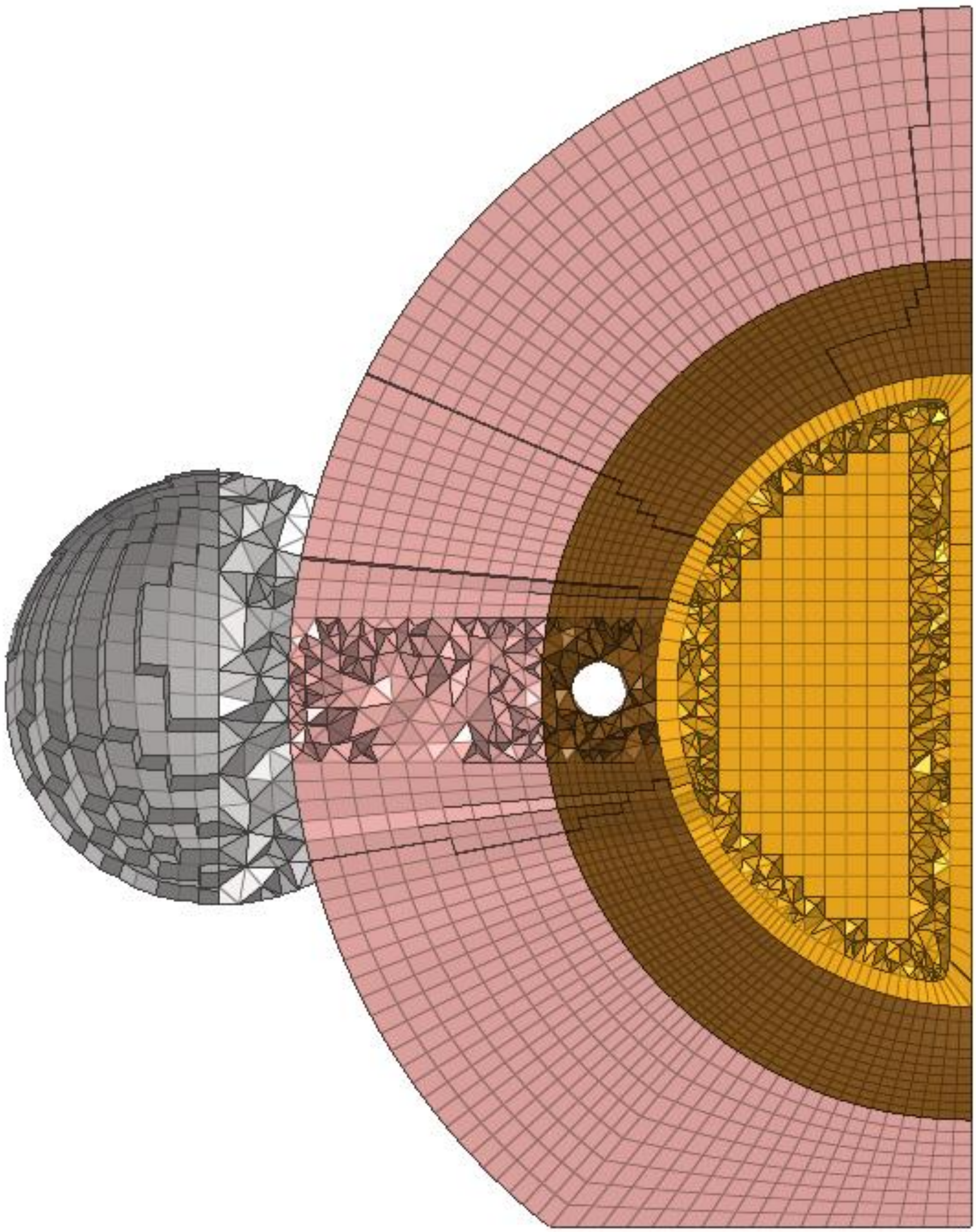


Figure 19: Section view from 'back' of turducken showing mesh

The total counts are <u>355,158 elements</u> and <u>159,417 nodes</u>.
All elements are exported as <u>SOLID185</u> type.

Mesh Quality

The quality of the mesh is assessed by the metrics like element Jacobian and aspect ratio.

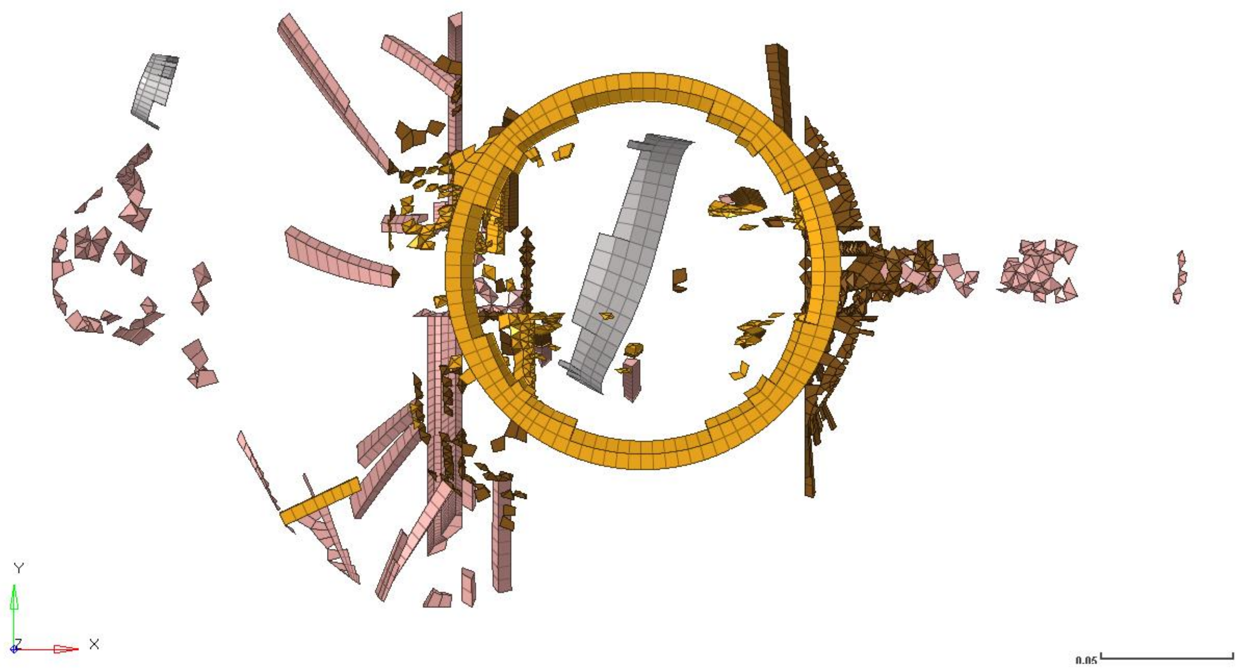


Figure 20: A masked plot of all elements with Jacobian < 0.7

There are 2,560 elements (0.7%) that have a Jacobian less than 0.7. Figure 20 shows a masked plot of only these elements. Figure 21 is a histogram of all elements by Jacobian.

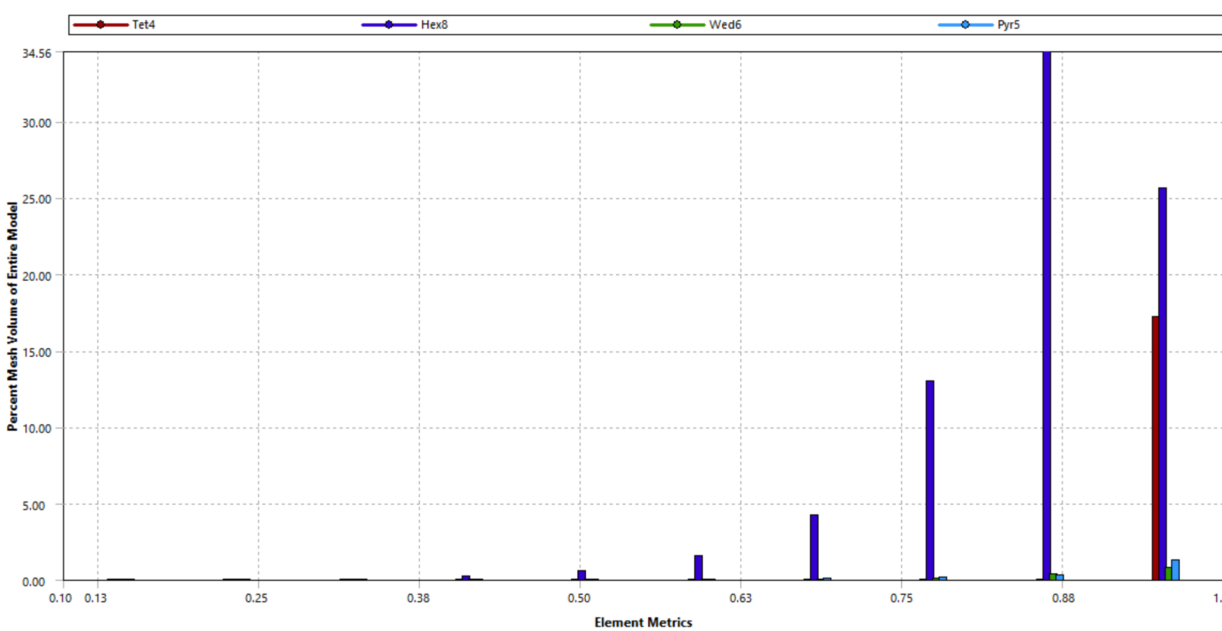


Figure 21: Histogram of element Jacobians, grouped by element shape.
The vast majority of the volume is meshed with elements that have a Jacobian greater than 0.7 or even 0.75.

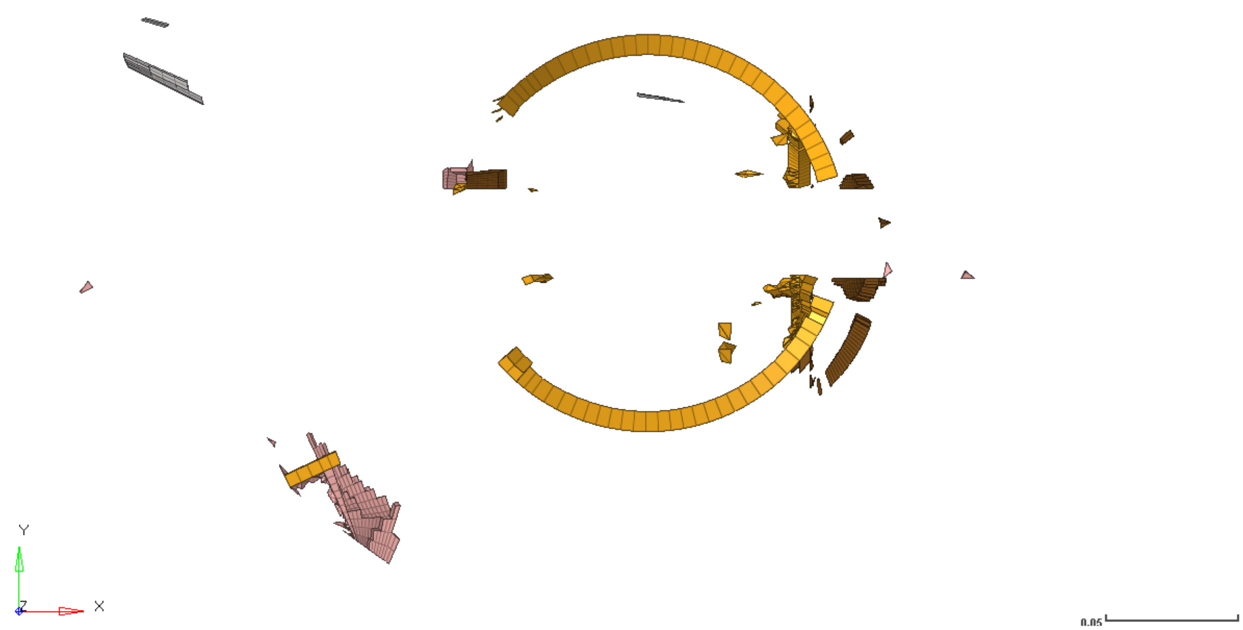


Figure 22: A masked plot of all elements with aspect ratio > 5.0

There are 1,183 elements (0.3%) with an aspect ratio greater than 5.0. Figure 22 shows a masked plot of only these elements. Figure 23 is a histogram of all elements by aspect ratio.

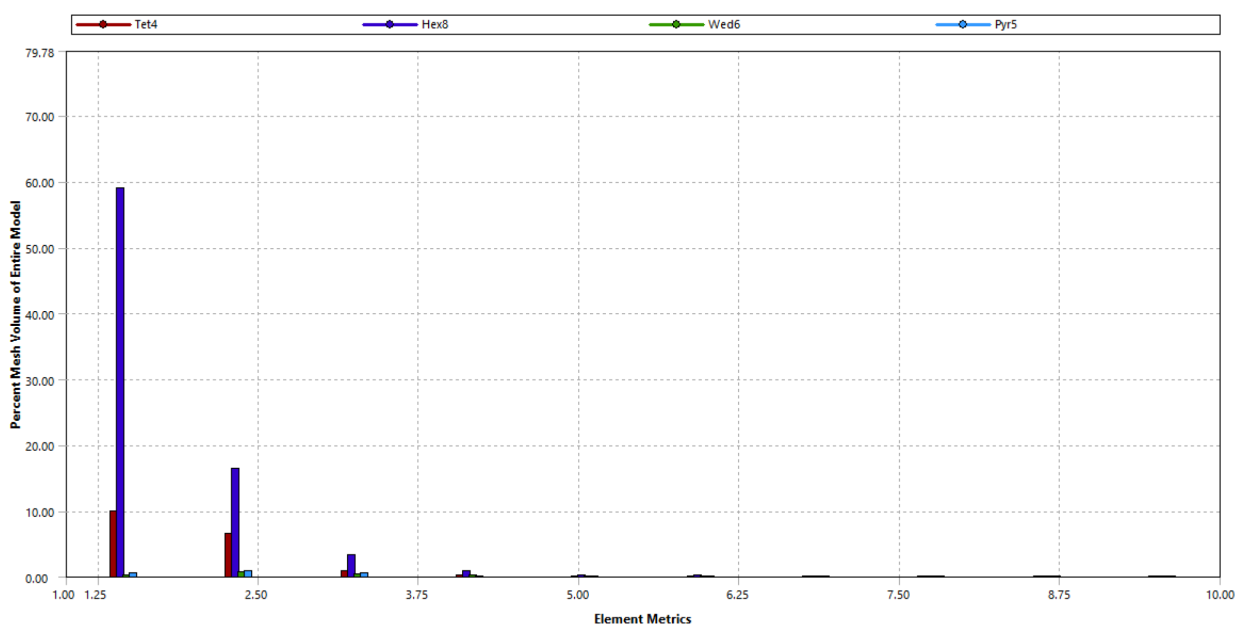


Figure 23: Histogram of elements by aspect ratio.
The vast majority of the volume is meshed with elements with aspect ratio less than 5.0.

Overall, the mesh metrics (Jacobian and aspect ratio) show that the quality of the mesh is very high. The element shapes (tetrahedral, pyramid, wedge and hexahedral) are all compatible with the chosen element type: SOLID185.

Results

With the previously reported boundary conditions and material data, the transient finite element model gives the following results:

The total cooking time was 6 hours and 6 minutes. At the end of the cooking time, approximately 41% of the volume was burned.

Summary of Plots

Figure 24 is a table summary of the result plots in the rest of this section.

Elapsed Cook Time (hh:mm)	*Oven Temp. (°F)*	*Skewer On?*	*Figure*	*Description*
0:00	500	On	-	Initial set-up, turkey fully refrigerated
0:30	500 → 350	On	24	Turkey 'skin' starts to burn
1:15	350	On	25	Skewer region is way too hot
2:00	350	On → Off	26	Skewer region is way, way too hot
4:00	350	Off	27	Cooked region has started to grow
6:00	350	Off	28	Almost done cooking
6:06	350	Off	29	Fully cooked, 41% burned volume

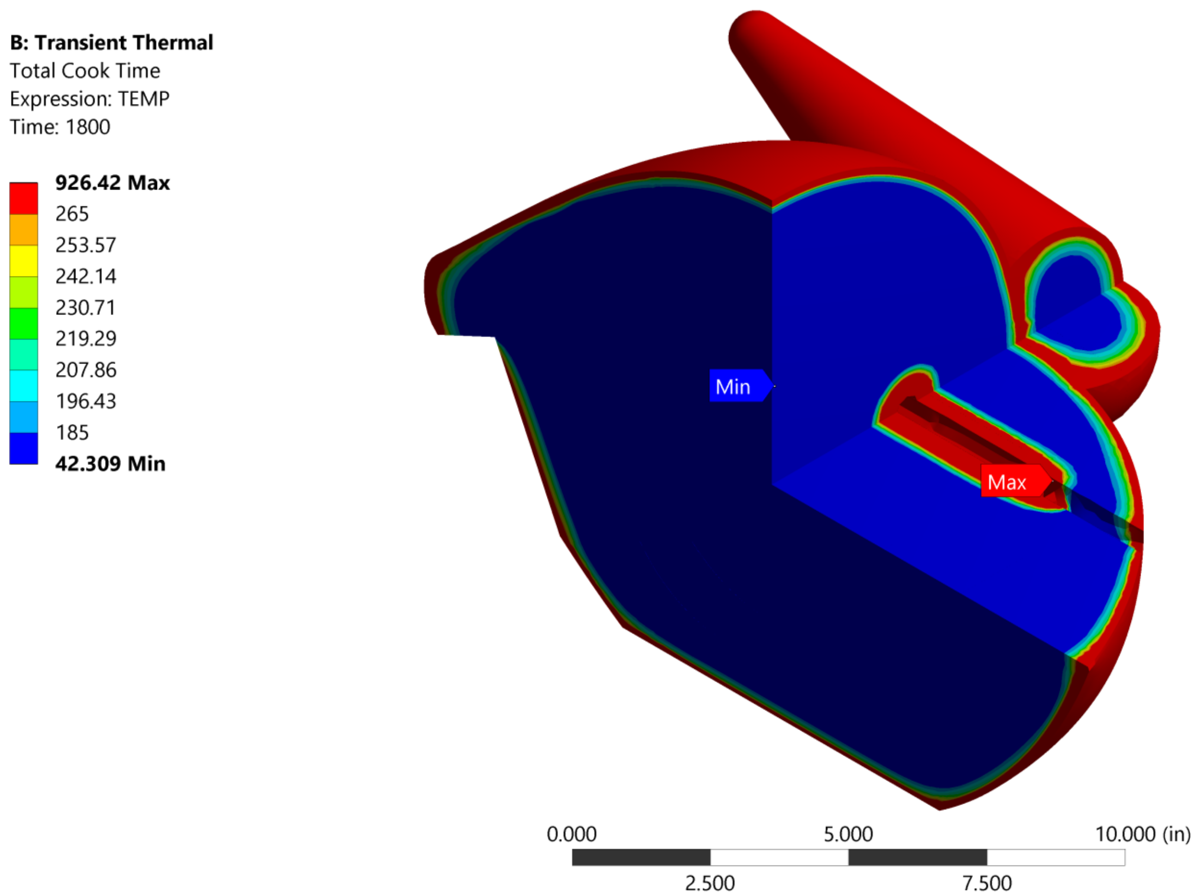


Figure 24: (above) t = 0h 30m

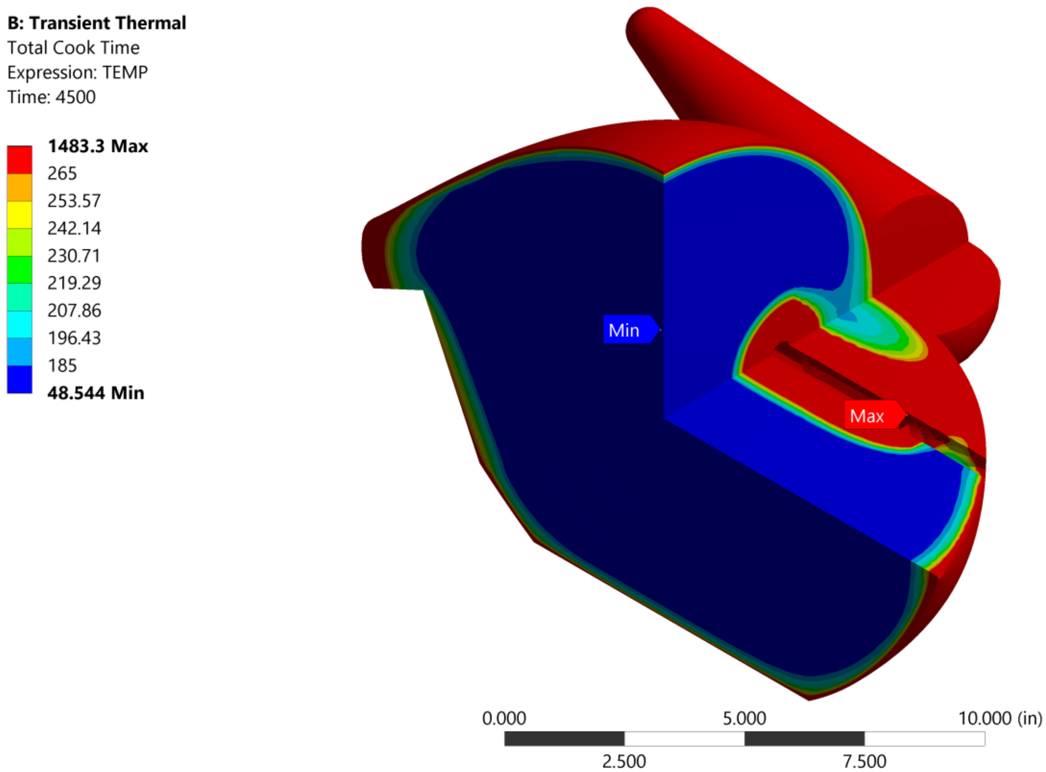


Figure 25: (above) t = 1h 15m

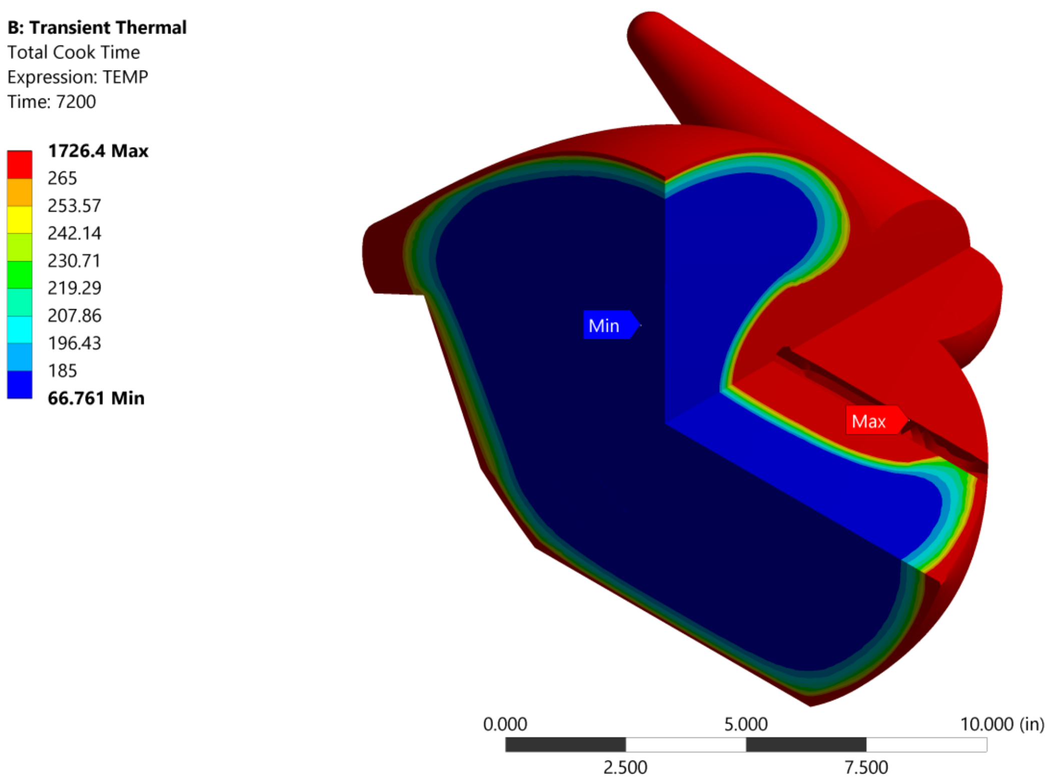


Figure 26: (above) t = 2h 00m

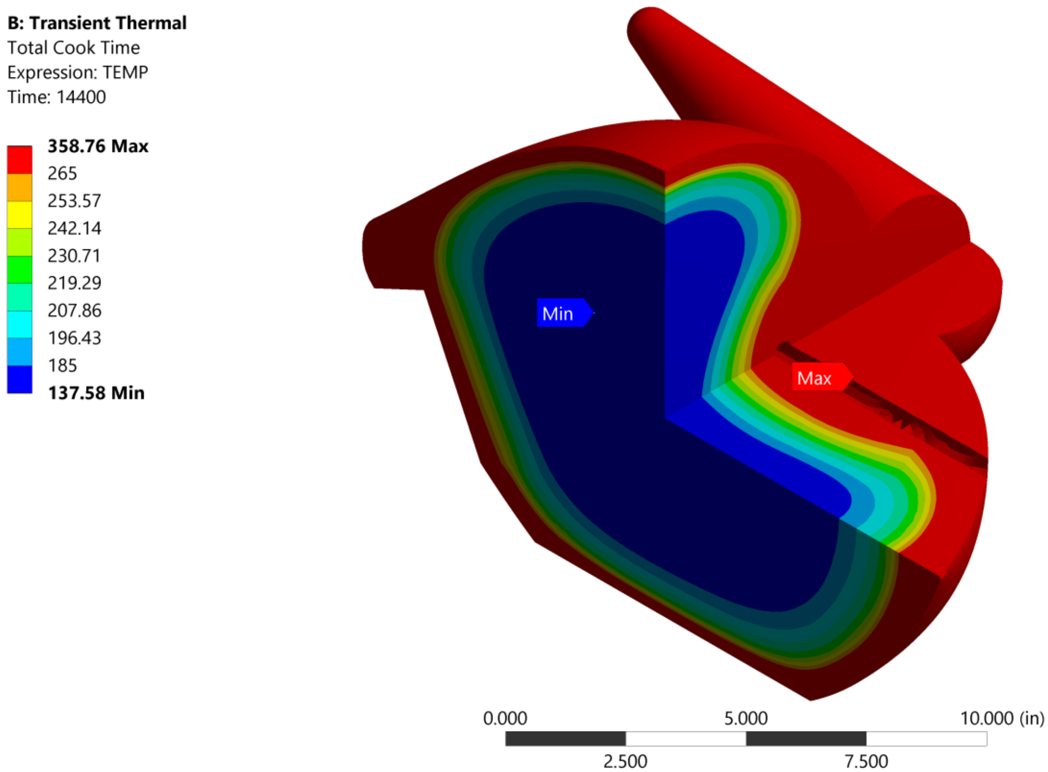


Figure 27: (above) t = 4h 00m

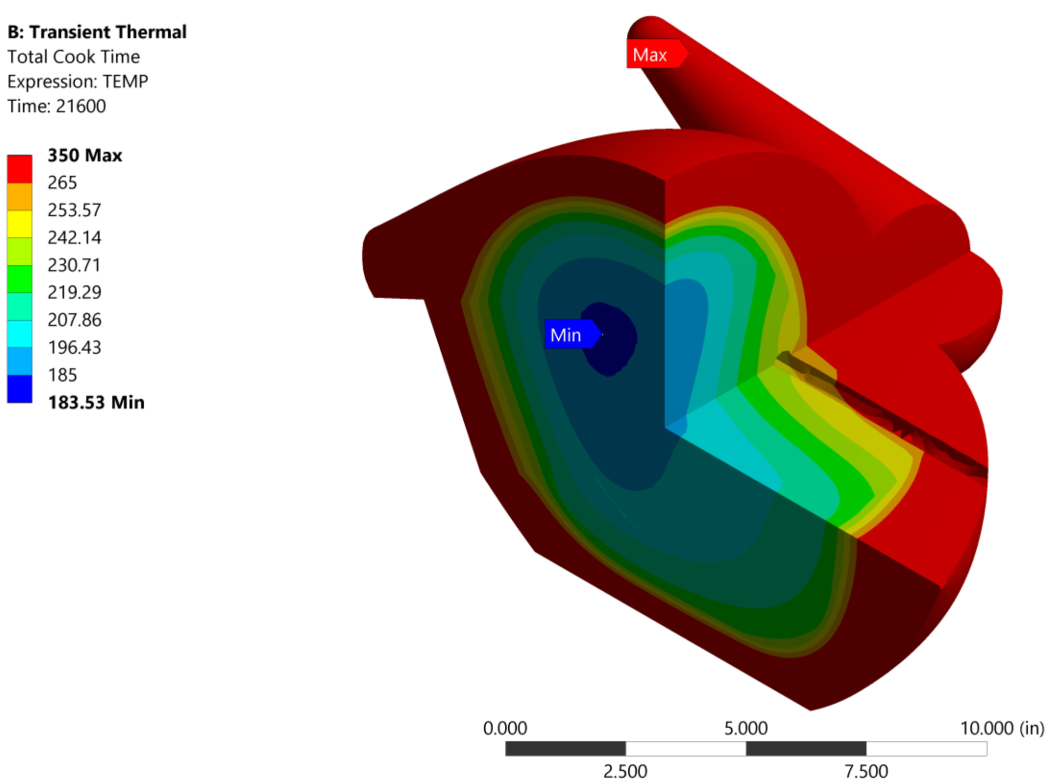


Figure 28: (above) 6h 00m

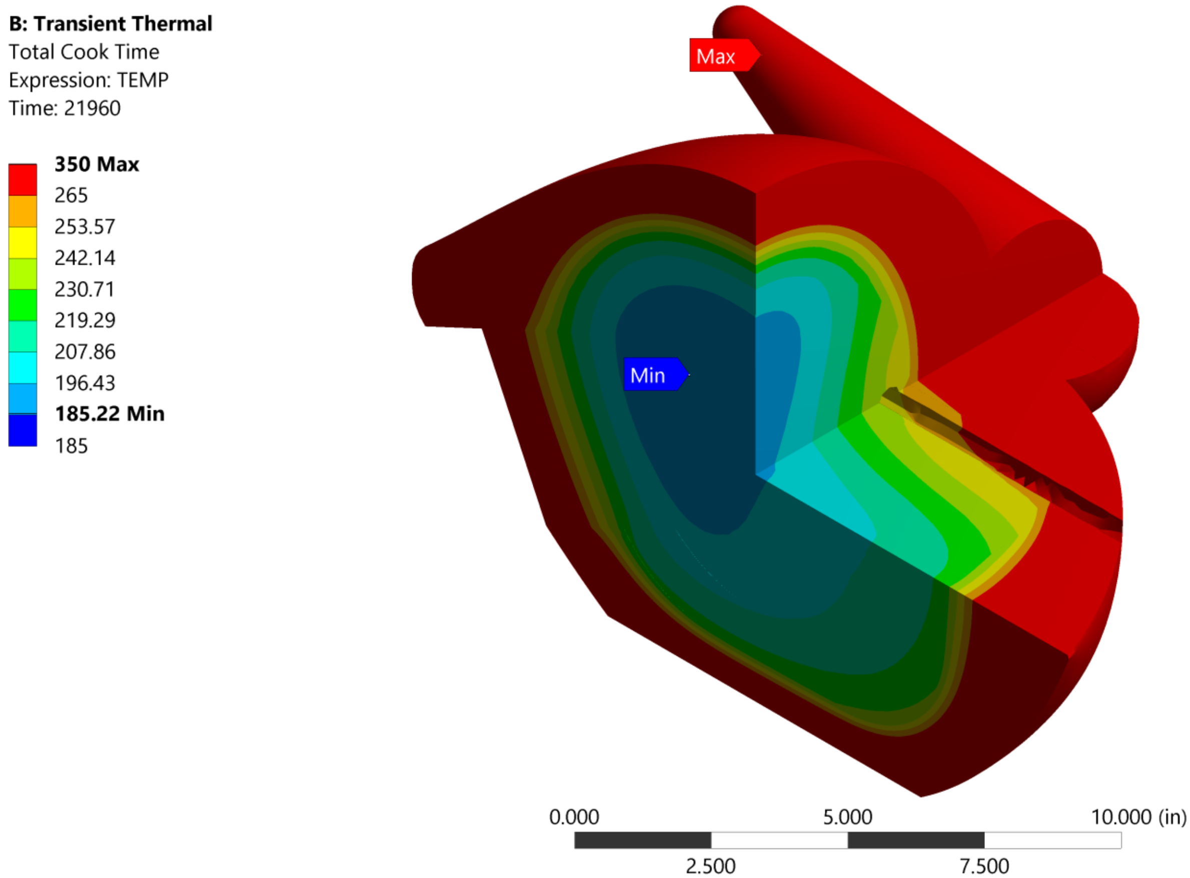


Figure 29: (above) t = 6h 06m

At the end of the cooking process, Figure 29 shows that the maximum temperature in the turducken of 350°F is located near the tip of the wing. This makes sense because at this final time step, the tip of the wing is entirely burned ash and has been receiving convection from the oven (set at 350°F also) for several hours.

Reviewing the results plots in sequence (or animated) shows that the transient response is smooth—there are no erratic changes in temperature—and no major discontinuities in the contours. There is one minor discontinuity near the contact region between the turkey and stuffing. This is the result of the discontinuous convection boundary condition identified earlier in this paper. The effect is noticeable only in the first hour of the transient response, after which it no longer has a significant impact on the cooking process simulation.

Moreover, the initial assumptions about the skewer being able to add a great deal of energy to the interior of the turducken was mostly correct. The skewer was able to greatly decrease the total cooking time by supplying a heat flux to a slower-cooking portion of the ducken. The issues with extremely high temperatures near the skewer and other more optimal skewer locations are discussed in the evaluation section at the end of this paper.

Thermal Gradient

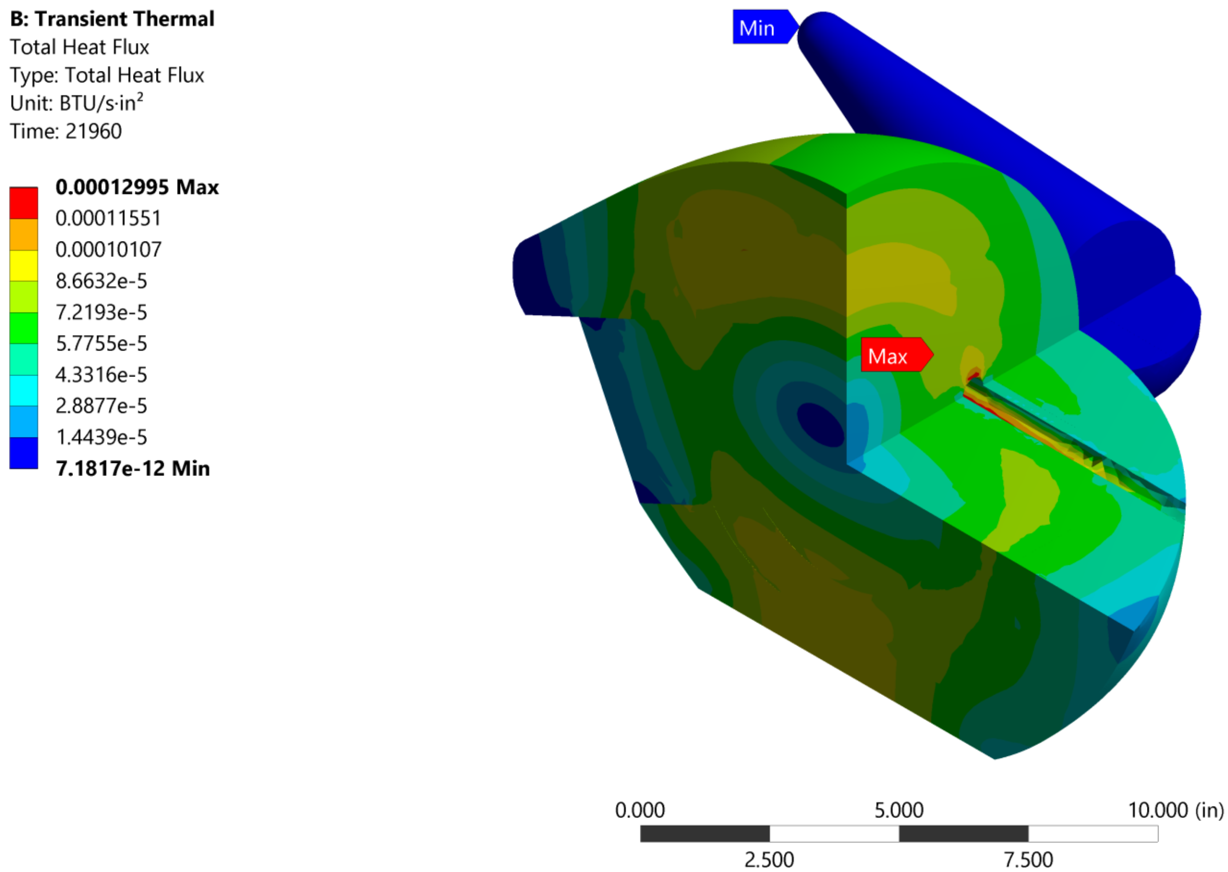


Figure 30: Thermal gradient within the turducken at the end of the cooking process (t = 6h 00m).

Figure 30 shows the thermal gradient within the interior of the turducken. The portions in blue have a lower thermal gradient, which means the rate of change of temperature in these regions is low. In simpler terms, in the next time-step (the final time-step), these regions will change temperature by a small amount. On the other hand, the portions in yellow (mostly through the ducken/turkey contact region) have a higher thermal gradient. This means the rate of change of temperature in these regions is higher. In the next time-step (the final time-step), these regions will change temperature by a larger margin.

The fact that the thermal gradient is continuous throughout the interior of the turducken means the finite element model is behaving correctly. More specifically, even though the material properties are non-linear, the difference in material properties from element to adjacent elements is small enough to keep the finite element model on track.

On the other hand, the very high thermal gradient at the skewer surface is more concerning, especially because the heat flux from the skewer has been switched off for four hours at this point. This should have been enough time for the thermal gradient to become more smooth in this region.

Burned Volume

To calculate the percent burned, the volume of turducken above a certain threshold temperature needed to be calculated. This was accomplished by exporting the temperature at each node, the volume of each element, and which nodes correspond to each element from ANSYS (for the result at the last time step only). Using a MATLAB script the average temperature in each element was calculated by taking the mean temperature of the nodes in each element. Next the elements over the burning temperature (265 F) were sorted and the total volume of all elements over the burning temperature was calculated. The quotient of the total volume burned divided by the total turducken volume represents.

The result of the MATLAB script is 41% of the turducken volume is burned at the end of the cooking process. Figure 31 is a visual representation of this volume using the isovolume feature in ANSYS Result Viewer.

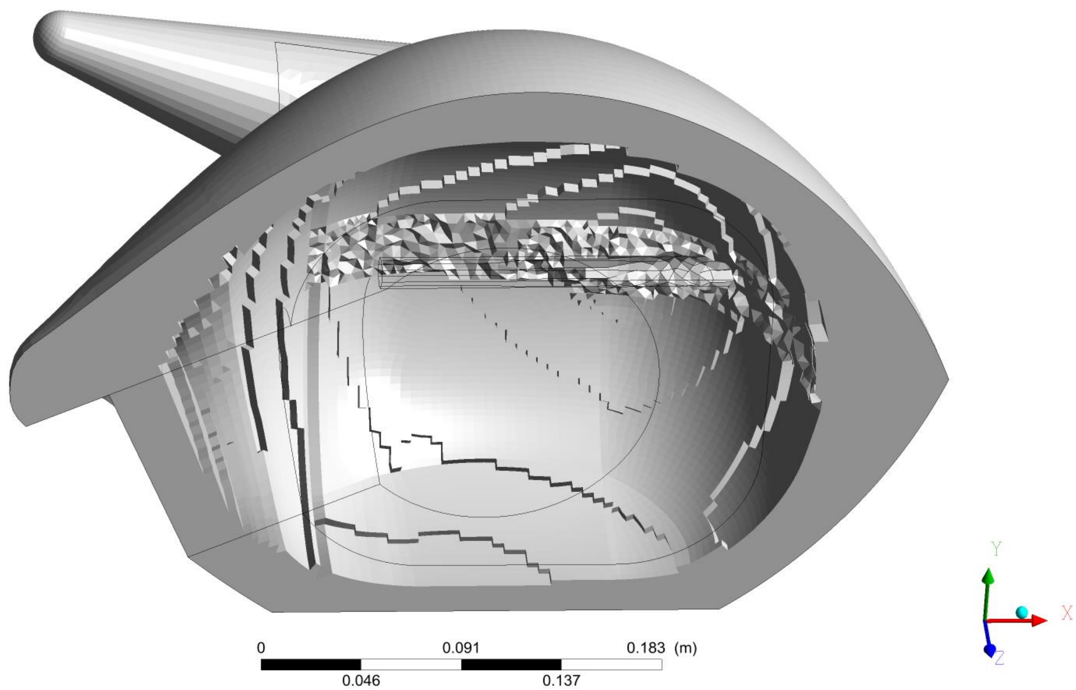


Figure 31: A visual representation of the burned volume of the turducken (isosurface with only elements with temperature greater than 265°F)

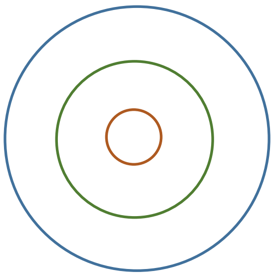
It's important to note that the burned volume calculation is carried out for the results of the last time step only. This means that any elements that 'burned' and then cooled off below the threshold temperature *during* the cooking time are not counted. For example, the elements around the skewer in Figure 24-25 (t = 1h to 2h) are shown to be well above the threshold temperature, but they cool off by the end of the cooking process and are not counted as 'burned' in this calculation.

Lacking an automatic way to compute burned volume at every time step within ANSYS, it is unfeasible to export each of the time steps individually for this MATLAB computation.

Hand Calculation

Three Concentric Sphere Model

The turducken as a whole can be simplified as three concentric spheres with turkey on the outside, ducken in the middle and stuffing in the center. The geometry of the three spheres are estimated from the turducken geometry provided:



Input Parameter	*Value*	*Unit*
Diameter (Turkey)	15.8	in
Diameter (Ducken)	7.9	in
Diameter (Stuffing)	5.5	in

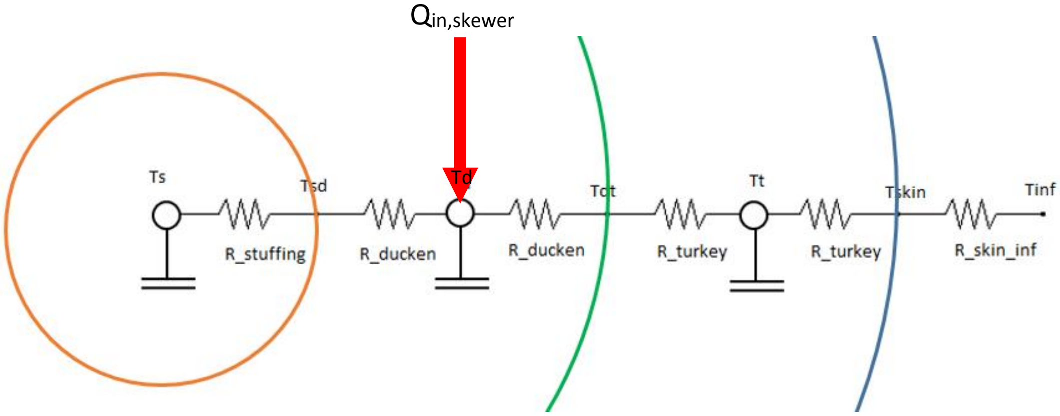


Figure 32: 3-node equivalent RC circuit (not drawn to scale)

Figure 32 shows the nodes of the model. In effect, there are 7 nodes total: 3 capacity-carrying nodes that represent the three concentric spheres and 4 non-capacity-carrying nodes that help with the computation at each time step.

The following additional assumptions were also made for this hand-calculation:

- The oven temperature is set to 500°F throughout
- The convection film coefficient is 1.0192×10^{-5} BTU/s-in²-°F
- Skewer heat flux is fixed at 400 BTU/hr and is attached to the ducken node only
- Skewer is switched on for 3 hours only
- Material properties are held constant at their initial values (at 42°F) shown below

	Thermal Conductivity ***k*** *(BTU/h.ft².F)*	***Density*** ***ρ*** *(lb/ft³)*	***Specific Heat Capacity*** ***c*** *(BTU/lb.°F)*
TURKEY	0.265	64.721	0.824
DUCKEN	0.224	63.282	0.710
STUFFING	0.224	63.282	0.710

Figure 33: Fixed material properties for the hand calculation

Derived Parameters

Capacitance

The capacitance is calculated using the following equation:

$$C_{section} = \rho \times c \times V$$

Where ρ is the density, *c* is specific heat, and *V* is the volume of the section

Resistance

The conductive resistance between the interior nodes of the food segments R_d is calculated using:

$$R_d = \frac{\Delta x}{k \times A}$$

Where Δx is the distance between two nodes, k is the conductive coefficient of the material, and A is the average area of the two sphere at adjacent nodes.

The convective resistance between node "skin" and the ambient R_v is calculated as follow:

$$R_v = \frac{1}{hA} = \frac{1}{h \times A_{Turkey}}$$

Where h is the convective coefficient of 1.0192×10^{-5} BTU/s-in²-°F

Excel Simulation Using Euler's Formula

For the transient response of the capacity-carrying nodes (s, d & t), the temperature can be simulated using:

$$T_i^{p+1} = T_i^p + \frac{\Delta t}{C_1}\left(q + \frac{T_{i+1} - T_i}{R_{i+1\ to\ i}} + \frac{T_{i-1} - T_i}{R_{i-1\ to\ i}}\right)$$

Where "i+1" is the node index to the adjacent right of node i. Heat-in "q" is applied when skewer is turned on, otherwise, $q = 0$. Thus, the temperature of nodes without added heat from the skewer can be simulated using:

$$T_i^{p+1} = T_i^p + \frac{\Delta t}{C_1}\left(\frac{T_{i+1} - T_i}{R_{i+1\ to\ i}} + \frac{T_{i-1} - T_i}{R_{i-1\ to\ i}}\right)$$

The transient response of the intermediate, non-capcacity-carrying (sd, dt, skin) is calculated using simple voltage division.

$$T_i = \frac{\left(\frac{T_{i-1}}{R_{i-1}} + \frac{T_{i+1}}{R_{i+1}}\right)}{\left(\frac{1}{R_{i-1}} + \frac{1}{R_{i+1}}\right)}$$

Results

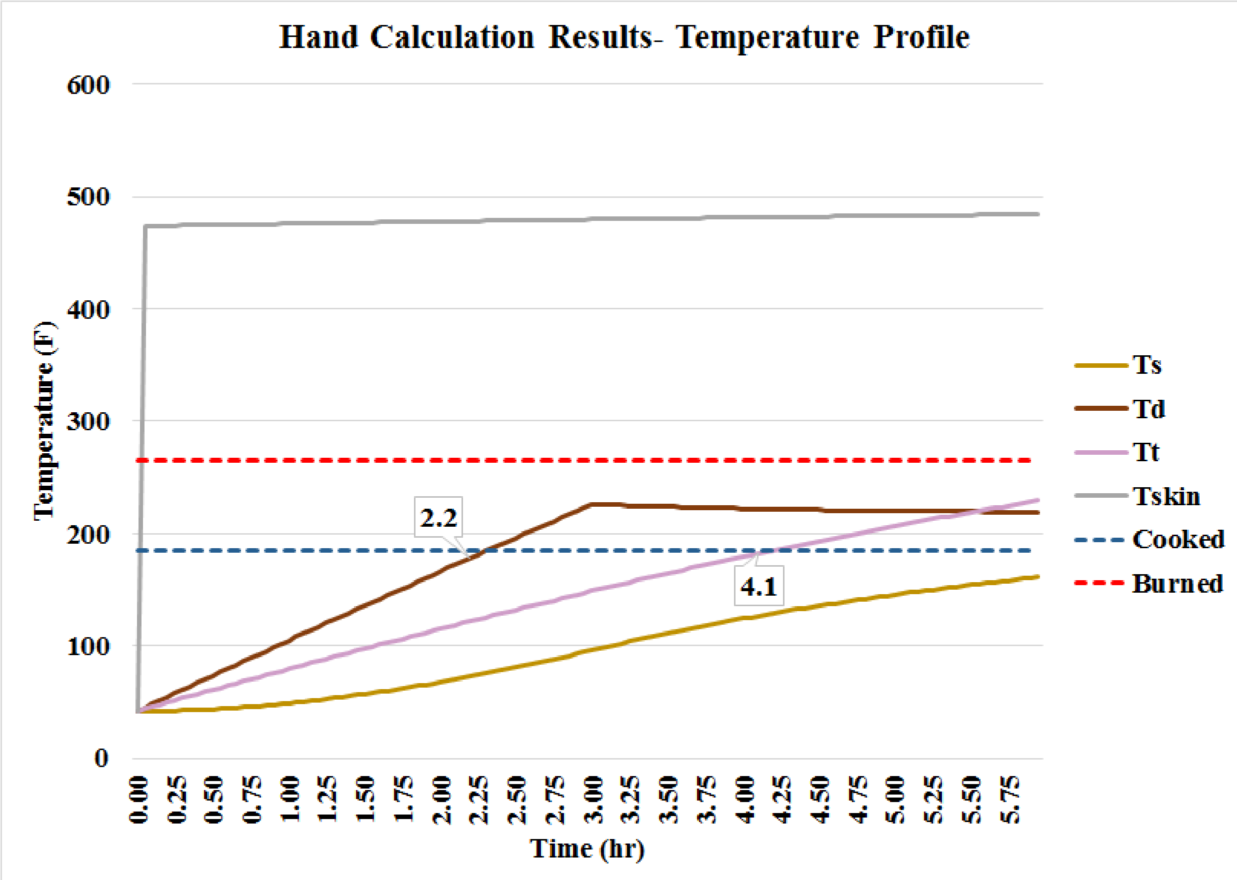


Figure 34: Results of the hand calculated three concentric sphere model – cooking with skewer

The transient response results of the hand calculation are plotted in Figure 34. The hand calculation shows that the ducken and turkey are able to reach the cooking temperature threshold. The ducken node is fully cooked after just 2.2 hours, while the turkey takes longer to cook, until 4.1 hours. Meanwhile, the stuffing never reaches the threshold of 185°F. However, since the stuffing is made of potatoes and peas, it can be argued that the stuffing can be considered pre-cooked.

This model helps to justify the placement of the skewer in the first place. Without the heat flux from the skewer applied to the ducken node, the hand calculation shows that the ducken would take never actually reach the cooking threshold.

The limitations of this model is that there are only three capacity-carrying nodes. As such, each of the turkey, ducken and stuffing are lumped into one node each, which means it is impossible to see that some portion of the turkey/ducken may be burning while a large other portion has barely changed temperature. It might have been worth the effort to make a 5- or 7- node model with multiple capacity-carrying nodes for each of the spheres.

Evaluation

After a closer look of the results of simulations, the skewer is likely not placed in the optimal location. The idea behind the skewer placement came from the hand calcs, which demonstrated that the cooking time would be optimized by adding heat to the ducken. However, the hand calculations were based on a 7 node model (one for each material, and boundary layer), and as such would model the heat flux as an even distribution of heat to the entire ducken, not limited by the location of the skewer as in the real model. Our skewer design placement meant that parts of the stuffing and the ducken were still quite far in distance from the heat input and as such were not optimally heated. A better design for skewer placement would have been to take a cross sectional plane in the ducken and find the axis which minimizes distance to all point in the ducken as this would mean that as whole the ducken would be heated the fastest due to heat expanding linearly radially outward from the skewer axis.

Another flaw in the simulations was caused by the boundary conditions of the heat flux on the mesh of the ducken. The skewer heat input was represented by a direct flow of energy onto the inside surface of the ducken "cut" by the skewer. Due to the relatively small cross section of the skewer and the relatively large energy input from the heat flux, a large amount of energy was applied to a very small volume represented by too few nodes, causing an artificially high temperature localized around the heat flux. This likely did not change the final temperature distribution to a large extent, as the skewer was turned off allowing the total heat entered into the system by the skewer to redistribute, giving reasonable temperatures throughout the entire turkey.

Allowing the heat of the skewer to redistribute, while more accurately simulating the reality of the turducken cooking process and the final temperature distribution, would however artificially decrease the final calculation for percent burned. Due to the redistribution of heat, nodes that were above the burning temperature were allowed to decrease back below the burning temperature, thus not being counted for in the percent burned calculation which was solely based on the final temperature distribution.

Lastly, heat of vaporization was not considered in the material properties in the model, meaning that while a step change in material properties was added from 212°F to 213°F, the model does not recognize that it takes energy to evaporate the water and thus change the nodal temperatures from 212°F to 213°F. If this effect were considered the consequences would likely be beneficial as it would not hinder the ability to reach 185° Fahrenheit, but would require a lot more energy for each node to reach the burning temperature of 265°F. Not taking into account heat of vaporization thus artificially increases the burn percentage.

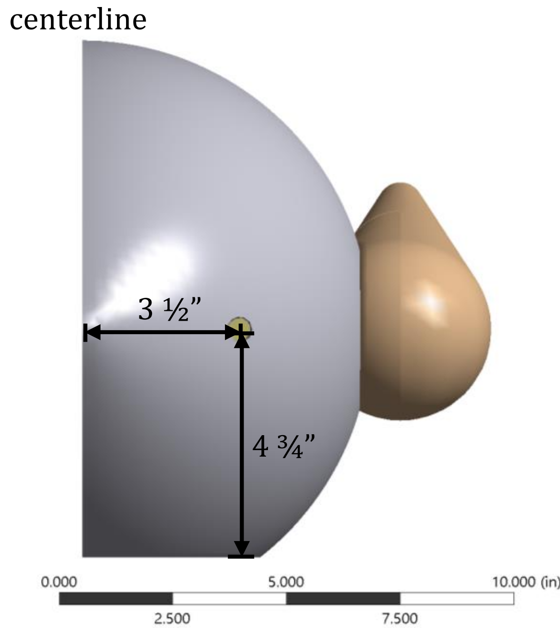
Design Time Estimate

	Time (hours)
Mesh	75
Simulations/Tuning BCs	60
Hand Calculations	10
Report	25
Total	*170*

Appendix: Simplified Cooking Process Instructions for Customer

Inserting the Skewers:

1. Position the turkey such that the opening for the stuffing faces away from you and such that the wings are on each side.
2. Approximate the centerline of the turkey as where the distance from the base of the turkey to where the top of the turkey is the largest.
3. Using a ruler, approximate the point along the turkey skin that is 4 ¾ inches from the base and 3 ½ inches from the center line.
4. The skewer is marked at the 10 inch mark Enter the skewer into the turkey parallel to the center plane and the base such that the mark along the skewer rests at the skin of the turkey



Cooking Instructions:

1. Preheat the oven to 500°F.
2. Enter the turkey in the oven. Set a timer for 30 minutes.
3. Once the turkey has been cooking for 30 minutes, turn the oven temperature down to 350°F. Set a timer for 90 minutes.
4. After the 90 minutes have passed, turn off the resistive heating skewers. Set a timer for 4 hour and 6 minutes.
5. Once the cook time is finished, take the turkey out of the oven.
6. Enjoy!

References

1. American Society of Heating, Refrigerating and Air-Conditioning Engineers. 2006 ASHRAE Handbook: Fundamentals. Inch-pound ed. Atlanta, Ga.

2. "Thermal Properties of Metals, Conductivity, Thermal Expansion, Specific Heat." Engineers Edge - Engineering and Manufacturing Solutions, www.engineersedge.com/properties_of_metals.htm.

3. "Measurements of Heat Transfer Coefficients within Convection Ovens." Journal of Food Engineering, www.sciencedirect.com/science/article/pii/S0260877405000130.

4. "How to Safely Thaw a Turkey." FoodSafety.gov, Department of Health and Human Services, 21 Nov. 2016, www.foodsafety.gov/blog/2016/11/defrost-turkey.html.