The Tippe Top

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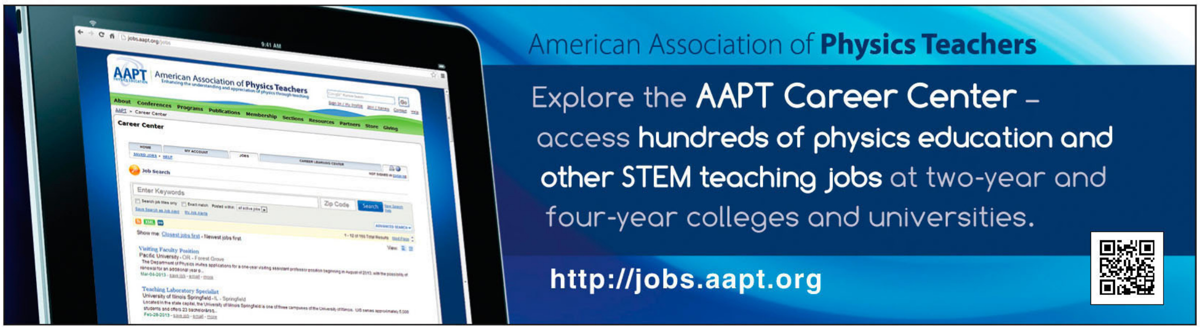

NOTES AND DISCUSSION

The Tippe Top

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WITH reference to a recent article[1] in this *Journal*, I would like to comment on different ways in which the motion of the tippe top can be described. Pliskin has shown that the two ways of approach to be discussed hereafter are nearly equivalent, once the equations are written down, but it might be useful to someone who is called upon to teach the physics behind these equations to consider the following.[2]

In Fig. 1 of reference 1, the torque of the friction will tend to decrease ω_z+s and increase ω_y. But before one can say that θ must therefore increase, it is necessary to show that the deviation of the axis of rotation from the vertical is limited. The explanation is, of course, that the effect of the torque of the friction, averaged over one period of ω, is zero, leaving the axis of angular momentum approximately vertical.

From here on, however, it takes only one more step to come to the alternative explanation, because what the axis of angular momentum really does during a period of ω is to describe a narrow cone about the vertical. The horizontal component of the torque of friction spins around with the top, always lying in the vertical plane through the axis of the top. Its time integral, the horizontal component of the angular momentum, is running 90 degrees behind, so that for example in Fig. 1 of reference 1, it is pointing towards the reader. This component of the angular momentum results in the component $\dot{\theta}$ of the angular velocity which makes the top turn over. In this way, it is very easy to obtain an approximate expression for $\dot{\theta}$:

If $L\ll r$, $|A-C|\ll C$, and $\omega\gg\sqrt{(g/r)}$:

$$\dot{\theta}=\frac{\mu Wr}{A\omega}, \tag{1}$$

which is the leading term in Pliskin's equation (5). Numerical values can be obtained by putting $A=\frac{2}{3}mr^2$ (hollow sphere), $W=mg$, and taking a reasonable value for μ ($\mu\approx 1$ for plastic on sandpaper or ≈ 0.2 for plastic on a hard table). When the top is spun by hand, ω may be of the order of 100 radians/second, not quite high enough to make Eq. (1) a very good approximation, but sufficient for a demonstration of the order of magnitude of the effect. Putting $\mu=0.2$, $r=1.5$ cm, and $\omega=100$ radians/second in Eq. (1), one obtains $\dot{\theta}=2$ radians/second.

The preceding discussion would seem to cover what most people want to know about the tippe top. For this purpose, the fixed coordinate system offers a very direct physical understanding, based upon the relation between torque and angular momentum. To achieve a more detailed description, one has to go through rather lengthy calculations,[3,4] and then the use of a coordinate system, as adopted by Hugenholz and Pliskin,[1] seems more satisfactory from a mathematical point of view.

Some remarks on the sliding of the point of contact may be added here. Both eccentricity of the center of gravity and inequality of the principal moments of inertia can create the effect. In the latter case, the instantaneous angular-velocity vector makes an angle with the nearly vertical angular-momentum vector and for that reason does not go through the point of contact. But, either way, θ has to assume a finite difference from 0 before the top actually starts to slip. What happens before this slipping angle is reached does not seem to be quite clear. Another point which is not clear is how accurate the basic assumption

$$F=\mu W=\text{constant}$$

really is; this makes detailed calculations of the motion in the case of sliding friction of rather academic interest.

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[1] W. A. Pliskin, Am. J. Phys. 22, 28 (1954).
[2] We shall assume that the top was started with its axis of symmetry and the axis of rotation both vertical and with no linear velocity. As was done by Pliskin, we will here neglect the circular motion of the center of gravity.
[3] N. M. Hugenholz, Physica 18, 515 (1952).
[4] C. M. Braams, Physica 18, 503 (1952).

The Uncertainty Principle and the Bohr Theory of the Hydrogen Atom

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THERE are several well-known illustrations of Heisenberg's uncertainty principle, e.g., determination of the position of a free electron (No. 1), diffraction of the electrons at a slit (No. 2), etc. In demonstrating the two results, *viz.*, $\Delta x\Delta p\sim h$ and $\Delta E\Delta t\sim h$, in some illustrations, one of the quantities (Δp in No. 2) is determined by the particle picture, and the other (Δx in No. 2) by wave picture. The multiplication of the two gives the result. It is found that similar considerations involving the hydrogen atom also yield the two results.

For the ground state of the hydrogen-like atoms, we have, in the usual notation,

$$\text{energy}\quad E=-\frac{2\pi^2me^4Z^2}{h^2},$$

$$\text{orbital radius}\quad a=\frac{h^2}{4\pi^2me^2Z},$$

$$\text{electronic velocity}\quad v=\frac{2\pi e^2}{h}Z.$$

(a) The circumference of the electronic orbit $=2\pi a=h^2/2\pi me^2Z$.

From any observations on the atom, e.g., the spectra, we cannot say at which place the electron exists in the orbit. Further in the wave-mechanical picture even the orbits are not defined. Hence,

$$\Delta x\sim\frac{h^2}{2\pi me^2Z}.$$