Unit 3 Exam: P versus NP

QUESTION 1: Circuits & Formulas

Prove that $y_1 = (x_1 \text{ AND } x_2)$ is equivalent to $(x_1 \vee \bar{y}_1) \wedge (x_2 \vee \bar{y}_1) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee y_1)$ in the Boolean circuit shown below.

Complete the following statements about the three clauses. You may use either natural language or logical statements according to the notation used above.

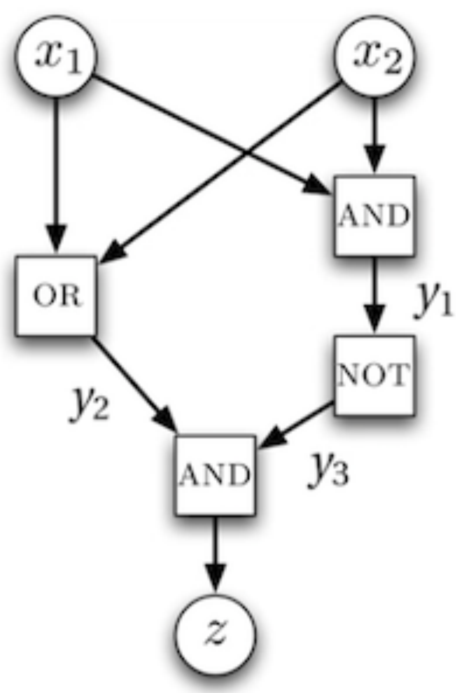
(a) Given than the three clauses are linked by logical AND statements, all three clauses must be **True for the entire clause to be True**

(b) If x_1 is false, y_1 must be **False**

(c) If x_2 is false, y_1 must be **False**

(d) If x_1 is true and x_2 is true, y_1 must be **True**

NOTE on notation: $\vee$ = logical OR; $\wedge$ = logical AND; $\bar{\ }$ = NOT (the statement is false)



The proof that $y_1 = (x_1 \text{ AND } x_2)$ is equivalent to $(x_1 \vee \bar{y}_1) \wedge (x_2 \vee \bar{y}_1) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee y_1)$ is given by the Truth Table below. It should be noted the very last column y1 = x1 ^ x2 should be thought of as a Boolean equality. That is y1 = x1 ^ x2 (if written in Python y1 == x1 and y2) asks if y1 has the same logical value as x1 ^ x2. Inspecting the last two columns highlighted in pink shows that the two Boolean expressions in question have the same truth values thus proving that $y_1 = (x_1 \text{ AND } x_2)$ is equivalent to $(x_1 \vee \bar{y}_1) \wedge (x_2 \vee \bar{y}_1) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee y_1)$.

x1	x2	y1	$(x_1 \vee \bar{y}_1)$	$(x_2 \vee \bar{y}_1)$	$(\bar{x}_1 \vee \bar{x}_2 \vee y_1)$	$(x_1 \vee \bar{y}_1) \wedge (x_2 \vee \bar{y}_1) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee y_1)$	y1 = x1^x2
F	F	F	T	T	T	T	T
F	F	T	F	F	T	F	F
F	T	F	T	T	T	T	T
F	T	T	F	T	T	F	F
T	F	F	T	T	T	T	T
T	F	T	T	F	T	F	F
T	T	F	T	T	F	F	F
T	T	T	T	T	T	T	T

QUESTION 2: Traveling Salesperson

A traveling salesperson needs to visit a series of cities connected by edges (roads). *D* is the distance of the **shortest** path through all of the vertices (cities) in the network. Which of these questions is in NP? Provide a brief explanation of your reasoning, eg. "It is easy to check if the solution is true by..."

Is *D* less than 10,000 miles?

This question **is in NP** because all that is required is to produce one path (call it P) with a distance less than 10,000 miles. Then a shortest path, by nature being the shortest, must have a distance less than or equal to P. Since P has distance less than 10,000 miles, which is easy to check, then so does any shortest path.

Is *D* more than 8,000 miles?

This question **is not in NP** because because even is one path of distance more than 8,000 is offered as the shortest path there still exists an exponentially large number of paths to be examined to verify that the shortest path is indeed greater than 8,000 miles.

Is *D* exactly 9,219 miles?

This question **is not in NP** because even is a path with exactly 9,219 miles is offered as the shortest path there still exists an exponentially large number of paths to be examined to verify this path is indeed the shortest.

QUESTION 3: Complexity Hierarchy

Considering a cellular automata with a state s at time t_n, what is the complexity class that each of the following questions belongs to?

- What will the state be at t_{n+x}?

  This question belongs to P.

- Does s have a predecessor?

  This question belongs to NP.

- On a lattice of size n, is s on a periodic orbit?

  This question belongs to PSPACE.

- On a lattice of infinite size, will s ever die out?

  This question is undecidable.

Your answers should indicate whether each question is in P, NP, PSPACE, or undecidable.