Unit 1 Exam: Easy & Hard

QUESTION 1: Polynomials & Exponentials
Today, your computer can do T steps in a week. According to Moore's law, next year, your computer will be able to do 2T steps in a week. How does doubling T change the n that can be computed in a week?

a) If $T = n^2$, what does doubling T correspond to in terms of n? In other words, by what arithmetic factor does n change when T doubles?

If $T = n^2$ then $2 \cdot T = (\sqrt(2)n)^2$ therefore n changes by a factor of $\sqrt(2)$.

This means the size of the problem that can be solved in a week increases by ~ 141 %

b) If T = 2^n, what does doubling T correspond to in terms of n? In other words, by what arithmetic factor does n change when T doubles?

If $T = 2^n$ then $2 \cdot T = 2 \cdot 2^n = 2^{(n+1)}$ or $n - > (n + 1)$

This means the size of the problem that can be solved in a single week increases by only 1.

Important! Your answer should include an algebraic solution as well as the reasoning/logical steps - either in words or equations - by which you arrived at your answer. The latter will form a part of the exam grade.

QUESTION 2: Divide & Conquer
For the Towers of Hanoi puzzle, there is a function f(n) that computes the total number of moves needed to move n disks. For f(0) = 0, but for n greater than 0, f(n) = 2f(n-1)+1. Additionally:

n	0	1	2	3
f(n)	0	1	3	7

a) What is the function that allows the equation to be true?
$f(n) = 2^n - 1$

$f(n) = 2f(n-1) + 1 = 2^n - 1$ can be proved by induction on n.

Proof:

Check $n = 0{,}1$

$f(0) = 0 \quad 2^0 - 1 = 0$

$f(1) = 2f(1-1) + 1 = 0 + 1 = 2^1 - 1 = 1$

assume true for all n less than or equal to some N prove true for n = N+1

$f(N+1) = 2f(N) + 1$ (recursive definition of $f(n)$)

$= 2(2^N - 1) + 1$ (induction hypothesis)

$= 2^{N+1} - 1$ (true for N+1 as required)

b) What is f(n) for n = 64?

$f(64) = 2^{64} - 1 = 18446744073709551615$

Python gives this $f(64) = 2**64 - 1 = 18446744073709551615$

Important! Your answer should include both an algebraic solution (the identity of function f) as well as a numeric solution, solving for n=64.