Unit 4 Exam: Worst-case, Natural and Random

QUESTION 1: Phase Transitions

Referring to the Ising Model interactive in this unit:

a) What is the critical temperature (± 0.1)?

Ising Model: a) The critical temperature is ~ 2.6

b) Describe the relationship between spin correlation and lattice length (L/2) at the critical point. (No equations needed, simply provide a phrase describing the general relationship.)

Ising Model: b) The spin correlation shows a **polynomial decay** at the critical temperature.

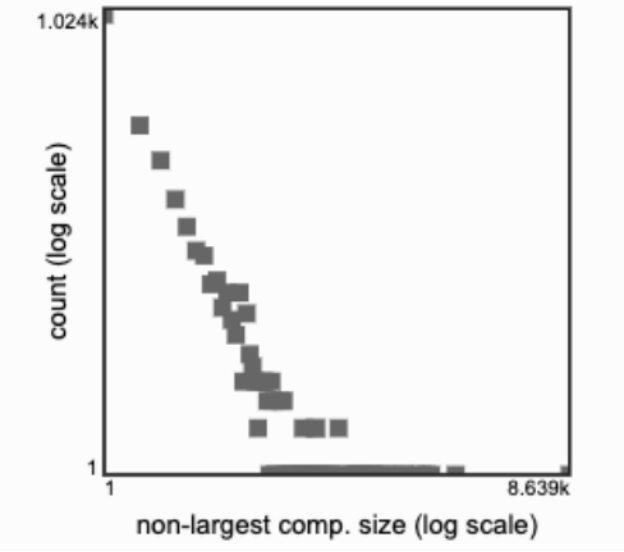
Referring to the Percolation interactive in this unit:

c) What is the critical site occupancy probability p (± 0.02)?

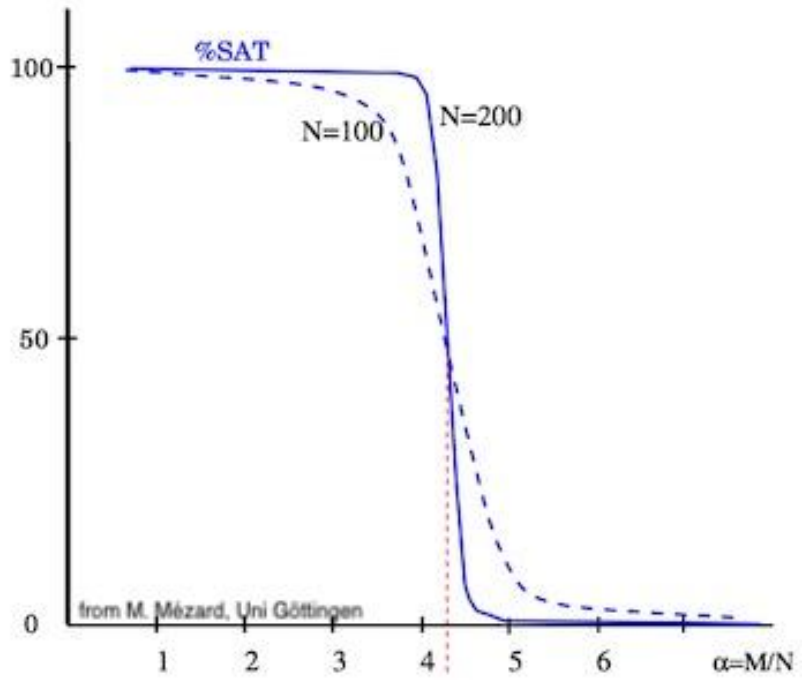
Percolation: c) The critical occupancy proability is ~ 0.605, at which p a spanning component appears with near complete reproducibility.

d) Describe the relationship of non-giant component sizes and their frequency at the critical point.

Percolation: d) There is a power-law relationship between the size of non-giant components and their frequency at the critical point.



QUESTION 2: Landscapes, Clustering, Freezing and Hardness



Recall SAT (SATisfiability) problems like this one: "You are hosting a party for your Computation in Complex Systems classmates. Cris wants you to invite either Priyanka or to exclude Esteban. John asks you to invite either Esteban or Xiaojie or both. Isa does not want Xiaojie or Priyanka or both to attend. Is there a guest list that will satisfy everyone?"

k-SAT for $k \geq 3$ is NP-complete but is computable. The satisfiability of a SAT problem relates to the ratio, **α**, of constraints, **M**, *eg.* "Priyanka OR not-Esteban" to variables, **N**, *eg.* Priyanka, Esteban, and Xiaojie.

Based on the plot above showing percent satisfiability (%SAT) relative to **α** (alpha) for two different **N**:

a) Generally speaking, what happens in the solvability space of SAT problems?

Generally speaking the solvability space of SAT problems exhibits a phase transition from solvability to unsolvability as the ratio, α, of constraints increases.

b) What does the vertical red dashed line indicate?

The vertical red dashed line is the critical density (ratio of number of clauses per number of variables) where the probability of a random k-SAT formula being satisfiable goes from almost always satisfiable to almost never satisfiable.

c) Where do you expect the computational time would be largest along the α axis?

The maximum search time occurs at the vertical red dashed line on the α axis.

d - BONUS*) Is the SAT party problem above satisfiable?

Yes the problem is satisfiable. The minizinc (https://www.minizinc.org/) constraint modeling program given below found a unique solution:

Priyanka is not invited, Esteban is not invited, Xiaojie is invited.

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%Recall SAT (SATisfiability) problems like this one:

%“You are hosting a party for your Computation in Complex Systems classmates.

%Cris wants you to invite either Priyanka or to exclude Esteban.

%John asks you to invite either Esteban or Xiaojie or both.

%Isa does not want Xiaojie or Priyanka or both to attend.

%Is there a guest list that will satisfy everyone?”

var 0..1: p;

var 0..1: e;

var 0..1: x;

constraint (p == 1 \/ e == 0) /\ (e == 1 \/ x == 1) /\ (x == 0 \/ p == 0);

solve satisfy;

%Finished in 409msec

%Compiling Unit_4.mzn

%Running Unit_4.mzn

%p = 0;

%e = 0;

%x = 1;
```