

# Ponder This

October 2014

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## Ponder This Challenge:

What are the last 10 decimal digits of  $2^{(3^{(4^{(5^{(6^{(7^{(8^9))}}))})})}$ ?

In our version of the game, the rules 01100;00010 mean that a cell is born if it has one or two neighbors, and stays alive if it has three. If we start with a single cell in the middle of an 11x11 torus board, then after 15 generations, you will have an alternating chess-like pattern, and after 16 steps, just the four corners.

Your task, this month, is to find rules for our version of the game and an initial input on an 11x11 torus board that will lead, after at least 100,000 generations, to a 72-long cycle.

This challenge is easily handled with Wolfram Alpha which gives the answer: 8170340352

This can also be done using Wolfram Alpha and a sequence of reductions mod  $10^{10}$  as follows:

$$7^{(8^9)} \bmod 10^{10} = 6763596801$$

$$6^{6763596801} \bmod 10^{10} = 6763596801$$

$$5^{6763596801} \bmod 10^{10} = 1064453125$$

$$4^{1064453125} \bmod 10^{10} = 8212890624$$

$$3^{8212890624} \bmod 10^{10} = 4919828481$$

$$2^{4919828481} \bmod 10^{10} = 8170340352$$

