December 2018 - Challenge

You get an integer number as a challenge and your goal is to make it a square of an integer number. To do that, you use two circles with four integers each.
You can change the number you got in steps. In each step we multiply it by the number in the bottom of one of two circles and rotate this circle clock-wise to its next number.

For example, if the two circles are [3,14,15,92] and [6,5,3,5] (the first number is initially at the bottom) then we can solve the input number 42 (i.e. make it a square) by multiplying as follows:

1st circle [3,14,15,92]   42*3=126
2nd circle [6,5,3,5]   126*6=756
2nd circle [5,3,5,6]   756*5=3,780
2nd circle [3,5,6,5]   3780*3=11,340
2nd circle [5,6,5,3]   11340*5=56,700
1st circle [14,15,92,3]   56700*14=793,800
2nd circle [6,5,3,5]   793800*6=4,762,800
1st circle [15,92,3,14]   4762800*15=71,442,000
2nd circle [5,3,5,6]   71442000*5=357,210,000
and getting a square (357,210,000 = 18,900**2).
BTW, There is a much simpler solution, can you find it?

Your challenge, this month, is to find two circles (with four numbers each) such that the same initial state will allow you to solve at least 2,187 different integers between 1 and 65,536.
To make the problem more interesting, you must not use "boring" circles. A circle is called boring if the set of solvable numbers using only this circle is closed under multiplication (i.e. if you can solve X and Y using only this circle, then you can also solve X*Y with it). In the example above, one of the circles (which?) is boring.

Solution

Hi Oded,

Perhaps you could give me some feedback on this month's challenge. According to my Python program, the circles [2, 10, 11, 30] and [3, 21 , 22 , 34] solve a few more than the required 3**7 different integers between 1 and 2**16. The first circle [2, 10, 11, 30] solves 5 and 8, but not 40. So it is not a boring circle. The second circle [3, 21 , 22 , 34] solves 3 and 7, but not 21. This circle is also not boring. Altogether, they appear (according to a computer check) to produce a boring solution.

Concerning my 2 circles: perfect squares like 4, 9, 16, etc. are counted in my total of solved integers, as any circle looped twice does no more than multiply the original number by a square integer. If an integer is a perfect a square to start, it will then automatically be counted. I feel my solution falls a bit short of the mark because it counts perfect squares, but the problem statement makes no mention of initially omitting perfect squares. Could you please clarify this?

Perhaps, the integers 42 and 2187 appeared in the challenge due to D. Adam's book and the Star War movies. Solvers might be interested in more background on 2187, which can be found here: https://www.scientificamerican.com/article/the-number-2-187-is-lucky-here-s-why/

Thank you for considering.

Charles Joscelyne

Dear Charles,

Your solution is correct - squares are (trivially) solvable.

Oded