January 2019 - Challenge

Alice and Bob are playing the following game: they start from a number N and each one of them in his or her turn (Alice starts) divides N by any divisor that is either a prime or a product of several distinct prime numbers. The winner is the one who gets to one - thus leaving the other player with no legal move.
To define the initial N, Alice chooses a number a from a set A, and a number b from a set B. The game is played with N=a+b. Charlie knows that Alice will start, and he wants to let Bob win. He does that by fixing the sets A and B.

He can do that, for example, by choosing A=[3,99] and B=[1,22]. (Why?)

Your challenge, this month, is to help Charlie find sets A and B with at least four different numbers each, that will allow Bob to win.

Bonus '*' for solutions with more than 4 elements in the set B.

Solution:

Hi Oded,

The sets A = [63, 363608, 46655, 1498175] and B = [29521, 506881, 150481, 1] will allow Bob to win no matter which pair of numbers a, b Alice chooses. The 16 possible (a+b) sum pairs given below are all perfect squares. As your example A=[3,99] and B=[1,22] illustrates, Bob can always win the game when N = a+b is perfect square. He does this in turn by matching each of Alice's division choices. This is always possible since the all primes appearing in the prime factorization of N will have an even exponent when N is a perfect square.

63 + 29521 = 29584 sqrt-> 172
63 + 506881 = 506944 sqrt-> 712
63 + 150481 = 150544 sqrt-> 388
63 + 1 = 64 sqrt-> 8
363608 + 29521 = 393129 sqrt-> 627
363608 + 506881 = 870489 sqrt-> 933
363608 + 150481 = 514089 sqrt-> 717
363608 + 1 = 363609 sqrt-> 603
46655 + 29521 = 76176 sqrt-> 276
46655 + 506881 = 553536 sqrt-> 744
46655 + 150481 = 197136 sqrt-> 444
46655 + 1 = 46656 sqrt-> 216
1498175 + 29521 = 1527696 sqrt-> 1236
1498175 + 506881 = 2005056 sqrt-> 1416
1498175 + 150481 = 1648656 sqrt-> 1284
1498175 + 1 = 1498176 sqrt-> 1224

Thanks for considering.

Charles Joscelyne

Minizinc model code with hard coded perfect squares:

```
include "alldifferent.mzn";

set of int: wn = {4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441, 484,
529, 576, 625, 676, 729, 784, 841, 900, 961, 1024, 1089, 1156, 1225, 1296, 1369, 1444, 1521, 1600, 1681, 1764,
1849, 1936, 2025, 2116, 2209, 2304, 2401, 2500, 2601, 2704, 2809, 2916, 3025, 3136, 3249, 3364, 3481, 3600,
3721, 3844, 3969, 4096, 4225, 4356, 4489, 4624, 4761, 4900, 5041, 5184, 5329, 5476, 5625, 5776, 5929, 6084,
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2002225, 2005056, 2007889, 2010724, 2013561, 2016400, 2019241, 2022084, 2024929};


array[1..4] of var 1..2024929: a;
array[1..4] of var 0..2024929: b;

constraint forall (i in 1..4,j in 1..4) (a[i] + b[j] in wn );

constraint alldifferent(a);
constraint alldifferent(b);

solve satisfy;
```

Minizinc output:

```
Compiling IBM_Jan_19.mzn
Running IBM_Jan_19.mzn
a = array1d(1..4 ,[63, 150543, 506943, 29583]);
b = array1d(1..4 ,[46593, 363546, 1498113, 1]);
----------
Finished in 2m 23s
```