April 2018 - Challenge

There are nine balloons in nine different colors. Each balloon has a different lifting capability, from one through nine grams (1, 2, 3...9) respectively. Alice knows the lifting capacities of each balloon, and she wants to use the smallest possible combinations of balloons to prove to Bob the identity of the 1-gram balloon. She can tie together any subset of balloons to a 9.5-gram weight that will stay put if and only if the total lifting power of the balloons is less than 9.5 grams. Otherwise the weight will lift off and fly away.
How does Alice prove to Bob the color of the 1-gram balloon by doing as few "experiments" as possible?
Each demonstration should use a subset of the balloons that does not lift the weight. Supply your answer as a list of weights per experiment.
For example, here is a (non-optimal) solution for a similar problem with only six balloons with 1-6 gram lifting capacities and a 6.5-gram weight:
1,2
1,3
1,4
1,5

In each one of the four experiments, Alice uses a subset of the balloons that does not lift the weight and that has a total lift capacity of six grams or less. Therefore, Bob can conclude that the balloon that is common to all the experiments must be the one with the 1-gram lift capacity.
Two wrong answers for the six-balloon example include the following:

A single experiment: "1,2,3" - this is the only combination of three balloons that won't lift the weight, but Bob can't know from this experiment alone which of the three balloons has the 1-gram lifting capacity.
Three experiments: "1,2" "1,3" and "1,5" - Bob can't distinguish between that and "2,1" "2,3" and "2,4". Both sets of experiments look exactly the same to him, so he can't be sure if the repeated balloon has a 1-gram or 2-gram lifting capacity.

Solution:

Label the balloons A, B, C, D, E, F, G, H, I. Suppose the weights of the balloons are A = 1g, B = 2g, …, I = 9g. Alice knows the balloons' weights and wants to convince Bob that balloon A weighs 1g.

Alice first selects 3 balloons the B, C, D "2,3,4". Alice ties the balloons to the 9.5g weight and Bob notes the weight does not fly away since B, C, D weigh a total of 9g.

After this Alice takes balloon A "1" and performs 4 more experiments:

A&E “1,5”   A&F”1,6”   A&G“1,7”   A&H “1,8” where the 9.5g again does not fly away.

Initially, Bob can only conclude that set B, C, D might be:
(1,2,3) (1,2,4) (1,2,5) (1,2,6) (1,3,4) (1,3,5) (2,3,4)

but Bob can reason that all (1,2,3) (1,2,4) (1,2,5) (1,2,6) (1,3,4) (1,3,5) are impossible if the subsequent 4 experiments all are to hold.

He then concludes the set that set B, C, D is (2,3,4) and A is the 1g balloon for a total of 5 experiments.

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Dear Charles,

There is a better solution, with fewer than 5 experiments.

Oded

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Dear Oded,

Thank you for your feedback. Below is my Minizinc discrete optimization program and its output. Alice can now convince Bob that balloon A weighs 1g in 4 experiments. For instance the output 13542
would entail the experiments "1,3,5" "1,4,2" "1,3,4" "1,5,2"

Minizinc gives other permutations of 13542, but in all instances Bob concludes that A balloon must weigh 1g.

Best regards,

Charles Joscelyne

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include "globals.mzn";

var 1..9: A;
var 1..9: B;
var 1..9: C;
var 1..9: D;
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var 1..9: E;
var 1..9: F;
var 1..9: G;
var 1..9: H;
var 1..9: I;

constraint A+B+C < 9.5;
constraint A+D+E < 9.5;
constraint A+B+D < 9.5;
constraint A+C+E < 9.5;

constraint alldifferent([A,B,C,D,E,F,G,H,I]);
solve satisfy;

output[show(A),show(B),show(C),show(D),show(E)];

Compiling IBM_April.mzn

Running IBM_April.mzn

13542

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13452

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12543

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12453

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15324

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15234

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14325
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14235

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Finished in 51msec