

MPMP15: PRIME PAIRS PUZZLE

Rearrange the numbers from 1-9, such that all adjacent pairs sum to a prime number.

```
/*
MPMP15: PRIME PAIRS PUZZLE
Rearrange the numbers from 1-9, such that all adjacent pairs
sum to a prime number.
*/
include "alldifferent.mzn";
int:n = 9;
set of int: primes;
primes = {2,3,5,7,11,13,17,19,23,29};

array[1..n] of var 1..n: s;
constraint forall(i in 1..n-1)((s[i]+s[i+1]) in primes);

constraint alldifferent(s);
solve satisfy;
output["\ (s)"];
```

Minizinc found 140 solutions, the first and last three are given below:

```
[7, 4, 9, 8, 5, 6, 1, 2, 3]
[7, 4, 3, 8, 5, 6, 1, 2, 9]
[7, 4, 1, 6, 5, 8, 3, 2, 9]
. . . . .
[7, 6, 5, 2, 1, 4, 9, 8, 3]
[3, 2, 1, 6, 7, 4, 9, 8, 5]
[7, 6, 1, 2, 3, 4, 9, 8, 5]
```

Further explorations with minizinc show for small $n \leq 20$ that solutions exist (actually many for each n). As n becomes larger minizinc has varying amounts of success in finding solutions depending on n . For example:

[45, 44, 39, 50, 47, 42, 37, 46, 43, 40, 49, 48, 41, 32, 35, 38, 33, 26, 27, 34, 25, 36, 31, 30, 29, 24, 23, 20, 21, 22, 19, 18, 11, 12, 17, 14, 15, 16, 13, 10, 9, 8, 5, 6, 7, 4, 3, 2, 1, 28]

is a solution for $n = 50$ found in msecs, while $n = 23$ yields no solution in any reasonable amount of time.

Lastly, included below is a solution for $n = 1000$.

[991, 982, 997, 1000, 979, 994, 985, 988, 999, 980, 993, 986, 987, 992, 981, 998, 975, 974, 977, 996, 983, 990, 989, 984, 995, 978, 973, 960, 971, 962, 969, 964, 967, 966, 965, 968, 963, 950, 951, 956, 945, 944, 957, 976, 955, 952, 949, 958, 943, 946, 961, 972, 959, 942, 947, 954, 953, 948, 941, 926, 935, 936, 925, 922, 939, 928, 933, 940, 927, 934, 937, 924, 923, 938, 929, 932, 915, 908, 903, 920, 911, 912, 919, 904, 907, 916, 931, 930, 917, 914, 909, 902, 921, 910, 913, 918, 905, 906, 895, 882, 901, 900, 889, 894, 883, 876, 871, 888, 899, 884, 893, 896, 881, 872, 887, 890, 897, 886, 891, 892, 885, 898, 879, 880, 873, 874, 867, 866, 875, 878, 863, 858, 865, 868, 853, 870, 877, 864, 859, 862, 861, 860, 849, 844, 855, 854, 869, 852, 845, 848, 851, 846, 847, 850, 843, 856, 837, 820, 817, 840, 857, 836, 827, 830, 833, 824, 839, 818, 819, 838, 825, 832, 831, 826, 841, 816, 821, 842, 815, 822, 835, 828, 829, 834, 823, 814, 813, 808, 811, 810, 809, 812, 807, 794, 803, 798, 799, 802, 795, 806, 791, 792, 805, 796, 801, 800, 797, 804, 793, 790, 789, 778, 781, 786, 785, 782, 777, 772, 787, 784, 783, 788, 779, 780, 773, 776, 767, 764, 759, 752, 771, 760, 763, 768, 755, 756, 775, 774, 769, 762, 749, 750, 761, 770, 753, 758, 765, 766, 757, 754, 745, 748, 733, 738, 743, 744, 739, 742, 747, 734, 737, 746, 735, 736, 751, 730, 741, 740, 731, 728, 719, 732, 727, 724, 729, 722, 725, 726, 721, 718, 705, 704, 723, 716, 707, 720, 703, 706, 717, 710, 713, 714, 709, 700, 667, 694, 715, 712, 711, 688, 673, 708, 701, 698, 669, 692, 675, 686, 681, 680, 687, 674, 693, 668, 699, 682, 679, 702, 697, 670, 691, 676, 685, 696, 671, 690, 677, 684, 683, 678, 689, 672, 695, 666, 655, 664, 657, 662, 659, 660, 661, 658, 663, 656, 665, 654, 653, 650, 651, 646, 645, 652, 649, 648, 641, 642, 637, 640, 643, 634, 625, 624, 635, 644, 633, 626, 623, 636, 647, 630, 629, 620, 639, 638, 621, 628, 631, 618, 619, 612, 617, 632, 627, 622, 615, 614, 609, 604, 613, 600, 601, 616, 607, 610, 603, 598, 589, 592, 595, 606, 611, 602, 599, 594, 593, 608, 605, 596, 597, 590, 591, 572, 579, 574, 577, 586, 585, 568, 583, 570, 581, 582, 571, 580, 573, 578, 575, 576, 587, 564, 559, 558, 565, 588, 563, 566, 557, 560, 569, 584, 567, 556, 561, 562, 555, 554, 537,

550, 541, 546, 545, 542, 549, 538, 553, 534, 535, 552, 539, 548, 543,
544, 547, 540, 551, 536, 533, 530, 531, 532, 529, 522, 527, 524, 525,
526, 523, 528, 521, 510, 511, 520, 513, 518, 515, 516, 517, 514, 519,
512, 501, 508, 505, 492, 499, 498, 493, 504, 509, 500, 497, 494, 503,
506, 507, 502, 495, 496, 487, 480, 491, 476, 477, 490, 481, 486, 485,
482, 489, 478, 475, 472, 457, 484, 483, 488, 479, 474, 467, 470, 471,
466, 445, 462, 449, 458, 453, 454, 465, 464, 473, 468, 469, 460, 451,
456, 455, 452, 459, 448, 463, 444, 433, 450, 461, 446, 431, 426, 427,
430, 447, 440, 441, 442, 439, 438, 443, 434, 419, 420, 437, 416, 423,
436, 417, 422, 435, 418, 421, 432, 425, 428, 429, 424, 415, 412, 411,
410, 413, 414, 407, 402, 395, 392, 405, 404, 393, 394, 403, 406, 391,
396, 401, 408, 389, 398, 399, 388, 409, 400, 397, 390, 383, 386, 387,
382, 379, 378, 373, 384, 385, 376, 381, 380, 377, 374, 369, 370, 363,
364, 375, 368, 371, 372, 367, 366, 361, 358, 351, 350, 359, 360, 349,
352, 357, 362, 365, 354, 355, 346, 345, 356, 353, 348, 343, 330, 347,
344, 333, 340, 337, 336, 341, 342, 335, 324, 323, 338, 339, 334, 325,
322, 319, 328, 313, 318, 329, 332, 327, 314, 317, 326, 315, 316, 331,
310, 321, 320, 311, 306, 307, 312, 305, 308, 309, 304, 303, 290, 297,
296, 291, 302, 299, 294, 293, 300, 301, 292, 295, 298, 289, 288, 283,
286, 285, 284, 287, 282, 265, 276, 271, 270, 277, 280, 267, 274, 273,
268, 279, 278, 269, 272, 275, 266, 281, 260, 263, 258, 251, 248, 261,
262, 259, 264, 257, 246, 253, 256, 247, 240, 239, 252, 235, 244, 243,
236, 255, 254, 249, 238, 241, 250, 237, 242, 245, 234, 229, 228, 233,
224, 225, 232, 231, 226, 223, 220, 219, 230, 227, 222, 221, 218, 213,
208, 211, 210, 209, 212, 207, 214, 217, 216, 203, 206, 215, 204, 205,
196, 201, 200, 197, 192, 191, 198, 199, 202, 195, 194, 189, 184, 183,
190, 193, 186, 187, 180, 179, 188, 185, 182, 177, 176, 161, 170, 167,
164, 173, 174, 175, 178, 169, 168, 181, 166, 171, 160, 147, 146, 165,
172, 159, 152, 155, 156, 151, 162, 145, 148, 163, 144, 149, 158, 153,
154, 157, 150, 143, 138, 139, 142, 141, 140, 137, 134, 129, 128, 135,
136, 133, 124, 127, 130, 121, 118, 115, 126, 131, 132, 119, 120, 103,
108, 125, 116, 107, 104, 123, 100, 111, 112, 117, 106, 105, 122, 101,
110, 113, 114, 109, 102, 89, 92, 99, 94, 97, 96, 95, 98, 93, 86, 87,
80, 77, 90, 91, 88, 85, 82, 81, 76, 75, 74, 83, 84, 79, 78, 71, 60,
67, 72, 59, 68, 63, 64, 73, 58, 69, 62, 65, 66, 61, 70, 57, 56, 51,
46, 55, 54, 47, 50, 53, 44, 45, 52, 49, 48, 41, 42, 37, 36, 43, 40,
39, 32, 35, 38, 33, 26, 27, 34, 25, 28, 31, 30, 29, 24, 23, 20, 21,
22, 19, 18, 11, 12, 17, 14, 15, 16, 13, 10, 9, 8, 5, 6, 7, 4, 3, 2, 1,
970]