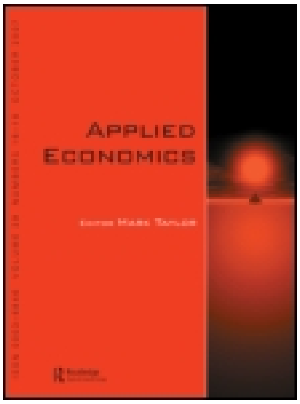

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Predicting bookmaker odds and efficiency for UK football

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Predicting bookmaker odds and efficiency for UK football

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The efficiency of gambling markets has frequently been questioned. In order to investigate the rationality of bookmaker odds, we use an ordered probit model to generate predictions for English football matches and compare these predictions with the odds of UK bookmaker William Hill. Further, we develop a model that predicts bookmaker odds. Combining a predictive model based on results and a bookmaker model based on previous quoted odds allows us to compare directly William Hill opinion of various teams with the team ratings generated by the predictive model. We also compare the objective value of individual home advantage and distance travelled with the value attributed to these factors by bookmakers. We show that there are systematic biases in bookmaker odds, and that these biases cannot be explained by William Hill odds omitting valuable, or excluding extraneous, information.

I. Introduction

The growing popularity of football and the continuing deregulation of the the gambling industry in the UK mean that more attention is placed on football results today than ever before.

There is a tradition of modelling football matches in the academic literature, in order to gauge the relative quality of teams and to predict next week's results. Similarly, there has been a long history of analyzing bookmaker odds: the gambling market is important from an economic perspective due to the parallels between gambling and financial markets. The possibility of profiting from badly-set odds as another incentive for studying bookmakers.[1]

In this article, we apply a simple ordered probit model to English football results. The ordered probit model gives an objective rating of quality for all English teams, and gives an indication of the size of the home advantage – that is, the greater likelihood that a team playing at home will win a match, *ceteris paribus*.

In order to compare model team ratings to bookmaker opinion of teams, we apply an ordered probit model of the same form to bookmaker odds. This allows us to compare directly the team ratings fit to bookmaker odds with the team ratings implied by previous results. Both models are extended to compare their ratings of other factors, such as individual team home advantage and distance travelled by the away team. By including these extra factors in both models, it becomes possible to determine whether bookmakers are omitting valuable or including extraneous information when they set their odds.

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[1] None of the work published has been particularly successful at beating the bookmakers. If it was successful, it would not have been published.

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The ordered probit model we use is based on one applied to Dutch football by Koning (2000), which in turn draws its inspiration from Neumann and Tamura (1996). The strength of each team is measured by a single parameter, and the model is fit by maximizing the likelihood of the match results (win, lose or draw). Two extra parameters fit the relative probabilities of a game between two average teams resulting in a home win, away win or draw.

Clarke and Norman (1995) employed a very similar model in order to investigate the home ground advantage of individual clubs in English soccer. Instead of fitting match results Clarke and Norman (1995) modelled the winning margin of the home team, and fit the model by least squares. Though this approach gives more information per match about the relative strengths of teams, it is unsuitable for use in our case because bookmakers assign probabilities to *results* rather than winning margins. In any case, Goddard (2005) found the difference in performance between results-based and goals-based probit models to be small.

Clarke and Norman (1995) pioneered some of the work on home advantage that we extend here: they allowed each team to possess its own home advantage. Although there was some variation in individual home advantage the effect was not significant. Further, they allowed each *pair* of clubs to have a mutual home advantage and found that this advantage had a small but significant positive correlation with the distance between the clubs, implying that this may be a factor in determining home advantage.

More sophisticated methods can be employed. A common technique is to use modified Poisson models to generate predictions for the exact score of a game, and then sum the exact scores to make result predictions. Dixon and Coles (1997) employ a Poisson-type model based on Maher (1982). They allow each team an attack and defence parameter, and modify a Poisson distribution of home and away goals so that it fits the observed distribution of match scores. Karlis and Ntzoufras (2003) discuss in more detail strategies for improving the fit of Poisson scores to actual match results.

Levitt (2004) provided evidence of bookmaker behaviour by investigating prices offered and volumes taken by bookmakers for spread bet markets in American Football. He finds that the prices quoted by bookmakers deviate systematically from those expected by supply and demand arguments, and that this deviation is explained by the fact that bookmakers are better than typical gamblers at predicting match outcomes. This means that, in bookmaking, profits can be maximized by taking positions on match outcomes, rather than by the traditionally accepted method of adjusting odds to lock in a risk-free profit. The work of Levitt (2004) hinged on a knowledge of the amount of money that had been bet on each outcome. In this article, we show that publicly available information can be used to uncover bias in bookmaker odds.

The subject of comparing predictions from an objective model with bookmaker odds has also been studied recently. Forrest *et al*. (2005) used an ordered probit model similar to the one employed in this article, and compared its predictions with bookmaker odds, but no analysis of intrinsic bookmaker opionion was made. In addition to recent results, they added to their model factors such as form over the past 24 months, FA cup involvement and match attendance relative to league position. They found many of the factors not based on results to have a significant, but small, effect.

The model of Forrest *et al*. (2005) failed to outperform bookmaker predictions, and was unable to make a profitable return on attractive bets. Intriguingly, they found that, for the two most recent seasons studied (2001–2002 and 2002–2003), adding bookmaker odds to the forecasting model improved its predictions, but the model predictions did not add information to a forecasting model based on bookmaker odds.

Dixon and Pope (2004) used the model of Dixon and Coles (1997) to compare bookmaker prices with results from a Poisson model. One result was that the bookmaker predictions for draws were very narrowly distributed compared to the model predictions. Because the model predictions are based on real data, this implies the bookmakers underestimated the variance in draw results. Another cfeature of bookmaker odds was that the Home win and Away win predictions were bimodally distributed, differing from the unimodal distributions generated by the Poisson model. This is significant because the Poisson model has the ability to display a bimodal distribution of probabilities. That it did not indicate an important difference between bookmaker predictions and model predictions.

The Poisson model was pitted against bookmaker odds by placing fictional bets on results for which the model assigned a greater probability than the bookmaker. A positive, but nonsignificant, return was found for odds discrepancies[2] of greater than 20%, indicating the model has predictive power compared to bookmaker prices.

[2] The discrepancy in odds is $p_m/p_b - 1$, where p_m is model probability and p_b bookmaker probability.

Dixon and Pope (2004) also found that odds were not priced efficiently, in the sense that bets on results predicted at less than 30% produced a higher return than bets on results predicted at more than 70%. This may be indicative of a 'favourite bias' in football betting, with bookmakers adjusting prices in response to the positions they have taken. These studies set the stage for our work, which employs a forecasting model to uncover in great detail the biases and patterns intrinsic in bookmaker odds.

In Section II we introduce the ordered probit model that we use to predict match outcomes. In Section III we introduce the model that we use to predict bookmaker odds. In Section IV we compare the predictions of the ordered probit model with bookmaker odds, and discuss whether bookmakers include extraneous or exclude vital information when tabulating their odds. If this is the case, the model should be able to identify where bookmaker prices are systematically incorrect. Alternatively, if the bookmakers include the correct information in their odds, it is an indication that their prices are efficient. We briefly discuss our conclusions in Section V.

II. Predictive Results Model

Description of the model

The model we employ is an ordered probit model, based on the one described in Koning (2000). Each team is modelled by a single parameter α_i. Two extra parameters, c_1 and c_2 control the proportion of matches that end in a home win and a draw. The model is defined by generating the random variable

$$D^*_{ij} = \alpha_i - \alpha_j + \eta_{ij}$$

with α_i the strength of the home team and α_j the strength of the away team. η_{ij} is zero-mean Gaussian noise that captures the variation in match results not attributable to differences in team strength. The random variable D^*_{ij} leads to the match outcome D_{ij} as follows:

$$D_{ij} = \begin{cases} 1 & D^*_{ij} > c_2, \\ 0 & c_1 < D^*_{ij} \le c_2, \\ -1 & D^*_{ij} \le c_1 \end{cases}$$

The outcome variable $D_{ij} = 1$ for a home win, $D_{ij} = 0$ for a draw and $D_{ij} = -1$ for an away win.

Assuming independent identically distributed Gaussion noise η_{ij} leads to the usual ordered probit model equations:

$$\begin{aligned} \mathrm{P}(D_{ij} = 1) &= 1 - \Phi\Big(\frac{c_2 - \alpha_i + \alpha_j}{\sigma}\Big), \\ \mathrm{P}(D_{ij} = 0) &= \Phi\Big(\frac{c_2 - \alpha_i + \alpha_j}{\sigma}\Big) - \Phi\Big(\frac{c_1 - \alpha_i + \alpha_j}{\sigma}\Big), \\ \mathrm{P}(D_{ij} = -1) &= \Phi\Big(\frac{c_2 - \alpha_i + \alpha_j}{\sigma}\Big) \end{aligned} \tag{1}$$

where Φ is the standardized normal cumulative distribution function, and c_1 and c_2 fix the proportion of home wins and draw, as described above.

Following Koning (2000), we make adjustments to ensure the model is correctly parameterized. The model as it stands is over-parameterized. In order to make the parameters identifiable the scale of the model is fixed by setting the variance $\sigma^2 = 1$. The scale of the α's is fixed by imposing the constraint

$$\sum_i \alpha_i = 0 \tag{2}$$

Therefore the average team has strength $\alpha = 0$.

The model is fit by maximum likelihood estimation, with the log-likelihood

$$\begin{aligned} \ln(L) = \sum_{\text{all matches}} & \delta_{D_{ij},1} \ln(1 - \Phi(c_2 - \alpha_i + \alpha_j)) \\ & + \delta_{D_{ij},0} \ln(\Phi(c_2 - \alpha_i + \alpha_j)) - \Phi(c_1 - \alpha_i + \alpha_j) \\ & + \delta_{D_{ij},-1} \ln(\Phi(c_2 - \alpha_i + \alpha_j)) \end{aligned}$$

where $\delta_{m,n} = 1$ if $m = n$ and $\delta_{m,n} = 0$ otherwise, and subject to constraint Equation 2.

Model extensions

The model can be extended in two important ways. First, the form of the probit model can be changed. We incorporated individual club home advantage by modifying the arguments of Equation 1 as follows:

$$\begin{aligned} c_2 - \alpha_i + \alpha_j &\to c_2 - h_i - \alpha_i + \alpha_j, \\ c_1 - \alpha_i + \alpha_j &\to c_1 - h_i - \alpha_i + \alpha_j \end{aligned}$$

with h_i the individual home advantage of the team playing at home i. Other parameters can be added to the model in a similar way.

The form of the model is further modified by estimating the distance between clubs' home grounds, and adding two 'distance' parameters. As in Clarke and Norman (1995), we use map co-ordinates to calculate the straight-line distance between clubs, and use this as an estimate of the distance actually travelled by the away team. In this model, Equation 1 is modified as follows:

$$\begin{aligned} c_2 - \alpha_i + \alpha_j &\to c_2 - d_2 x - \alpha_i + \alpha_j, \\ c_1 - \alpha_i + \alpha_j &\to c_1 - d_1 x - \alpha_i + \alpha_j \end{aligned} \tag{3}$$

where d_1 and d_2 are the distance-controlled parameters and x is the straight line distance between the clubs in kilometres, relative to the average distance between two clubs.

So far, the model is static and designed to fit past results. We allow for changes in team strength (and other parameters) by down-weighting past results in the maximum likelihood equation. This method is based on the one employed by Dixon and Coles (1997) in order to model fluctuating team abilities. The log-likelihood is modified by a time-dependent function

$$\begin{aligned} \ln(L) = \sum_{\text{all matches}} & f(t_{ij} - t_0)[\delta_{D_{ij},1} \ln(1 - \Phi(c_2 - \alpha_i + \alpha_j)) \\ & + \delta_{D_{ij},0} \ln(\Phi(c_2 - \alpha_i + \alpha_j)) - \Phi(c_1 - \alpha_i + \alpha_j) \\ & + \delta_{D_{ij},-1} \ln(\Phi(c_2 - \alpha_i + \alpha_j))] \end{aligned} \tag{4}$$

where $f(t - t_0)$ is the time-dependent function, t_{ij} is the date of the current match being evaluated and t_0 is the date of the most recent match.

Following Dixon and Coles (1997), we employ an exponential function

$$f(t - t_0) = \exp\left(\frac{t - t_0}{\tau}\right)$$

introducing a new parameter τ that controls how heavily past results are weighted. Since the intention of introducing τ is to maximize the predictiveness of the model, it is optimized by analyzing the model predictions on a holdout sample. The predictiveness of the model is calculated using the likelihood statistic

$$\begin{aligned} S(\tau) = \sum_{\text{holdout sample}} & (\delta_{D_{ij},1} \mathrm{P}(D_{ij} = 1) + \delta_{D_{ij},0} \mathrm{P}(D_{ij} = 0) \\ & + \delta_{D_{ij},-1} \mathrm{P}(D_{ij} = -1)) \end{aligned} \tag{5}$$

where the δ functions measure the match outcome and $\mathrm{P}(D_{ij} = k)$ is the model's predicted probability of outcome k for a particular match.

Data

We collected data from www.football-data.co.uk. It consists of results and William Hill bookmaker prices of 11 000 English league matches across four divisions (In descending order of quality: Premiership, Championship, League 1 and League 2) from 11th August 2001 to 26th November 2006.

In England, there are 20 teams in the Premiership and 24 teams in each of the Championship, League 1 and League 2. Each team plays each other team twice. Therefore there are 380 matches per season in the Premiership and 552 matches per season in the other three leagues. Our data comprises five compete seasons of 2036 league matches plus the 820 English league matches played between 5th August and 26th November 2006.

In order to calibrate the team strengths between divisions, and to ensure that all model parameters can be identified, at least two seasons worth of data are required. After one season teams are promoted and relegated between divisions, allowing inter-division calibration. The second season is required in order to ensure the inter-division calibration is reliable.

An alternative strategy would be to include cup matches, in which teams from different divisions play one another, in the model. However, bookmaker odds for cup matches could not be found and so we use league results only, in order to ensure the same data set is used for both the results prediction model and the bookmaker odds prediction model.

Results

The basic model was fit to seasons 2001–2002 and 2002–2003. The maximum log likelihood was −4100 for these 4072 matches, corresponding to a geometric average assigned probability of 0.365. This is comparable to the geometric average probability of 0.357 assigned to results by William Hill over this period, and the result illustrates the low predictability of football match results. The team strengths ordered by α are shown in Table 1. The parameters c_1 and c_2 give home/draw/away percentages of 45.1/28.7/26.2.

The next step was to fit the model of individual home advantages. The results are shown ordered by h in Table 2. Larger h indicates a larger individual home advantage. Incorporating individual home advantages into the model increased the maximum likelihood to −4052, corresponding to an average assigned probability of 0.370 for each match result.

The final model adjustment was to incorporate distance between clubs. The result gave small, distance-dependent parameters of $d_1 = -2.5 \times 10^{-4}$ and $d_2 = -3.9 \times 10^{-4}$ (see Equation 3).

For clubs an average distance of 180 km apart $x = 0$ and the win/lose/draw percentages are 45.1/26.2/28.8 and for clubs 380 km apart the percentages are 48.1/24.6/27.3, so the effect size of the distance parameter is small.

Before moving to the dynamic model, the optimum model should be selected. We use AIC to

Table 1. Probit model fit 2001–2003. $c_1 = -0.637$ and $c_2 = 0.124$

Team	α	Team	α	Team	α	Team	α
Arsenal	1.824	Norwich	0.453	Oldham	0.066	Bournemouth	−0.610
Man United	1.714	Sunderland	0.451	QPR	0.053	Mansfield	−0.613
Liverpool	1.514	Millwall	0.442	Sheffield Weds	0.035	Northampton	−0.658
Newcastle	1.456	Sheffield United	0.432	Tranmere	−0.047	Bury	−0.676
Chelsea	1.382	Reading	0.370	Grimsby	−0.071	Scunthorpe	−0.683
Leeds	1.129	Preston	0.350	Plymouth	−0.102	Rochdale	−0.790
Blackburn	1.064	Nott'm Forest	0.345	Brentford	−0.119	Kidderminster	−0.845
Man City	0.993	Wimbledon	0.344	Luton	−0.174	Cambridge	−0.869
Everton	0.991	Burnley	0.315	Barnsley	−0.220	York	−0.906
Tottenham	0.964	Gillingham	0.282	Huddersfield	−0.247	Hull	−0.907
Southampton	0.957	Crystal Palace	0.273	Swindon	−0.283	Torquay	−0.916
Aston Villa	0.955	Watford	0.203	Colchester	−0.311	Lincoln	−0.927
West Ham	0.927	Wigan	0.169	Wycombe	−0.328	Oxford	−0.965
Fulham	0.903	Derby	0.168	Blackpool	−0.339	Boston	−0.973
Middlesboro	0.901	Cardiff	0.168	Peterboro	−0.340	Macclesfield	−1.006
Charlton	0.898	Coventry	0.163	Port Vale	−0.394	Darlington	−1.009
Bolton	0.843	Rotherham	0.156	Notts County	−0.419	Shrewsbury	−1.020
Birmingham	0.738	Brighton	0.137	Stockport	−0.474	Southend	−1.051
Leicester	0.698	Crewe	0.116	Chesterfield	−0.476	Leyton Orient	−1.086
Wolves	0.650	Stoke	0.097	Cheltenham	−0.498	Exeter	−1.090
West Brom	0.590	Bradford	0.075	Rushden	−0.511	Carlisle	−1.111
Portsmouth	0.549	Bristol City	0.070	Wrexham	−0.560	Swansea	−1.126
Ipswich	0.511	Walsall	0.070	Hartlepool	−0.566	Bristol Rvs	−1.160
						Halifax	−1.443

select our model. The results in Table 3 indicate that the likelihood enhancements caused by adding individual home advantages and distance between clubs are not sufficient to compensate for the extra model complexity.

In order to measure the predictiveness of the model, it is necessary to move to the dynamic version. Predictions were made for the 2003–2004 season for various values of τ and the value of τ that maximized the predictive likelihood statistic Equation 5 used for all further predictions. Fig. 1 shows that the optimum decay time is $\tau = 600$ days. The predictive likelihood flattens out after the optimum $\tau = 600$ days because the model has only two seasons of data to maximize over. If the data window was longer we would expect a steeper decay in the likelihood statistic as found in Dixon and Coles (1997). This is similar to the value of 525 days reported by Dixon and Coles (1997) in their implementation of a dynamic Poisson model.

The predictive likelihood per match was 0.346, slightly inferior to William Hill's predictive likelihood of 0.350 and unsurprisingly smaller than the likelihood of 0.365 found in this section for fitting of past results. Bootstrapping the predictive likelihood for each match indicated that the probit model was not significantly less predictive than William Hill. The probit model has the disadvantage that new clubs which enter the league each season are unknown, and the value of τ implies the model needs about 2 seasons-worth of data to make reliable predictions.

III. Odds Forecasting Model

Description of the model

The data we use are the odds offered by William Hill for a home win, a draw and an away win in each match. William Hill is one of the largest high street bookmakers in the UK, and this is our main reason for choosing it as the basis for the odds forecasting model. In addition, Forrest *et al.* (2005) found that William Hill was the best-performing bookmaker in the period 1998–2003, so that their odds can be considered a benchmark for our ordered probit model.

Other bookmakers can be compared, but Dixon and Pope (2004) and Forrest *et al.* (2005) both found odds offered by major high street UK bookmakers to be similar, and that outcome prediction varies much more season-to-season than it does between bookmakers within a single season. Thus, little information is lost by concentrating on one bookmaker.

The William Hills prices θ are presented as decimal odds. An odd of $\theta = 3$ returns three units for a one

Table 2. Individual home advantage model fit 2001–2003. $c_1 = -0.650$ and $c_2 = 0.124$

Team	α	h	Team	α	h
Norwich	0.180	0.576	Nott'm Forest	0.243	0.225
Southend	−1.248	0.370	Barnsley	−0.318	0.190
Mansfield	−0.803	0.370	Hartlepool	−0.659	0.185
Middlesboro	0.724	0.369	Man City	0.925	0.175
Aston Villa	0.794	0.354	Wycombe	−0.415	0.163
Brentford	−0.289	0.349	Walsall	−0.005	0.160
Fulham	0.741	0.348	Millwall	0.376	0.157
Preston	0.187	0.340	Huddersfield	−0.325	0.148
Newcastle	1.325	0.330	Port Vale	−0.477	0.146
York	−1.077	0.316	Blackpool	−0.424	0.140
Stoke	−0.046	0.300	Rotherham	0.093	0.134
Cambridge	−1.035	0.295	Southampton	0.897	0.133
Boston	−1.136	0.291	Scunthorpe	−0.755	0.133
Rushden	−0.646	0.285	Notts County	−0.493	0.128
West Ham	0.798	0.279	Tranmere	−0.106	0.118
Hull	−1.061	0.269	Crystal Palace	0.222	0.113
Bournemouth	−0.750	0.268	Chelsea	1.350	0.106
Tottenham	0.849	0.264	Derby	0.114	0.106
Darlington	−1.158	0.258	Brighton	0.095	0.094
Bristol City	−0.054	0.256	Plymouth	−0.144	0.085
Everton	0.876	0.253	Birmingham	0.715	0.067
Leyton Orient	−1.229	0.245	Watford	0.176	0.066
Halifax	−1.608	0.245	Gillingham	0.261	0.057
Blackburn	1.048	0.052	Rochdale	−0.689	−0.220
Chesterfield	−0.509	0.052	Shrewsbury	−0.918	−0.232
Sunderland	0.435	0.036	West Brom	0.719	−0.243
Kidderminster	−0.863	0.019	Oxford	−0.854	−0.244
QPR	0.040	0.016	Bradford	0.209	−0.254
Swansea	−1.145	0.013	Lincoln	−0.810	−0.262
Northampton	−0.668	0.002	Coventry	0.304	−0.269
Swindon	−0.275	−0.020	Man United	1.871	−0.287
Peterboro	−0.331	−0.027	Liverpool	1.671	−0.290
Grimsby	−0.043	−0.050	Colchester	−0.170	−0.292
Bristol Rvs	−1.138	−0.065	Arsenal	1.991	−0.297
Bolton	0.883	−0.067	Carlisle	−0.975	−0.312
Wimbledon	0.396	−0.091	Cardiff	0.327	−0.326
Portsmouth	0.606	−0.102	Sheffield Weds	0.204	−0.337
Cheltenham	−0.440	−0.131	Reading	0.558	−0.372
Leicester	0.770	−0.132	Bury	−0.492	−0.381
Burnley	0.393	−0.140	Crewe	0.324	−0.399
Sheffield United	0.515	−0.147	Ipswich	0.727	−0.422
Torquay	−0.852	−0.153	Luton	0.043	−0.444
Oldham	0.137	−0.155	Macclesfield	−0.791	−0.455
Wrexham	−0.471	−0.189	Charlton	1.177	−0.548
Stockport	−0.374	−0.198	Leeds	1.421	−0.549
Exeter	−0.992	−0.208	Wolves	0.946	−0.569
			Wigan	0.445	−0.573

unit stake (two units profit plus the one unit stake), and implies an event probability of 1/3. This is equivalent to traditional English odd of 2-to-1. Bookmaker odds, when converted to probabilities, do not sum to one because of the take-out or over-round R. Given three odds on a match then

$$\frac{1}{\theta_h}+\frac{1}{\theta_d}+\frac{1}{\theta_a}=1+R\simeq 1.124 \qquad (6)$$

Table 3. Log-likelihoods and AICs of model variants

Model	ln(L)	Parameters	AIC
Basic	−4100	92	8384
Individual home advantages	−4052	184	8472
Distance effect	−4098	94	8384

for William Hill. Thus, to convert decimal odds into probabilities, $1/\theta$ is divided by $(1+R)$.

If the bookmaker receives volumes proportional to the implied probabilities a percentage profit of $R/(1+R) \simeq 11\%$ is achieved without risk. Consider a match ending in an away win. The bookmaker keeps all the money bet on home wins and draws, and pays out $\theta_a - 1$ on away win bets. If the volume received is proportional to implied probability $V_i = C\theta_i/(1+R)$, the profit is

$$\frac{C}{1+R}\left(\frac{1}{\theta_h}+\frac{1}{\theta_d}-\frac{1}{\theta_a}*(\theta_a-1)\right)=\frac{CR}{1+R}$$

and the same profit is achieved regardless of the match outcome. Levitt (2004) suggests that bookmakers may not set prices in this way in practice.

With the available implied bookmaker probabilities, we can model how bookmakers rate each team. The bookmaker forecasting model follows the ordered probit model of Section II closely, so that we can draw direct comparisons between bookmaker predictions and model predictions. Bookmakers are assumed to make a rating of the strength of each team and then compute odds, based on their perception of relative team strength.

We assume that the bookmaker's model of team strength follows our probit model exactly. The bookmaker estimates strengths α_i and α_j and also the parameters c_1 and c_2. The implied probabilities that the bookmaker generates are then given by Equation 1.

Fitting this model proceeds by comparing model predictions to posted bookmaker odds. The posted odds are assumed to be normally distributed around those predicted by our bookmaker model. Here is an example for home wins: the model prediction of bookmaker probability is

$$\mathrm{P}(D_{ij}=1)=1-\Phi\left(\frac{c_2-\alpha_i+\alpha_j}{\sigma}\right)=m_h$$

and is fit to the actual bookmaker probability by least squares. The close resemblance of this model with the results-based model allows comparison of bookmaker opinion with model predictions. We can compare the relative strength of teams, how much importance bookmakers place on home advantage and so on.

Model extensions

Because the odds forecasting model is defined in same way as the probit model, all of the extensions described in section II, 'Model Extansions' can be implemented in exactly the same way. We can therefore analyze the effect of individual club home advantage and distance between clubs on bookmaker odds. In addition, we can also generate a dynamic odds forecasting model using the same method as described by Equation 4.

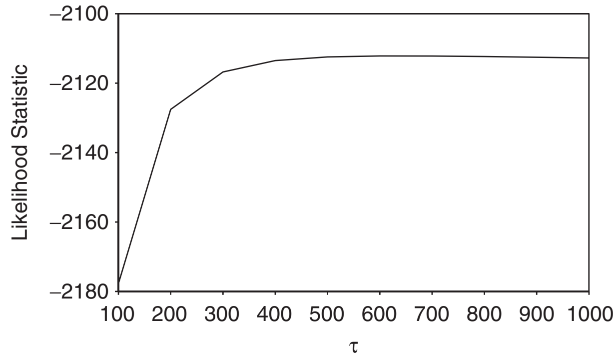

Fig. 1. Predictive likelihood statistic against decay parameter τ.

Data

The odds we use are for the same 11 000 English league matches described in section 'Data' of Section II, 99 sets of odds are missing from our data set, with 96 of the missing odds occurring in the 2001–2002 and 2002–2003 seasons. These missing odds should only have a minor effect on differences observed between bookmaker opinion and probit model opinion.

Results

Following the same procedure as in section 'Results' of section II, we first fit the static odds-forecasting model to seasons 2001–2003, then decide on a model to use, and then predict odds for seasons 2004–2006.

Table 4 shows the team-rankings of the bookmaker model and the probit model are similar, but there is more variation in the probit model parameters than there is in the odds-forecasting model. The values of c_1 and c_2 give home win/draw/away win percentages of 44.9/27.6/27.5. The home win chance is very similar to the result-prediction model chance of 45.1, indicating that William Hill opionion of home advantage matches that found from fitting previous results.

The individual home advantage model and the distance-based home advantage model were fit to seasons 2001–2003. Log-likelihoods and AICs relative to the basic model are shown in Table 5. The results show that adding individual home

Table 4. Basic odds-forecasting model fit 2001–2003. $c_1 = -0.599$ and $c_2 = 0.128$

Team	α	Team	α	Team	α	Team	α
Man United	1.179	West Brom	0.267	Walsall	−0.037	Cheltenham	−0.339
Arsenal	1.115	Derby	0.255	QPR	−0.039	Mansfield	−0.350
Liverpool	0.996	Coventry	0.238	Oldham	−0.061	Northampton	−0.363
Chelsea	0.874	Norwich	0.237	Grimsby	−0.090	Wrexham	−0.364
Newcastle	0.797	Millwall	0.208	Tranmere	−0.094	Rochdale	−0.382
Leeds	0.782	Sheffield United	0.189	Brentford	−0.109	Scunthorpe	−0.393
Tottenham	0.665	Nott'm Forest	0.186	Barnsley	−0.135	Bury	−0.414
Aston Villa	0.577	Watford	0.184	Luton	−0.144	Oxford	−0.446
Blackburn	0.577	Burnley	0.182	Huddersfield	−0.161	Cambridge	−0.452
Man City	0.570	Crystal Palace	0.172	Plymouth	−0.169	Kidderminster	−0.459
Fulham	0.553	Preston	0.167	Wycombe	−0.215	Shrewsbury	−0.490
Middlesboro	0.544	Reading	0.158	Blackpool	−0.224	York	−0.510
Everton	0.544	Milton Keynes Dons	0.126	Port Vale	−0.235	Bristol Rvs	−0.527
West Ham	0.510	Cardiff	0.095	Colchester	−0.262	Southend	−0.528
Southampton	0.460	Gillingham	0.078	Stockport	−0.263	Leyton Orient	−0.536
Charlton	0.441	Bradford	0.064	Swindon	−0.268	Torquay	−0.538
Sunderland	0.439	Bristol City	0.035	Notts County	−0.281	Darlington	−0.544
Wolves	0.400	Wigan	0.033	Peterboro	−0.299	Boston	−0.572
Ipswich	0.394	Stoke	0.030	Rushden	−0.302	Lincoln	−0.574
Bolton	0.382	Brighton	0.005	Chesterfield	−0.304	Swansea	−0.594
Birmingham	0.366	Crewe	0.00	Bournemouth	−0.311	Macclesfield	−0.604
Leicester	0.339	Sheffield Weds	−0.005	Hull	−0.311	Exeter	−0.628
Portsmouth	0.300	Rotherham	−0.014	Hartlepool	−0.338	Carlisle	−0.646
						Halifax	−0.765

advantages or distances is also not justified for the odds forecasting model.

For the dynamic model, the decay parameter τ was fit as in section 'Results' of Section II. A value of $\tau = 15$ days optimized the predictiveness of odds forecasting. This is significantly shorter than $\tau = 600$ days that was found to optimize the probit model predictiveness, but bookmaker odds encode much more information than match results, and so it is not surprising that only a few past odds per team are sufficient to determine future prices.

Table 5. Log-likelihoods and AICs of model variants

Model	Relative ln (L)	Parameters	Relative AIC
Basic	0.0	92	184.0
Individual home advantages	0.2	184	367.6
Distance effect	0.1	94	187.8

This result indicates that, if bookmakers are prone to bias and irrationality when setting odds, the *extra* information they possess more than makes up for this. The *extra* information that bookmakers possess relative to the probit model is a knowledge of player purchases, sales and injuries, a knowledge of teams new to the football league and a knowledge of goals scored and conceded. The overall effect is comparable predictiveness between the probit model and William Hill.

IV. Comparing Predictions with Odds

In this section we compare the results of the two models described in. Our models allow us to answer important questions concerning bookmaker odds.

Results prediction

Section 'Results' of Section II indicated that, from August 2004 to November 2006, William Hill implied probabilities out-performed a dynamic probit results model. The performance of William Hill probabilities was not significantly better, however. This result is not surprising, as it is in agreement with the findings of Forrest *et al*. (2005).

Rationality of prices

Do bookmakers give rational prices on average for football matches? Do the implied probabilities for each outcome match the empirical observations?

The odds-forecast model and the probit model give similar percentage chances to home win/draw/away win results (45.1/28.7/26.2 for the probit model compared to 44.9/27.6/27.5 for the odds model). For the whole data set, placing a one unit stake on all home win/draw/away win results gives profits of −11.1%/−10.3%/−16.0%. This indicates that, as in the 2001–2003 fit, William Hill assigns less probability to and offers better prices for home wins and draws. Betting on all results leads to an overall profit of −12.5%, consistent with Equation 6.

Are there inconsistencies within each bet type? Charting average returns for bets placed by probability indicates that this is the case. Figure 2 shows returns on bets placed on home wins and away wins of different probabilities. Home and away bets show significantly better returns at lower odds. Interestingly, this is precisely the opposite result to that found by Dixon and Pope (2004), who examined odds from three UK bookmakers in the period 1993–1996. This may be indicative of a change in strategy since 1996 for UK bookmakers, or may just illustrate that Dixon and Pope (2004) may not have been studying William Hill (they refer to the bookmaker as 'firm A').

The finding that bets with lower odds give higher returns is consistent with the results of the odds forecasting model and the probit model. In the odds-forecasting model, strong teams were consistently rated as worse than in the probit model, and weak teams as better than the probit model. For example, predictions for Arsenal V Sunderland in the 2001–2003 fit are shown in Table 6.

We can analyze this further by considering the odds posted for matches between strong and weak teams. Note that the odds-forecasting and probit models are in agreement about who is a strong team – ordering the 2001–2003 fits by rank gives a rank-difference SD of 3.96. The difference between the models, then, is in the quantitative rather than the ordinal rating of the teams.

We use the team rank difference between models to rate the ordinal difference between the teams, and study discrepancies between the outcome-forecating model and William Hill by team rank difference. The results are shown in Fig. 3. The figure clearly shows that when the rank difference is negative – the away team is stronger – the probit model assigns less probability than William Hill to the home team and more probability to the other outcomes. When the home team is ranked much higher, the outcome-forecasting model assigns more probability to the home team than do William Hill.

Three lines of evidence – larger losses for bets on all away wins, smaller losses for lower odds and under-rating of strong teams indicate that there is a long-shot bias in William Hill odds: the odds on long-shots are slightly short and the odds on favourites are slightly long. This irrationality is probably market-driven, and could be a case of bookmakers actively taking positions against punters, as suggested in Levitt (2004).

This irrationality in prices is not an artifact. We have shown that the probit model has similar predictive capability to William Hill, and the probit model is completely objective. Therefore we expect *a priori* a random difference in predictions. We have shown strong evidence for patterns in pricing, and this can be considered 'irrational' in the sense that the prices do not match the true event probabilities.

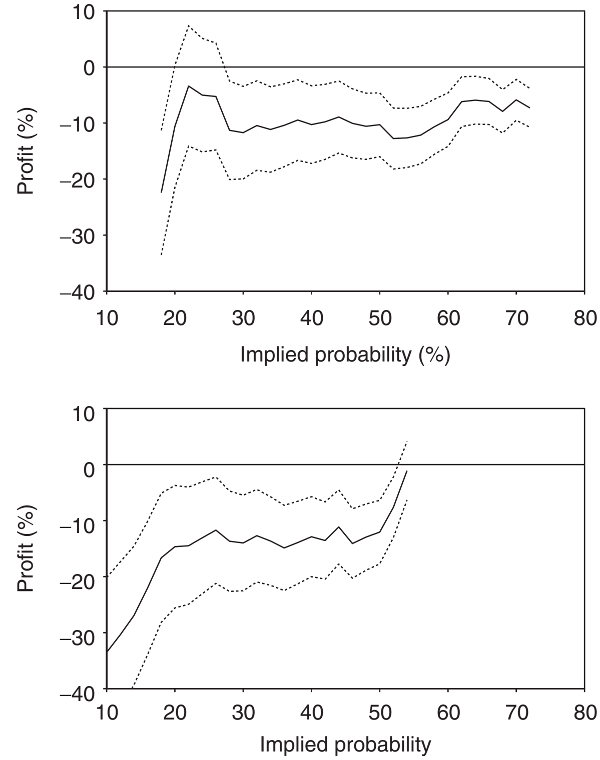

Fig. 2. Returns on betting on all home wins (top) and away wins (bottom) within 5% of a given probability. 95% confidence limits calculated by bootstrap are shown as dotted lines. Compare Hill over-round $R = 12.5\%$, the expected loss if bets were rationally priced.

Table 6. Predictions for Arsenal V Sunderland for 2001–2003 model fits

Model	Home Win%	Draw%	Away Win%
Outcome-forecasting	89.4	8.4	2.2
Odds-forecasting	71.0	19.6	9.4

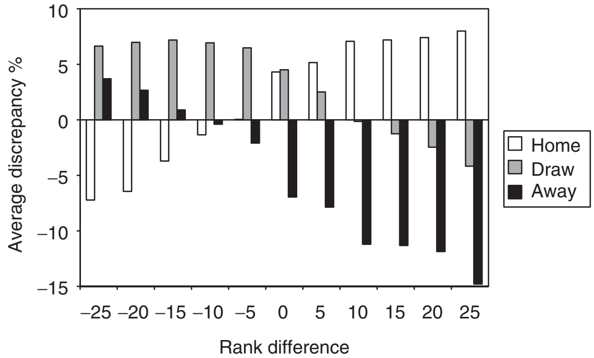

Fig. 3. Discrepancies between outcome-forecast predictions and William Hill predictions 2004–2006. Discrepancy is $p_{\text{Probit}}/p_{\text{Hill}} - 1$. Rank difference is difference in probit model ranking of home and away team.

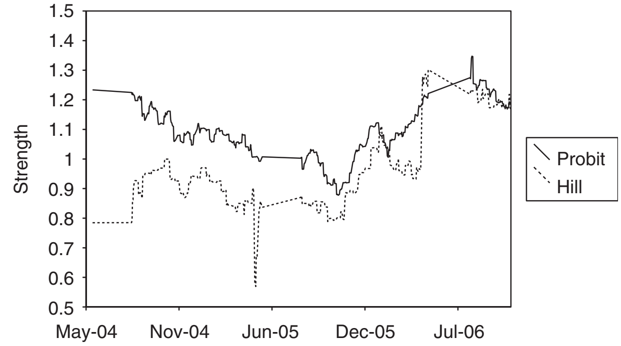

Fig. 4. Diffusion of Liverpool's team strength as calculated by the dynamic probit model and the dynamic odds-forecasting models. There are no matches in June or July each season.

Home advantage factors

In section 'Results' of Section II we found no evidence for the inclusion of individual club home advantages or the effect of distance in our model. Similarly, in the odds forecasting model, there was no need to include these factors. Thus, whether we are predicting results or bookmaker odds, we do not use this information.

The information is not useful in predicting William Hill odds and this, in turn, implies that bookmakers do not take account of individual home advantage or distances travelled. This is rational behaviour, in the sense that an objective, results-based model also rejects these factors.

Weighting of past results

We showed in sections 'Results' of Section II and III that the outcome-forecasting model has a decay time of 600 days and the bookie model a decay time of only 15 days. Do bookmakers weight past results heavily enough?

Bookmaker odds contain more information than results; therefore it is not surprising that the decay time is much shorter. An alternative interpretation is that William Hill already uses knowledge of past results to set prices, and so an inspection of past odds is not required to determine next week's odds.

In order to analyze weighting of past results, it is necessary to study the statistics of the underlying model parameters, because comparing τ's is not sufficient. Rather than comparing τ's we compare how quickly the underlying team strengths α diffuse in each model. For example, if the rate of diffusion is greater for the odds-forecasting model it is an indication that bookmakers have a shorter-term view of team quality than the outcome-forecasting model.

Figure 4 shows the dynamic rating of Liverpool for each model. Most teams diffuse around an average strength, with a few exceptions.[3] Therefore, a calculation of the average SD in team strength gives an indication of how much weight is given to past results by bookmakers.

The diffusion for each team in the odds-forecasting model is slightly smaller than the diffusion in the probit model. Average SD of team strength over 442 match dates is 0.096 for the bookmaker model and 0.101 for the probit model. A paired *t*-test indicates no significant difference in team strength diffusion between the models. This indicates that team quality diffuses at the same rate for the outcome-forecasting model and the odds-forecasting model, and that William Hill weights past results correctly.

Profiting from irrationality

We showed that William Hill prices have persistent, irrational patterns. Is it possible to profit from these patterns? Following Dixon and Pope (2004) we simulated bets on all matches where the probit model probabilities differed from the Hill probabilities by greater than a given percentage. The results are shown in Fig. 5.

It is clear that the inconsistent pricing of Hills cannot be exploited by the probit model using a simple strategy of betting on all outcomes above a certain discrepancy. Forrest *et al.* (2005) also showed that a probit model could not outperform

[3] In the past few seasons, Wigan Athletic have achieved a large increase in strength, and Leeds United have suffered a large decrease in strength.

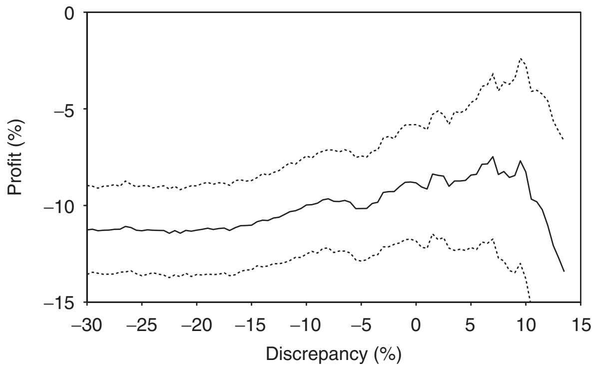

Fig. 5. Profit against odds discrepancy $p_{\mathrm{Probit}}/p_{\mathrm{Hill}} - 1$ for 2004–2006, shown together with bootstrap errors.

bookmakers. Our results are worse than those of Dixon and Pope (2004) because we used less sophisticated model, and becasue of the fact that bookmaker pricing may have improved since 1996 (Forrest *et al*., 2005).

As gambling markets continue to be de-regulated, bookmaker margins inexorably decrease. The betting exchange BetFair takes only a 5% commission, and in Asia, margins can be as small as 1%. In these small margin markets it is possible that the systematic biases shown in Fig. 3 can be exploited and that, as a result, must disappear if the bookmaker is to remain profitable.

V. Conclusions

We developed a results-based probit model and an odds forecasting model to compare the odds of William Hill to those generated by an objective model. We investigated whether home advantage factors played an important part in predictions, and discovered the time period over which past results were important in determining future probabilities. The probit model gave predictions of similar accuracy to William Hill probabilities, and the odds forecasting model produced reasonable predictions of William Hill odds.

We found that, for the most part, William Hill prices were rational: home advantage factors were not significant for either model, and team strength parameters diffused at similar rates for both models, indicating that William Hill's rating of the importance of past results is similar to that of the probit model.

We discovered one important difference between the probit and odds forecasting models. In the odds forecasting model, strong teams were rated worse than in the probit model, and weak teams were rated better. We found a systematic deviation in William Hill odds for games between weak teams and strong teams, with the probit model assigning more probability to the strong team winning and less to the weak team. The fact that William Hill predictions were as accurate as those of the probit model, despite systematic deviations, suggests that Hill may make good use of information not available to the probit model. An example of such information is news of player purchases and injuries.

William Hill prices are 'irrational' in the sense that they do not match event probabilities, but this irrationality cannot be exploited by a statistical model. The deviations observed are consistent with a 'long-shot bias' – Hill offers better odds on favourites. The next stage in research of this type would be an analysis of market forces, in order to understand why posted odds and the predictions of an objective model differ.

References

Clarke, S. R. and Norman, J. M. (1995) Home ground advantage of individual clubs in English soccer, *The Statistician*, **44**, 509–21.

Dixon, M. J. and Coles, S. G. (1997) Modelling association football scores and inefficiencies in the football betting market, *Applied Statistics*, **46**, 265–80.

Dixon, M. J. and Pope, P. F. (2004) The value of statistical forecasts in the UK association football betting market, *International Journal of Forecasting*, **20**, 697–711.

Forrest, D., Goddard, J. and Simmons, R. (2005) Odds-setters as forecasters: The case of English football, *International Journal of Forecasting*, **21**, 551–64.

Goddard, J. (2005) Regression models for forecasting goals and match results in association football, *International Journal of Forecasting*, **21**, 331–40.

Karlis, D. and Ntzoufras, I. (2003) Analysis of sports data by using bivariate Poisson models, *Journal of the Royal Statistical Society Series D(The Statistician)*, **52**, 381–93.

Koning, R. H. (2000) Balance in competition in Dutch soccer, *The Statistician*, **49**, 419–31.

Levitt, S. D. (2004) Why are gambling markets organised so differently from financial markets?, *The Economic Journal*, **114**, 223–46.

Maher, M. J. (1982) Modelling association football scores, *Statistica Neerlandica*, **36**, 109–18.

Neumann, G. R. and Tamura, R. F. (1996) *Managing Competition: The Case of the National Football League*, Dept. of Economics, College of Business Administration, University of Iowa.