# On an identity leading to the Fermat's Last Theorem in a short computation

Pawel Jan Piskorz[*]

September 5, 2023

## Introduction.

We write the equation

$$X^N + Y^N = Z^N \tag{1}$$

for which $X, Y, Z \in \mathbb{Z}_+$ are integers greater than zero and $N \in \mathbb{N}$ is a natural number.

Without loss of generality we rewrite Equation (1) as

$$(x + Mpx)^N + Y^N p = Z^N p \tag{2}$$

with $X = x + Mpx$. We have introduced a real number parameter $p \in \mathbb{R}$ the value of which will be later set to 1. The number $M \in \mathbb{N}$ is a natural number. With such assumptions we must have $x \in \{\frac{1}{M+1}, \frac{2}{M+1}, \frac{3}{M+1}, \ldots\}$ in order for the variable $X$ to assume integer values. E.g.: if $M = 1$, $p = 1$ then $x \in \{\frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \ldots\}$ and then $X \in \{1, 2, 3, \ldots\}$. We always arrive at $X \in \{1, 2, 3, \ldots\}$ for any natural number $M$ and $p = 1$, i.e. we have ensured that always $X \in \{1, 2, 3, \ldots\}$ no matter which natural number $M$ we take into account. Similarly we can ensure that $Y$ and $Z$ are also integers greater than zero.

## Computation.

We take partial derivative of both sides of Equation (2) with respect to $p$

$$\frac{\partial (x + Mpx)^N}{\partial p} + \frac{\partial Y^N p}{\partial p} = \frac{\partial Z^N p}{\partial p} \tag{3}$$

obtaining a new equation

$$NM(x + Mpx)^{N-1} x + Y^N = Z^N \tag{4}$$

[*]pjpxyz@protonmail.com, Krakowska 55, 31-066 Krakow, Poland

Now we set $p = 1$ in the Equation (4) receiving

$$NM(M+1)^{N-1}x^N + Y^N = Z^N \tag{5}$$

If we set $p = 1$ in Equation (2) on the other hand we obtain

$$(M+1)^N x^N + Y^N = Z^N \tag{6}$$

We compare the coefficients at the term with $x^N$ in Equations (5) and (6) receiving a constraint equation for $N$

$$NM(M+1)^N/(M+1) = (M+1)^N \tag{7}$$

and therefrom we obtain the values of exponent $N$ as a function of $M$

$$N = \frac{M+1}{M} \tag{8}$$

We find the values of $N(M)$ as

$$\begin{aligned} N(M=1) &= \frac{2}{1} = 2 \\ N(M=2) &= \frac{3}{2} \\ N(M=3) &= \frac{4}{3} \\ N(M=4) &= \frac{5}{4} \\ &\vdots \\ N(M=\infty) &= 1 \end{aligned} \tag{9}$$

We can state that with our assumption of having $N \in \mathbb{N}$ we have to reject all $N(M)$ solutions above which are not natural numbers.

## Conclusion.

We have started with the Equation (1) with assumption that $X, Y, Z$ are positive integers and $N$ is a natural number. We see from our computations that this equation is valid for natural exponents only if the exponents in it are either $N = 1$ or $N = 2$ what is in agreement with the Fermat's Last Theorem.

## Acknowledgment.

This article is in honor of American mathematician Kenneth S. Miller. Due to his technique of computing the expected value and the standard deviation

of number of successes in Bernoulli trials we were able to obtain our results. Author would also like to thank an anonymous student without whom it would take longer to write this paper and who suggested placing the number $p$ next to symbols $Y^N$ and $Z^N$ in Equation (2) while the author was working on the universal expression of the natural number $X$ using rational number $x$ and a real parameter $p$ for partial differentiation.

## References


[1] Fermat's Last Theorem
`https://en.wikipedia.org/wiki/Fermat%27s_Last_Theorem`