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Fermat's Last Theorem derived in a direct computation

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Abstract. We propose a procedure which allows to compute the only acceptable natural exponents of the positive integers X, Y, Z in the equation of the Fermat's Last Theorem. We use the approach similar to the one applied in computing of the expected value and the standard deviation of number of successes in Bernoulli trials presented by Kenneth S. Miller.

1. Introduction

We write the equation from the Fermat's Last Theorem [1]

$$X^N + Y^N = Z^N \tag{1.1}$$

in which $X, Y, Z \in \mathbb{Z}_+$ are integers greater than zero and $N \in \mathbb{N}$ is a natural number.

Without loss of generality we rewrite Equation (1.1) as

$$(x + Mpx)^N qr + (y + Lqy)^N rp = (z + Krz)^N pq \tag{1.2}$$

with $p, q, r \in \mathbb{R}$ and $M, L, K \in \mathbb{N}$. Now we set $q = 1$ and $r = 1$ with $Y = y + Ly$ and $Z = z + Kz$ obtaining

$$(x + Mpx)^N + Y^N p = Z^N p \tag{1.3}$$

with $X = x + Mpx$. We have introduced a real number parameter $p \in \mathbb{R}$ the value of which will be later set to 1. The number $M \in \mathbb{N}$ is a natural number. With such assumptions we must have $x \in \{\frac{1}{M+1}, \frac{2}{M+1}, \frac{3}{M+1}, \ldots\}$ in order for the variable X to assume integer values.

E.g.:

if $M = 1, p = 1$ $x \in \{\frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \ldots\}$ then $X = \{\frac{1}{2} + 1 \cdot \frac{1}{2}, \frac{2}{2} + 1 \cdot \frac{2}{2}, \frac{3}{2} + 1 \cdot \frac{3}{2}, \ldots\}$ what gives $X \in \{1, 2, 3, \ldots\}$;

if $M = 7, p = 1$ $x \in \{\frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \ldots\}$ then $X = \{\frac{1}{8} + 7 \cdot \frac{1}{8}, \frac{2}{8} + 7 \cdot \frac{2}{8}, \frac{3}{8} + 7 \cdot \frac{3}{8}, \ldots\}$ what gives again $X \in \{1, 2, 3, \ldots\}$.

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We always arrive at $X \in \{1, 2, 3, \ldots\}$ for any natural number M and $p = 1$, i.e. we have ensured that always $X \in \{1, 2, 3, \ldots\}$ no matter which natural number M we take into account. Similarly we can ensure that Y and Z are also integers greater than zero appropriately choosing natural number parameters L and K and appropriately real numbers q and r.

2. Computations

We take partial derivative of both sides of Equation (1.3) with respect to p

$$\frac{\partial(x+Mpx)^N}{\partial p} + \frac{\partial Y^N p}{\partial p} = \frac{\partial Z^N p}{\partial p} \tag{2.1}$$

We compute the partial derivtive of $(x + Mpx)^N$ as follows

$$\begin{aligned} \frac{\partial(x+Mpx)^N}{\partial p} &= N(x+Mpx)^{N-1}\frac{\partial(x+Mpx)}{\partial p} \\ = N(x+Mpx)^{N-1}Mx &= NM(Mp+1)^{N-1}x^{N-1}x \\ &= NM(Mp+1)^{N-1}x^N \end{aligned} \tag{2.2}$$

We receive a new equation

$$NM(Mp+1)^{N-1}x^N + Y^N = Z^N \tag{2.3}$$

in which we can set the prameter $p = 1$ obtaining

$$NM(M+1)^{N-1}x^N + Y^N = Z^N \tag{2.4}$$

If we set $p = 1$ in Equation (1.3) on the other hand we obtain

$$(M+1)^N x^N + Y^N = Z^N \tag{2.5}$$

We compare the coefficients at the term with x^N in Equations (2.4) and (2.5) receiving a constraint equation for N

$$NM(M+1)^N/(M+1) = (M+1)^N \tag{2.6}$$

and therefrom we obtain the values of exponent N as a function of M

$$N = \frac{M+1}{M} \tag{2.7}$$

We find the values of $N(M)$ as

$$\begin{aligned} N(M=1) &= \frac{2}{1} = 2 \\ N(M=2) &= \frac{3}{2} \\ N(M=3) &= \frac{4}{3} \\ N(M=4) &= \frac{5}{4} \\ &\vdots \\ N(M=\infty) &= 1 \end{aligned} \tag{2.8}$$

We can state that with our assumption of having $N \in \mathbb{N}$ we have to reject all $N(M)$ solutions above which are not natural numbers.

Quite similarly we can arrive at the formulas for N below

$$N = \frac{L+1}{L} \tag{2.9}$$

and

$$N = \frac{K+1}{K} \tag{2.10}$$

what gives the constraint equations for the functions $N(L)$ and $N(K)$ similar to the constraints in Equations 2.8.

3. Conclusion

We have started with the Equation (1.1) with assumption that X, Y, Z are positive integers and N is a natural number. We see from our computations that this equation is valid for natural exponents only if the exponents in it are either $N = 1$ or $N = 2$ what is in agreement with the Fermat's Last Theorem.

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References

[1] *Fermat's Last Theorem*, `https://en.wikipedia.org/wiki/Fermat%27s_Last_Theorem`.
[2] *Kenneth S. Miller*, Engineering Mathematics, Dover Publications, Inc. 1956.

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