On Allowed Natural Exponents in the Equation of the Fermat's Last Theorem

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Abstract

We propose a procedure which allows in a direct computation to determine the possible natural numbers as the exponents in the equation of the Fermat's Last Theorem.

I Introduction.

We write the equation

$$X^N + Y^N = Z^N \tag{1}$$

for which $X, Y, Z \in \mathbb{Z}$ are integers greater than zero and $N \in \mathbb{N}$ is a natural number.

Without loss of generality we rewrite Equation (1) as

$$(x + Mpx)^N + Y^N p = Z^N p \tag{2}$$

with $X = x + Mpx$. We have introduced a real number parameter $p \in \mathbb{R}$ the value of which will be later set to 1. The number $M \in \mathbb{N}$ is a natural number. With such assumptions we must have $x \in \{\frac{1}{M+1}, \frac{2}{M+1}, \frac{3}{M+1}, \ldots\}$ in order for the variable X to assume integer values. E.g.: if $M = 1$, $p = 1$ then $x \in \{\frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \ldots\}$ and $X \in \{1, 2, 3, \ldots\}$.

*This article is in honor of American mathematician Kenneth S. Miller. Due to his technique of computing the expected value and the standard deviation of number of successes in Bernoulli trials [1] we were able to obtain our results. Author would also like to thank an anonymous student who suggested placing the number p next to Y^N and Z^N in Equation (2) while the author was working on the expression for X^N.

II Computations.

We take partial derivative of both sides of Equation (2) with respect to p

$$\frac{\partial(x+Mpx)^N}{\partial p}+\frac{\partial Y^N p}{\partial p}=\frac{\partial Z^N p}{\partial p} \tag{3}$$

obtaining a new equation

$$NM(x+Mpx)^{N-1}x+Y^N=Z^N \tag{4}$$

Now we set $p=1$ in the Equation (4) receiving

$$NM(M+1)^{N-1}x^N+Y^N=Z^N \tag{5}$$

If we set $p=1$ in Equation (2) on the other hand we obtain

$$(M+1)^N x^N+Y^N=Z^N \tag{6}$$

We compare the coefficients at the term with x^N in Equations (5) and (6) receiving a constraint equation for N

$$NM(M+1)^N/(M+1)=(M+1)^N \tag{7}$$

and therefrom we obtain the values of exponent N as a function of M

$$N(M)=\frac{M+1}{M} \tag{8}$$

We find the values of $N(M)$ as

$$\begin{aligned} N(M=1) &= \frac{2}{1}=2 \\ N(M=2) &= \frac{3}{2} \\ N(M=3) &= \frac{4}{3} \\ N(M=4) &= \frac{5}{4} \\ &\vdots \\ N(M=\infty) &= 1 \end{aligned} \tag{9}$$

We can state that with our assumption of having $N\in\mathbb{N}$ we have to reject all $N(M)$ solutions above which are not natural numbers.

III Conclusion.

We have started with the Equation (1) with assumption that X, Y, Z are positive integers and N is a natural number. We see from our computations that this equation is valid for natural exponents only if the exponents N in it are all equal either to 1 or to 2.

References

[1] Miller, Kenneth S. (1956) *Engineering Mathematics* Dover Publications, Inc.

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