On an identity leading to the Fermat's Last Theorem in a short computation

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Abstract. We propose a procedure which allows in a direct computation to determine the possible natural numbers as the exponents in the equation of the Fermat's Last Theorem. We arrive at an identity from which the Fermat's Last Theorem reveals itself.

1. INTRODUCTION. We write the equation

$$X^N + Y^N = Z^N \tag{1}$$

for which $X, Y, Z \in \mathbb{Z}_+$ are integers greater than zero and $N \in \mathbb{N}$ is a natural number.

Without loss of generality we rewrite Equation (1) as

$$(x + Mpx)^N + Y^N p = Z^N p \tag{2}$$

with $X = x + Mpx$. We have introduced a real number parameter $p \in \mathbb{R}$ the value of which will be later set to 1. The number $M \in \mathbb{N}$ is a natural number. With such assumptions we must have $x \in \{\frac{1}{M+1}, \frac{2}{M+1}, \frac{3}{M+1}, \dots\}$ in order for the variable X to assume integer values. E.g.: if $M = 1$, $p = 1$ then $x \in \{\frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \dots\}$ and then $X \in \{1, 2, 3, \dots\}$. We always arrive at $X \in \{1, 2, 3, \dots\}$ for any natural number M and $p = 1$, i.e. we have ensured that always $X \in \{1, 2, 3, \dots\}$ no matter which natural number M we take into account. Similarly we can ensure that Y and Z are also integers greater than zero.

2. COMPUTATIONS. We take partial derivative of both sides of Equation (2) with respect to p

$$\frac{\partial (x + Mpx)^N}{\partial p} + \frac{\partial Y^N p}{\partial p} = \frac{\partial Z^N p}{\partial p} \tag{3}$$

obtaining a new equation

$$NM(x + Mpx)^{N-1}x + Y^N = Z^N \tag{4}$$

Now we set $p = 1$ in the Equation (4) receiving

$$NM(M + 1)^{N-1}x^N + Y^N = Z^N \tag{5}$$

If we set $p = 1$ in Equation (2) on the other hand we obtain

$$(M + 1)^N x^N + Y^N = Z^N \tag{6}$$

We compare the coefficients at the term with x^N in Equations (5) and (6) receiving a constraint equation for N

$$NM(M + 1)^N/(M + 1) = (M + 1)^N \tag{7}$$

and therefrom we obtain the values of exponent N as a function of M

$$N = \frac{M+1}{M} \tag{8}$$

We find the values of $N(M)$ as

$$\begin{aligned} N(M=1) &= \frac{2}{1} = 2 \\ N(M=2) &= \frac{3}{2} \\ N(M=3) &= \frac{4}{3} \\ N(M=4) &= \frac{5}{4} \\ &\vdots \\ N(M=\infty) &= 1 \end{aligned} \tag{9}$$

We can state that with our assumption of having $N \in \mathbb{N}$ we have to reject all $N(M)$ solutions above which are not natural numbers.

3. CONCLUSION. We have started with the Equation (1) with assumption that X, Y, Z are positive integers and N is a natural number. We see from our computations that this equation is valid for natural exponents only if the exponents in it are either $N = 1$ or $N = 2$ what is in agreement with the Fermat's Last Theorem.

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REFERENCES

1. `https://en.wikipedia.org/wiki/Fermat%27s_Last_Theorem` Fermat's Last Theorem

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