Obtaining Schrödinger Equation from Wave Equation

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January 2026 marks one hundred years since the Austrian physicist Erwin Schrödinger has published the equation later named after him. Schrödinger equation can be derived from wave equation, de Broglie hypothesis and Planck formula for energy of radiation quanta.

The wave equation in one dimension can be written as

$$\frac{1}{v_p^2}\frac{\partial^2\Psi}{\partial t^2} = \frac{\partial^2\Psi}{\partial x^2} \tag{1}$$

where v_p is the phase velocity of the wave, t denotes time and x is the coordinate of the poin on the Ox axis, where the wave of magnitude $\Psi(x,t)$ is being observed.

De Broglie hypothesis can be derived from Einstein formula

$$E = mc^2 \tag{2}$$

for energy E of the mass m. c denotes the speed of light in vacuum. We receive therefrom

$$E = mc^2 = mcc = pc = p\lambda/T \tag{3}$$

where p denotes particle momentum, λ is the wave length, and T is the period of the wave in time domain. On the other hand from the Planck formula we have

$$E = h\nu = h/T \tag{4}$$

where h is Planck constant, ν wave frequency. Comparing the two above equations for energy E we receive

$$p\lambda/T = h/T \tag{5}$$

and therefrom the de Broglie hypothesis

$$p = h/\lambda \tag{6}$$

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that is the formula relating the particle momentum p with the length λ of its wave. Such wave we call the *de Broglie wave.*

Now we derive the formula for the kinetic energy $E - V$ of a particle, where E is its total energy and V is its potential energy.

$$E - V = \frac{1}{2} m v_p^2 \tag{7}$$

We continue obtaining

$$2m(E - V) = m^2 v_p^2 = p^2 \tag{8}$$

$$p = [2m(E - V)]^{1/2} = h/\lambda \tag{9}$$

$$\lambda = \frac{h}{[2m(E - V)]^{1/2}} \tag{10}$$

$$v_p = \lambda/T = \lambda\nu = \frac{h\nu}{[2m(E - V)]^{1/2}} = \frac{E}{[2m(E - V)]^{1/2}} \tag{11}$$

Therefore

$$\frac{1}{v_p^2} = \frac{2m(E - V)}{E^2} \tag{12}$$

The solution of the wave equation can be presented in the form

$$\Psi = e^{i\omega t}\varphi(x) = e^{i(E/\hbar)t}\varphi(x) \tag{13}$$

where $E = \hbar\omega$ whereby $\hbar = h/2\pi$ and $\omega = 2\pi/T$

$$\frac{\partial\Psi}{\partial t} = i\frac{E}{\hbar} e^{i(E/\hbar)t}\varphi(x) \tag{14}$$

$$\frac{\partial^2\Psi}{\partial t^2} = -\frac{E^2}{\hbar^2} e^{i(E/\hbar)t}\varphi(x) \tag{15}$$

$$\frac{\partial^2\Psi}{\partial x^2} = e^{i(E/\hbar)t}\,\frac{\partial^2\varphi(x)}{\partial x^2} \tag{16}$$

$$\frac{2m(E - V)}{E^2}\frac{\partial^2\Psi}{\partial t^2} = \frac{\partial^2\Psi(x)}{\partial x^2} \tag{17}$$

$$-\frac{2m(E - V)}{E^2}\frac{E^2}{\hbar^2} e^{i(E/\hbar)t}\varphi(x) = e^{i(E/\hbar)t}\frac{\partial^2\varphi(x)}{\partial x^2} \tag{18}$$

$$-\frac{2m}{\hbar^2}(E - V)\varphi(x) = \frac{\partial^2\varphi(x)}{\partial x^2} \tag{19}$$

Therefrom we obtain the time independent Schrödinger equation in one dimension

$$-\frac{\hbar^2}{2m}\frac{\partial^2\varphi(x)}{\partial x^2} + V(x)\varphi(x) = E\varphi(x) \tag{20}$$

which opens up new possibilities for describing photons, electrons, atoms of elements, chemical molecules and other systems of the microworld.