On Fermat's Diophantine Equation and Its Only Allowed Natural Exponents

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Abstract

We propose a procedure which allows to compute the only possible natural exponents N of the positive integers X, Y, Z in the Fermat's Diophantine equation. We use approach similar to the one applied in computing of the expected value and the standard deviation of number of successes in Bernoulli trials presented by Kenneth S. Miller.

1 Introduction

We will deteremine the only allowed natural exponents N in the Fermat's Diophantine equation [1]

$$X^N + Y^N = Z^N \tag{1.1}$$

Let us write the equation

$$(x + Mx)^N + Y^N = Z^N \tag{1.2}$$

where $x+Mx = X$. We make the assumption that $M \in \mathbb{N}$ is a natural number $1, 2, 3, \ldots$ We assume also that $X, Y, Z \in \mathbb{Z}_+$ are positive integers greater than zero. In order to receive $x+Mx = X$ as positive integer greater than zero we must assume that $x \in \{\frac{1}{M+1}, \frac{2}{M+1}, \frac{3}{M+1}, \ldots\}$. Then indeed for any $M \in \{1, 2, 3, \ldots\}$ we receive $X \in \{1, 2, 3, \ldots\}$ if $x \in \{\frac{1}{M+1}, \frac{2}{M+1}, \frac{3}{M+1}, \ldots\}$. It is this way because $X = x(M + 1)$. For example, if $M = 1$, we must have $x \in \{\frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \ldots\}$ and $X \in \{\frac{1}{2} + \frac{1}{2}, \frac{2}{2} + \frac{2}{2}, \frac{3}{2} + \frac{3}{2}, \ldots\}$ what is equivalent to $X \in \{1, 2, 3, \ldots\}$. We will now determine the natural number N.

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2 Computations

Let us write the following equation with real parameter $p \in \mathbb{R}$

$$(x + Mpx)^N + Y^N p = Z^N p \tag{2.1}$$

We take partial derivative of both sides of Equation (2.1) with respect to p

$$\frac{\partial (x + Mpx)^N}{\partial p} + \frac{\partial Y^N p}{\partial p} = \frac{\partial Z^N p}{\partial p} \tag{2.2}$$

We compute the partial derivtive of $(x + Mpx)^N$ as follows

$$\begin{aligned} \frac{\partial (x + Mpx)^N}{\partial p} &= N(x + Mpx)^{N-1} \frac{\partial (x + Mpx)}{\partial p} \\ = N(x + Mpx)^{N-1} Mx &= NM(Mp + 1)^{N-1} x^{N-1} x \\ &= NM(Mp + 1)^{N-1} x^N \end{aligned} \tag{2.3}$$

We receive a new equation

$$NM(Mp + 1)^{N-1} x^N + Y^N = Z^N \tag{2.4}$$

in which we can set the prameter $p = 1$ obtaining

$$NM(M + 1)^{N-1} x^N + Y^N = Z^N \tag{2.5}$$

If we set $p = 1$ in Equation (2.1) on the other hand we obtain

$$(M + 1)^N x^N + Y^N = Z^N \tag{2.6}$$

We compare the coefficients at the term with x^N in Equations (2.5) and (2.6) receiving a constraint equation for N

$$NM(M + 1)^N / (M + 1) = (M + 1)^N \tag{2.7}$$

and therefrom we obtain the values of exponent N as a function of M

$$N(M) = \frac{M + 1}{M} \tag{2.8}$$

3 Conclusion

We have started with the Equation (1.1) with assumption that X, Y, Z are positive integers and N is a natural number.

For $N(M)$ we have

$$\begin{aligned} N(M=1) &= \frac{2}{1} = 2 \\ N(M=2) &= \frac{3}{2} \\ N(M=3) &= \frac{4}{3} \\ N(M=4) &= \frac{5}{4} \\ &\vdots \\ N(M=\infty) &= 1 \end{aligned} \tag{3.1}$$

We can state that with the assumption of having $N \in \mathbb{N}$ we have to reject all $N(M)$ solutions which are not natural numbers. Our natural exponent is always only either $N = 1$ or $N = 2$ what is in agreement with the Fermat's Last Theorem [1].

4 Acknowledgement

This article is in honor of American mathematician Kenneth S. Miller. Due to his technique of computing the expected value and the standard deviation of number of successes in Bernoulli trials [2] we were able to obtain our results. Author would also like to thank an anonymous student who suggested placing the number p next to symbols Y^N and Z^N in Equation (2.1) while the author was working on the universal expression of the natural number X using rational number x, integer number M and real parameter p for partial differentiation.

References

[1] Fermat's Last Theorem.
`https://en.wikipedia.org/wiki/Fermat%27s_Last_Theorem`

[2] Kenneth S. Miller, *Engineering Mathematics* (1956), Dover Publications, Inc., New York.