CS 476 A1Q1 ④

c) $r=0$, $d=0.25$ from a), $u=1.5$ from b)

Expected call option payoff $K=7, u=1.5$ $d=0.25$

$= p \cdot \max(uS_0 - K, 0) + (1-p) \cdot \max(dS_0 - K, 0)$

$= p \cdot \max(15-7, 0) + (1-p) \cdot \max(7.5-7, 0)$

$= 8p + (1-p) \cdot 0.5 = 0.5 + 7.5p$

Expected put option payoff $K=7$, $u=1.5$ $d=0.25$

$= p \cdot \max(K - uS_0, 0) + (1-p) \cdot \max(K - dS_0, 0)$

$= p \cdot \max(7-15, 0) + (1-p) \cdot \max(7-7.5, 0) = 0$

Prove that there are arbitrage opportunities when ① $p < q^* = \frac{1}{3}$ and ② $p > q^* = \frac{1}{3}$ ($q^* = \frac{1}{3}$ is from 1a))

if $p < q^* = \frac{1}{3}$, set-up a portfolio using put-call parity.

At T, we have that $\Pi_T : C_T - P_T - S_T + K = 0$ by put-call parity

but at 0, we have that $\Pi_0 : C_0 - P_0 - S_0 + K$ $(\because r=0)$

$= 0.5 + 7.5p - 0 - 10 + 7$

$= 7.5p - 2.5 < 0$ $(\because p < q^* = \frac{1}{3})$

So we have an arbitrage when $p < q^*$ as $\Pi_0 < 0$ but $\Pi_T = 0$

c) if $p > q^* = \frac{1}{3}$, set up a portfoilo using put-call parity.

At T, we have that $\Pi_T : P_T - C_T + S_T - K = 0$, by put-call parity.

Then at 0, we have the same portfolio such that

$$\Pi_0 : P_0 - C_0 + S_0 - K$$

$$= 0 - 0.5 - 7.5p + 10 - 7$$

$$= -7.5p + 2.5 < 0 \quad (\because p > q^* = \tfrac{1}{3})$$

So we have an arbitraje when $\Pi_T = 0$ but $\Pi_0 < 0$ for $p > q^* = \frac{1}{3}$.

As there exist arbitraje oppurtunities for both $p < q^* = \frac{1}{3}$ and $p > q^* = \frac{1}{3}$, we can construct an arbitrage based on the value of $p \neq q^* = \frac{1}{3}$.