CS 476 A3 Q1

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a)

The Euler-Maruyama formula for computing v_{n+1} at t_{n+1} is as follows:

$$\begin{aligned} v(n+1) = v(n) - \lambda(v(n) - \bar{v})\Delta t \\ + \eta \cdot \sqrt{v(n)} \cdot \sqrt{\Delta t} \cdot \phi_t \end{aligned}$$

Yes. v_{n+1} can be negative when the normal sample ϕ_t is less than $\frac{-v(n)+\lambda(v(n)-\bar{v})\Delta t}{\eta \cdot \sqrt{v(n)} \cdot \sqrt{\Delta t}}$.

b)

The Milstein method for computing v_{n+1} as follows:

$$\begin{aligned} v(n+1) &= v(n) - \lambda(v(n) - \bar{v})\Delta t + \eta \cdot \sqrt{v(n)} \cdot \sqrt{\Delta t} \cdot \phi_t \\ &+ \frac{1}{2}\eta \cdot \sqrt{v(n)} \cdot \eta \cdot \frac{1}{2 \cdot \sqrt{v(n)}} \cdot ((\sqrt{\Delta t} \cdot \phi_{t_n})^2 - \Delta t) \\ &= v(n) - \lambda(v(n) - \bar{v})\Delta t + \eta \cdot \sqrt{v(n)} \cdot \sqrt{\Delta t} \cdot \phi_t + \frac{\eta^2}{4}\Delta t(\phi_t^2 - 1) \\ &= \frac{\eta^2}{4}\Delta t \phi_t^2 + \eta\sqrt{v(n)} \cdot \sqrt{\Delta t} \cdot \phi_t + v(n) - \lambda(v(n) - \bar{v}) \cdot \Delta t - \frac{\eta^2}{4}\Delta t \end{aligned}$$

Assume $v_n = 0$, then we have:

$$v(n+1) = \frac{\eta^2}{4}\Delta t \phi_t^2 + \lambda\bar{v}\Delta t - \frac{\eta^2}{4}\Delta t$$

We want $v(n+1) > 0$ for any ϕ_t.

$$\begin{aligned} v(n+1) > 0 \\ \frac{\eta^2}{4}\Delta t \phi_t^2 + \lambda\bar{v}\Delta t - \frac{\eta^2}{4}\Delta t > 0 \\ \Delta t(\frac{\eta^2}{4}\phi_t^2 + \lambda\bar{v} - \frac{\eta^2}{4}) > 0 \end{aligned}$$

$$\frac{\eta^2}{4}\phi_t^2 + \lambda\bar{v} - \frac{\eta^2}{4} > 0 \qquad (\because \Delta t > 0)$$

$$\lambda\bar{v} - \frac{\eta^2}{4} > 0 \qquad (\because \eta^2 > 0, \phi_t^2 >= 0)$$

$$\lambda\bar{v} > \frac{\eta^2}{4}$$

Therefore, if we have $\lambda\bar{v} > \frac{\eta^2}{4}$, then $v_{n+1} > 0$ is guaranteed.