imageSegmentation_demo

May 14, 2023

1 Meanshift Algorithm

1.1 Imports and loading test data

Imports

[ ]:
```
import scipy as sp
import numpy as np
import matplotlib.pyplot as plt
from tqdm.notebook import tqdm
import time

%matplotlib inline
```

Load data

[ ]:
```
data_dict = sp.io.loadmat("data/pts.mat")
data = data_dict["data"]
data = data.T

print(data.shape)
```

```
(2000, 3)
```

1.2 Meanshift algorithm (slow version)

Define *findpeak* function

[ ]:
```
THRESHOLD = 0.01
```

[ ]:
```
def findpeak(data: np.ndarray, idx, r):
    center = data[idx]

    while True:
        # Collect points within radius
        points = data[np.linalg.norm(data - center, axis=1) <= r]

        # Compute mean of points within radius
        mean = np.mean(points, axis=0)
```

```
        # Check if shift > THRESHOLD
        if np.linalg.norm(center - mean) > THRESHOLD:
            center = mean
        else:
            break

    return center
```

[ ]:
```
print("Data point 0:", data[0])
print("Peak 0:", findpeak(data, 0, 2))
```

```
Data point 0: [-0.43256481 -1.66558438  0.12533231]
Peak 0: [-0.06756875  0.04788814  0.02778451]
```

Define *meanshift* function

[ ]:
```
def meanshift(data: np.ndarray, r):
    data_num = len(data)
    labels = np.empty((data_num,))
    peaks = []

    for i in tqdm(range(data_num)):
        # Find peak for current data point
        peak = findpeak(data, i, r)

        # Check if current peak can be identified
        # with a previous peak
        labeled = False
        for prev_label, prev_peak in enumerate(peaks):
            if np.linalg.norm(peak - prev_peak) <= r / 2:
                # Associate data point with previous peak
                labels[i] = prev_label
                labeled = True
                break

        # Otherwise, append the new peak and attach a new
        # label to the point
        if not labeled:
            labels[i] = len(peaks)
            peaks.append(peak)

    return labels, peaks
```

Apply the meanshift algorithm on test data

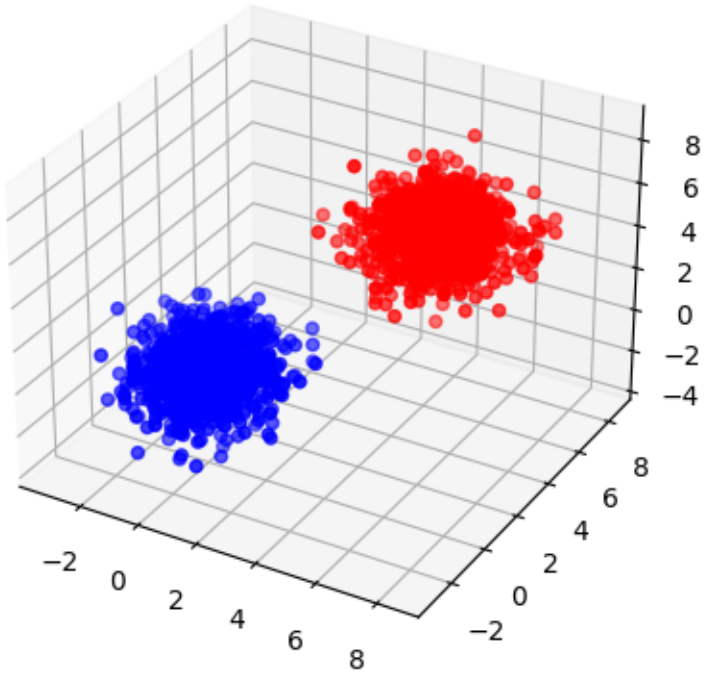
[ ]:
```
start = time.time()
labels, peaks = meanshift(data, 2)
end = time.time()
```

```
print("Labels:", np.unique(labels))
print("Peaks:", peaks)
print(f"Total time: {end - start:0.2f}s")
```

```
  0%|          | 0/2000 [00:00<?, ?it/s]
Labels: [0. 1.]
Peaks: [array([-0.06756875,  0.04788814,  0.02778451]), array([5.04987186,
4.98116882, 5.0104074 ])]
Total time: 5.58s
```

Plot the clusters found by the algorithm

[ ]:
```
def plot_test_clusters(data, labels):
    clusters = {}
    for i, prev_label in enumerate(labels):
        if prev_label not in clusters:
            clusters[prev_label] = []
        else:
            clusters[prev_label].append(data[i])

    xs = {}
    ys = {}
    zs = {}
    for prev_label in clusters:
        xs[prev_label] = [point[0] for point in clusters[prev_label]]
        ys[prev_label] = [point[1] for point in clusters[prev_label]]
        zs[prev_label] = [point[2] for point in clusters[prev_label]]

    ax = plt.axes(projection="3d")
    ax.scatter(xs[0], ys[0], zs[0], color="blue")
    ax.scatter(xs[1], ys[1], zs[1], color="red")
    plt.show()


plot_test_clusters(data, labels)
```



1.3 Meanshift algorithm (with speed-ups)

Define *findpeak_opt* function

[ ]:
```
def findpeak_opt(data: np.ndarray, idx, r, c):
    data_num = len(data)
    cpts = np.zeros((data_num,))
    center = data[idx]

    while True:
        # SPEEDUP 2
        # Collect points within distance r/c from the search path
        cpts[np.linalg.norm(data - center, axis=1) <= r / c] = 1

        # Collect points within radius
        points = data[np.linalg.norm(data - center, axis=1) <= r]

        # Compute mean of points within radius
        mean = np.mean(points, axis=0)

        # Check if shift > THRESHOLD
        if np.linalg.norm(center - mean) > THRESHOLD:
            center = mean
```

```
        else:
            break

    return center, cpts
```

Define *meanshift_opt* function

[ ]:
```
def meanshift_opt(data: np.ndarray, r, c):
    data_num = len(data)
    labels = np.zeros((data_num,)) - 1
    peaks = []

    pbar = tqdm(total=data_num)
    labeled_num = 0
    while True:
        i = np.argmax(labels < 0)
        peak, cpts = findpeak_opt(data, i, r, c)

        labeled = False
        for prev_label, prev_peak in enumerate(peaks):
            if np.linalg.norm(peak - prev_peak) <= r / 2:
                label = prev_label
                peak = prev_peak
                labeled = True
                break

        if not labeled:
            label = len(peaks)
            peaks.append(peak)

        # Change label of i'th data point
        labels[i] = label

        # SPEEDUP 1
        # Change the label of points within a distance of r from the peak
        labels[np.linalg.norm(data - peak, axis=1) <= r] = label

        # SPEEDUP 2
        # Change the label of points within a distance of r/c from the search␣
↪path
        labels[cpts > 0] = label

        # Update progress bar
        new_labeled_num = np.sum(labels >= 0)
        pbar.update(new_labeled_num - labeled_num)
        labeled_num = new_labeled_num
```

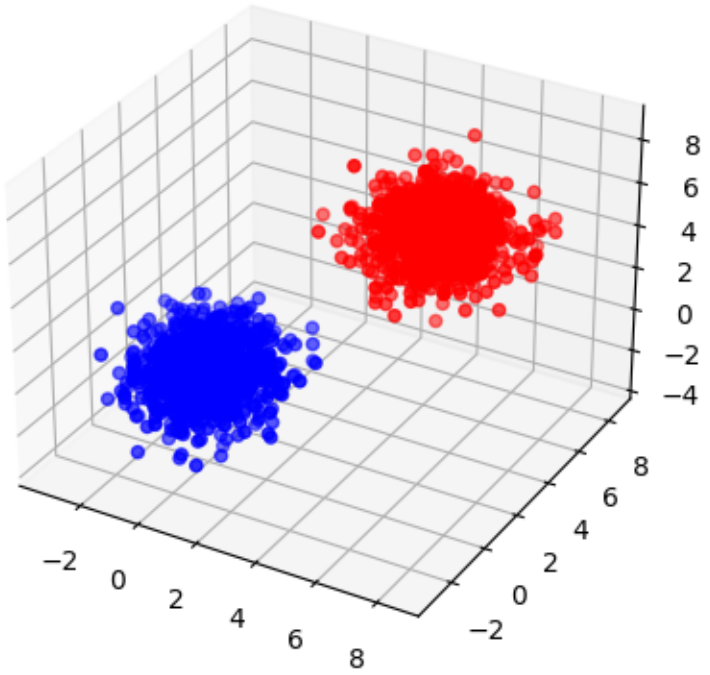
```
        # Check if all data has been labelled
        if labeled_num == data_num:
            break

    return labels, peaks
```

Apply the sped up meanshift algorithm on test data

[ ]:
```
start = time.time()
labels, peaks = meanshift_opt(data, 2, 4)
end = time.time()

print("Labels:", np.unique(labels))
print("Peaks:", peaks)
print(f"Total time: {end - start:0.2f}s")
```

```
  0%|          | 0/2000 [00:00<?, ?it/s]
Labels: [0. 1.]
Peaks: [array([-0.06756875,  0.04788814,  0.02778451]), array([5.04987186,
4.98116882, 5.0104074 ])]
Total time: 1.53s
```

[ ]:
```
plot_test_clusters(data, labels)
```



2 Image Segmentation

2.1 Imports

[ ]:
```
import cv2
from matplotlib import cm
from matplotlib.colors import ListedColormap
```

2.2 Pre-processing

Load the image in CIELAB color format

[ ]:
```
def load_image_lab(path):
    image_bgr = np.float32(cv2.imread(path)) / 255
    image_lab = cv2.cvtColor(image_bgr, cv2.COLOR_BGR2LAB)

    return image_lab
```

Create 3D/5D feature arrays from images

[ ]:
```
def image_to_3d_vector(image: np.ndarray):
    h, w, c = image.shape
    vec_image = image.reshape((h * w, c))

    return vec_image


def image_to_5d_vector(image: np.ndarray):
    h, w, c = image.shape
    vec_image = image.reshape((h * w, c))
    coords = np.array([[(i, j) for j in range(w)] for i in range(h)]).reshape(
        (h * w, 2)
    )
    vec_image_xy = np.hstack((vec_image, coords))

    return vec_image_xy
```

2.3 The segmentation algorithm

Reset the threshold

[ ]:
```
THRESHOLD = 1
```

[ ]:
```
def segmIm(path, vectorize_fun, r, c, verbose=False):
    """
    A function that uses the meanshift algorithm to perform image segmentation.
    """
    image = load_image_lab(path)
    vec_image = vectorize_fun(image)
```

```
    labels, peaks = meanshift_opt(vec_image, r, c)

    # Print results
    if verbose:
        print(f"##### Segmentation results for {path} #####")
        print()
        print(
            f"Parameters: {vectorize_fun.__name__[9:11].upper()} feature␣
↪vector, r = {r}, c = {c}"
        )
        print()
        print("-------------------------------------------------")
        print()
        print("Number of clusters found:", len(peaks))
        print()
        print("Labels -> Peaks:")
        for i, peak in enumerate(peaks):
            print(
                f"{str(i).rjust(6)} -> [ {'  '.join(f'{x:0.3f}'.rjust(8) for x␣
↪in peak)} ]"
            )

    return labels, peaks
```

2.4 Visualising the results of segmentation

[ ]:
```
def segm_image_peaks(
    im_shape: tuple, labels: np.ndarray, peaks: list[np.ndarray], save_as=None
):
    """
    A function to color each pixel according to the peak of its associated␣
↪cluster.
    """
    # Reshape labels to the shape of the original image
    labels = labels.reshape(im_shape)

    # Convert the CIELAB peaks to RGB colors
    colors_lab = np.float32([[peak[:3]] for peak in peaks])
    colors = cv2.cvtColor(colors_lab, cv2.COLOR_LAB2RGB)

    # Create colormap
    cmap = ListedColormap(colors)

    # Create and plot segmented image
    labels_num = len(peaks)
    if labels_num > 1:
        labels /= np.max(labels)
```

```
    rgb_image = cmap(labels)
    plt.imshow(rgb_image)
    plt.axis("off")

    if save_as:
        plt.savefig(save_as, bbox_inches="tight")

    plt.show()


def segm_image_clusters(
    im_shape: tuple, labels: np.ndarray, peaks: list[np.ndarray], save_as=None
):
    """
    A function to better visualise the distinct clusters (helpful in the case
    of 5D features).
    """
    # Reshape labels to the shape of the original image
    labels = labels.reshape(im_shape)

    # Create colormap
    labels_num = len(peaks)
    cmap = cm.get_cmap("Greys", labels_num)

    # Create and plot segmented image
    if labels_num > 1:
        labels /= np.max(labels)
    rgb_image = cmap(labels)
    plt.imshow(rgb_image)
    plt.axis("off")

    if save_as:
        plt.savefig(save_as, bbox_inches="tight")

    plt.show()
```

2.5 Testing on an example image

Load the image

[ ]:
```
image = "earth.jpg"
path = f"images/{image}"
image_lab = load_image_lab(path)
h, w, _ = image_lab.shape

image_rgb = cv2.cvtColor(image_lab, cv2.COLOR_LAB2RGB)
plt.imshow(image_rgb)
```

```
plt.axis("off")
plt.tight_layout()
plt.show()
```

Test parameters (r, c)

[ ]:
```
r = 20
c = 4
```

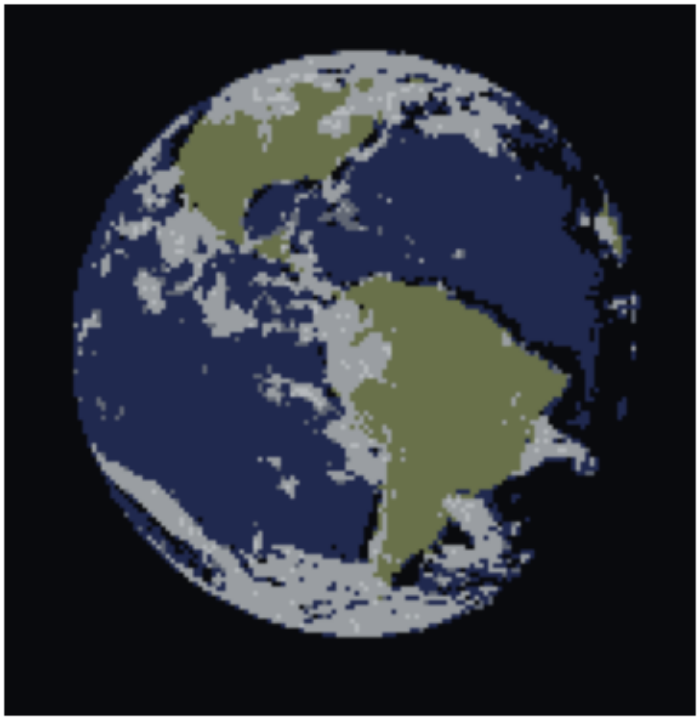
Perform segmentation (3D feature vectors)

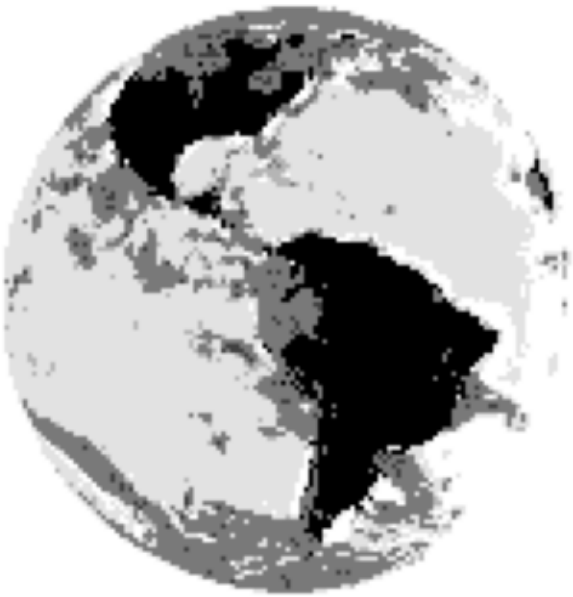
[ ]:
```
start = time.time()
labels_3d, peaks_3d = segmIm(path, image_to_3d_vector, r=r, c=c, verbose=True)
end = time.time()

print()
print("-------------------------------------------------")
print()
print(f"Total time: {end - start:0.2f}s")
```

```
  0%|          | 0/23100 [00:00<?, ?it/s]
##### Segmentation results for images/earth.jpg #####

Parameters: 3D feature vector, r = 20, c = 4

------------------------------------------------

Number of clusters found: 6

Labels -> Peaks:
     0 -> [    2.960     0.184    -1.314 ]
     1 -> [   17.955     9.285   -24.850 ]
     2 -> [   43.878     0.574    -7.986 ]
     3 -> [   65.226    -0.863    -2.406 ]
     4 -> [   75.599    -0.671    -2.115 ]
     5 -> [   46.296   -10.063    20.777 ]

------------------------------------------------

Total time: 2.10s
```

Plot and save the segmented image

[ ]:
```
segm_image_peaks(
    (h, w), labels_3d, peaks_3d, save_as=f"results/test/{image}_3D_r={r}_c={c}.
  ↪jpg"
)
segm_image_clusters(
    (h, w),
    labels_3d,
    peaks_3d,
    save_as=f"results/test/{image}_3D_clusters_r={r}_c={c}.jpg",
)
```





Perform segmentation (5D feature vectors)

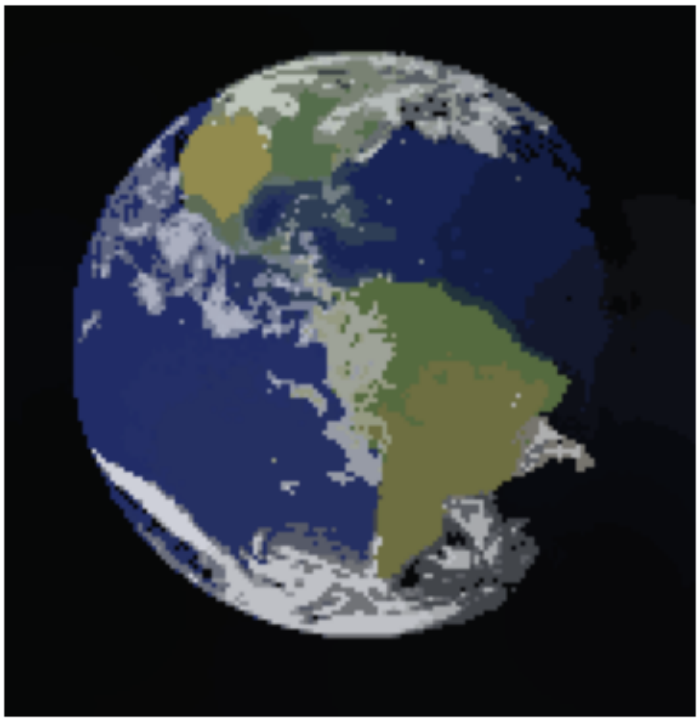
[ ]:
```
start = time.time()
labels_5d, peaks_5d = segmIm(path, image_to_5d_vector, r=r, c=c, verbose=True)
end = time.time()

print()
print("-------------------------------------------------")
print()
print(f"Total time: {end - start:0.2f}s")
```

```
  0%|          | 0/23100 [00:00<?, ?it/s]
##### Segmentation results for images/earth.jpg #####

Parameters: 5D feature vector, r = 20, c = 4

-------------------------------------------------

Number of clusters found: 68

Labels -> Peaks:
     0 -> [    1.918    -0.026     0.039    16.901    16.838 ]
     1 -> [    2.170    -0.329    -0.414     4.636    75.795 ]
     2 -> [    2.029    -0.032    -0.280    14.824   130.146 ]
     3 -> [   45.896    -2.060    -2.243    19.281    88.136 ]
     4 -> [   40.594    -0.538    -4.744    21.668    99.444 ]
     5 -> [   53.571    -6.385     6.966    20.095    72.582 ]
     6 -> [   70.076    -4.041     3.263    17.711    72.934 ]
     7 -> [   76.190    -2.248    -0.142    18.669    83.869 ]
     8 -> [   27.565    -1.986    -2.820    14.432    61.514 ]
     9 -> [   78.750    -4.185     3.750    18.479    65.665 ]
    10 -> [   67.009    -0.635    -2.820    20.394    95.076 ]
    11 -> [   43.799   -16.102    16.171    27.399    66.032 ]
    12 -> [   12.563    10.180   -25.787    51.042   111.887 ]
    13 -> [   20.690    15.959   -35.832    82.560    34.857 ]
    14 -> [   57.484    -6.448    33.573    33.854    47.803 ]
    15 -> [   60.530     2.483   -11.869    49.396    34.796 ]
    16 -> [   71.829     1.819    -9.144    52.769    37.057 ]
    17 -> [   43.564     3.854   -15.852    48.982    31.654 ]
    18 -> [   54.632    -8.756    21.455    28.671    54.569 ]
    19 -> [   16.364    13.404   -31.746    43.617    93.956 ]
    20 -> [   54.592    -0.875    -3.390    20.377    94.703 ]
    21 -> [   44.069   -10.655    10.904    47.041    52.605 ]
    22 -> [   14.252    12.402   -29.510    46.473   103.593 ]
    23 -> [   26.156     0.646   -16.090    48.735    68.157 ]
    24 -> [   62.575    -2.970     1.634    18.064    80.805 ]
    25 -> [    1.961    -0.032    -0.128    27.458    14.191 ]
    26 -> [   45.741    -7.255     0.106    50.280    61.253 ]
```

```
 27 -> [   51.959    -2.996    -4.542    56.785    64.825 ]
 28 -> [   59.972    -0.159    -6.931    62.362    64.575 ]
 29 -> [   40.750    -3.623    -7.348    51.404    65.266 ]
 30 -> [   54.347     2.712   -13.960    63.518    57.582 ]
 31 -> [   22.006    10.357   -28.928    53.079    61.470 ]
 32 -> [   66.410    -3.440     4.694    72.356    74.077 ]
 33 -> [    4.439     0.383    -2.049    60.175   140.791 ]
 34 -> [   42.643   -16.422    21.767    77.234    93.513 ]
 35 -> [   72.481     1.684    -9.475    64.906    56.006 ]
 36 -> [   62.996    -6.266    12.161    78.523    77.749 ]
 37 -> [   20.768    -4.601    -7.347    66.960   105.660 ]
 38 -> [    2.069     0.219    -1.189    63.873     7.174 ]
 39 -> [    4.703     1.243    -5.253    84.630   137.522 ]
 40 -> [   21.690    15.098   -34.859    89.438    43.573 ]
 41 -> [    9.059     5.111   -15.260    71.112   127.342 ]
 42 -> [    2.245     0.160    -1.392    75.232     7.148 ]
 43 -> [   45.820    -8.066    25.591    90.139   100.161 ]
 44 -> [   21.717    13.166   -32.079    96.333    50.548 ]
 45 -> [   71.526     0.053    -0.362    95.246   117.391 ]
 46 -> [    1.956    -0.028    -0.209   119.648    14.166 ]
 47 -> [    2.828     0.393    -2.247   109.313   135.706 ]
 48 -> [    3.456     0.561    -3.277    96.394   137.862 ]
 49 -> [   83.726     1.085    -4.545   109.763    35.093 ]
 50 -> [   40.846     0.324    -3.753   121.968   100.583 ]
 51 -> [   47.058     1.648    -8.410   119.831    45.275 ]
 52 -> [   64.181     0.656    -6.324    94.242    73.188 ]
 53 -> [   49.351    -0.599     3.748   100.000   111.986 ]
 54 -> [    2.660     0.450    -1.847   120.495   131.500 ]
 55 -> [   60.218     0.445    -3.639   126.077    64.584 ]
 56 -> [   29.370    -0.016    -3.739   121.459   101.323 ]
 57 -> [   78.182     0.377    -2.957   126.370    62.524 ]
 58 -> [    1.971    -0.029    -0.188   128.460    18.908 ]
 59 -> [   38.496     1.667    -8.506   121.770    65.775 ]
 60 -> [   69.766    -0.358    -1.780   121.456    96.089 ]
 61 -> [    2.179    -0.035    -0.518   138.702   117.301 ]
 62 -> [    1.989    -0.019    -0.143   135.527    28.042 ]
 63 -> [    2.164     0.148    -0.771   131.460   128.000 ]
 64 -> [    2.037     0.059    -0.261   139.505    38.106 ]
 65 -> [   48.509     0.271    -2.756   126.791    82.609 ]
 66 -> [    2.148     0.047    -0.512   143.986    63.910 ]
 67 -> [    2.247     0.017    -0.567   143.996    89.451 ]

------------------------------------------------

Total time: 73.25s
```

Plot and save the segmented image

[ ]:
```
segm_image_peaks(
    (h, w), labels_5d, peaks_5d, save_as=f"results/test/{image}_5D_r={r}_c={c}.
 ↪jpg"
)
segm_image_clusters(
    (h, w),
    labels_5d,
    peaks_5d,
    save_as=f"results/test/{image}_5D_clusters_r={r}_c={c}.jpg",
)
```



3 Experiments

Images to be tested (at 75% of original height and width)

[ ]:
```
IMAGES = ("55075_75.jpg", "181091_75.jpg", "368078_75.jpg")
```

Test parameters (r, c)

[ ]:
```
R = (8, 12, 16, 24)
C = (2, 3, 4, 5)
```

3.1 3D feature vectors

[ ]:
```
results_3d = {
    image: {r: {c: {"labels": None, "peaks": None, "time": None} for c in C}␣
 ↪for r in R}
    for image in IMAGES
}

for image in IMAGES:
    for r in R:
        for c in C:
            print(f"{image}, 3D, r={r}, c={c}")
```

```
            path = "images/" + image

            # Perform and time segmentation
            start = time.time()
            labels, peaks = segmIm(path, image_to_3d_vector, r, c)
            end = time.time()

            print("Clusters found:", len(peaks))
            print(f"Total time: {end - start:0.2f}s")
            print()

            # Save the results
            results_3d[image][r][c]["labels"] = labels
            results_3d[image][r][c]["peaks"] = peaks
            results_3d[image][r][c]["time"] = end - start
```

```
55075_75.jpg, 3D, r=8, c=2
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 26
Total time: 16.76s

55075_75.jpg, 3D, r=8, c=3
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 26
Total time: 39.05s

55075_75.jpg, 3D, r=8, c=4
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 27
Total time: 60.75s

55075_75.jpg, 3D, r=8, c=5
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 27
Total time: 89.36s

55075_75.jpg, 3D, r=12, c=2
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 6
Total time: 5.82s

55075_75.jpg, 3D, r=12, c=3
```

```
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 7
Total time: 13.36s

55075_75.jpg, 3D, r=12, c=4
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 8
Total time: 23.34s

55075_75.jpg, 3D, r=12, c=5
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 8
Total time: 30.51s

55075_75.jpg, 3D, r=16, c=2
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 4
Total time: 2.95s

55075_75.jpg, 3D, r=16, c=3
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 5
Total time: 6.76s

55075_75.jpg, 3D, r=16, c=4
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 5
Total time: 11.85s

55075_75.jpg, 3D, r=16, c=5
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 5
Total time: 19.26s

55075_75.jpg, 3D, r=24, c=2
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 3
Total time: 0.79s

55075_75.jpg, 3D, r=24, c=3
```

```
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 3
Total time: 1.45s

55075_75.jpg, 3D, r=24, c=4
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 3
Total time: 2.72s

55075_75.jpg, 3D, r=24, c=5
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 3
Total time: 3.68s

181091_75.jpg, 3D, r=8, c=2
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 59
Total time: 25.16s

181091_75.jpg, 3D, r=8, c=3
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 64
Total time: 48.20s

181091_75.jpg, 3D, r=8, c=4
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 67
Total time: 73.46s

181091_75.jpg, 3D, r=8, c=5
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 69
Total time: 100.84s

181091_75.jpg, 3D, r=12, c=2
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 9
Total time: 12.20s

181091_75.jpg, 3D, r=12, c=3
```

```
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 10
Total time: 25.20s

181091_75.jpg, 3D, r=12, c=4
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 11
Total time: 43.26s

181091_75.jpg, 3D, r=12, c=5
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 12
Total time: 67.91s

181091_75.jpg, 3D, r=16, c=2
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 7
Total time: 4.03s

181091_75.jpg, 3D, r=16, c=3
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 6
Total time: 10.93s

181091_75.jpg, 3D, r=16, c=4
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 7
Total time: 18.42s

181091_75.jpg, 3D, r=16, c=5
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 7
Total time: 26.71s

181091_75.jpg, 3D, r=24, c=2
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 3
Total time: 1.34s

181091_75.jpg, 3D, r=24, c=3
```

```
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 4
Total time: 2.20s

181091_75.jpg, 3D, r=24, c=4
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 4
Total time: 3.97s

181091_75.jpg, 3D, r=24, c=5
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 4
Total time: 7.08s

368078_75.jpg, 3D, r=8, c=2
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 99
Total time: 24.13s

368078_75.jpg, 3D, r=8, c=3
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 106
Total time: 42.19s

368078_75.jpg, 3D, r=8, c=4
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 113
Total time: 62.59s

368078_75.jpg, 3D, r=8, c=5
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 114
Total time: 81.34s

368078_75.jpg, 3D, r=12, c=2
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 16
Total time: 18.32s

368078_75.jpg, 3D, r=12, c=3
```

```
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 18
Total time: 41.79s

368078_75.jpg, 3D, r=12, c=4
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 19
Total time: 67.28s

368078_75.jpg, 3D, r=12, c=5
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 20
Total time: 87.16s

368078_75.jpg, 3D, r=16, c=2
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 7
Total time: 7.21s

368078_75.jpg, 3D, r=16, c=3
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 8
Total time: 19.00s

368078_75.jpg, 3D, r=16, c=4
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 9
Total time: 32.65s

368078_75.jpg, 3D, r=16, c=5
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 9
Total time: 42.36s

368078_75.jpg, 3D, r=24, c=2
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 3
Total time: 2.48s

368078_75.jpg, 3D, r=24, c=3
```

```
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 4
Total time: 5.90s

368078_75.jpg, 3D, r=24, c=4
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 4
Total time: 10.46s

368078_75.jpg, 3D, r=24, c=5
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 4
Total time: 16.26s
```

Save the results

[ ]:
```
np.save("results/arrays/results_3d.npy", results_3d)
```

Load the results

[ ]:
```
results_3d = np.load("results/arrays/results_3d.npy", allow_pickle=True).item()
```

Plot and save the segmented images

[ ]:
```
for image in IMAGES:
    for r in R:
        for c in C:
            print(f"{image}, 3D, r={r}, c={c}")
            path = "images/" + image
            h, w, _ = load_image_lab(path).shape
            labels = results_3d[image][r][c]["labels"]
            peaks = results_3d[image][r][c]["peaks"]
            print("Clusters found:", len(peaks))

            # Plot and save peaks
            filename_peaks = f"results/segmented_images/{image}_3D_r={r}_c={c}.
 ↪jpg"
            segm_image_peaks((h, w), labels, peaks, save_as=filename_peaks)

            # Plot and save clusters
            filename_clusters = (
                f"results/segmented_images/{image}_3D_clusters_r={r}_c={c}.jpg"
            )
            segm_image_clusters((h, w), labels, peaks,␣
 ↪save_as=filename_clusters)
```

55075_75.jpg, 3D, r=8, c=2
Clusters found: 26

55075_75.jpg, 3D, r=8, c=3
Clusters found: 26

55075_75.jpg, 3D, r=8, c=4
Clusters found: 27

55075_75.jpg, 3D, r=8, c=5
Clusters found: 27

55075_75.jpg, 3D, r=12, c=2
Clusters found: 6

55075_75.jpg, 3D, r=12, c=3
Clusters found: 7

55075_75.jpg, 3D, r=12, c=4
Clusters found: 8

55075_75.jpg, 3D, r=12, c=5
Clusters found: 8

55075_75.jpg, 3D, r=16, c=2
Clusters found: 4

55075_75.jpg, 3D, r=16, c=3
Clusters found: 5

55075_75.jpg, 3D, r=16, c=4
Clusters found: 5

55075_75.jpg, 3D, r=16, c=5
Clusters found: 5

55075_75.jpg, 3D, r=24, c=2
Clusters found: 3

55075_75.jpg, 3D, r=24, c=3
Clusters found: 3

55075_75.jpg, 3D, r=24, c=4
Clusters found: 3

55075_75.jpg, 3D, r=24, c=5
Clusters found: 3

181091_75.jpg, 3D, r=8, c=2
Clusters found: 59

181091_75.jpg, 3D, r=8, c=3
Clusters found: 64

181091_75.jpg, 3D, r=8, c=4
Clusters found: 67

181091_75.jpg, 3D, r=8, c=5
Clusters found: 69

181091_75.jpg, 3D, r=12, c=2
Clusters found: 9

181091_75.jpg, 3D, r=12, c=3
Clusters found: 10

181091_75.jpg, 3D, r=12, c=4
Clusters found: 11

181091_75.jpg, 3D, r=12, c=5
Clusters found: 12

181091_75.jpg, 3D, r=16, c=2
Clusters found: 7

181091_75.jpg, 3D, r=16, c=3
Clusters found: 6

181091_75.jpg, 3D, r=16, c=4
Clusters found: 7

181091_75.jpg, 3D, r=16, c=5
Clusters found: 7

181091_75.jpg, 3D, r=24, c=2
Clusters found: 3

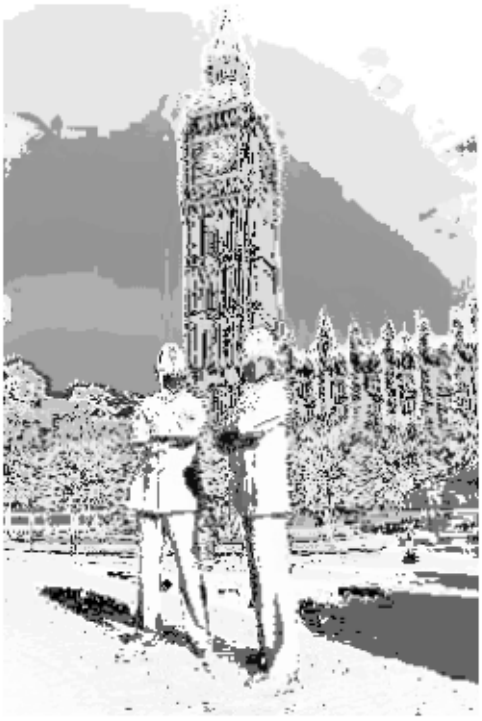
181091_75.jpg, 3D, r=24, c=3
Clusters found: 4

181091_75.jpg, 3D, r=24, c=4
Clusters found: 4

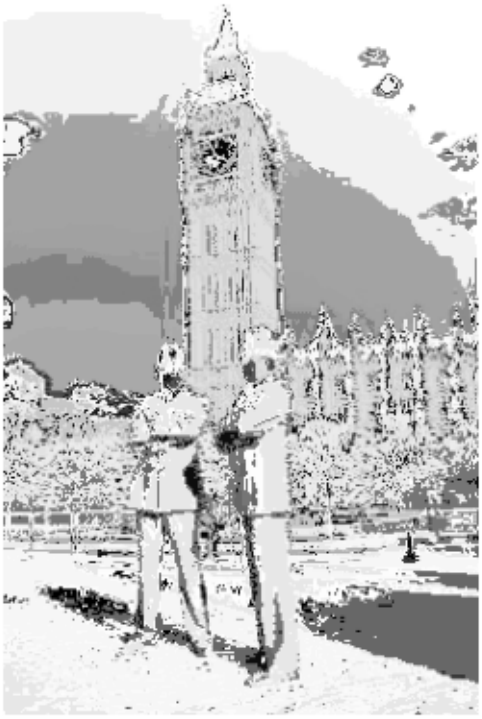
181091_75.jpg, 3D, r=24, c=5
Clusters found: 4

368078_75.jpg, 3D, r=8, c=2
Clusters found: 99

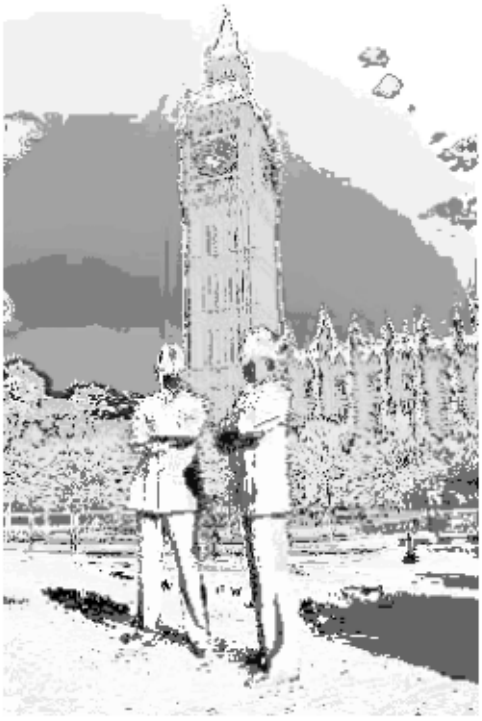
368078_75.jpg, 3D, r=8, c=3
Clusters found: 106



368078_75.jpg, 3D, r=8, c=4
Clusters found: 113

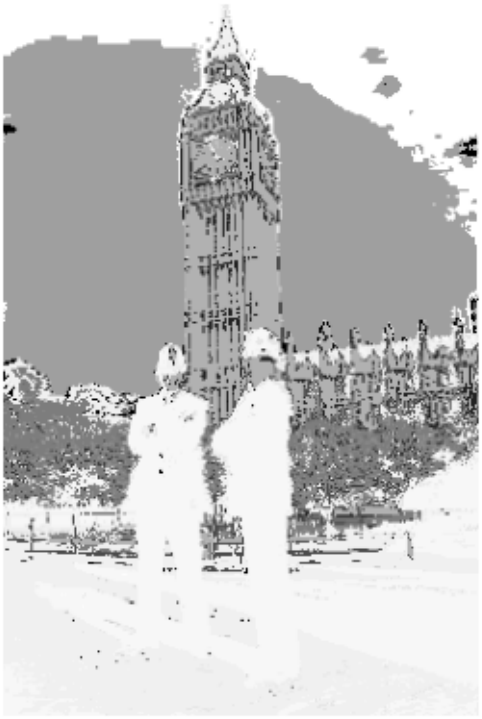


368078_75.jpg, 3D, r=8, c=5
Clusters found: 114



368078_75.jpg, 3D, r=12, c=2
Clusters found: 16

368078_75.jpg, 3D, r=12, c=3
Clusters found: 18

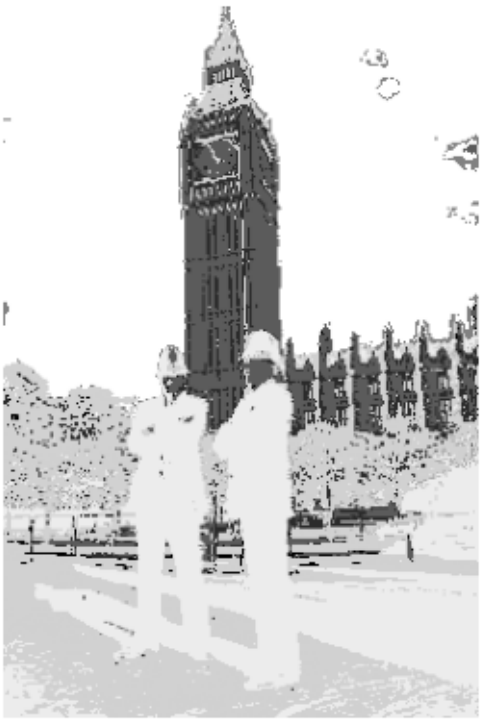


368078_75.jpg, 3D, r=12, c=4
Clusters found: 19

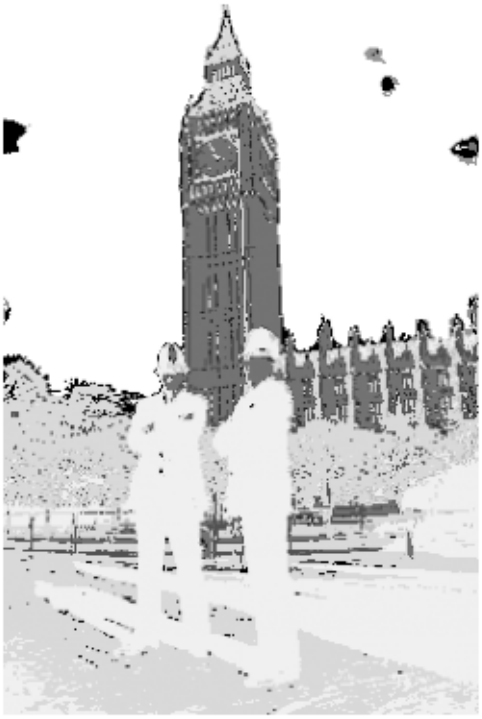
368078_75.jpg, 3D, r=12, c=5
Clusters found: 20

368078_75.jpg, 3D, r=16, c=2
Clusters found: 7

368078_75.jpg, 3D, r=16, c=3
Clusters found: 8



368078_75.jpg, 3D, r=16, c=4
Clusters found: 9



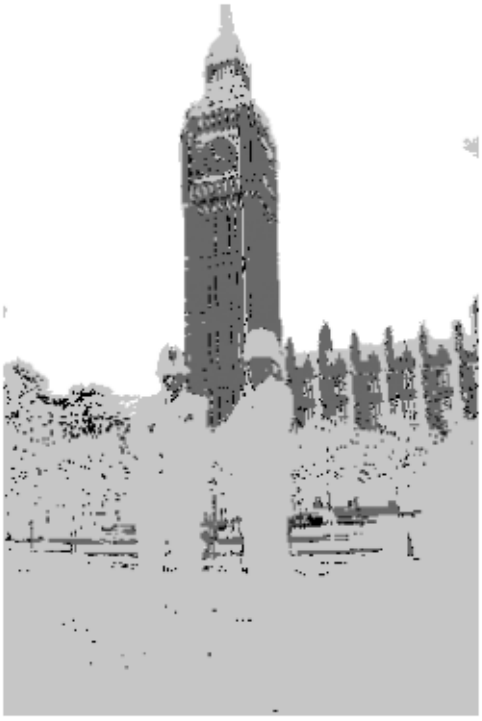
368078_75.jpg, 3D, r=16, c=5
Clusters found: 9

368078_75.jpg, 3D, r=24, c=2
Clusters found: 3

368078_75.jpg, 3D, r=24, c=3
Clusters found: 4

368078_75.jpg, 3D, r=24, c=4
Clusters found: 4

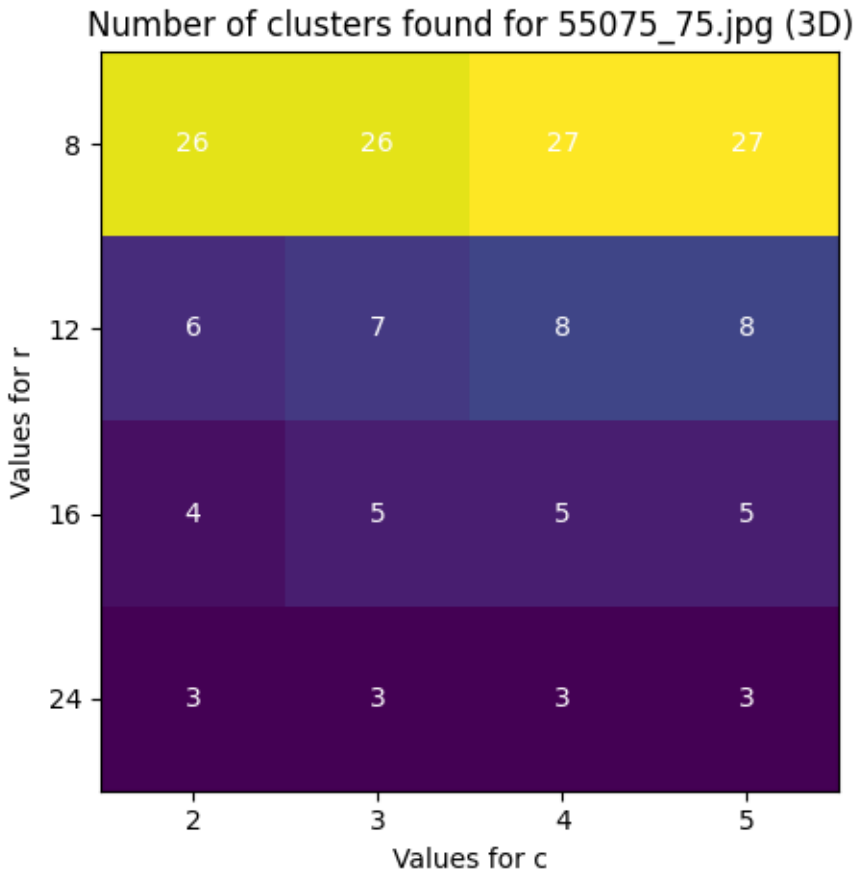
368078_75.jpg, 3D, r=24, c=5
Clusters found: 4

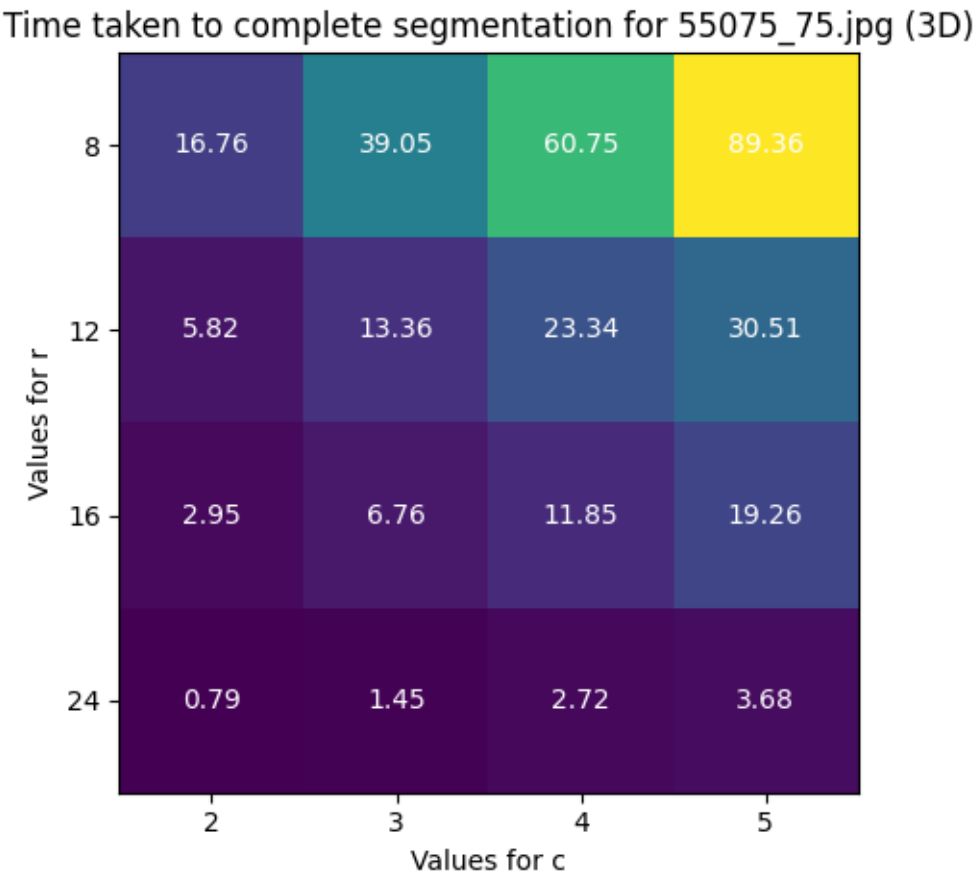


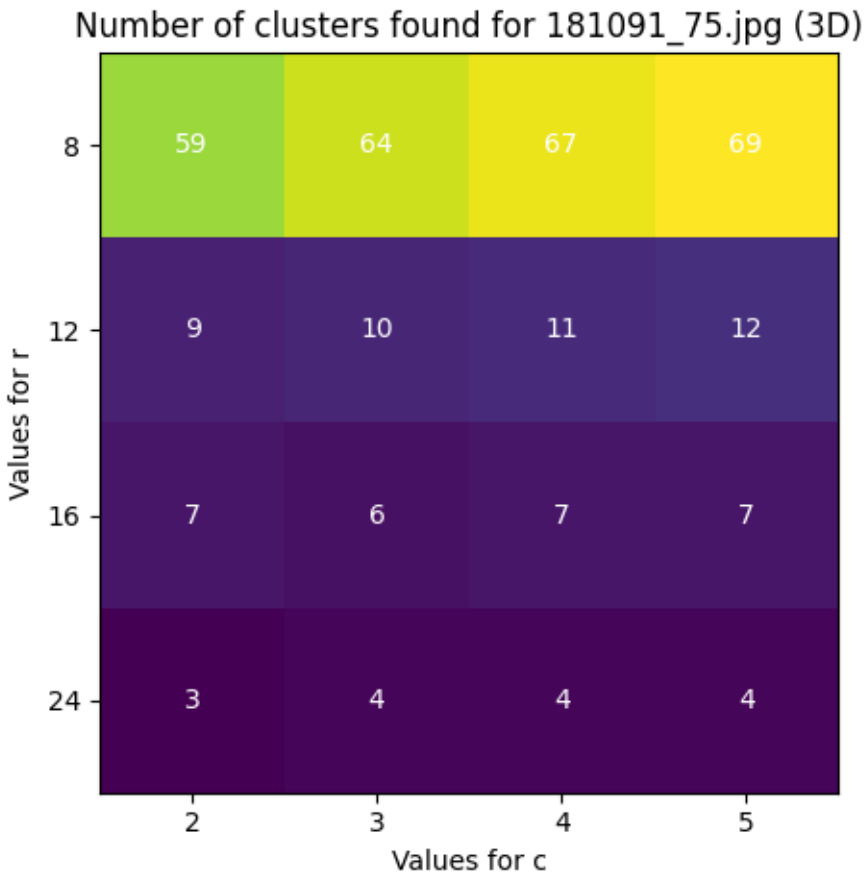
Plot number of clusters found and total time against the different values of r, c

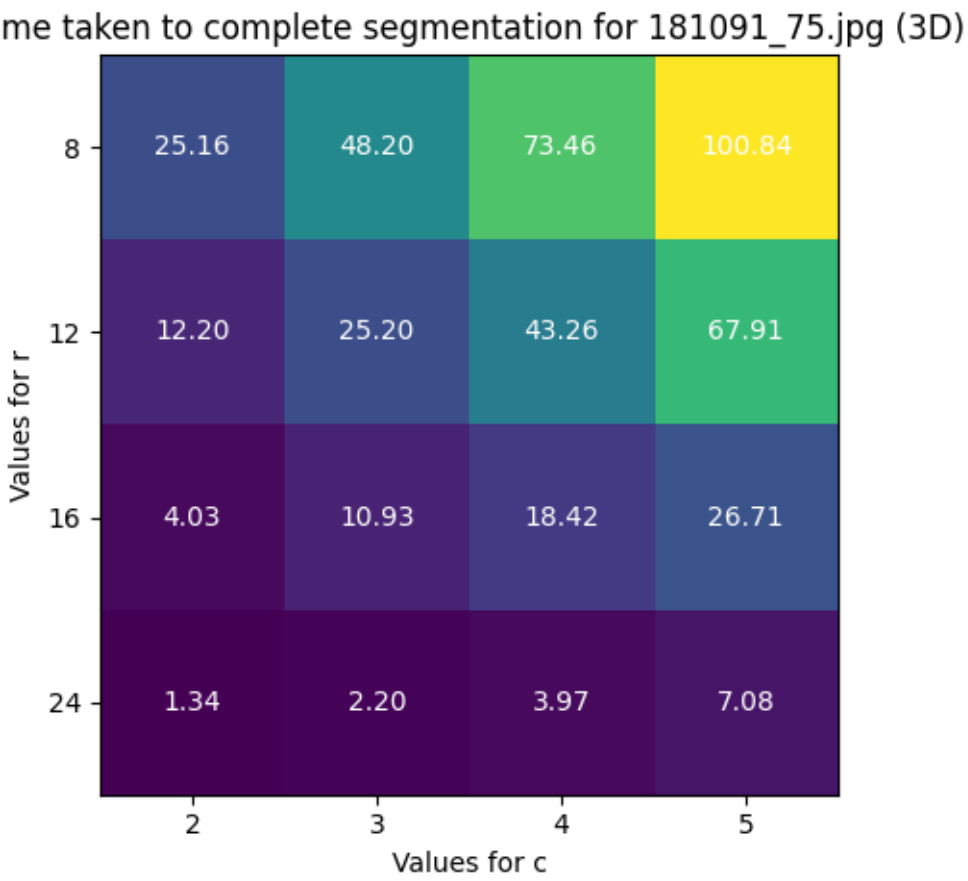
[ ]:
```
def heatmap_clusters_num(results, image, feature_type, save_as=None):
    clusters_num = np.array(
        [[len(results[image][r][c]["peaks"]) for c in C] for r in R]
    )
    fig, ax = plt.subplots()
    ax.imshow(clusters_num)
    ax.set_xticks(np.arange(len(C)), labels=C)
    ax.set_xlabel("Values for c")
    ax.set_yticks(np.arange(len(R)), labels=R)
    ax.set_ylabel("Values for r")
    ax.set_title(f"Number of clusters found for {image} ({feature_type})")
    fig.tight_layout()
    for i in range(len(R)):
        for j in range(len(C)):
            ax.text(j, i, clusters_num[i, j], ha="center", va="center",␣
 ↪color="w")

    if save_as:
        plt.savefig(save_as, bbox_inches="tight")

    plt.show()


def heatmap_time(results, image, feature_type, save_as=None):
    time = np.array([[results[image][r][c]["time"] for c in C] for r in R])
    fig, ax = plt.subplots()
    ax.imshow(time)
    ax.set_xticks(np.arange(len(C)), labels=C)
    ax.set_xlabel("Values for c")
    ax.set_yticks(np.arange(len(R)), labels=R)
    ax.set_ylabel("Values for r")
    ax.set_title(f"Time taken to complete segmentation for {image}␣
 ↪({feature_type})")
    fig.tight_layout()
    for i in range(len(R)):
        for j in range(len(C)):
            ax.text(j, i, f"{time[i, j]:0.2f}", ha="center", va="center",␣
 ↪color="w")

    if save_as:
        plt.savefig(save_as, bbox_inches="tight")

    plt.show()
```

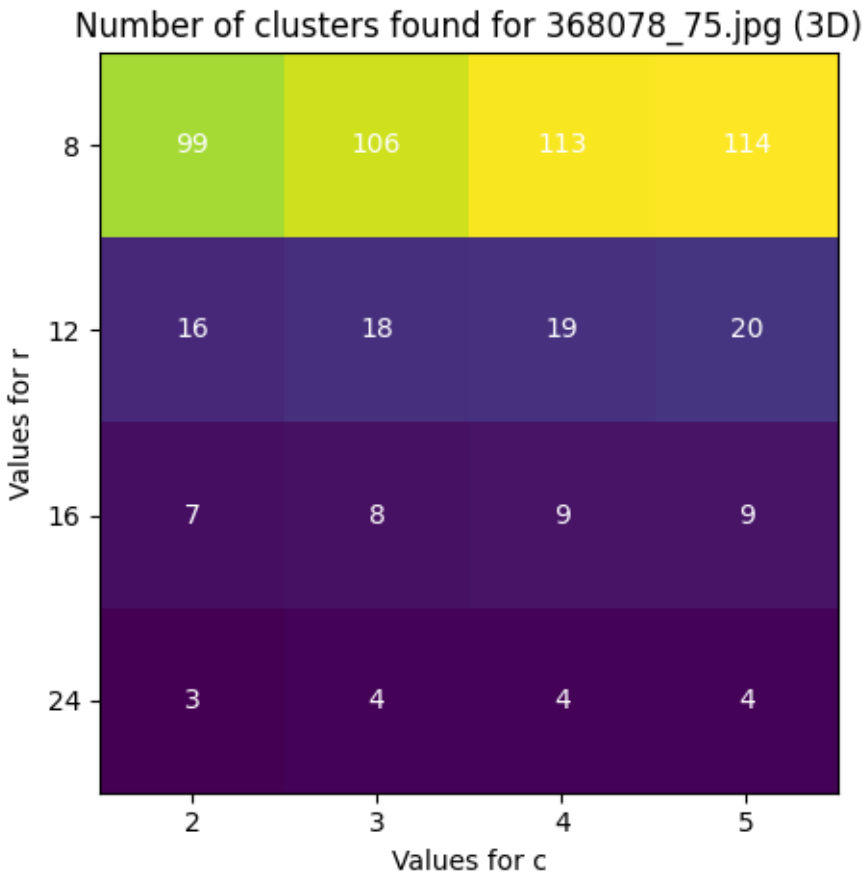
[ ]:
```
for image in IMAGES:
    heatmap_clusters_num(
        results_3d, image, "3D", save_as=f"results/heatmaps/
 ↪{image}_3D_clusters_num.jpg"
    )
    heatmap_time(
        results_3d, image, "3D", save_as=f"results/heatmaps/{image}_3D_time.jpg"
    )
```

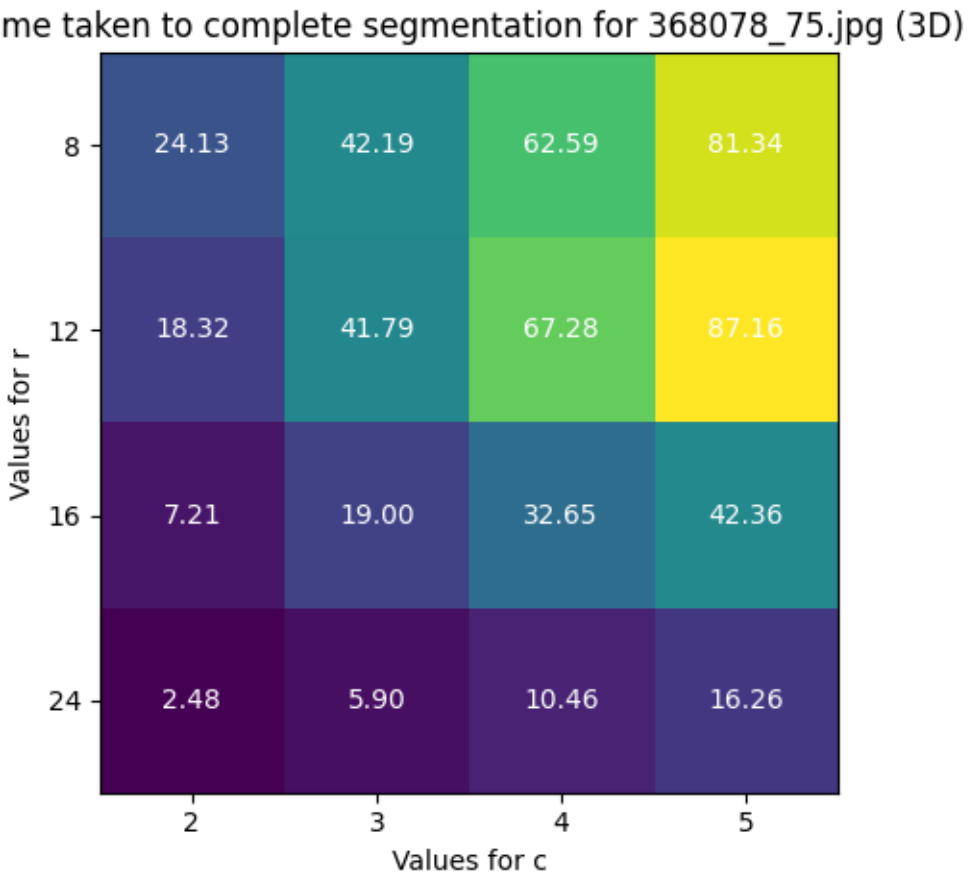



Time taken to complete segmentation for 55075_75.jpg (3D)
Values for r
8
12
16
24
16.76
39.05
60.75
89.36
5.82
13.36
23.34
30.51
2.95
6.76
11.85
19.26
0.79
1.45
2.72
3.68
2
3
4
5
Values for c


Number of clusters found for 181091_75.jpg (3D)
Values for r
8
12
16
24
59
64
67
69
9
10
11
12
7
6
7
7
3
4
4
4
2
3
4
5
Values for c


Time taken to complete segmentation for 181091_75.jpg (3D)
Values for r
8
12
16
24
25.16
48.20
73.46
100.84
12.20
25.20
43.26
67.91
4.03
10.93
18.42
26.71
1.34
2.20
3.97
7.08
2
3
4
5
Values for c


Number of clusters found for 368078_75.jpg (3D)
Values for r
8
12
16
24
99
106
113
114
16
18
19
20
7
8
9
9
3
4
4
4
2
3
4
5
Values for c



3.2 5D feature vectors

[ ]:
```
results_5d = {
    image: {r: {c: {"labels": None, "peaks": None, "time": None} for c in C}␣
  ↪for r in R}
    for image in IMAGES
}

for image in IMAGES:
    for r in R:
        for c in C:
            print(f"{image}, 5D, r={r}, c={c}")
            path = "images/" + image

            # Perform and time segmentation
            start = time.time()
            labels, peaks = segmIm(path, image_to_5d_vector, r, c)
            end = time.time()
```

```
            print("Clusters found:", len(peaks))
            print(f"Total time: {end - start:0.2f}s")
            print()

            # Save the results
            results_5d[image][r][c]["labels"] = labels
            results_5d[image][r][c]["peaks"] = peaks
            results_5d[image][r][c]["time"] = end - start
```

```
55075_75.jpg, 5D, r=8, c=2
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 2733
Total time: 225.01s

55075_75.jpg, 5D, r=8, c=3
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 2827
Total time: 272.33s

55075_75.jpg, 5D, r=8, c=4
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 2861
Total time: 307.51s

55075_75.jpg, 5D, r=8, c=5
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 2874
Total time: 329.67s

55075_75.jpg, 5D, r=12, c=2
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 870
Total time: 150.80s

55075_75.jpg, 5D, r=12, c=3
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 910
Total time: 224.01s

55075_75.jpg, 5D, r=12, c=4
  0%|          | 0/87001 [00:00<?, ?it/s]
```

```
Clusters found: 938
Total time: 293.94s

55075_75.jpg, 5D, r=12, c=5
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 951
Total time: 351.89s

55075_75.jpg, 5D, r=16, c=2
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 403
Total time: 103.02s

55075_75.jpg, 5D, r=16, c=3
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 431
Total time: 176.37s

55075_75.jpg, 5D, r=16, c=4
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 452
Total time: 243.02s

55075_75.jpg, 5D, r=16, c=5
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 451
Total time: 315.55s

55075_75.jpg, 5D, r=24, c=2
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 151
Total time: 55.30s

55075_75.jpg, 5D, r=24, c=3
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 163
Total time: 107.25s

55075_75.jpg, 5D, r=24, c=4
  0%|          | 0/87001 [00:00<?, ?it/s]
```

```
Clusters found: 168
Total time: 179.77s

55075_75.jpg, 5D, r=24, c=5
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 168
Total time: 257.77s

181091_75.jpg, 5D, r=8, c=2
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 3399
Total time: 297.02s

181091_75.jpg, 5D, r=8, c=3
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 3503
Total time: 344.49s

181091_75.jpg, 5D, r=8, c=4
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 3546
Total time: 386.12s

181091_75.jpg, 5D, r=8, c=5
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 3563
Total time: 434.13s

181091_75.jpg, 5D, r=12, c=2
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 1089
Total time: 207.95s

181091_75.jpg, 5D, r=12, c=3
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 1128
Total time: 285.23s

181091_75.jpg, 5D, r=12, c=4
  0%|          | 0/87001 [00:00<?, ?it/s]
```

```
Clusters found: 1165
Total time: 369.22s

181091_75.jpg, 5D, r=12, c=5
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 1174
Total time: 430.63s

181091_75.jpg, 5D, r=16, c=2
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 462
Total time: 140.87s

181091_75.jpg, 5D, r=16, c=3
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 493
Total time: 247.64s

181091_75.jpg, 5D, r=16, c=4
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 501
Total time: 352.73s

181091_75.jpg, 5D, r=16, c=5
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 511
Total time: 448.46s

181091_75.jpg, 5D, r=24, c=2
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 149
Total time: 80.90s

181091_75.jpg, 5D, r=24, c=3
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 152
Total time: 167.37s

181091_75.jpg, 5D, r=24, c=4
  0%|          | 0/87001 [00:00<?, ?it/s]
```

```
Clusters found: 158
Total time: 269.75s

181091_75.jpg, 5D, r=24, c=5
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 161
Total time: 384.44s

368078_75.jpg, 5D, r=8, c=2
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 4281
Total time: 327.26s

368078_75.jpg, 5D, r=8, c=3
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 4339
Total time: 380.08s

368078_75.jpg, 5D, r=8, c=4
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 4372
Total time: 413.08s

368078_75.jpg, 5D, r=8, c=5
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 4383
Total time: 427.36s

368078_75.jpg, 5D, r=12, c=2
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 1294
Total time: 249.58s

368078_75.jpg, 5D, r=12, c=3
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 1354
Total time: 359.56s

368078_75.jpg, 5D, r=12, c=4
  0%|          | 0/87001 [00:00<?, ?it/s]
```

```
Clusters found: 1374
Total time: 439.78s

368078_75.jpg, 5D, r=12, c=5
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 1397
Total time: 500.60s

368078_75.jpg, 5D, r=16, c=2
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 527
Total time: 203.27s

368078_75.jpg, 5D, r=16, c=3
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 551
Total time: 337.16s

368078_75.jpg, 5D, r=16, c=4
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 572
Total time: 475.73s

368078_75.jpg, 5D, r=16, c=5
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 584
Total time: 809.23s

368078_75.jpg, 5D, r=24, c=2
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 141
Total time: 156.70s

368078_75.jpg, 5D, r=24, c=3
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 156
Total time: 237.92s

368078_75.jpg, 5D, r=24, c=4
  0%|          | 0/87001 [00:00<?, ?it/s]
```

```
Clusters found: 160
Total time: 371.94s

368078_75.jpg, 5D, r=24, c=5
  0%|          | 0/87001 [00:00<?, ?it/s]
Clusters found: 165
Total time: 503.64s
```

Save the results

[ ]:
```python
np.save("results/arrays/results_5d.npy", results_5d)
```

Load the results

[ ]:
```python
results_5d = np.load("results/arrays/results_5d.npy", allow_pickle=True).item()
```

Plot and save the segmented images

[ ]:
```python
for image in IMAGES:
    for r in R:
        for c in C:
            print(f"{image}, 5D, r={r}, c={c}")
            path = "images/" + image
            h, w, _ = load_image_lab(path).shape
            labels = results_5d[image][r][c]["labels"]
            peaks = results_5d[image][r][c]["peaks"]
            print("Clusters found:", len(peaks))

            # Plot and save peaks
            filename_peaks = f"results/segmented_images/{image}_5D_r={r}_c={c}.
 ↪jpg"
            segm_image_peaks((h, w), labels, peaks, save_as=filename_peaks)

            # Plot and save clusters
            filename_clusters = (
                f"results/segmented_images/{image}_5D_clusters_r={r}_c={c}.jpg"
            )
            segm_image_clusters((h, w), labels, peaks,␣
 ↪save_as=filename_clusters)
```

```
55075_75.jpg, 5D, r=8, c=2
Clusters found: 2733
```

55075_75.jpg, 5D, r=8, c=3
Clusters found: 2827

55075_75.jpg, 5D, r=8, c=4
Clusters found: 2861

55075_75.jpg, 5D, r=8, c=5
Clusters found: 2874

55075_75.jpg, 5D, r=12, c=2
Clusters found: 870

55075_75.jpg, 5D, r=12, c=3
Clusters found: 910

55075_75.jpg, 5D, r=12, c=4
Clusters found: 938

55075_75.jpg, 5D, r=12, c=5
Clusters found: 951

55075_75.jpg, 5D, r=16, c=2
Clusters found: 403

55075_75.jpg, 5D, r=16, c=3
Clusters found: 431

55075_75.jpg, 5D, r=16, c=4
Clusters found: 452

55075_75.jpg, 5D, r=16, c=5
Clusters found: 451

55075_75.jpg, 5D, r=24, c=2
Clusters found: 151

55075_75.jpg, 5D, r=24, c=3
Clusters found: 163

55075_75.jpg, 5D, r=24, c=4
Clusters found: 168

55075_75.jpg, 5D, r=24, c=5
Clusters found: 168

181091_75.jpg, 5D, r=8, c=2
Clusters found: 3399

181091_75.jpg, 5D, r=8, c=3
Clusters found: 3503

181091_75.jpg, 5D, r=8, c=4
Clusters found: 3546

181091_75.jpg, 5D, r=8, c=5
Clusters found: 3563

181091_75.jpg, 5D, r=12, c=2
Clusters found: 1089

181091_75.jpg, 5D, r=12, c=3
Clusters found: 1128

181091_75.jpg, 5D, r=12, c=4
Clusters found: 1165

181091_75.jpg, 5D, r=12, c=5
Clusters found: 1174

181091_75.jpg, 5D, r=16, c=2
Clusters found: 462

181091_75.jpg, 5D, r=16, c=3
Clusters found: 493

181091_75.jpg, 5D, r=16, c=4
Clusters found: 501

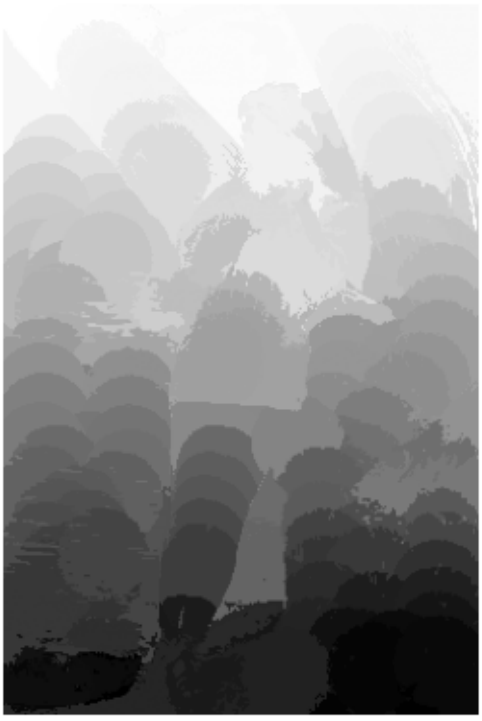
181091_75.jpg, 5D, r=16, c=5
Clusters found: 511

181091_75.jpg, 5D, r=24, c=2
Clusters found: 149

181091_75.jpg, 5D, r=24, c=3
Clusters found: 152

181091_75.jpg, 5D, r=24, c=4
Clusters found: 158

181091_75.jpg, 5D, r=24, c=5
Clusters found: 161



368078_75.jpg, 5D, r=8, c=2
Clusters found: 4281

368078_75.jpg, 5D, r=8, c=3
Clusters found: 4339

368078_75.jpg, 5D, r=8, c=4
Clusters found: 4372

368078_75.jpg, 5D, r=8, c=5
Clusters found: 4383

368078_75.jpg, 5D, r=12, c=2
Clusters found: 1294

368078_75.jpg, 5D, r=12, c=3
Clusters found: 1354

368078_75.jpg, 5D, r=12, c=4
Clusters found: 1374

368078_75.jpg, 5D, r=12, c=5
Clusters found: 1397

368078_75.jpg, 5D, r=16, c=2
Clusters found: 527

368078_75.jpg, 5D, r=16, c=3
Clusters found: 551

368078_75.jpg, 5D, r=16, c=4
Clusters found: 572

368078_75.jpg, 5D, r=16, c=5
Clusters found: 584

```
368078_75.jpg, 5D, r=24, c=2
Clusters found: 141
```

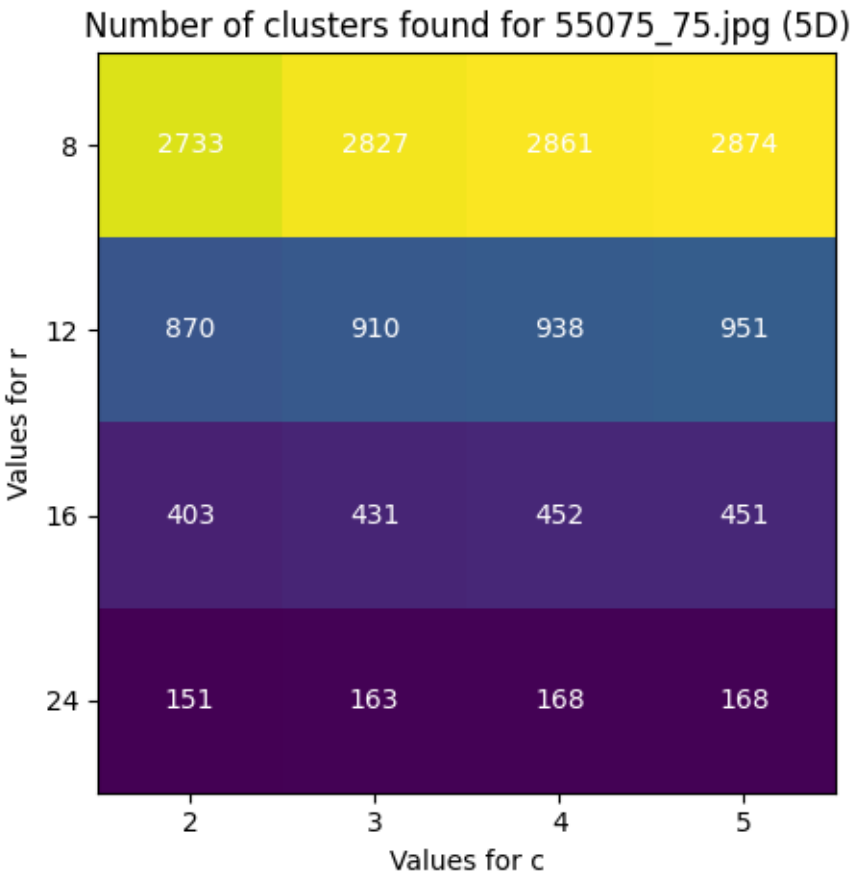
368078_75.jpg, 5D, r=24, c=3
Clusters found: 156

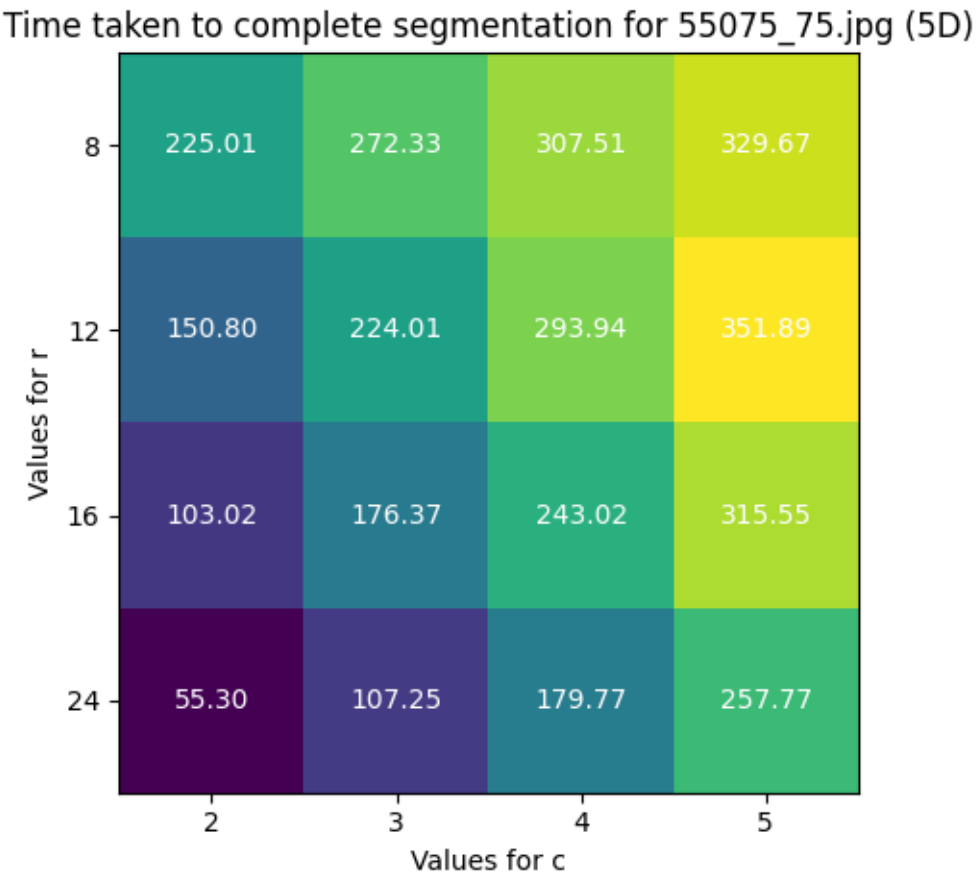
368078_75.jpg, 5D, r=24, c=4
Clusters found: 160

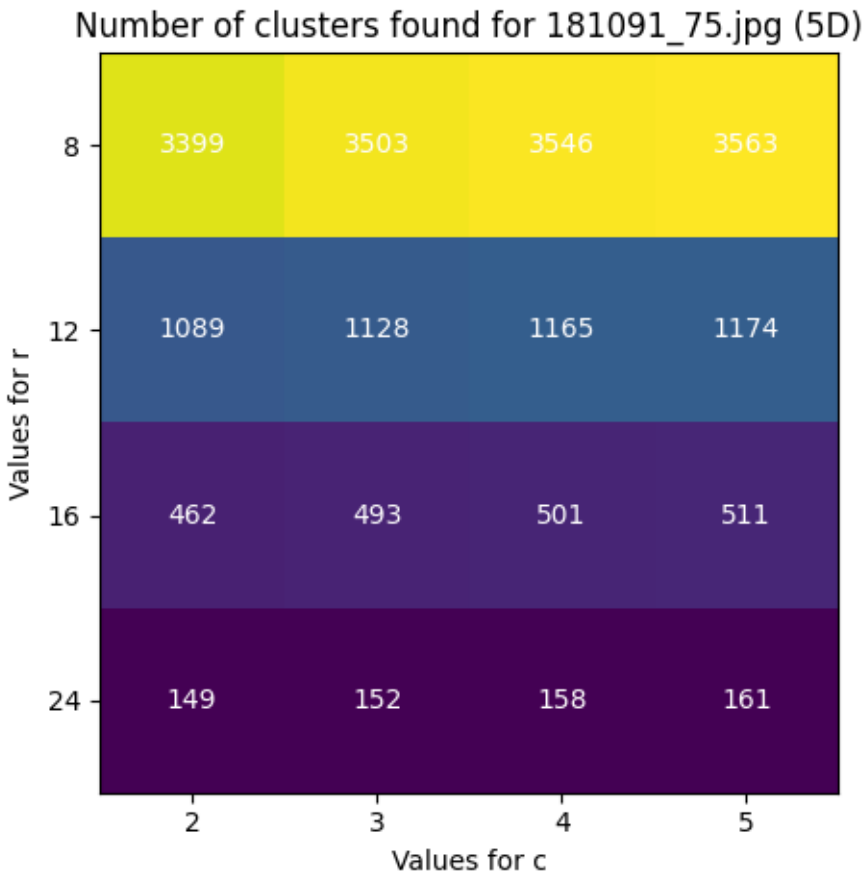
368078_75.jpg, 5D, r=24, c=5
Clusters found: 165

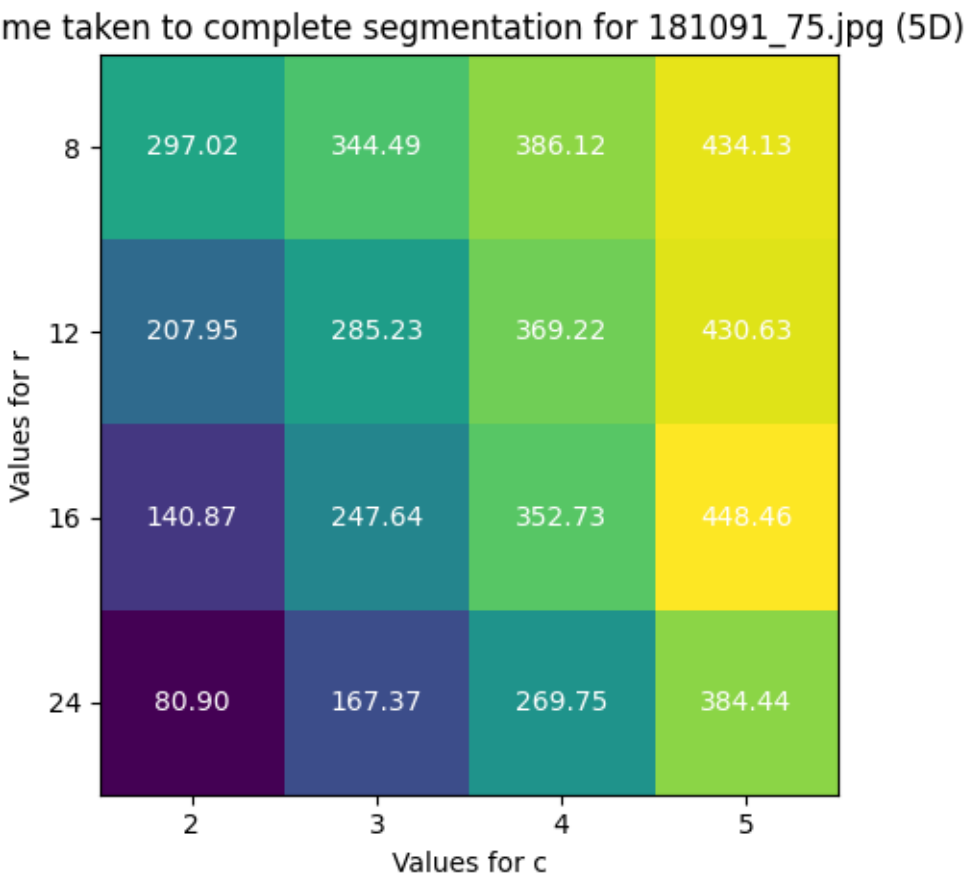
Plot number of clusters found and total time against different values of r, c

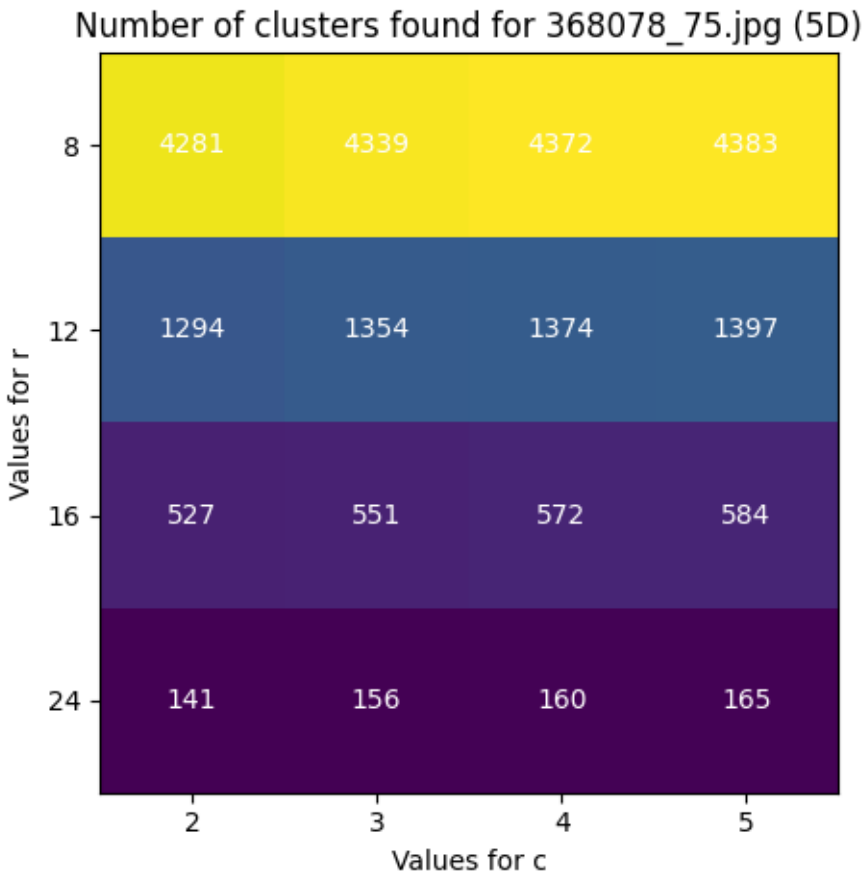
[ ]:
```
for image in IMAGES:
    heatmap_clusters_num(
        results_5d, image, "5D", save_as=f"results/heatmaps/
 ↪{image}_5D_clusters_num.jpg"
    )
    heatmap_time(
        results_5d, image, "5D", save_as=f"results/heatmaps/{image}_5D_time.jpg"
    )
```

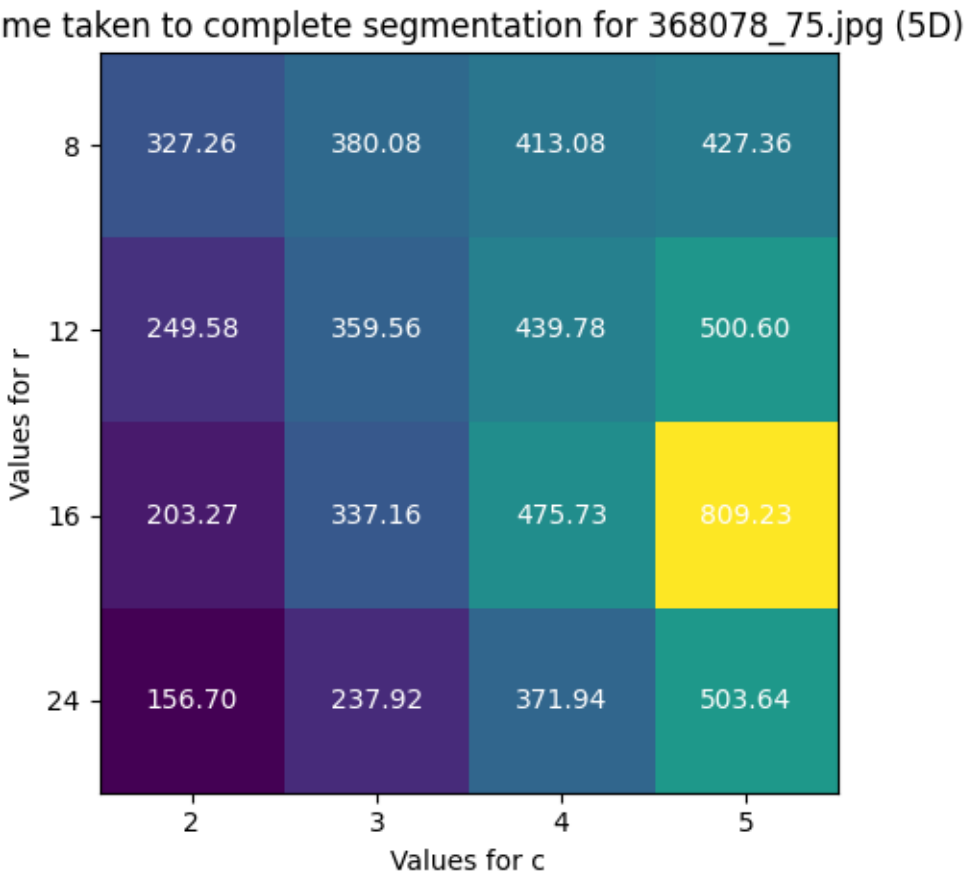

Number of clusters found for 55075_75.jpg (5D)
Values for r
8
12
16
24
2733
2827
2861
2874
870
910
938
951
403
431
452
451
151
163
168
168
2
3
4
5
Values for c


Time taken to complete segmentation for 55075_75.jpg (5D)
225.01
272.33
307.51
329.67
150.80
224.01
293.94
351.89
103.02
176.37
243.02
315.55
55.30
107.25
179.77
257.77
8
12
16
24
Values for r
2
3
4
5
Values for c


Number of clusters found for 181091_75.jpg (5D)
Values for r
8
12
16
24
3399
3503
3546
3563
1089
1128
1165
1174
462
493
501
511
149
152
158
161
2
3
4
5
Values for c


Time taken to complete segmentation for 181091_75.jpg (5D)
297.02
344.49
386.12
434.13
207.95
285.23
369.22
430.63
140.87
247.64
352.73
448.46
80.90
167.37
269.75
384.44
8
12
16
24
Values for r
2
3
4
5
Values for c


Number of clusters found for 368078_75.jpg (5D)
Values for r
8
12
16
24
4281
4339
4372
4383
1294
1354
1374
1397
527
551
572
584
141
156
160
165
2
3
4
5
Values for c


Time taken to complete segmentation for 368078_75.jpg (5D)
Values for r
8
12
16
24
327.26
380.08
413.08
427.36
249.58
359.56
439.78
500.60
203.27
337.16
475.73
809.23
156.70
237.92
371.94
503.64
2
3
4
5
Values for c