



Module 29

Partha Pratim
Das

Objectives &
Outline

Multivalued
Dependency

Definition

Example

Use

Theory

Decomposition to
4NF

Module Summary

Database Management Systems

Module 29: Relational Database Design/9: MVD and 4NF

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- Using the specification for a Library Information System, we have illustrated how a schema can be designed and then refined for finalization



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Module Summary

- To understand multi-valued dependencies arising out of attributes that can have multiple values
- To define Fourth Normal Form and learn the decomposition algorithm to 4NF



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Module Summary

- Multivalued Dependencies
- Decomposition to 4NF



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Module Summary

Multivalued Dependency



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Module Summary

- Persons(Man, Phones, Dog_Like)

Person :			Meaning of the tuples
Man(M)	Phones(P)	Dogs_Like(D)	
M1	P1/P2	D1/D2	Man M have phones P1 and P2, and likes the dogs D1 and D2.
M2	P3	D2	M2 have phones P3, and likes the dog D2.
Key : MPD			

There are no non trivial FDs because all attributes are combined forming Candidate Key, that is, MDP. In the above relation, two multivalued dependencies exists:

- Man \twoheadrightarrow Phones
- Man \twoheadrightarrow Dogs_Like

A man's phone are independent of the dogs they like. But after converting the above relation in Single Valued Attribute, each of a man's phones appears with each of the dogs they like in all combinations.

Source: <http://www.edugrabs.com/multivalued-dependency-mvd/>

Post 1NF Normalization

Man(M)	Phones(P)	Dogs_Likes(D)
M1	P1	D1
M1	P2	D2
M2	P3	D2
M1	P1	D2
M1	P2	D1



- If two or more independent relations are kept in a single relation, then Multivalued Dependency is possible. For example, Let there are two relations :
 - **Student(SID, Sname)** where $(SID \rightarrow Sname)$
 - **Course(CID, Cname)** where $(CID \rightarrow Cname)$
- There is no relation defined between Student and Course. If we kept them in a single relation named **Student_Course**, then MVD will exists because of m:n Cardinality
- If two or more MVDs exist in a relation, then while converting into SVAs, MVD exists.

Student:	
SID	Sname
S1	A
S2	B

Course:	
CID	Cname
C1	C
C2	B

SID	Sname	CID	Cname
S1	A	C1	C
S1	A	C2	B
S2	B	C1	C
S2	B	C2	B
2 MVDs exist: 1. $SID \twoheadrightarrow CID$ 2. $SID \twoheadrightarrow Cname$			

Source: <http://www.edugrabs.com/multivalued-dependency-mvd/>



MVD (3)

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- Suppose we record names of children, and phone numbers for instructors:
 - *inst_child*(*ID*, *child_name*)
 - *inst_phone*(*ID*, *phone_number*)
- If we were to combine these schema to get
 - *inst_info*(*ID*, *child_name*, *phone_number*)
 - Example data:
 - (99999, David, 512-555-1234)
 - (99999, David, 512-555-4321)
 - (99999, William, 512-555-1234)
 - (99999, William, 512-555-4321)
- This relation is in BCNF
 - Why?



- Let R be a relation schema and let $\alpha \subseteq R$ and $\beta \subseteq R$. The **multivalued dependency**

$$\alpha \twoheadrightarrow \beta$$

holds on R if in any legal relation $r(R)$, for all pairs for tuples t_1 and t_2 in r such that $t_1[\alpha] = t_2[\alpha]$, there exist tuples t_3 and t_4 in r such that:

$$t_1[\alpha] = t_2[\alpha] = t_3[\alpha] = t_4[\alpha]$$

$$t_3[\beta] = t_1[\beta]$$

$$t_3[R - \beta] = t_2[R - \beta]$$

$$t_4[\beta] = t_2[\beta]$$

$$t_4[R - \beta] = t_1[R - \beta]$$

Example: A relation of university courses, the books recommended for the course, and the lecturers who will be teaching the course:

- **course** \twoheadrightarrow **book**
- **course** \twoheadrightarrow **lecturer**

Test: **course** \twoheadrightarrow **book**

<u>Course</u>	<u>Book</u>	<u>Lecturer</u>	<u>Tuples</u>
AHA	Silberschatz	John D	t1
AHA	Nederpelt	William M	t2
AHA	Silberschatz	William M	t3
AHA	<u>Nederpelt</u>	John D	t4
AHA	Silberschatz	Christian G	
AHA	Nederpelt	Christian G	
OSO	Silberschatz	John D	
OSO	Silberschatz	William M	



MVD: Example

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- Let R be a relation schema with a set of attributes that are partitioned into 3 nonempty subsets.
 Y, Z, W
- We say that $Y \twoheadrightarrow Z$ (Y **multidetermines** Z) if and only if for all possible relations $r(R)$
 $\langle y_1, z_1, w_1 \rangle \in r$ and $\langle y_1, z_2, w_2 \rangle \in r$
then
 $\langle y_1, z_1, w_2 \rangle \in r$ and $\langle y_1, z_2, w_1 \rangle \in r$
- Note that since the behavior of Z and W are identical it follows that
 $Y \twoheadrightarrow Z$ if $Y \twoheadrightarrow W$



MVD: Example (2)

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- In our example:
 $ID \twoheadrightarrow child_name$
 $ID \twoheadrightarrow phone_number$
- The above formal definition is supposed to formalize the notion that given a particular value of $Y(ID)$ it has associated with it a set of values of $Z (child_name)$ and a set of values of $W (phone_number)$, and these two sets are in some sense independent of each other.
- Note:
 - If $Y \rightarrow Z$ then $Y \twoheadrightarrow Z$
 - Indeed we have (in above notation) $Z_1 = Z_2$
The claim follows.



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Module Summary

- We use multivalued dependencies in two ways:
 - a) To test relations to **determine** whether they are legal under a given set of functional and multivalued dependencies
 - b) To specify **constraints** on the set of legal relations. We shall thus concern ourselves only with relations that satisfy a given set of functional and multivalued dependencies.
- If a relation r fails to satisfy a given multivalued dependency, we can construct a relations r' that does satisfy the multivalued dependency by adding tuples to r .



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	Name	Rule
C-	Complementation	If $X \twoheadrightarrow Y$, then $X \rightarrow (R - (X \cup Y))$.
A-	Augmentation	If $X \twoheadrightarrow Y$ and $W \supseteq Z$, then $WX \twoheadrightarrow YZ$.
T-	Transitivity	If $X \twoheadrightarrow Y$ and $Y \twoheadrightarrow Z$, then $X \twoheadrightarrow (Z - Y)$.
	Replication	If $X \rightarrow Y$, then $X \twoheadrightarrow Y$ but the reverse is not true.
	Coalescence	If $X \twoheadrightarrow Y$ and there is a W such that $W \cap Y$ is empty, $W \rightarrow Z$ and $Y \supseteq Z$, then $X \rightarrow Z$.

- A MVD $X \twoheadrightarrow Y$ in R is called a trivial MVD is
 - Y is a subset of X ($X \supseteq Y$) or
 - $X \cup Y = R$. Otherwise, it is a non trivial MVD and we have to repeat values redundantly in the tuples.



MVD: Theory (2)

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Module Summary

- From the definition of multivalued dependency, we can derive the following rule:

- If $\alpha \rightarrow \beta$, then $\alpha \twoheadrightarrow \beta$

That is, every functional dependency is also a multivalued dependency

- The **closure** D^+ of D is the set of all functional and multivalued dependencies logically implied by D.
 - We can compute D^+ from D, using the formal definitions of functional dependencies and multivalued dependencies.
 - We can manage with such reasoning for very simple multivalued dependencies, which seem to be most common in practice
 - For complex dependencies, it is better to reason about sets of dependencies using a system of inference rules



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Fourth Normal Form

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Module Summary

- A relation schema R is in **4NF** with respect to a set D of functional and multivalued dependencies if for all multivalued dependencies in D^+ of the form $\alpha \twoheadrightarrow \beta$, where $\alpha \subseteq R$ and $\beta \subseteq R$, at least one of the following hold:
 - $\alpha \twoheadrightarrow \beta$ is trivial (that is, $\beta \subseteq \alpha$ or $\alpha \cup \beta = R$)
 - α is a superkey for schema R
- If a relation is in 4NF it is in BCNF



Restriction of Multivalued Dependencies

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Module Summary

- The restriction of D to R_i is the set D_i consisting of
 - All functional dependencies in D^+ that include only attributes of R_i
 - All multivalued dependencies of the form
$$\alpha \twoheadrightarrow (\beta \cap R_i)$$
where $\alpha \subseteq R_i$ and $\alpha \twoheadrightarrow \beta$ is in D^+



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Module Summary

- a) For all dependencies $A \twoheadrightarrow B$ in D^+ , check if A is a superkey
 - By using attribute closure
- b) If not, then
 - Choose a dependency in F^+ that breaks the 4NF rules, say $A \twoheadrightarrow B$
 - Create $R1 = A \ B$
 - Create $R2 = (R - (B - A))$
 - Note that: $R1 \cap R2 = A$ and $A \twoheadrightarrow AB (= R1)$, so this is lossless decomposition
- c) Repeat for $R1$, and $R2$
 - By defining $D1^+$ to be all dependencies in F that contain only attributes in $R1$
 - Similarly $D2^+$



4NF Decomposition Algorithm

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Module Summary

```
result := {R};  
done := false;  
compute  $D^+$ ;  
Let  $D_i$  denote the restriction of  $D^+$  to  $R_i$   
while ( not done)  
    if (there is a schema  $R_i$  in result that is not in 4NF) then  
        begin  
            let  $\alpha \twoheadrightarrow \beta$  be a nontrivial multivalued dependency that holds  
            on  $R_i$  such that  $\alpha \rightarrow R_i$  is not in  $D_i$ , and  $\alpha \cap \beta = \phi$  ;  
            result := (result -  $R_i$ )  $\cup$  ( $R_i - \beta$ )  $\cup$  ( $\alpha, \beta$ );  
        end  
    else done := true;
```

Note: each R_i is in 4NF, and decomposition is lossless-join



Example of 4NF Decomposition

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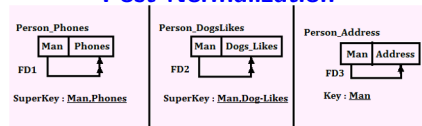
Decomposition to
4NF

Module Summary

- Example:
 - **Person_Modify**(Man(M), Phones(P), Dog_Likes(D), Address(A))
 - FDs:
 - ▷ FD1 : Man \twoheadrightarrow Phones
 - ▷ FD2 : Man \twoheadrightarrow Dogs_Like
 - ▷ FD3 : Man \rightarrow Address
 - Key = MPD
 - All dependencies violate 4NF

Man(M)	Phones(P)	Dogs_Likes(D)	Address(A)
M1	P1	D1	49-ABC,Bhiwani(HR.)
M1	P2	D2	49-ABC,Bhiwani(HR.)
M2	P3	D2	36-XYZ,Rohtak(HR.)
M1	P1	D2	49-ABC,Bhiwani(HR.)
M1	P2	D1	49-ABC,Bhiwani(HR.)

Post Normalization



In the above relations for both the MVD's – '**X**' is **Man**, which is again not the super key, but as **X** \cup **Y** = **R** i.e. (Man & Phones) together make the relation. So, the above MVD's are trivial and in FD 3, Address is functionally dependent on Man, where **Man** is the key in **Person_Address**, hence all the three relations are in 4NF.



Example of 4NF Decomposition

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Module Summary

- $R = (A, B, C, G, H, I)$
 $F = A \twoheadrightarrow B$
 $B \twoheadrightarrow HI$
 $CG \twoheadrightarrow H$
- R is not in 4NF since $A \twoheadrightarrow B$ and A is not a superkey for R
- Decomposition
 - a) $R_1 = (A, B)$ (R_1 is in 4NF)
 - b) $R_2 = (A, C, G, H, I)$ (R_2 is not in 4NF, decompose into R_3 and R_4)
 - c) $R_3 = (C, G, H)$ (R_3 is in 4NF)
 - d) $R_4 = (A, C, G, I)$ (R_4 is not in 4NF, decompose into R_5 and R_6)
 - $A \twoheadrightarrow B$ and $B \twoheadrightarrow HI \rightarrow A \twoheadrightarrow HI$, (MVD transitivity), and
 - and hence $A \twoheadrightarrow I$ (MVD restriction to R_4)
 - e) $R_5 = (A, I)$ (R_5 is in 4NF)
 - f) $R_6 = (A, C, G)$ (R_6 is in 4NF)



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Module Summary

- Understood multi-valued dependencies to handle attributes that can have multiple values
- Learnt Fourth Normal Form and decomposition to 4NF

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