



Module 22

Partha Pratim  
Das

Objectives &  
Outline

Functional  
Dependencies

Armstrong's Axioms

Closure of FDs

Module Summary

# Database Management Systems

## Module 22: Relational Database Design/2

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Module Summary

- Identified the features of good relational design
- Familiarized with the First Normal Form



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Module Summary

- To Introduce Functional Dependencies



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Module Summary

- Functional Dependencies



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# Functional Dependencies



# Goal: Devise a Theory for Good Relations

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Module Summary

- Decide whether a particular relation  $R$  is in “good” form.
- In the case that a relation  $R$  is not in “good” form, decompose it into a set of relations  $\{R_1, R_2, \dots, R_n\}$  such that
  - each relation is in good form
  - the decomposition is a lossless-join decomposition
- The theory is based on:
  - Functional dependencies
  - Multivalued dependencies
  - Other dependencies



# Functional Dependencies

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Module Summary

- Constraints on the set of legal relations
- Require that the value for a certain set of attributes determines uniquely the value for another set of attributes
- A functional dependency is a generalization of the notion of a *key*



# Functional Dependencies (2)

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Module Summary

- Let  $R$  be a relation schema

$$\alpha \subseteq R \text{ and } \beta \subseteq R$$

- The **functional dependency** or **FD**

$$\alpha \rightarrow \beta$$

**holds on**  $R$  if and only if for any legal relations  $r(R)$ , whenever any two tuples  $t_1$  and  $t_2$  of  $r$  agree on the attributes  $\alpha$ , they also agree on the attributes  $\beta$ . That is,

$$t_1[\alpha] = t_2[\alpha] \Rightarrow t_1[\beta] = t_2[\beta]$$

- Example: Consider  $r(A, B)$  with the following instance of  $r$ .

A	B
1	4
1	5
3	7

- On this instance,  $A \rightarrow B$  does **NOT** hold, but  $B \rightarrow A$  does hold. So we cannot have tuples like (2, 4), or (3, 5), or (4, 7) added to the current instance.





# Functional Dependencies (3)

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Module Summary

- $K$  is a superkey for relation schema  $R$  if and only if  $K \rightarrow R$
- $K$  is a candidate key for  $R$  if and only if
  - $K \rightarrow R$  and
  - for no  $\alpha \subset K$ ,  $\alpha \rightarrow R$
- Functional dependencies allow us to express constraints that cannot be expressed using superkeys. Consider the schema:  
*inst\_dept*( $ID$ , *name*, *salary*, *dept\_name*, *building*, *budget*)
- We expect these functional dependencies to hold:  
 $dept\_name \rightarrow building$   
 $dept\_name \rightarrow budget$   
 $ID \rightarrow budget$   
but would not expect the following to hold:  
 $dept\_name \rightarrow salary$



# Functional Dependencies (4)

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Module Summary

- We use functional dependencies to:
  - test relations to see if they are legal under a given set of functional dependencies.
    - ▷ If a relation  $r$  is legal under a set  $F$  of functional dependencies, we say that  $r$  **satisfies**  $F$
  - specify constraints on the set of legal relations
    - ▷ We say that  $F$  **holds on**  $R$  if all legal relations on  $R$  satisfy the set of functional dependencies  $F$
- **Note:** A specific instance of a relation schema may satisfy a functional dependency even if the functional dependency does not hold on all legal instances
  - For example, a specific instance of instructor may, by chance, satisfy
$$name \rightarrow ID$$
  - In such cases we do not say that  $F$  holds on  $R$



# Functional Dependencies (5)

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Module Summary

- A functional dependency is trivial if it is satisfied by all instances of a relation
  - Example:
    - ▷  $ID, name \rightarrow ID$
    - ▷  $name \rightarrow name$
- In general,  $\alpha \rightarrow \beta$  is trivial if  $\beta \subseteq \alpha$ .



# Functional Dependencies (6)

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Module Summary

- Functional dependencies are:

StudentID	Semester	Lecture	TA
1234	6	Numerical Methods	John
1221	4	Numerical Methods	Smith
1234	6	Visual Computing	Bob
1201	2	Numerical Methods	Peter
1201	2	Physics II	Simon

- $StudentID \rightarrow Semester$   
 $StudentID, Lecture \rightarrow TA$   
 $\{StudentID, Lecture\} \rightarrow \{TA, Semester\}$



# Functional Dependencies (7)

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Module Summary

- Functional dependencies are:

Employee ID	Employee Name	Department ID	Department Name
0001	John Doe	1	Human Resources
0002	Jane Doe	2	Marketing
0003	John Smith	1	Human Resources
0004	Jane Goodall	3	Sales

- $EmployeeID \rightarrow EmployeeName$   
 $EmployeeID \rightarrow DepartmentID$   
 $DepartmentID \rightarrow DepartmentName$



# Functional Dependencies (8): Armstrong's Axioms

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Module Summary

- Given a set of Functional Dependencies  $F$ , we can infer new dependencies by the **Armstrong's Axioms**:
  - Reflexivity**: if  $\beta \subseteq \alpha$ , then  $\alpha \rightarrow \beta$
  - Augmentation**: if  $\alpha \rightarrow \beta$ , then  $\gamma\alpha \rightarrow \gamma\beta$
  - Transitivity**: if  $\alpha \rightarrow \beta$  and  $\beta \rightarrow \gamma$ , then  $\alpha \rightarrow \gamma$
- These axioms can be repeatedly applied to generate new FDs and added to  $F$
- A new FD obtained by applying the axioms is said to be **logically implied** by  $F$
- The process of generations of FDs terminate after finite number of steps and we call it the **Closure Set  $F^+$**  for FDs  $F$ . This is the set of **all** FDs logically implied by  $F$
- Clearly,  $F \subseteq F^+$
- These axioms are
  - Sound** (generate only functional dependencies that actually hold), and
  - Complete** (eventually generate all functional dependencies that hold)
- Prove the axioms from definitions of FDs
- Prove the soundness and completeness of the axioms



# Functional Dependencies (9): Closure of a Set of FDs

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Module Summary

- $F = \{A \rightarrow B, B \rightarrow C\}$
- $F^+ = \{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$



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Module Summary

- Introduced the notion of Functional Dependencies

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