Objectives & Outline

Algorithms for FDs

Attribute Set Closure

Extraneous Attributes

Equivalence of FD Sets

Canonical Cover of FDs

Practice Problems

Module Summary

Database Management Systems

Module 24: Relational Database Design/4

Partha Pratim Das

Department of Computer Science and Engineering
Indian Institute of Technology, Kharagpur

ppd@cse.iitkgp.ac.in

Module Recap

Module 24

Partha Pratim Das

Objectives & Outline

Algorithms for FDs

Attribute Set Closure

Extraneous Attributes

Equivalence of FD Sets

Canonical Cover of FDs

Practice Problems

Module Summary

- Introduced the theory of functional dependencies
- Discussed issues in "good" design in the context of functional dependencies

Module Objectives

Module 24

Partha Pratim Das

Objectives & Outline

Algorithms for FDs

Attribute Set Closure

Extraneous Attributes

Equivalence of FD Sets

Canonical Cover of FDs

Practice Problems

Module Summary

- To Learn Algorithms for Properties of Functional Dependencies

Module Outline

Module 24

Partha Pratim Das

Objectives & Outline

Algorithms for FDs

Attribute Set Closure

Extraneous Attributes

Equivalence of FD Sets

Canonical Cover of FDs

Practice Problems

Module Summary

- Algorithms for Functional Dependencies

Module 24

Partha Pratim Das

Objectives & Outline

Algorithms for FDs

Attribute Set Closure

Extraneous Attributes

Equivalence of FD Sets

Canonical Cover of FDs

Practice Problems

Module Summary

Algorithms for Functional Dependencies

Attribute Set Closure

Module 24

Partha Pratim Das

Objectives & Outline

Algorithms for FDs

Attribute Set Closure

Extraneous Attributes

Equivalence of FD Sets

Canonical Cover of FDs

Practice Problems

Module Summary

- $R = (A, B, C, G, H, I)$
- $F = \{A \rightarrow B,\ A \rightarrow C,\ CG \rightarrow H,\ CG \rightarrow I,\ B \rightarrow H\}$
- $(AG)^+$
  a) $result = AG$
  b) $result = ABCG$ $\quad (A \rightarrow C$ and $A \rightarrow B)$
  c) $result = ABCGH$ $\quad (CG \rightarrow H$ and $CG \subseteq AGBC)$
  d) $result = ABCGHI$ $\quad (CG \rightarrow I$ and $CG \subseteq AGBCH)$
- Is AG a candidate key?
  a) Is AG a super key?
    i) Does $AG \rightarrow R? ==$ Is $(AG)^+ \supseteq R$
  b) Is any subset of AG a superkey?
    i) Does $A \rightarrow R? ==$ Is $(A)^+ \supseteq R$
    ii) Does $G \rightarrow R? ==$ Is $(G)^+ \supseteq R$

Attribute Set Closure: Uses

There are several uses of the attribute closure algorithm:

- Testing for superkey:
  - To test if α is a superkey, we compute α^+, and check if α^+ contains all attributes of R.
- Testing functional dependencies
  - To check if a functional dependency $\alpha \rightarrow \beta$ holds (or, in other words, is in F^+), just check if $\beta \subseteq \alpha^+$.
  - That is, we compute α^+ by using attribute closure, and then check if it contains β.
  - Is a simple and cheap test, and very useful
- Computing closure of F
  - For each $\gamma \subseteq R$, we find the closure γ^+, and for each $S \subseteq \gamma^+$, we output a functional dependency $\gamma \rightarrow S$.

Extraneous Attributes

Module 24

Partha Pratim Das

Objectives & Outline

Algorithms for FDs

Attribute Set Closure

Extraneous Attributes

Equivalence of FD Sets

Canonical Cover of FDs

Practice Problems

Module Summary

- Consider a set F of FDs and the FD $\alpha \rightarrow \beta$ in F.
  - Attribute A is **extraneous** in α if $A \in \alpha$ and F logically implies $(F - \{\alpha \rightarrow \beta\}) \cup \{(\alpha - A) \rightarrow \beta\}$.
  - Attribute A is **extraneous** in β if $A \in \beta$ and the set of FDs $(F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$ logically implies F.
- *Note:* Implication in the opposite direction is trivial in each of the cases above, since a "stronger" functional dependency always implies a weaker one
- Example: Given $F = \{A \rightarrow C, AB \rightarrow C\}$
  - B is extraneous in $AB \rightarrow C$ because $\{A \rightarrow C, AB \rightarrow C\}$ logically implies $A \rightarrow C$ (that is, the result of dropping B from $AB \rightarrow C$).
  - $A^+ = AC$ in $\{A \rightarrow C, AB \rightarrow C\}$
- Example: Given $F = \{A \rightarrow C, AB \rightarrow CD\}$
  - C is extraneous in $AB \rightarrow CD$ since $AB \rightarrow C$ can be inferred even after deleting C
  - $AB^+ = ABCD$ in $\{A \rightarrow C, AB \rightarrow D\}$

Extraneous Attributes (2): Tests

Module 24

Partha Pratim Das

Objectives & Outline

Algorithms for FDs

Attribute Set Closure

Extraneous Attributes

Equivalence of FD Sets

Canonical Cover of FDs

Practice Problems

Module Summary

- Consider a set F of functional dependencies and the functional dependency $\alpha \rightarrow \beta$ in F.
- To test if attribute $A \in \alpha$ is extraneous in α
  a) Compute $(\{\alpha\} - A)^+$ using the dependencies in F
  b) Check that $(\{\alpha\} - A)^+$ contains β; if it does, A is extraneous in α
- To test if attribute $A \in \beta$ is extraneous in β
  a) Compute α^+ using only the dependencies in
  $F' = (F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$,
  b) Check that α^+ contains A; if it does, A is extraneous in β

Equivalence of Sets of Functional Dependencies

Module 24

Partha Pratim Das

Objectives & Outline

Algorithms for FDs

Attribute Set Closure

Extraneous Attributes

Equivalence of FD Sets

Canonical Cover of FDs

Practice Problems

Module Summary

- Let F & G are two functional dependency sets.
  - These two sets F & G are equivalent if $F^+ = G^+$. That is: $(F^+ = G^+) \Leftrightarrow (F^+ \Rightarrow G \textit{ and } G^+ \Rightarrow F)$
  - Equivalence means that every functional dependency in F can be inferred from G, and every functional dependency in G an be inferred from F
- F and G are equal only if
  - F covers G: Means that all functional dependency of G are logically numbers of functional dependency set $F \Rightarrow F^+ \supseteq G$.
  - G covers F: Means that all functional dependency of F are logically members of functional dependency set $G \Rightarrow G^+ \supseteq F$.

Condition	CASES			
F Covers G	True	True	False	False
G Covers F	True	False	True	False
Result	F=G	F⊃G	G⊃F	No Comparison

Canonical Cover

- Sets of FDs may have redundant dependencies that can be inferred from the others
- Can we have some kind of "optimal" or "minimal" set of FDs wto work with?
- A **Canonical Cover** for F is a set of dependencies F_c such that ALL the following properties are satisfied:
  - $F^+ = F_c^+$. Or,
    - F logically implies all dependencies in F_c
    - F_c logically implies all dependencies in F
  - No functional dependency in F_c contains an extraneous attribute
  - Each left side of functional dependency in F_c is unique. That is, there are no two dependencies $\alpha_1 \to \beta_1$ and $\alpha_2 \to \beta_2$ in such that $\alpha_1 \to \alpha_2$
- Intuitively, a **Canonical cover** of F is a **minimal** set of FDs
  - Equivalent to F
  - Having no redundant FDs
  - No redundant parts of FDs
- **Minimal / Irreducible Set of Functional Dependencies**

Canonical Cover (2): Example

- For example: $A \rightarrow C$ is redundant in: $\{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$
- Parts of a functional dependency may be redundant
  - For example: on RHS: $\boldsymbol{\{A \rightarrow B, B \rightarrow C, A \rightarrow CD\}}$ can be simplified to $\boldsymbol{\{A \rightarrow B, B \rightarrow C, A \rightarrow D\}}$
  - In the forward: (1) $A \rightarrow CD \Rightarrow A \rightarrow C$ and $A \rightarrow D$
    (2) $A \rightarrow B, B \rightarrow C \Rightarrow A \rightarrow C$
  - In the reverse: (1) $A \rightarrow B, B \rightarrow C \Rightarrow A \rightarrow C$
    (2) $A \rightarrow C, A \rightarrow D \Rightarrow A \rightarrow CD$
  - For example: on LHS: $\boldsymbol{\{A \rightarrow B, B \rightarrow C, AC \rightarrow D\}}$ can be simplified to $\boldsymbol{\{A \rightarrow B, B \rightarrow C, A \rightarrow D\}}$
  - In the forward: (1) $A \rightarrow B, B \rightarrow C \Rightarrow A \rightarrow C \Rightarrow A \rightarrow AC$
    (2) $A \rightarrow AC, AC \rightarrow D \Rightarrow A \rightarrow D$
  - In the reverse: $A \rightarrow D \Rightarrow AC \rightarrow D$

Canonical Cover (3): RHS

Module 24

Partha Pratim Das

Objectives & Outline

Algorithms for FDs

Attribute Set Closure

Extraneous Attributes

Equivalence of FD Sets

Canonical Cover of FDs

Practice Problems

Module Summary

- $\{A \rightarrow B, B \rightarrow C, A \rightarrow CD\} \Rightarrow \{A \rightarrow B, B \rightarrow C, A \rightarrow D\}$
  - (1) $A \rightarrow CD \Rightarrow A \rightarrow C$ and $A \rightarrow D$
    (2) $A \rightarrow B, B \rightarrow C \Rightarrow A \rightarrow C$
  - $A^+ = ABCD$
- $\{A \rightarrow B, B \rightarrow C, A \rightarrow D\} \Rightarrow \{A \rightarrow B, B \rightarrow C, A \rightarrow CD\}$
  - $A \rightarrow B, B \rightarrow C \Rightarrow A \rightarrow C$
  - $A \rightarrow C, A \rightarrow D \Rightarrow A \rightarrow CD$
  - $A^+ = ABCD$

Canonical Cover (4): LHS

- $\{A \rightarrow B, B \rightarrow C, AC \rightarrow D\} \Rightarrow \{A \rightarrow B, B \rightarrow C, A \rightarrow D\}$
  - $A \rightarrow B, B \rightarrow C \Rightarrow A \rightarrow C \Rightarrow A \rightarrow AC$
  - $A \rightarrow AC, AC \rightarrow D \Rightarrow A \rightarrow D$
  - $A^+ = ABCD$
- $\{A \rightarrow B, B \rightarrow C, A \rightarrow D\} \Rightarrow \{A \rightarrow B, B \rightarrow C, AC \rightarrow D\}$
  - $A \rightarrow D \Rightarrow AC \rightarrow D$
  - $AC^+ = ABCD$

Canonical Cover (5)

- To compute a canonical cover for F:
  repeat
  Use the union rule to replace any dependencies in F
  $\alpha_1 \rightarrow \beta_1$ and $\alpha_1 \rightarrow \beta_2$ with $\alpha_1 \rightarrow \beta_1\beta_2$
  Find a functional dependency $\alpha \rightarrow \beta$ with an
  extraneous attribute either in α or in β
  /* Note: test for extraneous attributes done using F_c, not F */
  If an extraneous attribute is found, delete it from $\alpha \rightarrow \beta$
  until F does not change
- Note: Union rule may become applicable after some extraneous attributes have been deleted, so it has to be re-applied

Canonical Cover (6): Example

Module 24

Partha Pratim Das

Objectives & Outline

Algorithms for FDs

Attribute Set Closure

Extraneous Attributes

Equivalence of FD Sets

Canonical Cover of FDs

Practice Problems

Module Summary

- $R = (A, B, C)$
  $F = \{A \rightarrow BC, B \rightarrow C, A \rightarrow B, AB \rightarrow C\}$
- Combine $A \rightarrow BC$ and $A \rightarrow B$ into $A \rightarrow BC$
  - Set is now $\{A \rightarrow BC, B \rightarrow C, AB \rightarrow C\}$
- A is extraneous in $AB \rightarrow C$
  - Check if the result of deleting A from $AB \rightarrow C$ is implied by the other dependencies
    - Yes: in fact, $B \rightarrow C$ is already present!
  - Set is now $\{A \rightarrow BC, B \rightarrow C\}$
- C is extraneous in $A \rightarrow BC$
  - Check if $A \rightarrow C$ is logically implied by $A \rightarrow B$ and the other dependencies
    - Yes: using transitivity on $A \rightarrow B$ and $B \rightarrow C$.
      - Can use attribute closure of A in more complex cases
- The canonical cover is: $A \rightarrow B$, $B \rightarrow C$

Practice Problems on Functional Dependencies

- **Find if a given functional dependency is implied from a set of Functional Dependencies:**
  a) For: $A \rightarrow BC, CD \rightarrow E, E \rightarrow C, D \rightarrow AEH, ABH \rightarrow BD, DH \rightarrow BC$
     i) Check: $BCD \rightarrow H$
     ii) Check: $AED \rightarrow C$
  b) For: $AB \rightarrow CD, AF \rightarrow D, DE \rightarrow F, C \rightarrow G, F \rightarrow E, G \rightarrow A$
     i) Check: $CF \rightarrow DF$
     ii) Check: $BG \rightarrow E$
     iii) Check: $AF \rightarrow G$
     iv) Check: $AB \rightarrow EF$
  c) For: $A \rightarrow BC, B \rightarrow E, CD \rightarrow EF$
     i) Check: $AD \rightarrow F$

Practice Problems on Functional Dependencies (2)

Module 24

Partha Pratim Das

Objectives & Outline

Algorithms for FDs

Attribute Set Closure

Extraneous Attributes

Equivalence of FD Sets

Canonical Cover of FDs

Practice Problems

Module Summary

- **Find Super Key using Functional Dependencies:**
  a) Relational Schema $R(ABCDE)$. Functional dependencies: $AB \rightarrow C, DE \rightarrow B, CD \rightarrow E$
  b) Relational Schema $R(ABCDE)$. Functional dependencies: $AB \rightarrow C, C \rightarrow D, B \rightarrow EA$

Practice Problems on Functional Dependencies (3)

Module 24
Partha Pratim Das
Objectives & Outline
Algorithms for FDs
Attribute Set Closure
Extraneous Attributes
Equivalence of FD Sets
Canonical Cover of FDs
Practice Problems
Module Summary

- **Find Candidate Key using Functional Dependencies:**
  a) Relational Schema $R(ABCDE)$. Functional dependencies: $AB \rightarrow C, DE \rightarrow B, CD \rightarrow E$
  b) Relational Schema $R(ABCDE)$. Functional dependencies: $AB \rightarrow C, C \rightarrow D, B \rightarrow EA$

Practice Problems on Functional Dependencies (4)

Module 24
Partha Pratim Das
Objectives & Outline
Algorithms for FDs
Attribute Set Closure
Extraneous Attributes
Equivalence of FD Sets
Canonical Cover of FDs
Practice Problems
Module Summary

- **Find Prime and Non Prime Attributes using Functional Dependencies:**
  a) $R(ABCDEF)$ having FDs $\{AB \rightarrow C, C \rightarrow D, D \rightarrow E, F \rightarrow B, E \rightarrow F\}$
  b) $R(ABCDEF)$ having FDs $\{AB \rightarrow C, C \rightarrow DE, E \rightarrow F, C \rightarrow B\}$
  c) $R(ABCDEFGHIJ)$ having FDs $\{AB \rightarrow C, A \rightarrow DE, B \rightarrow F, F \rightarrow GH, D \rightarrow IJ\}$
  d) $R(ABDLPT)$ having FDs $\{B \rightarrow PT, A \rightarrow D, T \rightarrow L\}$
  e) $R(ABCDEFGH)$ having FDs $\{E \rightarrow G, AB \rightarrow C, AC \rightarrow B, AD \rightarrow E, B \rightarrow D, BC \rightarrow A\}$
  f) $R(ABCDE)$ having FDs $\{A \rightarrow BC, CD \rightarrow E, B \rightarrow D, E \rightarrow A\}$
  g) $R(ABCDEH)$ having FDs $\{A \rightarrow B, BC \rightarrow D, E \rightarrow C, D \rightarrow A\}$

- **Prime Attributes**: Attribute set that belongs to any candidate key are called Prime Attributes
  - It is union of all the candidate key attribute: $\{CK_1 \cup CK_2 \cup CK_3 \cup \cdots\}$
  - If Prime attribute determined by other attribute set, then more than one candidate key is possible.
  - For example, If A is Candidate Key, and $X \rightarrow A$, then, X is also Candidate Key.
- **Non Prime Attribute**: Attribute set does not belong to any candidate key are called Non Prime Attributes

Practice Problems on Functional Dependencies (5)

- **Check the Equivalence of a Pair of Sets of Functional Dependencies:**
  a) Consider the two sets F and G with their FDs as below :
     i) $F : A \rightarrow C, AC \rightarrow D, E \rightarrow AD, E \rightarrow H$
     ii) $G : A \rightarrow CD, E \rightarrow AH$
  b) Consider the two sets P and Q with their FDs as below :
     i) $P : A \rightarrow B, AB \rightarrow C, D \rightarrow ACE$
     ii) $Q : A \rightarrow BC, D \rightarrow AE$

Practice Problems on Functional Dependencies (6)

Module 24

Partha Pratim Das

Objectives & Outline

Algorithms for FDs

Attribute Set Closure

Extraneous Attributes

Equivalence of FD Sets

Canonical Cover of FDs

Practice Problems

Module Summary

- **Find the Minimal Cover or Irreducible Sets or Canonical Cover of a Set of Functional Dependencies:**
  a) $AB \rightarrow CD, BC \rightarrow D$
  b) $ABCD \rightarrow E, E \rightarrow D, AC \rightarrow D, A \rightarrow B$

Module Summary

Module 24

Partha Pratim Das

Objectives & Outline

Algorithms for FDs

Attribute Set Closure

Extraneous Attributes

Equivalence of FD Sets

Canonical Cover of FDs

Practice Problems

Module Summary

- Studied Algorithms for Properties of Functional Dependencies

Slides used in this presentation are borrowed from http://db-book.com/ with kind permission of the authors.
Edited and new slides are marked with "PPD".