IIT Madras
BSc Degree

Module 42

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Database Management Systems

Module 42: Indexing and Hashing/2: Indexing/2

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Module Recap

- Appreciated the reasons for indexing database tables
- Understood the ordered indexes

Module Objectives

- To recap Balanced Binary Search Trees as options for optimal in-memory search data structures
- To understand the issues relating to external search data structures for persistent data
- To study 2-3-4 Tree as a precursor to B/B+-Tree for an efficient external data structure for database and index tables

Module Outline

- Balanced Binary Search Trees
- 2-3-4 Tree

Balanced Binary Search Trees

Search Data Structures

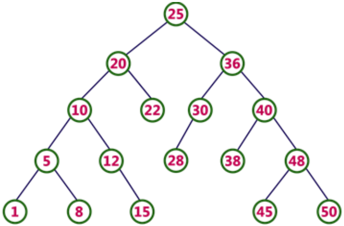
- How to search a key in a list of n data items?
  - Linear Search: $O(n)$: Find 28 $\Rightarrow$ 16 comparisons
    - ▷ Unordered items in an array – search sequentially
    - ▷ Unordered / Ordered items in a list – search sequentially

22	50	20	36	40	15	08	01	45	48	30	10	38	12	25	28	05	END

  - Binary Search: $O(\lg n)$: Find 28 $\Rightarrow$ 4 comparisons – 25, 36, 30, 28
    - ▷ Ordered items in an array – search by divide-and-conquer

01	05	08	10	12	15	20	22	25	28	30	36	38	40	45	48	50	END

    - ▷ Binary Search Tree – recursively on left / right



Search Data Structures (2)

- Worst case time (n data items in the data structure):

Data Structure	Search	Insert	Delete	Remarks
Unordered Array	O(n)	O(1)	O(1)	The time to Insert / Delete an item is the time after the location of the item has been ascertained by Search.
Ordered Array	O(log n)	O(n)	O(n)	
Unordered List	O(n)	O(1)	O(1)	
Ordered List	O(n)	O(1)	O(1)	
Binary Search Tree	O(h)	O(1)	O(1)	

- Between an array and a list, there is a trade-off between search and insert/delete complexity
- For a BST of n nodes, $\lg n \leq h < n$, where h is the height of the tree
- A BST is balanced if $h \sim O(\lg n)$: this what we desire

Search Data Structures (3): BST

- In the worst case, searching a key in a BST is $O(h)$, where h is the height of the key
- **Bad Tree:** $h \sim O(n)$
  - The BST is a skewed binary search tree (all the nodes except the leaf would have only one child)
  - This can happen if keys are inserted in sorted order
  - Height (h) of the BST having n elements becomes $n - 1$
  - Time complexity of search in BST becomes $O(n)$
- **Good Tree:** $h \sim O(\lg n)$
  - The BST is a balanced binary search tree
  - This is possible if
    - If keys are inserted in purely randomized order, Or
    - If the tree is explicitly balanced after every insertion
  - Height (h) of the binary search tree becomes $\lg n$
  - Time complexity of search in BST becomes $O(\lg n)$

Balanced Binary Search Trees

- A BST is balanced if $h \sim O(\lg n)$
- **Balancing Guarantees** may be of various types:
  - Worst-case
    - AVL Tree: Self-balancing BST
      - Named after inventors Adelson-Velsky-Landis
      - Heights of the two child subtrees of any node differ by at most one: $|h_L - h_R| \leq 1$
      - If they differ by more than one, rebalancing is done rotation
  - Randomized
    - Randomized BST
      - A BST on n keys is random if either it is empty ($n = 0$), or the probability that a given key is at the root is $\frac{1}{n}$, and the left and right subtrees are random
    - Skip List
      - A skip list is built (probabbilistically) in layers of ordered linked lists
  - Amortized
    - Splay
      - A BST where recently accessed elements are quick to access again

Balanced Binary Search Trees (2)

- These data structures have optimal complexity for the required operations:
  - Search: $O(\lg n)$
  - Insert: Search + $O(1)$: $O(\lg n)$
  - Delete: Search + $O(1)$: $O(\lg n)$
- And they are:
  - Good for in-memory operations
  - Work well for small volume of data
  - Has complex rotation and / or similar operations
  - Do not scale for external data structures

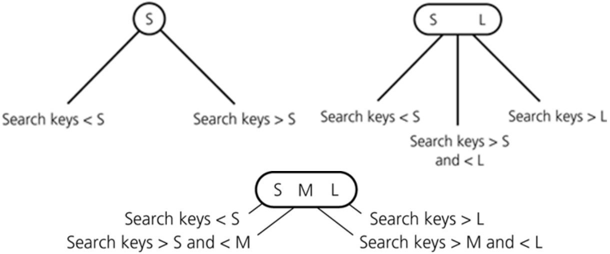
2-3-4 Tree

2-3-4 Trees

- All leaves are at the same depth (the bottom level).
  - Height, h, of all leaf nodes are same
    - $h \sim O(\lg n)$
    - Complexity of search, insert and delete: $O(h) \sim O(\lg n)$
- All data is kept in sorted order
- Every node (leaf or internal) is a 2-node, 3-node or a 4-node (based on the number of links or children), and holds one, two, or three data elements, respectively
- Generalizes easily to larger nodes
- Extends to external data structures

2-3-4 Trees

- Uses 3 kinds of nodes satisfying key relationships as shown below:
  - A 2-node must contain a single data item (S) and two links
  - A 3-node must contain two data items (S, L) and three links
  - A 4-node must contain three data items (S, M, L) and four links
  - A leaf may contain either one, two, or three data items



2-3-4 Trees: Search

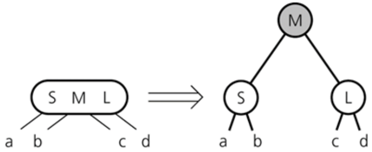
- Search
  - Simple and natural extension of search in BST

2-3-4 Trees: Insert

- Insert
  - Search to find expected location
    - ▷ If it is a 2 node, change to 3 node and insert
    - ▷ If it is a 3 node, change to 4 node and insert
    - ▷ If it is a 4 node, split the node by moving the middle item to parent node, then insert
  - Node Splitting
    - ▷ A 4-node is split as soon as it is encountered during a search from the root to a leaf
    - ▷ The 4-node that is split will
      - Be the root, or
      - Have a 2-node parent, or
      - Have a 3-node parent

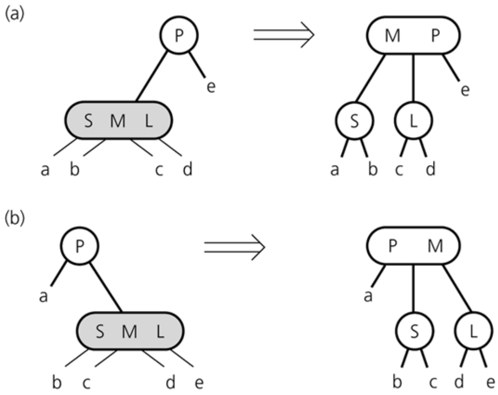
2-3-4 Trees: Insert

- Splitting at Root



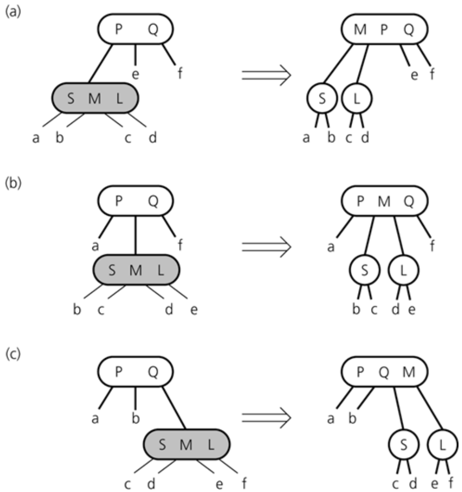
2-3-4 Trees: Insert

- Splitting with 2 Node parent



2-3-4 Trees: Insert

- Splitting with 3 Node parent

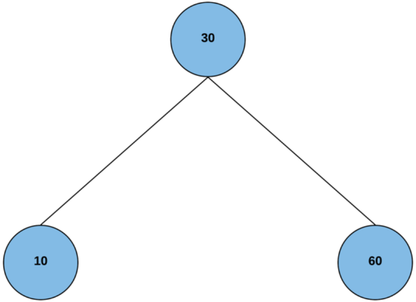


2-3-4 Trees: Insert

- Node Splitting: There are two strategies:
  - *Early*: Split a 4-node as soon as you cross on in traversal. It ensures that the tree does not have a path with multiple 4-nodes at any point
  - *Late*: Split a 4-node only when you need to insert an item in it. This might lead to cases where for one insert we may need to perform $O(h)$ splits going till up to the root
- Both are valid and has the same complexity $O(h)$. However, they lead to different results. Different texts and sites follow different strategies.
- Here we are following early strategy

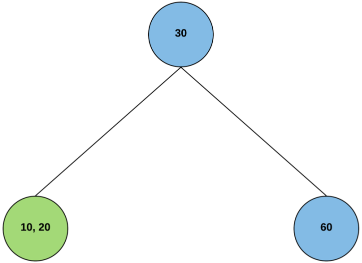
2-3-4 Trees: Insert: Example

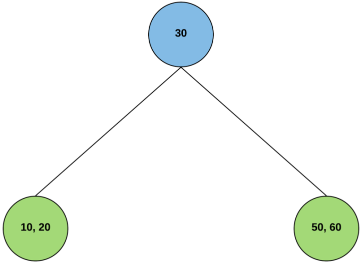
- Insert 10, 30, 60, 20, 50, 40, 70, 80, 15, 90, 100
- 10
- 10, 30
- 10, 30, 60
- Split for 20



2-3-4 Trees: Insert: Example

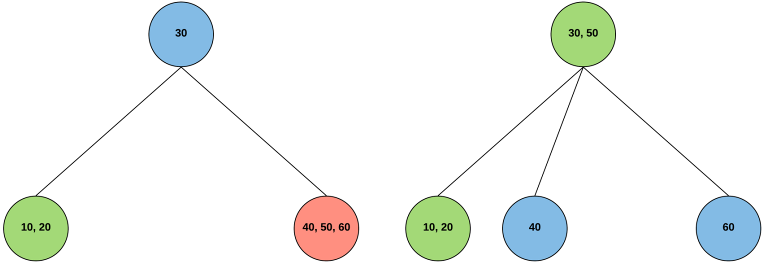
- 10, 30, 60, 20
- 10, 30, 60, 20, 50





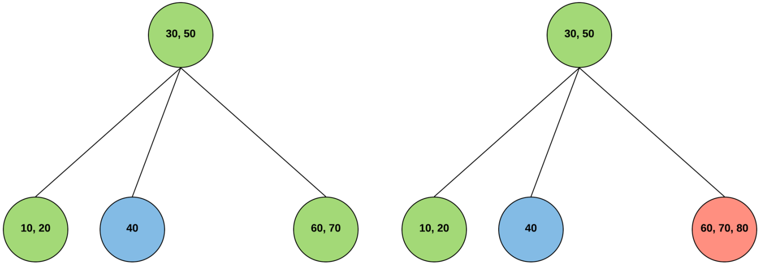
2-3-4 Trees: Insert: Example

- 10, 30, 60, 20, 50, 40
- Split for 70



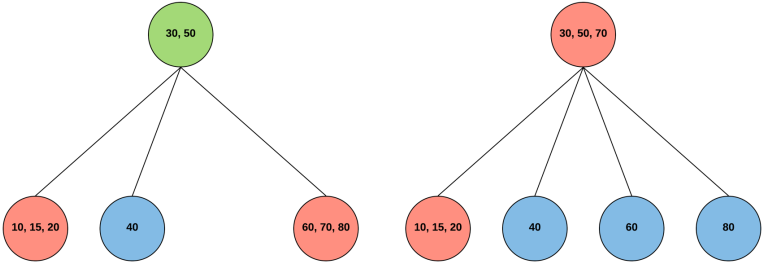
2-3-4 Trees: Insert: Example

- 10, 30, 60, 20, 50, 40, 70
- 10, 30, 60, 20, 50, 40, 70, 80



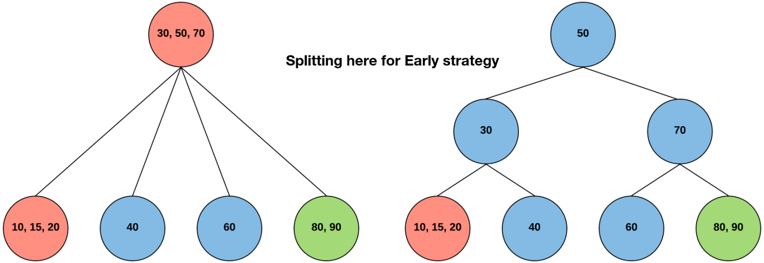
2-3-4 Trees: Insert: Example

- 10, 30, 60, 20, 50, 40, 70, 80, 15
- Split for 90



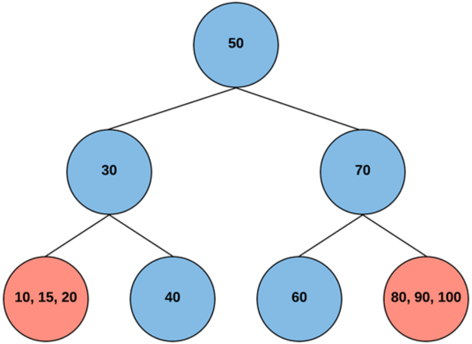
2-3-4 Trees: Insert: Example

- 10, 30, 60, 20, 50, 40, 70, 80, 15, 90
- Split for 100



2-3-4 Trees: Insert: Example

- 10, 30, 60, 20, 50, 40, 70, 80, 15, 90, 100



2-3-4 Trees: Delete

- Delete
  - Locate the node n that contains the item *theItem*
  - Find *theItem*'s inorder successor and swap it with *theItem* (deletion will always be at a leaf)
  - If that leaf is a 3-node or a 4-node, remove *theItem*
  - To ensure that *theItem* does not occur in a 2-node
    - ▷ Transform each 2-node encountered into a 3-node or a 4-node
    - ▷ Reverse different cases illustrated for splitting

2-3-4 Tree

- Advantages
  - All leaves are at the same depth (the bottom level): Height, $h \sim O(\lg n)$
  - Complexity of search, insert and delete: $O(h) \sim O(\lg n)$
  - All data is kept in sorted order
  - Generalizes easily to larger nodes
  - Extends to external data structures
- Disadvantages
  - Uses variety of node types – need to destruct and construct multiple nodes for converting a 2 Node to 3 Node, a 3 Node to 4 Node, for splitting etc.

2-3-4 Trees

- Consider only one node type with space for 3 items and 4 links
  - Internal node (non-root) has 2 to 4 children (links)
  - Leaf node has 1 to 3 items
  - Wastes some space, but has several advantages for external data structure
- Generalizes easily to larger nodes
  - All paths from root to leaf are of the same length
  - Each node that is not a root or a leaf has between $\lceil \frac{n}{2} \rceil$ and n children.
  - A leaf node has between $\left\lceil \frac{(n-1)}{2} \right\rceil$ and $n-1$ values
  - Special cases:
    - If the root is not a leaf, it has at least 2 children.
    - If the root is a leaf, it can have between 0 and $(n-1)$ values.
- Extends to external data structures
  - B-Tree
  - 2-3-4 Tree is a B-Tree where $n = 4$

Module Summary

- Recapitulated the notions of Balanced Binary Search Trees as options for optimal in-memory search data structures
- Understood the issues relating to external data structures for persistent data
- Explored 2-3-4 Tree in depth as a precursor to B/B+-Tree for an efficient external data structure for database and index tables

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