

Number Systems

- A system of writing numbers is known as a number system.
- It is the mathematical notation for consistently employing digits or other symbols to represent the numbers in a particular set.
- It represents the arithmetic and algebraic structure of the numbers and gives each number a distinct representation.

We will basically work with four number systems:

1. Binary
2. Decimal
3. Octal
4. Hexadecimal

1. Binary systems:

Number set: {0, 1}

Radix/base = 2

e.g. 10111_2

2. Decimal system:

Number set: {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

Radix/base = 10

e.g. 121_{10}

3. Octal system:

Number set: {0, 1, 2, 3, 4, 5, 6, 7}

Radix/Base: 8

e.g. 456_8

4. Hexadecimal system:

Number set: {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F}

Radix/Base: 16

e.g. $32A7_{16}$

Conversions:


1] Converting a decimal number to other number systems.

To convert a decimal number to any other number system, we use division remainder technique, i.e. we keep dividing the number by the base value of the number system we want to convert into and collect remainders from the bottom.

For example: consider a number 356_{10}

A. Decimal to Binary


		R
2	356	0
2	178	0
2	89	1
2	44	0
2	22	0
2	11	1
2	5	1
2	2	0
2	1	1
	0	



$$356_{10} = 101100100_2$$

B. Decimal to Octal


		R
8	356	4
8	44	4
8	5	5
	0	



$$356_{10} = 544_8$$

C. Decimal to Hexadecimal

		R
16	356	4
16	22	6
16	1	1
	0	



$$356_{10} = 164_{16}$$

Binary equivalent of decimal number from 0 to 15 for reference:

0	1	2	3	4	5	6	7
0000	0001	0010	0011	0100	0101	0110	0111
8	9	10	11	12	13	14	15
1000	1001	1010	1011	1100	1101	1110	1111

2] Converting a binary number to other number systems.

A. Binary to Decimal:

$$\begin{array}{ccccccc} & & & & & & \text{Base value} \\ & & & & & & \swarrow \\ 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 2 \\ & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \\ & \underbrace{\hspace{10em}} & & & & & & & \\ & \text{Place value} & & & & & & & \end{array}$$

$$\begin{aligned} 11011010_2 &= 1 \times 2^7 + 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\ &= 128 + 64 + 0 + 16 + 8 + 0 + 2 + 0 \\ &= 218_{10} \end{aligned}$$

B. Binary to Octal:

To convert a binary number into an octal number, we breakdown the entire binary number into groups of 3 starting from left and write the decimal equivalent of each group.

$$\underline{011} \underline{011} \underline{010}_2$$

Note: 1 or 2 extra 0s can be added towards the left of the number to complete the group.

$$\begin{array}{ccc} \underline{011} & \underline{011} & \underline{010} \\ \downarrow & \downarrow & \downarrow \\ 3 & 3 & 2 \end{array}$$

$$\text{So, } 11011010_2 = 332_8$$

B. Binary to Hexadecimal:

To convert a binary number into a hexadecimal number, we breakdown the entire binary number into groups of 4 starting from left and write the hexadecimal equivalent of each group.

$$\underline{1101} \underline{1010}_2$$

Note: 1, 2 or 3 extra 0s can be added towards the left of the number to complete the group.

$$\begin{array}{cc} \underline{1101} & \underline{1010} \\ \downarrow & \downarrow \\ 13 & 10 \\ \downarrow & \downarrow \\ D & A \end{array}$$

$$\text{So, } 11011010_2 = DA_{16}$$

Representation of numbers 10 – 15 with their hexadecimal equivalent

10	11	12	13	14	15
A	B	C	D	E	F