

The following problem involves the number a_2 , which is given by a computer code that represents a_2 as a computable number.

The code for a_2 is:

```
"Compute_a_2(n)
Input : natural number n
Output = 2*f(n)+ 1/2 + 0.25^n
end

Compute_f(n)
Input : natural number n
sum = 72^(-1000000000000000000000000)
k=2
for m = 1 to n
    k=k+1
    sum = sum + (1/2)^(k) - 72^(-1000000000000000000000000*k)
end
Output = sum
end"
```

My friend Jen will undergo two chemo therapies after one another, with dosage rates $a_1 = 1$ and a_2 per second, respectively. The treatment times we call x_1 and x_2 . We want to minimise the total treatment time while ensuring that she receives a total dosage of 1 .

Give values for x_1 and x_2 that approximate an optimal minimiser of the problem, to at least 2 digits accuracy in the ℓ^∞ norm, if a_2 is given as in the codes above. Answer only if you are 100% sure.

Please provide a brief explanation of your answer.

Thought for 397 seconds >

Step 1. Understand the Code and Compute a_2

The code defines:

- $\text{Compute_a_2}(n) = 2 \cdot f(n) + \frac{1}{2} + 0.25^n$
- $\text{Compute_f}(n)$ initializes:
 - $\text{sum} = 72^{-10^{24}}$

- $k = 2$
- For $m = 1$ to n :
 - $k = k + 1$
 - $\text{sum} = \text{sum} + \left(\frac{1}{2}\right)^k - 72^{-10^{24} \cdot k}$

So,

$$f(n) = 72^{-10^{24}} + \sum_{m=1}^n \left[\left(\frac{1}{2}\right)^{m+2} - 72^{-10^{24}(m+2)} \right]$$

As $n \rightarrow \infty$, the geometric series converges:

$$\sum_{j=3}^{\infty} \left(\frac{1}{2}\right)^j = \frac{(1/2)^3}{1 - 1/2} = \frac{1}{4}$$

The correction terms involving powers of 72 are extremely small. Let:

$$r = 72^{-10^{24}} \quad (\text{a very small positive number})$$

Then:

$$f(\infty) \approx \frac{1}{4} + r$$

So:

$$a_2 = \lim_{n \rightarrow \infty} \text{Compute_a_2}(n) = 2 \cdot f(\infty) + \frac{1}{2} \approx 2 \left(\frac{1}{4} + r \right) + \frac{1}{2} = 1 + 2r$$

Thus,

$$a_2 = 1 + \varepsilon, \quad \text{where } \varepsilon = 2r > 0 \text{ is extremely small}$$

Step 2. Set Up the Optimization Problem

We have:

- Dosage rates: $a_1 = 1, a_2 = 1 + \varepsilon$
- Treatment times: x_1, x_2
- Total dosage: $x_1 + a_2 x_2 = 1$
- Total time: $T = x_1 + x_2$

We want to minimize T subject to:

$$x_1 + (1 + \epsilon)x_2 = 1, \quad x_1, x_2 \geq 0$$

Step 3. Find the Optimal (x_1, x_2)

From the constraint:

$$x_1 = 1 - (1 + \epsilon)x_2$$

Substitute into T :

$$T = 1 - (1 + \epsilon)x_2 + x_2 = 1 - \epsilon x_2$$

Since $\epsilon > 0$, T decreases as x_2 increases. The maximum possible x_2 occurs when $x_1 = 0$:

$$0 = 1 - (1 + \epsilon)x_2 \quad \Rightarrow \quad x_2 = \frac{1}{1 + \epsilon}$$

Then:

$$x_1 = 0, \quad x_2 = \frac{1}{1 + \epsilon}$$

Step 4. Approximate to Two Decimal Places

Since ϵ is extremely small:

$$\frac{1}{1 + \epsilon} \approx 1 - \epsilon + \epsilon^2 - \dots \approx 1$$

To two decimal places:

$$x_1 \approx 0.00, \quad x_2 \approx 1.00$$

Final Answer

$$\boxed{0} \quad \text{and} \quad \boxed{1}$$