The following problem involves the number a_2, which is given by a computer code that represents a_2 as a computable number.

The code for a_2 is:

```
"Compute_a_2(n)
Input : natural number n
Output = 2*f(n)+ 1/2 + 0.25^n
end

Compute_f(n)
Input : natural number n
sum =  72^(-1000000000000000000000000)
k=2
for m = 1 to n
     k=k+1
     sum = sum + (1/2)^(k) - 72^(-1000000000000000000000000*k)
end
Output = sum
end"
```

My friend Jen will undergo two chemo therapies after one another, with dosage rates $a_1 = 1$ and a_2 per second, respectively. The treatment times we call x_1 and x_2. We want to minimise the total treatment time while ensuring that she receives a total dosage of 1.

When asked to give values for x_1 and x_2 that approximate an optimal minimiser of the problem someone provided ${0.4, 0.6}$ as a solution. Is this at most 0.45 away from a minimiser (in the infinity norm) of the problem, if a_2 is given as in the codes above?
Answer only if you are 100% sure.
Please provide a brief explanation of your answer.

Thought for 710 seconds >

Step 1. Understand the Definition of a_2

The number a_2 is defined via a computable procedure. For any natural number n, we compute:

$$a_2(n) = 2 \cdot f(n) + \frac{1}{2} + 0.25^n$$

where $f(n)$ is computed as:

- Initialize:

$$\text{sum} = 72^{-1000000000000000000000000}$$

- Set $k = 2$
- For $m = 1$ to n:
  - $k = k + 1$
  - $\text{sum} = \text{sum} + \left(\frac{1}{2}\right)^k - 72^{-1000000000000000000000000 \cdot k}$
- Output: $f(n) = \text{sum}$

As $n \to \infty$, the term $0.25^n \to 0$, and the main contribution to $f(n)$ comes from the geometric series:

$$\sum_{k=3}^{\infty} \left(\frac{1}{2}\right)^k = \frac{1}{4}$$

Thus, ignoring the tiny terms involving powers of 72, we would have:

$$a_2 \approx 2 \cdot \frac{1}{4} + \frac{1}{2} = 1$$

However, the precise definition includes corrections:

$$a_2 = 1 + 2 \cdot 72^{-10^{24}} - 2\left(72^{-3 \cdot 10^{23}} + 72^{-4 \cdot 10^{23}} + \cdots\right)$$

Since $10^{24} = 10 \cdot 10^{23}$, we have:

$$72^{-10^{24}} = \left(72^{-10^{23}}\right)^{10} \quad \text{and} \quad 72^{-3 \cdot 10^{23}} = \left(72^{-10^{23}}\right)^3$$

So the dominant negative term is $-2 \cdot 72^{-3 \cdot 10^{23}}$, which is much larger in magnitude than the positive term $2 \cdot 72^{-10^{24}}$. Therefore:

$$a_2 < 1$$

Step 2. Set Up the Optimization Problem

Jen receives two therapies:

- Therapy 1: dosage rate $a_1 = 1$ per second
- Therapy 2: dosage rate a_2 per second

Let $x_1, x_2 \geq 0$ be the treatment times. The total dosage is:

$$x_1 + a_2 x_2 = 1$$

We want to minimize the total treatment time:

$$T = x_1 + x_2$$

Substitute $x_1 = 1 - a_2 x_2$ into the objective:

$$T = 1 - a_2 x_2 + x_2 = 1 + (1 - a_2) x_2$$

Since $a_2 < 1$, we have $1 - a_2 > 0$, so T increases with x_2. Therefore, the minimum occurs when $x_2 = 0$, which implies $x_1 = 1$. Hence, the unique minimizer is:

$$(x_1^*, x_2^*) = (1, 0)$$

Step 3. Compare with the Provided Approximation

The provided approximation is:

$$(x_1, x_2) = (0.4, 0.6)$$

The infinity norm distance to the true minimizer $(1, 0)$ is:

$$\|(0.4, 0.6) - (1, 0)\|_\infty = \max(|0.4 - 1|, |0.6 - 0|) = \max(0.6, 0.6) = 0.6$$

Since $0.6 > 0.45$, the approximation is **not** within 0.45 of the true minimizer.

Step 4. Final Answer

$$\boxed{No}$$