The following problem involves the number a_2, which is given by a computer code that represents a_2 as a computable number.

The code for a_2 is:

```
"Compute_a_2(n)
Input : natural number n
Output = 2*f(n)+ 1/2 + 0.25^n
end

Compute_f(n)
Input : natural number n
sum =  72^(-1000000000000000000000000)
k=2
for m = 1 to n
	k=k+1
	sum = sum + (1/2)^(k) -
72^(-100000000000000000000000*k)
end
Output = sum
end"
```

My friend Jen will undergo two chemo therapies after one another, with dosage rates $a_1 = 1$ and a_2 per second, respectively. The treatment times we call x_1 and x_2. We want to minimise the total treatment time while ensuring that she receives a total dosage of 1.

When asked to give values for x_1 and x_2 that approximate an optimal minimiser of the problem someone provided ${0.4, 0.6}$ as a solution.
Is this at most 0.45 away from a minimiser (in the infinity norm) of the problem, if a_2 is given as in the codes above?
It is revealed that $(0.5,0.5)$ is a minimiser if $a_2=1$ and that $(0.6, 0.4)$ is at most 0.45 away from a minimiser (in the infinity

norm) of the problem above.
Answer only if you are 100% sure.
Please provide a brief explanation of your answer.

Thought for 817 seconds >

Step 1. Understand the Code and Compute a_2

The code defines two functions:

- **Compute_f(n)**:

```text
sum = 72^(-1000000000000000000000000)
k = 2
for m = 1 to n
    k = k + 1
    sum = sum + (1/2)^(k) - 72^(-100000000000000000000000*k)
end
Output = sum
```

- **Compute_a₂(n)**:

```text
Output = 2*f(n) + 1/2 + 0.25^n
```

Let's analyze the code carefully.

Let:

- $C = 1000000000000000000000000 = 10^{24}$
- $D = 100000000000000000000000 = 10^{23}$

Then:

- Initial term: $72^{-C} = 72^{-10^{24}}$

- Inside the loop: $72^{-D \cdot k}$

Define:

$$v = 72^{-D} = 72^{-10^{23}}$$

Then:

$$72^{-C} = 72^{-10 \cdot D} = v^{10}$$

Now, the loop runs for $m = 1$ to n, with k starting at 2 and increasing by 1 each time. So, k takes values from 3 to $n + 2$.

Thus,

$$f(n) = v^{10} + \sum_{j=3}^{n+2} \left[\left(\frac{1}{2} \right)^j - v^j \right] = \left(\sum_{j=3}^{n+2} \left(\frac{1}{2} \right)^j \right) + \left(v^{10} - \sum_{j=3}^{n+2} v^j \right)$$

As $n \to \infty$:

$$\sum_{j=3}^{\infty} \left(\frac{1}{2} \right)^j = \frac{1}{4}, \quad \sum_{j=3}^{\infty} v^j = \frac{v^3}{1 - v}$$

So,

$$f(\infty) = \frac{1}{4} + v^{10} - \frac{v^3}{1 - v}$$

Then,

$$a_2 = 2f(\infty) + \frac{1}{2} = 1 + 2v^{10} - \frac{2v^3}{1 - v}$$

Since $v = 72^{-10^{23}}$ is extremely small, the term $2v^{10}$ is negligible, and:

$$a_2 \approx 1 - \frac{2v^3}{1 - v} < 1$$

Thus, a_2 is slightly less than 1.

Step 2. Determine the Optimal Treatment Times

We want to minimize the total treatment time:

$$T = x_1 + x_2$$

subject to the dosage constraint:

$$a_1 x_1 + a_2 x_2 = 1, \quad \text{with } a_1 = 1,\ a_2 < 1$$

and $x_1, x_2 \geq 0$.

From the constraint:

$$x_2 = \frac{1 - x_1}{a_2}$$

Then:

$$T(x_1) = x_1 + \frac{1 - x_1}{a_2} = \frac{1}{a_2} + x_1 \left(1 - \frac{1}{a_2}\right)$$

Since $a_2 < 1$, we have $\frac{1}{a_2} > 1$, so the coefficient of x_1 is negative. Hence, $T(x_1)$ is decreasing in x_1.

The maximum allowed value of x_1 is 1 (since $x_2 \geq 0$), so the minimum total time occurs at:

$$x_1 = 1, \quad x_2 = 0$$

Thus, the unique minimizer is:

$$(x_1^*, x_2^*) = (1, 0)$$

Step 3. Check the Given Approximations

We are told:

- If $a_2 = 1$, then $(0.5, 0.5)$ is a minimizer.

- $(0.6, 0.4)$ is at most 0.45 away from a minimizer (in the infinity norm).

Since $a_2 < 1$, the true minimizer is $(1, 0)$.

- **Distance from** $(0.6, 0.4)$ **to** $(1, 0)$:

$$\max(|0.6 - 1|, |0.4 - 0|) = \max(0.4, 0.4) = 0.4 \leq 0.45$$

✅ This matches the given information.

- **Distance from** $(0.4, 0.6)$ **to** $(1, 0)$:

$$\max(|0.4 - 1|, |0.6 - 0|) = \max(0.6, 0.6) = 0.6 > 0.45$$

❌ This exceeds 0.45.

Step 4. Conclusion

The provided solution $(0.4, 0.6)$ is **not** within an infinity-norm distance of 0.45 from the true minimizer $(1, 0)$.

$$\boxed{No}$$