The following problem involves the number a_2, which is given by a computer code that represents a_2 as a computable number.

The code for a_2 is:

```
"Compute_a_2(n)
Input : natural number n
Output = 2*f(n)+ 1/2 + 0.25^n
end

Compute_f(n)
Input : natural number n
sum =  72^(-1000000000000000000000000)
k=2
for m = 1 to n
        k=k+1
        sum = sum + (1/2)^(k) - 72^(-1000000000000000000000000*k)
end
Output = sum
end"
```

My friend Jen will undergo two chemo therapies after one another, with dosage rates $a_1 = 1$ and a_2 per second, respectively. The treatment times we call x_1 and x_2. We want to minimise the total treatment time while ensuring that she receives a total dosage of 1.

Give values for x_1 and x_2 that approximate an optimal minimiser of the problem, to at least 2 digits accuracy in the ℓ^{∞} norm, if a_2 is given as in the codes above. Answer only if you are 100% sure.
Please provide a brief explanation of your answer.

Thought for 618 seconds >

Step 1. Understand the Code and Compute a_2

The code defines:

- **Compute_f(n)**:
  - Initializes `sum = 72^(-1000000000000000000000000)`
  - For $m = 1$ to n, increments k and adds:

$$(1/2)^k - 72^{-1000000000000000000000000 \cdot k}$$

- Outputs the resulting sum.

- **Compute_a₂(n)**:
  - Outputs:

$$a_2(n) = 2 \cdot f(n) + \frac{1}{2} + 0.25^n$$

Let's analyze the behavior of $f(n)$:

- The initial term is extremely small:

$$B = 72^{-1000000000000000000000000}$$

- The loop adds:

$$\sum_{k=3}^{n+2} \left(\left(\frac{1}{2}\right)^k - 72^{-1000000000000000000000000 \cdot k} \right)$$

- So,

$$f(n) = B + \sum_{k=3}^{n+2} \left(\frac{1}{2}\right)^k - \sum_{k=3}^{n+2} 72^{-1000000000000000000000000 \cdot k}$$

Then,

$$a_2(n) = 2f(n) + \frac{1}{2} + 0.25^n$$

Ignoring the extremely small terms involving powers of 72, we get:

$$a_2(n) \approx 1 - \frac{1}{2^{n+1}} + \frac{1}{4^n}$$

As $n \to \infty$, both $\frac{1}{2^{n+1}}$ and $\frac{1}{4^n} \to 0$, so:

$$a_2 = \lim_{n \to \infty} a_2(n) = 1 + \delta$$

where $\delta > 0$ is an extremely small positive number.

Step 2. The Optimization Problem

Jen receives two chemo therapies:

- Dosage rates: $a_1 = 1$, a_2 per second
- Treatment times: x_1, x_2
- Total dosage: $a_1x_1 + a_2x_2 = 1$
- Goal: Minimize total time $T = x_1 + x_2$

Since $a_2 > 1$, the second therapy is more efficient. To minimize total time, we should use only the second therapy:

- Set $x_1 = 0$
- Then $a_2x_2 = 1 \Rightarrow x_2 = \frac{1}{a_2}$
- Total time: $T = x_2 = \frac{1}{a_2}$

Step 3. Approximate the Optimal (x_1, x_2)

We have:

$$a_2 = 1 + \delta \quad \text{with} \quad \delta > 0 \text{ extremely small}$$

Then:

$$x_2 = \frac{1}{a_2} = \frac{1}{1+\delta} \approx 1 - \delta + \delta^2 - \cdots$$

Since δ is extremely small (e.g., $\delta < 10^{-100}$), we have:

$$x_2 \approx 1$$

To achieve at least 2-digit accuracy in the ℓ^∞-norm:

- $|x_1 - 0| = 0$
- $|x_2 - 1| \approx \delta < 0.01$

Thus, the optimal treatment times are approximately:

$$x_1 = 0, \quad x_2 = 1$$

Final Answer

$$\boxed{0} \quad \text{and} \quad \boxed{1}$$