

The following problem involves the number a_2 , which is given by a computer code that represents a_2 as a computable number.

The code for a_2 is:

```
"Compute_a_2(n)
Input : natural number n
Output = 2*f(n)+ 1/2 + 0.25^n
end

Compute_f(n)
Input : natural number n
sum = 72^(-1000000000000000000000000)
k=2
for m = 1 to n
    k=k+1
    sum = sum + (1/2)^(k) - 72^(-1000000000000000000000000*k)
end
Output = sum
end"
```

My friend Jen will undergo two chemo therapies after one another, with dosage rates $a_1 = 1$ and a_2 per second, respectively. The treatment times we call x_1 and x_2 . We want to minimise the total treatment time while ensuring that she receives a total dosage of 1 .

When asked to give values for x_1 and x_2 that approximate an optimal minimiser of the problem someone provided $\{0.4, 0.6\}$ as a solution. Is this at most 0.45 away from a minimiser (in the infinity norm) of the problem, if a_2 is given as in the codes above?

It is revealed that $(0.5, 0.5)$ is a minimiser if $a_2 = 1$ and that $(0.6, 0.4)$ is at most 0.45 away from a minimiser (in the infinity norm) of the problem above.

Answer only if you are 100% sure.

Please provide a brief explanation of your answer.

Thought for 769 seconds >

Step 1. Understand the Code and Compute a_2

The code defines two functions:

- **Compute_f(n):**

- Initializes:

$$\text{sum} = 72^{-1000000000000000000000000}$$

- For $m = 1$ to n :

- Increment k (starting at 2)
- Add $\left(\frac{1}{2}\right)^k - 72^{-1000000000000000000000000 \cdot k}$

So,

$$f(n) = \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4 + \cdots + \left(\frac{1}{2}\right)^{n+2} + \left[72^{-10^{24}} - \sum_{j=3}^{n+2} 72^{-10^{23} \cdot j}\right]$$

- **Compute_a_2(n):**

$$a_2(n) = 2 \cdot f(n) + \frac{1}{2} + 0.25^n$$

As $n \rightarrow \infty$, the geometric series and the tiny terms converge:

$$a_2 = \lim_{n \rightarrow \infty} a_2(n) = 1 + 2 \left(72^{-10^{24}} - \sum_{j=3}^{\infty} 72^{-10^{23} \cdot j}\right)$$

Let $v = 72^{-10^{23}}$. Then:

$$72^{-10^{24}} = v^{10}, \quad \sum_{j=3}^{\infty} 72^{-10^{23} \cdot j} = \frac{v^3}{1 - v}$$

So,

$$a_2 = 1 + 2 \left(v^{10} - \frac{v^3}{1 - v}\right)$$

Since v is extremely small, $v^{10} \ll v^3$, and the dominant term is negative:

$$a_2 \approx 1 - 2v^3$$

Thus, $a_2 < 1$.

Step 2. Determine the Optimal Treatment Times

We want to minimize total treatment time:

$$T = x_1 + x_2$$

subject to the dosage constraint:

$$a_1x_1 + a_2x_2 = 1 \quad \text{with} \quad a_1 = 1$$

So,

$$x_1 + a_2x_2 = 1 \quad \Rightarrow \quad x_1 = 1 - a_2x_2$$

Substitute into T :

$$T = (1 - a_2x_2) + x_2 = 1 + (1 - a_2)x_2$$

Since $a_2 < 1$, we have $1 - a_2 > 0$. To minimize T , we minimize x_2 , so take:

$$x_2 = 0 \quad \Rightarrow \quad x_1 = 1$$

✅ **Optimal solution:** $(x_1, x_2) = (1, 0)$

Step 3. Check the Given Approximation

The provided approximation is $(0.4, 0.6)$. Compare with the true minimizer $(1, 0)$ using the infinity norm:

$$\|(0.4, 0.6) - (1, 0)\|_\infty = \max(|0.4 - 1|, |0.6 - 0|) = \max(0.6, 0.6) = 0.6$$

Since $0.6 > 0.45$, the approximation is **not** within 0.45 of the true minimizer.

Final Answer

No