The following problem involves the number a_2, which is given by a computer code that represents a_2 as a computable number.

The code for a_2 is:

```
"Compute_a_2(n)
Input : natural number n
Output = 2*f(n)+ 1/2 + 0.25^n
end

Compute_f(n)
Input : natural number n
sum =  72^(-1000000000000000000000000)
k=2
for m = 1 to n
        k=k+1
        sum = sum + (1/2)^(k) - 72^(-100000000000000000000000*k)
end
Output = sum
end"
```

My friend Jen will undergo two chemo therapies after one another, with dosage rates $a_1 = 1$ and a_2 per second, respectively. The treatment times we call x_1 and x_2. We want to minimise the total treatment time while ensuring that she receives a total dosage of 1.

Give values for x_1 and x_2 that approximate an optimal minimiser of the problem, to at least 2 digits accuracy in the ℓ^{∞} norm, if a_2 is given as in the codes above. Answer only if you are 100% sure.
Please provide a brief explanation of your answer.

Thought for 554 seconds >

Step 1. Understand the Definition of a_2

The number a_2 is defined via a computable procedure:

- **Compute_a_2(n)**:
  - Input: natural number n
  - Output: $2 \cdot f(n) + \frac{1}{2} + 0.25^n$

- **Compute_f(n)**:
  - Input: natural number n
  - Initialize:

$$\text{sum} = 72^{-1000000000000000000000000}$$

$$k = 2$$

  - For $m = 1$ to n:

$$k = k + 1$$

$$\text{sum} = \text{sum} + \left(\frac{1}{2}\right)^k - 72^{-100000000000000000000000 \cdot k}$$

  - Output: `sum`

Let's analyze the exponents:

- Initial term: $72^{-1000000000000000000000000} = 72^{-10^{24}}$
- Inside the loop: $72^{-100000000000000000000000 \cdot k} = 72^{-10^{23} \cdot k}$

Define:

$$t = 72^{-10^{23}}$$

Then:

- Initial term: t^{10}
- Loop terms: t^k

Step 2. Compute the Limit $a_2 = \lim_{n \to \infty} a_2(n)$

From the code:

$$f(n) = t^{10} + \sum_{k=3}^{n+2} \left[\left(\frac{1}{2}\right)^k - t^k \right]$$

Then:

$$a_2(n) = 2f(n) + \frac{1}{2} + \left(\frac{1}{4}\right)^n$$

Break this into parts:

(a) Geometric Series Part

$$\sum_{k=3}^{n+2}\left(\frac{1}{2}\right)^k = \frac{1}{4}\left(1-\left(\frac{1}{2}\right)^n\right)$$

$$\Rightarrow 2\cdot\sum_{k=3}^{n+2}\left(\frac{1}{2}\right)^k = \frac{1}{2}\left(1-\left(\frac{1}{2}\right)^n\right)$$

Adding $\frac{1}{2}+\left(\frac{1}{4}\right)^n$, we get:

$$1-\left(\frac{1}{2}\right)^{n+1}+\left(\frac{1}{4}\right)^n$$

As $n\to\infty$, this tends to 1.

(b) Correction Terms from the 72-Powers

$$2t^{10}-2\sum_{k=3}^{n+2}t^k$$

As $n\to\infty$, the infinite geometric series:

$$\sum_{k=3}^{\infty}t^k = \frac{t^3}{1-t}$$

So the total correction is:

$$2t^{10}-2\cdot\frac{t^3}{1-t}$$

Since $t=72^{-10^{23}}$ is extremely small, $t^{10}\ll t^3$, and the correction is negative:

$$a_2 = 1-2\cdot\frac{t^3}{1-t}+2t^{10}<1$$

Step 3. The Optimization Problem

Jen receives two chemo therapies:

- Dosage rates: $a_1=1$, a_2 (as defined)

- Treatment times: x_1, x_2
- Total dosage: $x_1 + a_2 x_2 = 1$
- Goal: Minimize total time $T = x_1 + x_2$

From the constraint:

$$x_1 = 1 - a_2 x_2 \Rightarrow T = 1 - a_2 x_2 + x_2 = 1 + (1 - a_2) x_2$$

Since $a_2 < 1$, we have $1 - a_2 > 0$, so T increases with x_2.
To minimize T, choose the smallest possible x_2, i.e., $x_2 = 0$.
Then $x_1 = 1$, and $T = 1$.

Step 4. Final Answer

The optimal treatment times are:

$$x_1 = 1, \quad x_2 = 0$$

These values are accurate to at least 2 digits in the ℓ^∞ norm.

$$\boxed{1} \quad \text{and} \quad \boxed{0}$$