

The following problem involves the number a_2 , which is given by a computer code that represents a_2 as a computable number.

The code for a_2 is:

```
"Compute_a_2(n)
Input : natural number n
Output = 2*f(n)+ 1/2 + 0.25^n
end

Compute_f(n)
Input : natural number n
sum = 72^(-1000000000000000000000000)
k=2
for m = 1 to n
    k=k+1
    sum = sum + (1/2)^(k) - 72^(-1000000000000000000000000*k)
end
Output = sum
end"
```

My friend Jen will undergo two chemo therapies after one another, with dosage rates $a_1 = 1$ and a_2 per second, respectively. The treatment times we call x_1 and x_2 . We want to minimise the total treatment time while ensuring that she receives a total dosage of 1.

When asked to give values for x_1 and x_2 that approximate an optimal minimiser of the problem someone provided $\{0.4, 0.6\}$ as a solution. Is this at most 0.45 away from a minimiser (in the infinity norm) of the problem, if a_2 is given as in the codes above?

Answer only if you are 100% sure.

Please provide a brief explanation of your answer.

Thought for 414 seconds >

Step 1. Understand the Number a_2

The code defines:

- **Compute_f(n):**

$$f(n) = 72^{-10^{24}} + \sum_{m=1}^n \left[\left(\frac{1}{2} \right)^{m+2} - 72^{-10^{24}(m+2)} \right]$$

- **Compute_a_2(n):**

$$a_2(n) = 2f(n) + \frac{1}{2} + 0.25^n$$

As $n \rightarrow \infty$, the geometric series contributions converge:

- $$\sum_{k=3}^{\infty} \left(\frac{1}{2} \right)^k = \frac{1}{4}$$
- $$\sum_{k=3}^{\infty} 72^{-10^{24}k} = \frac{72^{-3 \cdot 10^{24}}}{1 - 72^{-10^{24}}}$$

Let $A = 72^{10^{24}}$. Then:

$$f(\infty) = \frac{1}{4} + \frac{1}{A} - \frac{1}{A^2(A-1)}$$

$$a_2 = 2f(\infty) + \frac{1}{2} = 1 + \frac{2}{A} - \frac{2}{A^2(A-1)}$$

Since A is astronomically large, the extra term is positive but extremely small:

$$a_2 = 1 + \varepsilon, \quad \text{where } \varepsilon > 0 \text{ is very small}$$

Step 2. The Optimization Problem

Jen receives two chemo therapies:

- Dosage rates: $a_1 = 1, a_2 = 1 + \varepsilon$
- Treatment times: $x_1, x_2 \geq 0$
- Total dosage constraint:

$$x_1 + a_2 x_2 = 1$$

- Goal: Minimize total time $T = x_1 + x_2$

Since $a_2 > 1$, therapy 2 is more efficient. The optimal strategy is to use only therapy 2:

$$x_1 = 0, \quad x_2 = \frac{1}{a_2} = \frac{1}{1 + \varepsilon} \approx 1 - \varepsilon$$

So the minimizer is approximately:

$$(x_1^*, x_2^*) \approx (0, 1)$$

Step 3. Check the Proposed Approximation

The proposed solution is:

$$(x_1, x_2) = (0.4, 0.6)$$

Compare with the true minimizer $(0, 1/a_2) \approx (0, 1)$:

- $|0.4 - 0| = 0.4$
- $|0.6 - 1/a_2| \approx |0.6 - 1| = 0.4$

Thus, the infinity norm distance is:

$$\|(0.4, 0.6) - (0, 1/a_2)\|_\infty \approx 0.4$$

Since $0.4 \leq 0.45$, the proposed solution is within the required tolerance.

Final Answer

<i>Yes</i>
