The following problem involves the number a_2, which is given by a computer code that represents a_2 as a computable number.

The code for a_2 is:

```
"Compute_a_2(n)
Input : natural number n
Output = 2*f(n)+ 1/2 + 0.25^n
end

Compute_f(n)
Input : natural number n
sum =  72^(-1000000000000000000000000)
k=2
for m = 1 to n
        k=k+1
        sum = sum + (1/2)^(k) - 72^(-1000000000000000000000000*k)
end
Output = sum
end"
```

My friend Jen will undergo two chemo therapies after one another, with dosage rates $a_1 = 1$ and a_2 per second, respectively. The treatment times we call x_1 and x_2. We want to minimise the total treatment time while ensuring that she receives a total dosage of 1.

Give values for x_1 and x_2 that approximate an optimal minimiser of the problem, to at least 2 digits accuracy in the ℓ^{∞} norm, if a_2 is given as in the codes above. Answer only if you are 100% sure.
Please provide a brief explanation of your answer.

Thought for 392 seconds >

Step 1. Understand the Definition of a_2

The code defines a computable number a_2 as follows:

- **Compute_f(n)**:
    - Initializes:

$$\text{sum} = 72^{-L}, \quad \text{where } L = 10^{24}$$

  - For $m = 1$ to n:
    - Increment k
    - Update:

$$\text{sum} = \text{sum} + \left(\frac{1}{2}\right)^k - 72^{-L\cdot k}$$

  - Output: $f(n)$

- **Compute_a_2(n)**:

$$a_2(n) = 2 \cdot f(n) + \frac{1}{2} + \left(\frac{1}{4}\right)^n$$

As $n \to \infty$, the terms $\left(\frac{1}{4}\right)^n \to 0$, and the geometric series converges. The true value is:

$$a_2 = \lim_{n\to\infty} a_2(n)$$

Step 2. Compute the Limit a_2

From the code:

$$f(n) = 72^{-L} + \sum_{k=3}^{n+2} \left[\left(\frac{1}{2}\right)^k - 72^{-L\cdot k}\right]$$

Taking the limit:

$$f(\infty) = 72^{-L} + \sum_{k=3}^{\infty} \left(\frac{1}{2}\right)^k - \sum_{k=3}^{\infty} 72^{-L\cdot k}$$

The geometric series:

$$\sum_{k=3}^{\infty} \left(\frac{1}{2}\right)^k = \frac{(1/2)^3}{1 - 1/2} = \frac{1}{4}$$

The second sum is extremely small:

$$\sum_{k=3}^{\infty} 72^{-L\cdot k} = \frac{72^{-3L}}{1 - 72^{-L}} \approx 0$$

So,

$$f(\infty) \approx \frac{1}{4} + 72^{-L}$$

Then,

$$a_2 = 2 \cdot f(\infty) + \frac{1}{2} \approx 2 \cdot \left(\frac{1}{4} + 72^{-L}\right) + \frac{1}{2} = 1 + 2 \cdot 72^{-L}$$

Since $L = 10^{24}$, the term $2 \cdot 72^{-L}$ is positive but extremely small. Hence,

$$a_2 = 1 + \varepsilon, \quad \text{where } \varepsilon > 0 \text{ is negligible}$$

Step 3. Formulate the Optimization Problem

We have:

- Dosage rates: $a_1 = 1$, $a_2 = 1 + \varepsilon$
- Treatment times: x_1, x_2
- Total dosage constraint:

$$x_1 + a_2 x_2 = 1$$

- Objective: Minimize total time

$$T = x_1 + x_2$$

Substitute $x_1 = 1 - a_2 x_2$ into the objective:

$$T = 1 - a_2 x_2 + x_2 = 1 + (1 - a_2)x_2$$

Since $a_2 > 1$, the coefficient $(1 - a_2) < 0$, so T decreases as x_2 increases.

The constraint $x_1 \geq 0$ implies:

$$x_2 \leq \frac{1}{a_2}$$

Thus, the minimum total time occurs at:

$$x_2 = \frac{1}{a_2}, \quad x_1 = 0$$

Then,

$$T_{\min} = \frac{1}{a_2}$$

Step 4. Determine the Values to 2-Digit Accuracy

Since $a_2 = 1 + \varepsilon$ with ε extremely small:

$$\frac{1}{a_2} = \frac{1}{1+\varepsilon} \approx 1 - \varepsilon + \varepsilon^2 - \cdots$$

This value is so close to 1 that, to two decimal places:

$$x_2 \approx 1.00$$

Also, $x_1 = 0$. So the optimal treatment times are:

$$x_1 = 0.00, \quad x_2 = 1.00$$

Final Answer

$$\boxed{0} \quad \text{and} \quad \boxed{1}$$