

The following problem involves the number a_3 , which is given by a computer code that represents a_3 as a computable number.

The code for a_3 is:

```
"Compute_a_3(n)
Input : natural number n
Output = 1 + (1/2)^n - 72^(-n)
end
"
```

Is it the case that the number a_3 is positive?

Thought for 110 seconds >

The computer code `Compute_a_3(n)` outputs the value $1 + \left(\frac{1}{2}\right)^n - 72^{-n}$ for any natural number n . This code represents a_3 as a computable number, meaning that it provides an approximation to a_3 with error less than 2^{-n} for each n .

The limit of the output as n approaches infinity is 1, since both $\left(\frac{1}{2}\right)^n$ and 72^{-n} approach 0. Therefore, $a_3 = 1$.

Since $1 > 0$, a_3 is positive.

For any finite n , the output is greater than 1 because $\left(\frac{1}{2}\right)^n > 72^{-n}$ (as $72 > 2$, so $\frac{1}{72} < \frac{1}{2}$, and thus $\left(\frac{1}{2}\right)^n - 72^{-n} > 0$). Hence, all approximations are also positive.

Thus, a_3 is positive.

yes