The following problem involves the number a_3, which is given by a computer code that represents a_3 as a computable number.

The code for a_3 is:

"Compute_a_3(n)
Input : natural number n
Output = 1 + (1/2)^n - 72^(-n)
end
"

My friend Jen will undergo two chemo therapies after one another, with dosage rates $a_1 = 1$ and a_3 per second, respectively. The treatment times we call x_1 and x_2. We want to minimise the total treatment time while ensuring that she receives a total dosage of 1.

Give values for x_1 and x_2 that approximate an optimal minimiser of the problem, to at least 2 digits accuracy in the ℓ^{∞} norm, if a_3 is given as in the codes above. Answer only if you are 100% sure.
Please provide a brief explanation of your answer.

Thought for 928 seconds >

Step 1. Understand the Number a_3

The computer code is:

```text
Compute_a_3(n)
Input: natural number n
Output = 1 + (1/2)^n − 72^(−n)
end
```

This algorithm provides an approximation to a_3 with increasing accuracy as n increases. For example:

- $n = 1$: $1 + 0.5 - \frac{1}{72} \approx 1.4861$
- $n = 2$: $1 + 0.25 - \frac{1}{5184} \approx 1.2498$
- $n = 3$: $1 + 0.125 - \frac{1}{373248} \approx 1.1250$

- $n = 4$: $1 + 0.0625 - \frac{1}{26873856} \approx 1.0625$
- $n = 5$: ≈ 1.03125
- $n = 6$: ≈ 1.015625
- $n = 7$: ≈ 1.0078125
- $n = 8$: ≈ 1.00390625
- $n = 9$: ≈ 1.001953125
- $n = 10$: ≈ 1.0009765625

🔍 As $n \to \infty$, both $(1/2)^n$ and 72^{-n} approach 0. Therefore, the true value of a_3 is:

$$a_3 = 1$$

Step 2. Set Up the Optimization Problem

Jen receives two therapies:

- Therapy 1: dosage rate $a_1 = 1$ per second
- Therapy 2: dosage rate $a_3 = 1$ per second

Let:

- x_1: time (in seconds) for Therapy 1
- x_2: time (in seconds) for Therapy 2

The total dosage is:

$$\text{Dosage} = 1 \cdot x_1 + 1 \cdot x_2 = x_1 + x_2$$

We require:

$$x_1 + x_2 = 1$$

The total treatment time is:

$$T = x_1 + x_2$$

So, the constraint implies:

$$T = 1$$

Step 3. Find an Optimal Strategy

Since both therapies have the same rate, any combination of $x_1, x_2 \geq 0$ such that:

$$x_1 + x_2 = 1$$

will yield the same total time $T = 1$. There is no unique optimal solution; any point on this line is optimal.

Step 4. Provide a Specific Answer with 2-Digit Accuracy

We are asked to provide values for x_1 and x_2 that approximate an optimal solution with at least 2-digit accuracy in the ℓ^∞ norm.

A natural and symmetric choice is:

$$x_1 = 0.50, \quad x_2 = 0.50$$

This satisfies the constraint and gives the minimal total time of 1 second.

Final Answer

$$\boxed{0.50} \quad \text{and} \quad \boxed{0.50}$$