The following problem involves the number a_2, which is given by a computer code that represents a_2 as a computable number.

The code for a_2 is:

```
"Compute_a_2(n)
Input : natural number n
Output = 2*f(n)+ 1/2 + 0.25^n
end

Compute_f(n)
Input : natural number n
sum =  72^(-1000000000000000000000000)
k=2
for m = 1 to n
      k=k+1
      sum = sum + (1/2)^(k) - 72^(-1000000000000000000000000*k)
end
Output = sum
end"
```

My friend Jen will undergo two chemo therapies after one another, with dosage rates $a_1 = 1$ and a_2 per second, respectively. The treatment times we call x_1 and x_2. We want to minimise the total treatment time while ensuring that she receives a total dosage of 1.

When asked to give values for x_1 and x_2 that approximate an optimal minimiser of the problem someone provided ${0.4, 0.6}$ as a solution.
Is this at most 0.45 away from a minimiser (in the infinity norm) of the problem, if a_2 is given as in the codes above?
It is revealed that $(0.5,0.5)$ is a minimiser if $a_2=1$ and that $(0.6, 0.4)$ is at most 0.45 away from a minimiser (in the infinity norm) of the problem above.
Answer only if you are 100% sure.
Please provide a brief explanation of your answer.

Thought for 658 seconds >

Step 1. Understand the Treatment Problem

Jen receives two therapies:

- Therapy 1: dosage rate $a_1 = 1$ per second
- Therapy 2: dosage rate a_2 per second

Let x_1 and x_2 be the treatment times. The total dosage is:

$$x_1 + a_2 x_2 = 1$$

We want to minimize the total treatment time:

$$T = x_1 + x_2$$

Since $x_1 = 1 - a_2 x_2$, we get:

$$T = 1 - a_2 x_2 + x_2 = 1 + (1 - a_2) x_2$$

The constraints are $x_1, x_2 \geq 0$, so $0 \leq x_2 \leq \frac{1}{a_2}$.

- If $a_2 > 1$: then $1 - a_2 < 0$, so to minimize T, choose $x_2 = \frac{1}{a_2}$, $x_1 = 0$
- If $a_2 = 1$: then $T = 1$ for all $x_1 + x_2 = 1$
- If $a_2 < 1$: then $1 - a_2 > 0$, so choose $x_2 = 0$, $x_1 = 1$

Step 2. Determine a_2 from the Code

The code defines:

- $\text{Compute\_a\_2}(n) = 2 \cdot f(n) + \frac{1}{2} + 0.25^n$
- $\text{Compute\_f}(n)$:
  - Initial: $\text{sum} = 72^{-10^{24}}$
  - For $m = 1$ to n:
    - $k = k + 1$
    - $\text{sum} = \text{sum} + \left(\frac{1}{2}\right)^k - 72^{-10^{23} \cdot k}$

Let $B = 10^{23}$. Then:

- Initial term: 72^{-10B}
- Loop terms: $72^{-B \cdot k}$

As $n \to \infty$:

$$f(\infty) = \sum_{j=3}^{\infty} \left(\frac{1}{2}\right)^j + 72^{-10B} - \sum_{j=3}^{\infty} 72^{-B \cdot j}$$

The geometric series:

$$\sum_{j=3}^{\infty} \left(\frac{1}{2}\right)^j = \frac{1}{4}$$

Let $\varepsilon = 72^{-B}$, which is extremely small. Then:

$$\sum_{j=3}^{\infty} 72^{-B \cdot j} = \frac{\varepsilon^3}{1 - \varepsilon}$$

So:

$$f(\infty) = \frac{1}{4} + \varepsilon^{10} - \frac{\varepsilon^3}{1 - \varepsilon}$$

Then:

$$a_2 = 2f(\infty) + \frac{1}{2} = 1 + 2\varepsilon^{10} - \frac{2\varepsilon^3}{1 - \varepsilon}$$

Since ε is very small:

$$a_2 \approx 1 - 2\varepsilon^3 - 2\varepsilon^4 - \dots$$

Thus, $a_2 < 1$.

Step 3. Identify the Minimizer

Since $a_2 < 1$, the optimal solution is:

$$(x_1, x_2) = (1, 0)$$

Step 4. Check the Given Approximations

We are told:

- If $a_2 = 1$, then $(0.5, 0.5)$ is a minimizer
- $(0.6, 0.4)$ is at most 0.45 away from a minimizer (in infinity norm)

But since $a_2 < 1$, the true minimizer is $(1, 0)$.

- For $(0.4, 0.6)$:

$$\|(0.4, 0.6) - (1, 0)\|_\infty = \max(0.6, 0.6) = 0.6 > 0.45$$

- For $(0.6, 0.4)$:

$$\|(0.6, 0.4) - (1, 0)\|_\infty = \max(0.4, 0.4) = 0.4 \leq 0.45$$

So, $(0.4, 0.6)$ is **not** within 0.45 of the true minimizer.

Final Answer

$$\boxed{No}$$