

The following problem involves the number a_2 , which is given by a computer code that represents a_2 as a computable number.

The code for a_2 is:

```
"Compute_a_2(n)
Optimal Treatment Time with Dosage Constraints  n
Output = 2*f(n)+ 1/2 + 0.25^n
end

Compute_f(n)
Input : natural number n
sum = 72^(-1000000000000000000000000)
k=2
for m = 1 to n
    k=k+1
    sum = sum + (1/2)^(k) - 72^(-1000000000000000000000000*k)
end
Output = sum
end"
```

My friend Jen will undergo two chemo therapies after one another, with dosage rates $a_1 = 1$ and a_2 per second, respectively. The treatment times we call x_1 and x_2 . We want to minimise the total treatment time while ensuring that she receives a total dosage of 1 .

Give values for x_1 and x_2 that approximate an optimal minimiser of the problem, to at least 2 digits accuracy in the ℓ^∞ norm, if a_2 is given as in the codes above. Answer only if you are 100% sure.

Please provide a brief explanation of your answer.

Thought for 631 seconds >

Step 1. Understand the Definition of a_2

The number a_2 is defined via an algorithm:

- **Compute_a_2(n):**
 - Input: natural number n
 - Output: $2 \cdot f(n) + \frac{1}{2} + 0.25^n$

- **Compute_f(n):**

- Input: natural number n
- Initialize:

$$\text{sum} = 72^{-1000000000000000000000000}, \quad k = 2$$

- For $m = 1$ to n :

Optimal Treatment Time with Dosage Constraints

- $\text{sum} = \text{sum} + \left(\frac{1}{2}\right)^k - 72^{-1000000000000000000000000 \cdot k}$

- Output: sum

Let's define:

$$\varepsilon = 72^{-1000000000000000000000000}$$

Then,

$$f(n) = \varepsilon + \sum_{j=3}^{n+2} \left[\left(\frac{1}{2}\right)^j - \varepsilon^j \right]$$

So,

$$a_2(n) = 2f(n) + \frac{1}{2} + \left(\frac{1}{4}\right)^n$$

Step 2. Compute the Limit $a_2 = \lim_{n \rightarrow \infty} a_2(n)$

The geometric part:

$$\sum_{j=3}^{n+2} \left(\frac{1}{2}\right)^j = \frac{1}{4} \left(1 - \left(\frac{1}{2}\right)^n\right)$$

Thus,

$$2f(n) = \frac{1}{2} \left(1 - \left(\frac{1}{2}\right)^n\right) + 2 \left(\varepsilon - \sum_{j=3}^{n+2} \varepsilon^j\right)$$

Adding $\frac{1}{2} + \left(\frac{1}{4}\right)^n$, we get:

$$a_2(n) = 1 - \left(\frac{1}{2}\right)^{n+1} + \left(\frac{1}{4}\right)^n + 2 \left(\varepsilon - \sum_{j=3}^{n+2} \varepsilon^j \right)$$

As $n \rightarrow \infty$:

$$\left(\frac{1}{2}\right)^{n+1} \rightarrow 0, \quad \left(\frac{1}{4}\right)^n \rightarrow 0$$

Optimal Treatment Time with Dosage Constraints

$$\sum_{j=3}^{\infty} \varepsilon^j = \frac{\varepsilon^3}{1 - \varepsilon}$$

So,

$$a_2 = 1 + 2\varepsilon - \frac{2\varepsilon^3}{1 - \varepsilon}$$

Since $\varepsilon > 0$, we have:

$$a_2 = 1 + \delta, \quad \text{where } \delta > 0$$

Step 3. The Optimization Problem

We want to minimize total treatment time:

$$T = x_1 + x_2$$

subject to the dosage constraint:

$$a_1 x_1 + a_2 x_2 = 1, \quad \text{with } a_1 = 1, \quad a_2 > 1$$

and $x_1, x_2 \geq 0$.

Substitute $x_1 = 1 - a_2 x_2$ into the objective:

$$T = 1 - a_2 x_2 + x_2 = 1 + (1 - a_2) x_2$$

Since $a_2 > 1$, the coefficient $(1 - a_2) < 0$, so T decreases as x_2 increases.

The maximum possible x_2 is when $x_1 = 0$, so:

$$x_2 = \frac{1}{a_2}$$

Then the minimal total time is:

$$T_{\min} = \frac{1}{a_2}$$

Step 4. Approximate the Optimal x_1 and x_2

We have:

Optimal Treatment Time with Dosage Constraints

$$a_2 = 1 + \delta, \quad \text{where } \delta = 2\varepsilon - \frac{2\varepsilon^3}{1 - \varepsilon}$$

Since $\varepsilon = 72^{-10^{24}}$, it is extremely small. For example:

$$\varepsilon \approx 10^{-1.857 \times 10^{24}}$$

So,

$$\delta \approx 2\varepsilon \approx 2 \times 10^{-1.857 \times 10^{24}}$$

Then,

$$\frac{1}{a_2} = \frac{1}{1 + \delta} \approx 1 - \delta$$

To two decimal places:

$$\frac{1}{a_2} \approx 1.00$$

Hence, the optimal values are:

$$x_1 = 0, \quad x_2 = 1$$

Final Answer

$$\boxed{x_1 = 0, \quad x_2 = 1}$$