

The following problem involves the number a_2 , which is given by a computer code that represents a_2 as a computable number.

The code for a_2 is:

```
"Compute_a_2(n)
Input : natural number n
Output = 2*f(n)+ 1/2 + 0.25^n
end

Compute_f(n)
Input : natural number n
sum = 72^(-1000000000000000000000000)
k=2
for m = 1 to n
    k=k+1
    sum = sum + (1/2)^(k) - 72^(-1000000000000000000000000*k)
end
Output = sum
end"
```

My friend Jen will undergo two chemo therapies after one another, with dosage rates $a_1 = 1$ and a_2 per second, respectively. The treatment times we call x_1 and x_2 . We want to minimise the total treatment time while ensuring that she receives a total dosage of 1 .

Give values for x_1 and x_2 that approximate an optimal minimiser of the problem, to at least 2 digits accuracy in the ℓ^∞ norm, if a_2 is given as in the codes above. Answer only if you are 100% sure.

Please provide a brief explanation of your answer.

Thought for 644 seconds >

Step 1. Understand the Definition of a_2

The number a_2 is defined via two algorithms:

- **Compute_f(n):**
 - Initialize:

$$\text{sum} = 72^{-1000000000000000000000000}$$

- For $m = 1$ to n :
 - $k = k + 1$
 - Add $\left(\frac{1}{2}\right)^k - 72^{-1000000000000000000000000 \cdot k}$
- Output: sum
- **Compute_a_2(n):**
 - Output:

$$2 \cdot \text{Compute_f}(n) + \frac{1}{2} + 0.25^n$$

The true value of a_2 is the limit:

$$a_2 = \lim_{n \rightarrow \infty} \text{Compute_a_2}(n)$$

Step 2. Simplify the Expression for a_2

Let's analyze the contributions:

- The term $0.25^n = \left(\frac{1}{4}\right)^n \rightarrow 0$ as $n \rightarrow \infty$.
- The terms involving powers of 72 are extremely small and negligible. For example:

$$72^{-1000000000000000000000000} \approx 0$$

- The main contribution comes from:

$$\text{Compute_f}(n) \approx \sum_{j=3}^{n+2} \left(\frac{1}{2}\right)^j = \frac{1}{4} - \frac{1}{4} \cdot \left(\frac{1}{2}\right)^n$$

- Then:

$$\begin{aligned}\text{Compute_a_2}(n) &\approx 2 \cdot \left(\frac{1}{4} - \frac{1}{4} \cdot \left(\frac{1}{2} \right)^n \right) + \frac{1}{2} + \left(\frac{1}{4} \right)^n \\ &= 1 - \left(\frac{1}{2} \right)^{n+1} + \left(\frac{1}{4} \right)^n\end{aligned}$$

As $n \rightarrow \infty$, both $\left(\frac{1}{2}\right)^{n+1}$ and $\left(\frac{1}{4}\right)^n$ vanish, so:

$$a_2 \approx 1$$

However, the tiny negative corrections from the 72 terms imply:

$$a_2 < 1$$

For practical purposes, especially to two-digit accuracy:

$$a_2 \approx 1.00$$

Step 3. Set Up the Optimization Problem

Jen receives two therapies:

- Therapy 1: rate $a_1 = 1$ per second
- Therapy 2: rate $a_2 \approx 1$ per second

Let:

- x_1 : time (in seconds) of therapy 1
- x_2 : time (in seconds) of therapy 2

Total dosage:

$$a_1x_1 + a_2x_2 = 1 \cdot x_1 + a_2x_2 = 1$$

Total treatment time:

$$T = x_1 + x_2$$

We want to minimize T subject to:

$$x_1 + a_2x_2 = 1, \quad x_1, x_2 \geq 0$$

Step 4. Determine the Optimal Treatment Strategy

Since $a_2 < 1$, therapy 1 is more efficient. To minimize total time, we should use only therapy 1:

- Set $x_2 = 0$
- Then $x_1 = 1$
- Total time: $T = 1$

Using any therapy 2 would increase total time because:

$$T = 1 + (1 - a_2)x_2 > 1 \quad \text{since } 1 - a_2 > 0$$

Step 5. Final Answer

To two-digit accuracy in the ℓ^∞ norm:

$$x_1 = 1.00, \quad x_2 = 0.00$$

$$\boxed{1} \quad \text{and} \quad \boxed{0}$$