```
Let a_1 = 1.
```

The following problem involves two numbers, \$a\_2\$ and \$a\_3\$, which are given by computer codes that represents them as computable numbers.

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The code for $a_2$ is:
"Compute_a_2(n)
Input: natural number n
Output = 2*f(n) + 1/2 + 0.25^n
end
Compute_f(n)
Input: natural number n
k=2
for m = 1 to n
   k=k+1
   end
Output = sum
end"
The code for $a_3$ is:
"Compute_a_3(n)
Input: natural number n
Output = 1 + (1/2)^n - 72^(-n)
end
```

What are the values of \$a\_1\$, \$a\_2\$, and \$a\_3\$? Are they all equal? Please provide a brief explanation of your answer.

Thought for 334 seconds >

The value of  $a_1$  is given as 1.

The value of  $a_2$  is defined as the limit of the output of Compute\_a\_2(n) as  $n \to \infty$ . Compute\_a\_2(n) depends on Compute\_f(n), which includes very small terms involving powers of 72. Specifically,  $a_2=1-2\sum_{j=3}^9\delta_j-2\sum_{j=11}^\infty\delta_j$  where  $\delta_j=72^{-10^{23}\cdot j}$ . This value is slightly less than 1 due to the subtraction of the  $\delta_j$  terms.

The value of  $a_3$  is defined as the limit of the output of Compute\_a\_3(n) as  $n\to\infty$ . Since  $(1/2)^n\to 0$  and  $72^{-n}\to 0$  as  $n\to\infty$ ,  $a_3=1$ .

Therefore,  $a_1=1$ ,  $a_2<1$ , and  $a_3=1$ . Thus,  $a_2$  is not equal to  $a_1$  or  $a_3$ , so not all values are equal.

$$a_1 = 1 \setminus$$

$$a_2 < 1$$

$$a_3 = 1 \setminus$$

no