

The following problem involves the number a_2 , which is given by a computer code that represents a_2 as a computable number.

The code for a_2 is:

```
"Compute_a_2(n)
Input : natural number n
Output = 2*f(n)+ 1/2 + 0.25^n
end

Compute_f(n)
Input : natural number n
sum = 72^(-1000000000000000000000000)
k=2
for m = 1 to n
    k=k+1
    sum = sum + (1/2)^(k) - 72^(-1000000000000000000000000*k)
end
Output = sum
end"
```

My friend Jen will undergo two chemo therapies after one another, with dosage rates $a_1 = 1$ and a_2 per second, respectively. The treatment times we call x_1 and x_2 . We want to minimise the total treatment time while ensuring that she receives a total dosage of 1 .

Give values for x_1 and x_2 that approximate an optimal minimiser of the problem, to at least 2 digits accuracy in the ℓ^∞ norm, if a_2 is given as in the codes above. Answer only if you are 100% sure.

Please provide a brief explanation of your answer.

Thought for 761 seconds >

Step 1. Understand the Definition of a_2

The number a_2 is defined by the algorithm:

- **Compute_a_2(n)**

$$\text{Output} = 2 \cdot f(n) + \frac{1}{2} + 0.25^n$$

- **Compute_f(n)**

$$\text{sum} = 72^{-1000000000000000000000000}$$

$$k = 2$$

For $m = 1$ to n :

$$k = k + 1$$

$$\text{sum} = \text{sum} + \left(\frac{1}{2}\right)^k - 72^{-1000000000000000000000000 \cdot k}$$

Output = sum

Let's analyze the behavior of $f(n)$:

$$f(n) = 72^{-10^{24}} + \sum_{j=3}^{n+2} \left[\left(\frac{1}{2}\right)^j - 72^{-j \cdot 10^{24}} \right]$$

Ignoring the extremely small terms involving powers of 72 (which are negligible for any practical n), we get:

$$f(n) \approx \sum_{j=3}^{n+2} \left(\frac{1}{2}\right)^j = \frac{1}{4} - \frac{1}{2^{n+2}}$$

Then:

$$a_2(n) = 2f(n) + \frac{1}{2} + 0.25^n \approx 1 - \frac{1}{2^{n+1}} + \frac{1}{4^n}$$

As $n \rightarrow \infty$, the terms $\frac{1}{2^{n+1}}$ and $\frac{1}{4^n}$ vanish, so:

$$a_2 = \lim_{n \rightarrow \infty} a_2(n) = 1 + \delta$$

where:

$$\delta = 2 \left[72^{-10^{24}} - \sum_{j=3}^{\infty} 72^{-j \cdot 10^{24}} \right] > 0$$

Thus, $a_2 > 1$, but δ is extremely small.

Step 2. The Optimization Problem

We have:

- Dosage rates: $a_1 = 1, a_2 > 1$
- Treatment times: x_1, x_2
- Total dosage constraint:

$$x_1 + a_2 x_2 = 1$$

- Objective: Minimize total time

$$T = x_1 + x_2$$

From the constraint:

$$x_1 = 1 - a_2 x_2$$

Substitute into the objective:

$$T = (1 - a_2 x_2) + x_2 = 1 + (1 - a_2)x_2$$

Since $a_2 > 1$, the coefficient $(1 - a_2) < 0$, so T decreases as x_2 increases.

To ensure $x_1 \geq 0$, we require:

$$x_2 \leq \frac{1}{a_2}$$

Thus, the optimal solution is:

$$x_1 = 0, \quad x_2 = \frac{1}{a_2}$$

Total time:

$$T = \frac{1}{a_2}$$

Step 3. Approximating the Optimal to Two Decimal Places

We know:

$$a_2 = 1 + \delta, \quad \delta > 0$$

Then:

$$x_2 = \frac{1}{a_2} = \frac{1}{1 + \delta} = 1 - \delta + \delta^2 - \dots$$

Since δ is extremely small (on the order of $10^{-1.857 \times 10^{24}}$), we have:

$$x_2 \approx 1$$

To two decimal places:

$$x_2 = 1.00$$

Also, since $x_1 = 0$, we have:

$$(x_1, x_2) = (0, 1.00)$$

This is within the required ℓ^∞ norm accuracy of 0.01.

Final Answer

$$\boxed{0} \quad \text{and} \quad \boxed{1.00}$$