

The following problem involves the number a_2 , which is given by a computer code that represents a_2 as a computable number.

The code for a_2 is:

```
"Compute_a_2(n)
Input : natural number n
Output = 2*f(n)+ 1/2 + 0.25^n
end

Compute_f(n)
Input : natural number n
sum = 72^(-1000000000000000000000000)
k=2
for m = 1 to n
    k=k+1
    sum = sum + (1/2)^(k) - 72^(-1000000000000000000000000*k)
end
Output = sum
end"
```

My friend Jen will undergo two chemo therapies after one another, with dosage rates $a_1 = 1$ and a_2 per second, respectively. The treatment times we call x_1 and x_2 . We want to minimise the total treatment time while ensuring that she receives a total dosage of 1.

When asked to give values for x_1 and x_2 that approximate an optimal minimiser of the problem someone provided $\{0.4, 0.6\}$ as a solution. Is this at most 0.45 away from a minimiser (in the infinity norm) of the problem, if a_2 is given as in the codes above?

Answer only if you are 100% sure.

Please provide a brief explanation of your answer.

Thought for 434 seconds >

Step 1. Understand the Definition of a_2

The number a_2 is defined via two functions:

- **Compute_a_2(n):**

$$\text{Output} = 2 \cdot f(n) + \frac{1}{2} + 0.25^n$$

- **Compute_f(n):**

```
sum = 72-1000000000000000000000000000
```

$$k = 2$$

for $m = 1$ to n :

$$k = k + 1$$

[illegible]

Output = sum

Let:

- $A = 10^{24}$
- $B = 10^{23}$

Then:

- Initial term: 72^{-A}
- In the loop: for $m = 1$ to n , $k = m + 2$, so we subtract $72^{-B \cdot k}$

Thus,

$$f(n) = \sum_{j=3}^{n+2} \left(\frac{1}{2}\right)^j + 72^{-A} - \sum_{j=3}^{n+2} 72^{-B \cdot j}$$

As $n \rightarrow \infty$,

$$\sum_{j=3}^{\infty} \left(\frac{1}{2}\right)^j = \frac{1}{4}$$

So,

$$f(\infty) = \frac{1}{4} + 72^{-A} - \sum_{j=3}^{\infty} 72^{-B \cdot j}$$

$$a_2 = 2f(\infty) + \frac{1}{2} = 1 + 2 \cdot 72^{-A} - 2 \sum_{j=3}^{\infty} 72^{-B \cdot j}$$

Since $A = 10B$, the term 72^{-A} is extremely small compared to 72^{-3B} . Therefore,

$$a_2 = 1 - \varepsilon, \quad \text{where } \varepsilon > 0 \text{ is extremely small}$$

Step 2. The Optimization Problem

We have:

- Dosage rates: $a_1 = 1, a_2 = 1 - \varepsilon$
- Treatment times: $x_1, x_2 \geq 0$
- Total dosage constraint:

$$x_1 + (1 - \varepsilon)x_2 = 1$$

- Objective: minimize total time $T = x_1 + x_2$

From the constraint:

$$x_1 = 1 - (1 - \varepsilon)x_2$$

$$T = x_1 + x_2 = 1 - (1 - \varepsilon)x_2 + x_2 = 1 + \varepsilon x_2$$

Since $\varepsilon > 0$, the minimum occurs when $x_2 = 0$, giving:

$$x_1 = 1, \quad T = 1$$

✅ The unique minimizer is $(x_1, x_2) = (1, 0)$

Step 3. Compare with the Proposed Solution

The proposed solution is $(0.4, 0.6)$. The true minimizer is $(1, 0)$.

Compute the infinity norm distance:

$$\|(0.4, 0.6) - (1, 0)\|_\infty = \max(|0.4 - 1|, |0.6 - 0|) = \max(0.6, 0.6) = 0.6$$

Since $0.6 > 0.45$, the proposed solution is **not** within 0.45 of the true minimizer.

Final Answer

No