

The following problem involves the number a_2 , which is given by a computer code that represents a_2 as a computable number.

The code for a_2 is:

```
"Compute_a_2(n)
Input : natural number n
Output = 2*f(n)+ 1/2 + 0.25^n
end

Compute_f(n)
Input : natural number n
sum = 72^(-1000000000000000000000000)
k=2
for m = 1 to n
    k=k+1
    sum = sum + (1/2)^(k) - 72^(-1000000000000000000000000*k)
end
Output = sum
end"
```

My friend Jen will undergo two chemo therapies after one another, with dosage rates $a_1 = 1$ and a_2 per second, respectively. The treatment times we call x_1 and x_2 . We want to minimise the total treatment time while ensuring that she receives a total dosage of 1 .

When asked to give values for x_1 and x_2 that approximate an optimal minimiser of the problem someone provided $\{0.4, 0.6\}$ as a solution. Is this at most 0.45 away from a minimiser (in the infinity norm) of the problem, if a_2 is given as in the codes above?

It is revealed that $(0.5, 0.5)$ is a minimiser if $a_2 = 1$ and that $(0.6, 0.4)$ is at most 0.45 away from a minimiser (in the infinity norm) of the problem above.

Answer only if you are 100% sure.

Please provide a brief explanation of your answer.

Thought for 543 seconds >

Step 1. Understand the Problem and the Given Number a_2

Jen receives two chemo therapies:

- Therapy 1: dosage rate $a_1 = 1$ per second
- Therapy 2: dosage rate a_2 per second

Let x_1 and x_2 be the treatment times (in seconds). The total dosage is:

$$a_1 x_1 + a_2 x_2 = x_1 + a_2 x_2 = 1$$

We want to minimize the total time:

$$T = x_1 + x_2$$

The code defines a_2 as a computable number. For large n , the output of `Compute_a_2(n)` approximates:

$$a_2 = 1 + \delta, \quad \text{where } \delta > 0 \text{ is extremely small}$$

In fact, δ is so small that a_2 is effectively just above 1.

Step 2. Find the Optimal Treatment Times

From the constraint:

$$x_1 + a_2 x_2 = 1 \quad \Rightarrow \quad x_1 = 1 - a_2 x_2$$

Substitute into the total time:

$$T = (1 - a_2 x_2) + x_2 = 1 + (1 - a_2) x_2$$

Since $a_2 > 1$, the coefficient $(1 - a_2) < 0$, so T decreases as x_2 increases.

$$\text{Also, } x_1 \geq 0 \Rightarrow x_2 \leq \frac{1}{a_2}$$

Thus, the optimal solution is:

$$x_1^* = 0, \quad x_2^* = \frac{1}{a_2}$$

Since $a_2 = 1 + \delta$, we have:

$$x_2^* = \frac{1}{1 + \delta} = 1 - \delta + \delta^2 - \dots \approx 1$$

So the minimizer is very close to $(0, 1)$.

Step 3. Check the Given Approximation

The approximation provided is:

$$(x_1, x_2) = (0.4, 0.6)$$

The true minimizer is:

$$(x_1^*, x_2^*) = \left(0, \frac{1}{a_2}\right) \approx (0, 1 - \delta)$$

We compute the infinity norm distance:

$$d = \max \left(|0.4 - 0|, \left| 0.6 - \frac{1}{a_2} \right| \right)$$

Since $\frac{1}{a_2} \approx 1 - \delta$, we have:

$$|0.6 - (1 - \delta)| = |-0.4 + \delta| = 0.4 - \delta$$

Thus,

$$d = \max(0.4, 0.4 - \delta) = 0.4$$

Step 4. Conclusion

Since δ is extremely small, the distance is:

$$d = 0.4 \leq 0.45$$

✅ Therefore, the point $(0.4, 0.6)$ is within 0.45 (in the infinity norm) of a minimizer.

Yes