PROJECT REPORT ON

IMPLEMENTATION OF FIS ON INVERTED PENDULUM using MATLAB

In Partial Fulfillment of the Requirements for the Degree

of

BACHELOR OF TECHNOLOGY

in

Electronics and Communication Engineering

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Abstract

PURPOSE: FINDING THE SUITABLE DEFUZZ METHOD TO MAINTAIN EQUILIBRIUM OF A INVERTED PENDULUM THROUGH FIS

Abstract: The Inverted Pendulum is a classical control problem, which involves developing a system to balance a pendulum using a fuzzy system approach. This paper describes the setup and case study of the rotating base inverted pendulum. The inverted pendulum is a classical control system problem because of its nonlinear characteristics and unstable behavior. The rotating inverted pendulum is an excellent test bed for nonlinear control theory. This paper describes the hardware and software used and the different control schemes that were implemented and the results are presented. To study this problem, this project incorporated a full system design using matlab.

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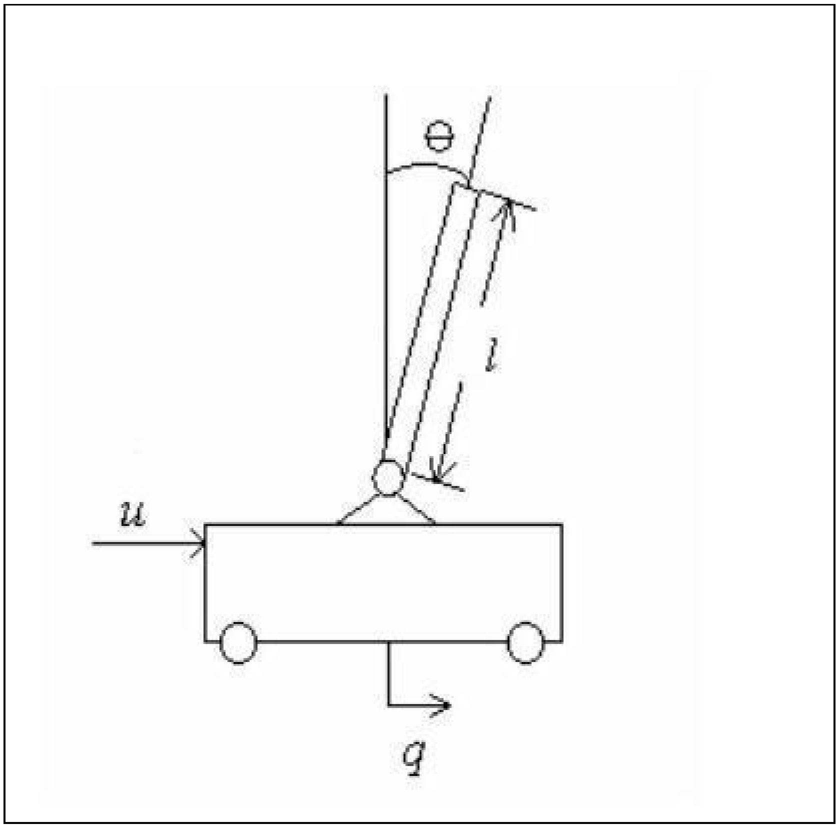
1. Introduction

1.1. Motivation

Our educational experience at Kalyani Government Engineering College has given us a broad background in many of the fields in Electronics and Computer Engineering such as microcontroller design, digital signal processing (DSP), control theory, etc. For a Master's of Engineering project, I hoped to integrate Design techniques from multiple fields in creating a system. Since my current interests lie within Microcontroller design, control theory, and DSP, I decided to design and build an unstable System and then a controller that would stabilize it using feedback control techniques. Most of My control experience had been simulating mathematical models using Matlab. My hopes were To design a controller that would work in theory and then figure out how to translate a Mathematical representation of that controller into a working model. After much thought, I Decided the classic control problem of the inverted pendulum was the perfect problem to do this With. I had dealt with this problem in ECE 472, Feedback Control Systems, in two of the Laboratory experiments, but the extent of the design was to come up with a mathematical model Of a controller and then upload it into an existing system to test the controller. On the other hand, this project will be the complete system design including all of the mechanical, hardware, And software design that can be done at a minimal cost.

1.2 Background

The inverted pendulum is a system that has a cart that is programmed to balance a pendulum as shown by a basic block diagram in Figure 1. This system is inherently unstable since even the slightest disturbance would cause the pendulum to start falling. Thus some sort of control is necessary to maintain a balanced pendulum. An ideal controller would keep the pendulum balanced with very little change in the angle, θ, or cart displacement, q. Obviously, limitations would be imposed based on the actual parameters of the system as well as the method for Implementing a controller. Thus designing a controller that is close to ideal is a challenging design program.



Basic diagram of a cart with a pendulum and a generic force being applied to the system.

2. Design Problem and Requirements

2.1. The Problem

The goal of this project is to design a mechanical system for the inverted pendulum problem and then implement a feasible controller as the main processing unit. The controller should minimize both the displacement of the cart and the angle of the pendulum. The system should be standalone and easy to use such that other controllers can be implemented as desired.

2.2 The Constraints

The assembly of the mechanical system requires access to a variety of tools and the assistance of certified users of various machine shops. Since assistance is required, the ability to machine parts is limited, and thus the design has to be relatively simple such that the number of iterations of building and testing the system are relatively few.

Another limiting factor in this design is the budget. Since this project does not support any research at Kalyani Government Engineering College, it is not funded by the university. Thus the project needs to be low cost. To minimize cost, some parts were sampled and others were chosen not because they were the best choice, but the best choice that was affordable. Components such as the motor suffer because of this constraint.

2.3. The Requirements

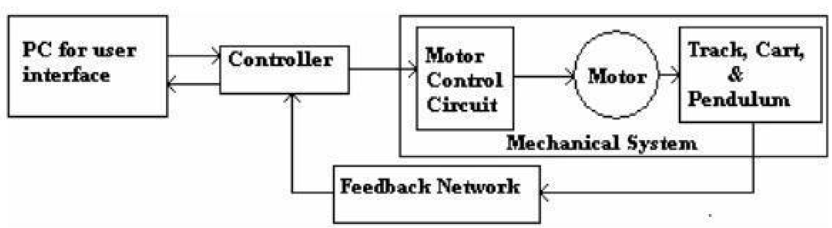
To design any system, a set of requirements is necessary to have guidelines when making decisions about implementation. This project is no different. Taking the constraints into consideration, a set of requirements and goals was established for this project as follows:

- Working mechanical system.
- Well designed accurate sensors.
- Matlab interface (Visual and data extraction).
- Easily programmable through user interface to allow other solutions.
- Main solution should balance for an extended period of time (at least 10 seconds).
- System is self-contained except for programming purposes and data extraction.
- Nonlinear Control (if time permits).
- Double inverted pendulum (if time permits).
- Wireless Sensors (if time permits).

As the project progresses it is expected that some of these requirements will change as necessary. A modified list of requirements will be included at the end of the design section outlining what was actually feasible

3. The Range of Possible Solutions

Due to the modularity of this project, the system can be broken up into many subsystems, as shown in the Figure below , that can each be solved in a variety of ways.



Block Diagram of the overall system

It is necessary to realize that each module can be solved in its own way, but to design a solution for a specific module, it is imperative to have some knowledge of the other modules. Thus this design process must take a gray box approach. For this discussion, each module will be discussed with a variety of solutions presented. The final design choices will be outlined at the end of this section explaining the reasoning for the choice and demonstrating how the dependence between subsystems had an effect on these decisions. Note that the solutions given in this section are not inclusive. One could easily imagine many other solutions to this problem.

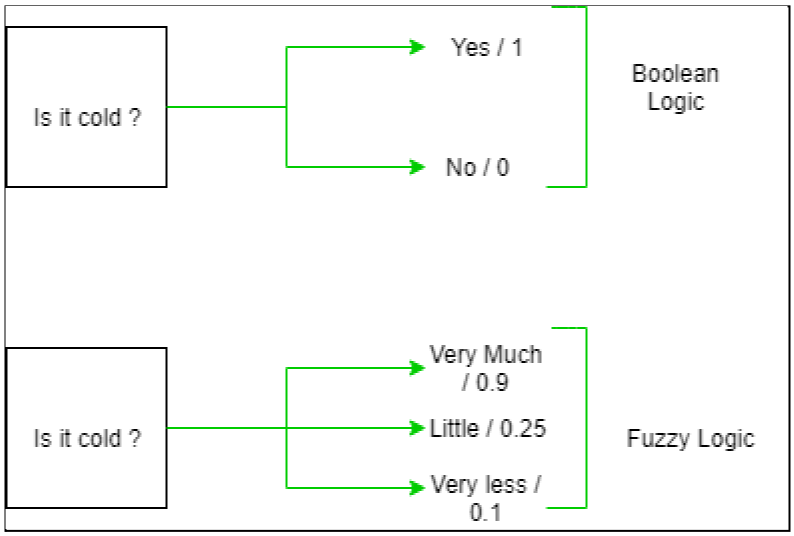
3.1 Fuzzy Logic

Fuzzy logic is an approach to computing based on "degrees of truth" rather than the usual "true or false" (1 or 0) Boolean logic on which the modern computer is based.

The idea of fuzzy logic was first advanced by Lotfi Zadeh of the University of California at Berkeley in the 1960s. Zadeh was working on the problem of computer understanding of natural language. Natural language -- like most other activities in life and indeed the universe -- is not easily translated into the absolute terms of 0 and 1. Whether everything is ultimately describable in binary terms is a philosophical question worth pursuing, but in practice, much data we might want to feed a computer is in some state in between and so, frequently, are the results of computing. It may help to see fuzzy logic as the way reasoning really works and binary, or Boolean, logic is simply a special case of it.

The term fuzzy refers to things which are not clear or are vague. In the real world many times we encounter a situation when we can’t determine whether the state is true or false, their fuzzy logic provides a very valuable flexibility for reasoning. In this way, we can consider the inaccuracies and uncertainties of any situation.

In the boolean system truth value, 1.0 represents absolute truth value and 0.0 represents absolute false value. But in the fuzzy system, there is no logic for absolute truth and absolute false value. But in fuzzy logic, there is an intermediate value to present which is partially true and partially false.

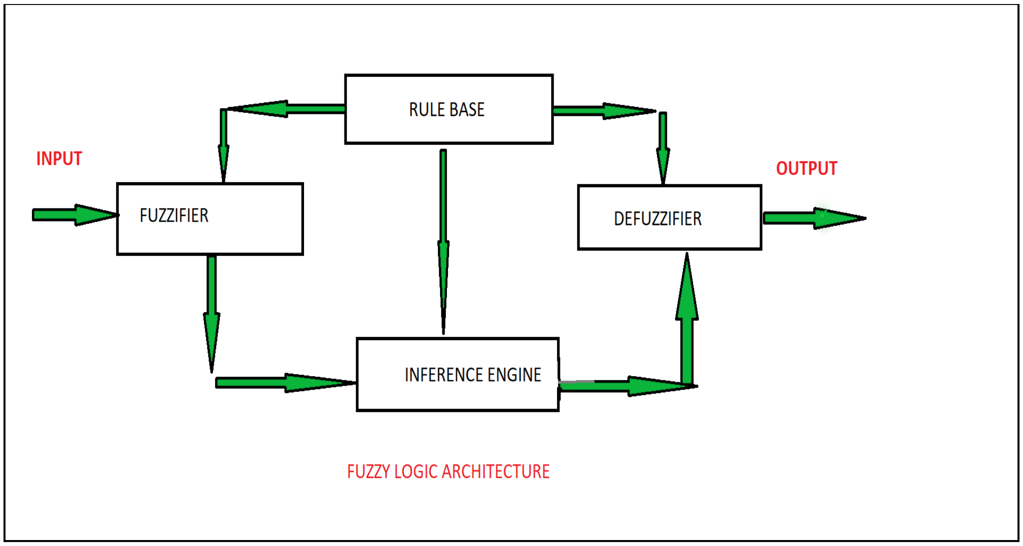


3.1.1 Architecture

Its Architecture contains four parts :

- RULE BASE: It contains the set of rules and the IF-THEN conditions provided by the experts to govern the decision making system, on the basis of linguistic information. Recent developments in fuzzy theory offer several effective methods for the design and tuning of fuzzy controllers. Most of these developments reduce the number of fuzzy rules.
- FUZZIFICATION: It is used to convert inputs i.e. crisp numbers into fuzzy sets. Crisp inputs are basically the exact inputs measured by sensors and passed into the control system for processing, such as temperature, pressure, rpm's, etc.
- INFERENCE ENGINE: It determines the matching degree of the current fuzzy input with respect to each rule and decides which rules are to be fired according to the input field. Next, the fired rules are combined to form the control actions.
- DEFUZZIFICATION: It is used to convert the fuzzy sets obtained by inference engine into a crisp value. There are several defuzzification methods available and the best suited one is

used with a specific expert system to reduce the error.



3.1.2 Membership function

Definition: A graph that defines how each point in the input space is mapped to membership value between 0 and 1. Input space is often referred as the universe of discourse or universal set (u), which contain all the possible elements of concern in each particular application.

There are largely three types of fuzzifiers:

- Singleton fuzzifier
- Gaussian fuzzifier
- Trapezoidal or triangular fuzzifier

3.1.3 What is Fuzzy Control?

- It is a technique to embody human-like thinkings into a control system.
- It may not be designed to give accurate reasoning but it is designed to give acceptable reasoning.

- It can emulate human deductive thinking, that is, the process people use to infer conclusions from what they know.
- Any uncertainties can be easily dealt with the help of fuzzy logic.

3.1.4 Advantages of Fuzzy Logic System

- This system can work with any type of inputs whether it is imprecise, distorted or noisy input information.
- The construction of Fuzzy Logic Systems is easy and understandable.
- Fuzzy logic comes with mathematical concepts of set theory and the reasoning of that is quite simple.
- It provides a very efficient solution to complex problems in all fields of life as it resembles human reasoning and decision making.
- The algorithms can be described with little data, so little memory is required.

3.1.5 Disadvantages of Fuzzy Logic Systems

- Many researchers proposed different ways to solve a given problem through fuzzy logic which lead to ambiguity.There is no systematic approach to solve a given problem through fuzzy logic.
- Proof of its characteristics is difficult or impossible in most cases because every time we do not get mathematical description of our approach.
- As fuzzy logic works on precise as well as imprecise data so most of the time accuracy is compromised.

3.1.6 Application

- It is used in the aerospace field for altitude control of spacecraft and satellite.
- It is used in the automotive system for speed control, traffic control.

- It is used for decision making support systems and personal evaluation in the large company business.
- It has application in the chemical industry for controlling the pH, drying, chemical distillation process.
- Fuzzy logic is used in Natural language processing and various intensive applications in Artificial Intelligence.
- Fuzzy logic is extensively used in modern control systems such as expert systems.
- Fuzzy Logic is used with Neural Networks as it mimics how a person would make decisions, only much faster. It is done by Aggregation of data and changing into more meaningful data by forming partial truths as Fuzzy sets.

3.2 Relations and composition in fuzzy logic

3.2.1 Crisp relation

Crisp relation is defined on the Cartesian product of two universal sets determined as X Y xy x X y Y {(,) | , } The crisp relation R is defined by its membership function 1, , , 0, , R x y R x y x y R Here "1" implies complete truth degree for the pair to be in relation and "0" implies no relation. When the sets are finite the relation is represented by a matrix R called a relation matrix.

3.2.2 Fuzzy relation

Let X,Y subset of R, be universal sets then;

$$R=\{((x,y),\mu_R(x,y))\,|\,(x,y)\in X\times Y\}$$

Is called a fuzzy relation in X *Y subset of R

Or X and Y are two universal sets, the fuzzy relation R(x,y) is given as

$$R(x,y)=\left\{\frac{\mu_R(x,y)}{(x,y)}\,|\,(x,y)\in X\times Y\right\}$$

Fuzzy relations are often presented in the form of two dimensional tables. A $m\times n$ matrix represents a contented way of entering the fuzzy relation R .

$$R=\begin{array}{c} \\ x_1 \\ \vdots \\ x_m \end{array}\begin{array}{c} \begin{array}{ccc} y_1 & \cdots & y_n \end{array} \\ \begin{bmatrix} \mu_R(x_1,y_1) & \cdots & \mu_R(x_1,y_n) \\ \vdots & \ddots & \vdots \\ \mu_R(x_m,y_1) & \cdots & \mu_R(x_m,y_n) \end{bmatrix} \end{array}$$

3.2.2.1 The maximum-minimum composition of relations

Let X , Y and Z be universal sets and let R be a relation that relates elements from X to Y , i.e

And

$$R=\{((x,y),\mu_R(x,y))\} \quad x\in X, y\in Y, R\subset X\times Y$$

$$Q=\{((y,z),\mu_Q(y,z))\} \quad y\in Y, z\in Z, Q\subset Y\times Z$$

$$S=R\circ Q$$

Then Swill be a relation that relates elements in X that R contains to the elements In Z that Q contains, i.e.

Here "o" means the composition of membership degrees of R and Q in the max-min Sense.

Max-min composition is then defined as

$$S=\{((x,z),\mu_s(x,z))\} \quad x\in X, z\in Z, S\subset X\times Z$$

$$\mu_S(x,z) = \max_{y \in Y}\left(min\left(\mu_R(x,y), \mu_Q(y,z)\right)\right)$$

And max product composition is then defined

$$\mu_S(x,z) = \max_{y \in Y}\left(min\left(\mu_R(x,y).\mu_Q(y,z)\right)\right)$$

3.2.2.2 Fuzzy max-min composition operation

Let us consider two fuzzy relations R1 and R2 defined on a Cartesian space X×Y And Y ×Z respectively. The max-min composition of R1 and R2 is a fuzzy set defined on a Cartesian spaces X × Z a

Where R1°R2 is the max-min composition of fuzzy relations R1 and R2 and max product composition is

$$R_1 \circ R_2 = \left[(x,z), \max\left\{\min\left\{\mu_{R_1}(x,y), \mu_{R_2}(y,z)\right\}\right\}\right] \mid x \in X, y \in Y, z \in Z$$

defined as

$$\mu_{R_1 \circ R_2} = \max\left[\mu_{R_1}(x,y) \cdot \mu_{R_2}(y,z)\right] \mid x \in X, y \in Y, z \in Z$$

3.2.2.3 Projection of Fuzzy Relation

Let $R=\{((x,y),\mu_R(x,y))\,|\,(x,y)\in X\times Y\}$ be a fuzzy relation. The projection of $R(x,y)$ on X denoted by R_1 is given by

$$R_1=\left\{\left(x,\max_y \mu_R(x,y)\right)\,|\,(x,y)\in X\times Y\right\}$$

and the projection of $R(x,y)$ on Y denoted by R_2 is given by

$$R_2=\left\{\left(y,\max_x \mu_R(x,y)\right)\,|\,(x,y)\in X\times Y\right\}$$

3.2.2.4 Cylindrical extension of fuzzy relation

The extension on X ×Y of a fuzzy set A of X is a fuzzy relation cylA whose membership function is equal to

Cylindrical extension from X-projection means filling all the columns of the related matrix by the X

$$cylA(x,y)=A(x), \qquad \forall x\in X, \qquad \forall y\in Y$$

-projection. Similarly cylindrical extension from Y projection means filling all the rows of the relational matrix by the Y -proj.

3.2.2.5 Reflexive Relation

Let R be a fuzzy relation in X × X then R is called reflexive if,

$$\mu_R(x,x)=0, x\in X$$

3.2.2.6 Anti Reflexive relations

Fuzzy relation R sub set of X X is anti reflexive if

$$\mu_R(x,x)=1 \qquad \forall x \in X$$

3.2.2.7 Symmetric Relation

A fuzzy relation R is called symmetric if,

$$\mu_R(x,y)=\mu_R(y,x) \quad \forall x,y \in X$$

3.2.2.8 Antisymmetric Relation

$$\text{if } \mu_R(x,y)>0 \text{ then } \mu_R(y,x)=0 x,y \in X, x \neq y$$

3.2.2.9 Transitive Relation

Fuzzy relation $R \subset X \times X$ is transitive in the sense of max-min iff

$$\mu_R(x,z) \geq \max_{y \in X}\left(\min\left(\mu_R(x,y),\mu_R(y,z)\right)\right) x,z \in X$$

since $R^2 = R \circ R$ if

$$\mu_{R^2}(x,z) = \max_{y \in X}\left(\mu_R(x,y),\mu_R(y,z)\right)$$

then R is transitive if $R \circ R = R(R \circ R \subseteq R)$

and $R^2 \subset R$ means that $\mu_{R^2}(x,y) \leq \mu_R(y,x)$

3.2.2.10 Similarity Relations

R sub set of × X which is reflexive, symmetric and transitive is called the similarity relation

$$R = \begin{array}{c|cccccc} & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\ \hline x_1 & 1 & 0.2 & 1 & 0.6 & 0.2 & 0.6 \\ x_2 & 0.2 & 1 & 0.2 & 0.2 & 0.8 & 0.2 \\ x_3 & 1 & 0.2 & 1 & 0.6 & 0.2 & 0.6 \\ x_4 & 0.6 & 0.2 & 0.6 & 1 & 0.2 & 0.8 \\ x_5 & 0.2 & 0.8 & 0.2 & 0.2 & 1 & 0.2 \\ x_6 & 0.6 & 0.2 & 0.6 & 0.8 & 0.2 & 1 \end{array} \quad \text{is a similarity relation.}$$

3.2.2.11 Anti Similarity Relation

If R is a similarity relation then the complement of R is anti similarity relation. R subset of X×X is a anti similarity relation if

$$\mu_{R'}(x, y) = 1 - \mu_R(x, y)$$

The anti similarity relation is anti reflexive, symmetric and transitive in the sense of maxmin, i.e.

3.2.2.12 Weak Similarity

$$\mu_{R'}(x,z) \geq \min_{y\in X}\Big(\max\big(\mu_{R'}(x,y),\mu_{R'}(y,z)\big)\Big) x,z \in R$$

R subset of X × X which is reflexive and symmetric is called the relation of weak similarity (not transitive).

$$R = \begin{bmatrix} 1 & 0.1 & 0.8 & 0.2 & 0.3 \\ 0.1 & 1 & 0 & 0.3 & 1 \\ 0.8 & 0 & 1 & 0.7 & 0 \\ 0.2 & 0.3 & 0.7 & 1 & 0.6 \\ 0.3 & 1 & 0 & 0.6 & 1 \end{bmatrix} \text{ is weak similarity relation}$$

3.2.2.13 Order Relation

An order relation R sub set of X × X is transitive relation in the sense of max-min; i.e

$$\mu_R(x,z) \geq \max_{y\in X}\big(\min(\mu_R(x,y),\mu_R(y,z))\big), x,z \in X$$

3.2.2.14 Pre Order Relations

A pre order relation R sub set of X × X is reflexive and transitive in the max-min sense e

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{bmatrix} 1 & 0.7 & 0.8 & 0.5 & 0.5 \\ 0 & 1 & 0.3 & 0 & 0.2 \\ 0 & 0.7 & 1 & 0 & 0.2 \\ 0.6 & 1 & 0.9 & 1 & 0.6 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

3.2.2.15 Half Order Relation

A fuzzy half order is a relation R sub set of X × X which is reflexive

$$\mu_R(x, x) = 1 \qquad \forall x \in X$$

and weakly antisymmetric, i.e.

$$\text{if} \quad \mu_R(x, y) > 0 \text{ and } \mu_R(y, x) > 0 \quad \text{then} \quad x = y$$

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} \!\!\begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \begin{bmatrix} 1 & 0.8 & 0.2 & 0.6 & 0.6 & 0.4 \\ 0 & 1 & 0 & 0 & 0.6 & 0 \\ 0 & 0 & 1 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 1 & 0.6 & 0.4 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \text{ is half order relation}$$

3.2.3 Fuzzy Graph

In 1975, Rosenfeld considered fuzzy relations on fuzzy sets. He developed the theory of fuzzy graphs. Bang and Yeh during the same time introduced various connectedness concepts in the fuzzy graphs. Inexact information is used in expressing or describing human behaviors and mental processes. The information depends upon a person subjectively and it is difficult to process objectively.

Fuzzy information can be analyzed by using a fuzzy graph. The fuzzy graph is an expression of fuzzy relation and thus the fuzzy graph is frequently expressed in the fuzzy matrix.

Mathematically a graph is defined as $G = (V, E)$ where V denotes the set of vertices and E denotes the set of edges. A graph is called a crisp graph if all the values of arcs are 1 or 0 and a graph is called fuzzy graph if its values is between 0 and 1.Fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : S \to [0,1]$ where S is the set of vertices and $\mu : S \times S \to [0,1]$, $\forall x, y \in S$.

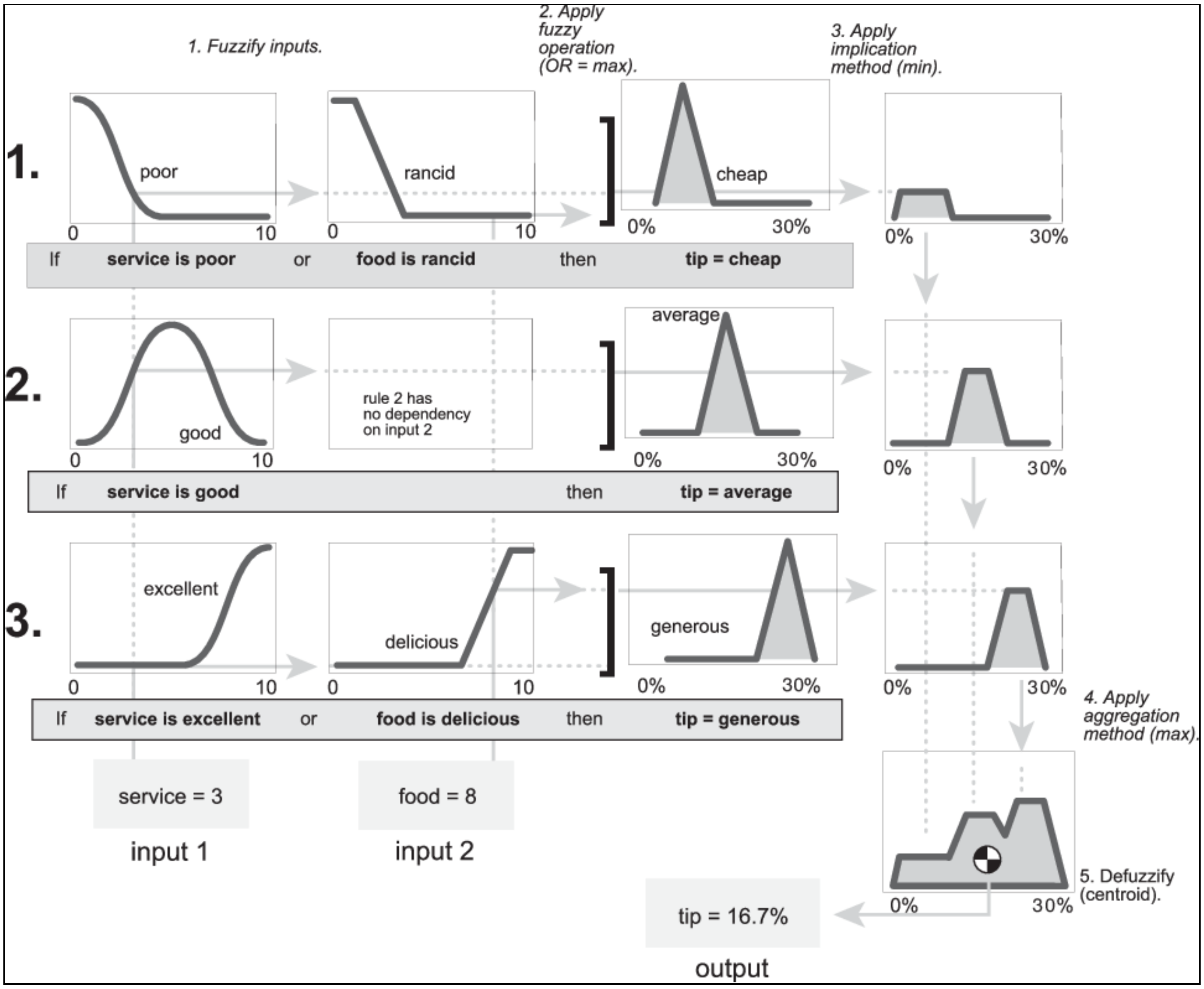
Fuzzy graph $H = (\tau, v)$ is called a fuzzy subgraph of G if

$$\tau(x) \leq \sigma(x), \forall x \in S \quad \text{and} \quad v(x, y) \leq \mu(x, y) \forall x, y \in S$$

Mamdani inference was first introduced as a method to create a control system by synthesizing a set of linguistic control rules obtained from experienced human operators. In a Mamdani system, the output of each rule is a fuzzy set.

Since Mamdani systems have more intuitive and easier to understand rule bases, they are well-suited to expert system applications where the rules are created from human expert knowledge, such as medical diagnostics.

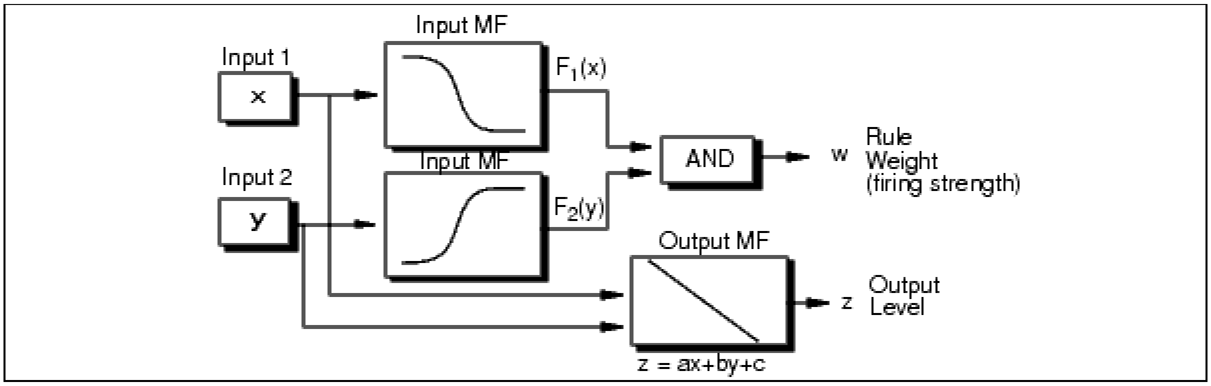
The inference process of a Mamdani system is described in the Fuzzy Inference Process and summarized in the following figure.



3.3 Sugeno Fuzzy Inference Systems

Sugeno fuzzy inference, also referred to as Takagi-Sugeno-Kang fuzzy inference, uses *singleton* output membership functions that are either constant or a linear function of the input values. The defuzzification process for a Sugeno system is more computationally efficient compared to that of a Mamdani system, since it uses a weighted average or weighted sum of a few data points rather than compute a centroid of a two-dimensional area.

Each rule in a Sugeno system operates as shown in the following diagram, which shows a two-input system with input values *x* and *y*.



Each rule generates two values:

z_i — Rule output level, which is either a constant value or a linear function of the input values:

$$z_i = a_i x + b_i y + c_i$$

Here, x and y are the values of input 1 and input 2, respectively, and a_i, b_i, and c_i are constant coefficients. For a zero-order Sugeno system, z_i is a constant ($a = b = 0$).

w_i — Rule firing strength derived from the rule antecedent

$$w_i = AndMethod(F_1(x), F_2(y))$$

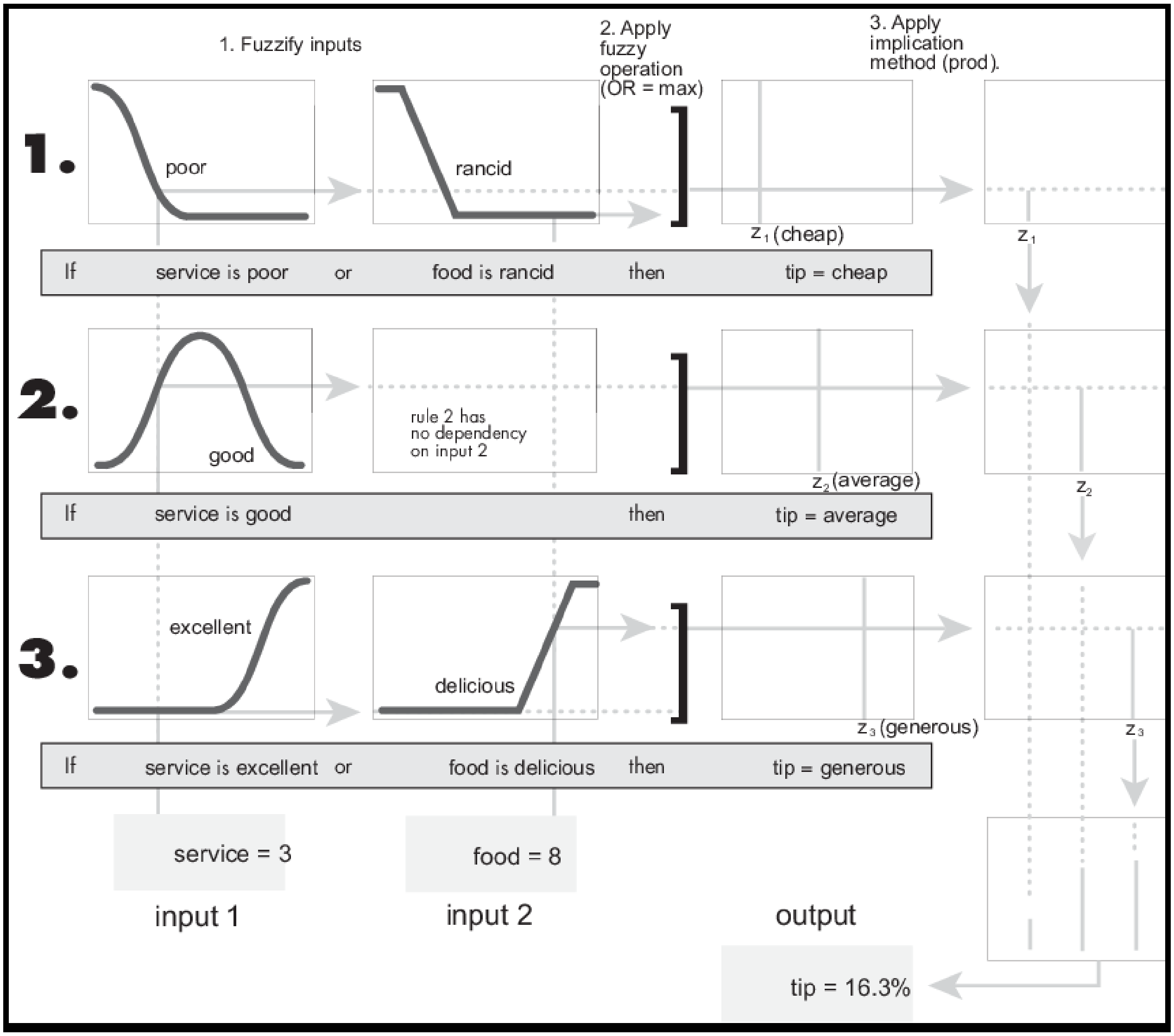
Here, $F_1(...)$ and $F_2(...)$ are the membership functions for inputs 1 and 2, respectively.

The output of each rule is the weighted output level, which is the product of w_i and z_i.

The easiest way to visualize first-order Sugeno systems (*a* and *b* are nonzero) is to think of each rule as defining the location of a moving singleton. That is, the singleton output spikes can move around in a linear fashion within the output space, depending on the input values. The rule firing strength then defines the size of the singleton spike.

The final output of the system is the weighted average over all rule outputs:

Final Output = *weighted mean *formula



3.4 Defuzzification Methods

- ***Centroid*** : Centroid defuzzification returns the center of gravity of the fuzzy set along the x-axis. If you think of the area as a plate with uniform thickness and density, the centroid is the point along the x-axis about which the fuzzy set would balance.
- ***Bisector*** : The bisector method finds the vertical line that divides the fuzzy set into two sub-regions of equal area. It is sometimes, but not always, coincident with the centroid line.
- ***Middle, Smallest, and Largest of Maximum*** : MOM, SOM, and LOM stand for middle, smallest, and largest of maximum, respectively. In this example, since the aggregate fuzzy set has a plateau at its maximum value, the MOM, SOM, and LOM defuzzification results have distinct values. If the aggregate fuzzy set has a unique maximum, then MOM, SOM, and LOM all produce the same value.

3.5 Mechanical System

The design of a mechanical system for this project involves integrating four main components: (1) the cart, (2) the pendulum, (3) the track, and (4) the mechanism used to move the cart. There are many ways to implement these, though each component is quite dependent on the other three. These components also have to meet some basic requirements such that it is possible to design a controller to balance the pendulum. These requirements are as follows.

• The cart motion needs to be limited to one degree of freedom which is in the horizontal plane.

• The pendulum motion needs to be limited to two degrees of freedom, one of which is the same as the cart's degree of freedom.

• The friction that impedes the cart and pendulum motion must be reduced as much as possible.

3.5.1 Track, Cart, and Pendulum Solution

One solution would be to have a U-channel track within which the cart would move. Ideally, the cart would fit well in the track and slide smoothly with little friction. Also needed in this design is a mechanism to apply force on the cart in either direction. This could be done in one of two ways. The first way would be to mount a motor at the end of the track with a drive wheel/sprocket attached to the shaft and another wheel/sprocket free-spinning at the opposite end of the track. Around the two wheels would be a pulley or chain with which the cart can be fixed to. The second way would be to have a motor directly drive wheels on the bottom of the cart. This solution will not be considered since the design of adding a motor to the cart for a reasonable size U-channel (2 inch width) would be too complex and intricate. Below is a simple rating of this solution based primarily on the basic requirements outlined at the beginning of this section.

• **Pros**

a)2 degrees of freedom for the pendulum.

b) 1 degree of freedom for the cart.

c) Simple cart design.

• **Cons**

a) Friction could be a problem.

b)Drive Mechanism could be complicated.

c) More room for error with the integration of multiple systems (drive mechanism can be considered its own system).

A second option for the mechanical design could have the motor as the main component of the cart. In this design, depending on the size of the motor, the cart can be built around the motor. The motor would have a toothed drive wheel mounted on its shaft which would fit into a teethed track that is flat on the surface. In order to guarantee that the drive wheel does not come off the track, a guide will need to be implemented such that it is parallel to the track that the cart would be attached to. One possible guide could be a long steel rod with which the cart could have an arm to wrap around.

• **Pros**

a)Overall System is compact.

b) Could be self-contained if the controller is mounted on the cart as well.

c) Reduced Friction.

d) Good ability to change direction quickly.

• **Cons**

a)Cart design may be complex.

b)Guide may be hard to integrate.

c) Possible slip condition with drive wheel and track.

A variation of the second option would be to not have a track- just have a cart with the pendulum still mounted on top. This cart would be self contained with the whole system integrated in the cart itself. The motor would control a set of wheels that would move to keep the pendulum balanced. This design would not need a guide since the center of mass of this design would be low to the ground. A gear assembly may be necessary to drive to wheels simultaneously. It could be possible to have one wheel centered on one side with free spinning wheels on the other that could work satisfactorily without any gear assembly.

• **Pros**

a) Overall System is compact.

b) Self Contained.

• **Cons**

a) Reaction time is dependent on motor torque and the strength of the friction between the wheels and ground surface.
b) may have more than 1 degree of freedom.
c) Pendulum may have more than 2 degrees of freedom.
d) Displacement may be hard to measure since there is no reference point to measure again.

One other intriguing solution would be to have the motor with a rod mounted on the shaft such that when the motor turns, the rod is parallel to the ground. Attached to the end of this shaft would be the pendulum. Thus this system would have a circular motion instead of the linear displacement previously discussed.

• **Pros**

a) Cart is simply a rod.

• **Cons**

a) Strong motor is needed.
b) must be well built such that the rod is strongly attached to the motor shaft.
c) Nonlinearities of the motion of the cart will be difficult to model.

Other variations of the mechanical system include using any of the suggested solutions with pendulums of different shapes, lengths, mass distributions, etc. The ideal case for the inverted pendulum problem, which is the easiest to model, is the pendulum with all the mass located at the tip. This allows for the approximation of the moment of inertia of the pendulum to be zero.

3.5.2 The Motor And The Control Circuit

There are three main choices to use for motors for this project: (1) a DC motor, (2) a stepper motor, and (3) a servo motor. The two main considerations in choosing a motor are the needs for high torque and high speed. The torque is necessary for the cart to change direction quickly in order to keep the pendulum balanced. High speed is needed such that the cart can move faster than the pendulum can fall.

The DC motor could have high torque and high speed, but it comes at a cost. First of all, when the torque and speed of a DC motor increase, it requires more power to run the motor. This will be limited by the circuitry used to control the motor. The control circuitry for a DC motor is typically an H-bridge which controls the direction of current across the motor based on the directional signal. Another input to the H-bridge controls the speed of the motor. The second cost of having a good DC motor is that they can be quite expensive.

The stepper motor could provide high torque, but it would lack sufficient speed. A bi-polar stepper would be necessary to ensure that the motor turns both directions. For this motor, there are four inputs lines that need to be toggled in the correct order to have the motor turn in a certain direction and in the opposite order for the motor to move in the reverse direction. This control can be done externally through the use of digital logic components that would require just a directional signal and a speed control, but this would make the design more complex. Instead a simpler solution would have four control lines come directly out of the MCU. Stepper motors are also costly and consume a great deal of power.

The servo motor could supply high speed, but would suffer with the torque. It would also be harder to incorporate into this design. First off, servo motors typically have the ability to turn only 360°. In order to have such a motor, the drive wheel attached to the motor shaft would have to be sized such that one rotation could cause the cart to travel the length of the track. This would require a large wheel would decrease the amount of torque provided by the motor and could possibly damage the motor. Also note that controlling a servo motor could be quite difficult in this application since the voltage level applied to the motor tells the motor which angle to be at. There is less intuition in designing controllers that operate in this way.

Regardless of which motor is chosen, a separate power supply will be necessary just for the motor. The large power consumption by the motors and the inductive spikes created each time the motor changes direction could be harmful to any other circuit hooked up to the same power supply. The only way to control the motor would be through opto-isolation which completely separated circuits with different power supplies.

3.5.3 Feedback Network

Designing an accurate feedback network is essential to stabilizing the system. Thus the sensors need to be relatively noiseless and have a fast response such that the information retrieved from the sensors accurately reflects the state of the system. Determining the variables of the system to measure can be difficult. In this case there are four parameters that govern the inverted pendulum system (which will be derived in a later section). They are (1) the angle, (2) the angle's velocity, (3) the displacement of the cart, and (4) the velocity of the cart. Thus there are four measurable parameters that could be used for feedback, which would determine the control necessary to stabilize the system. Most conventional approaches to this problem only measure the angle and displacement and derive the other two parameters from these. This project follows In suit since those two parameters are the easiest to measure and give the most information about the system.

3.5.4 Displacement Sensor

One way to measure displacement would be to attach a potentiometer to either the drive sprocket orr the free-spinning one. The voltage on the wiper of the potentiometer would then be converted to a digital signal by an ADC. It is then possible to determine the displacement of the cart by using the diameter of the sprocket, the measured voltage, and the number of turns the potentiometer is capable of.

- **Pros**

  a) Easy Implementation.

- **Cons**

  a) Potentiometer output voltage may be nonlinear.
  b) Sampling frequency may be limited by ADC conversion time thus reaction time by the controller is limited.
  c) Accuracy is limited by two factors.:

     i) Potentiometer is accurate to within a tolerance.

     ii) ADC is accurate to within a tolerance

Another option could be a linear potentiometer. This could be mounted to the track with the slider attached to the cart. Similar sampling methods would then be implemented using an ADC.

- **Pros**

  a) Easy Implementation.

• **Cons**

a)Expensive since most linear potentiometers are only tens of centimeters long and this project may have tracks approximately 1.5 meters long.

b) Sampling frequency may be limited by ADC conversion time thus reaction time By the controller is limited.

c) Accuracy is limited by two factors.

i)Potentiometer is accurate to within a tolerance.

ii) ADC is accurate to within a tolerance.

Another possibility would be to have light sensors spaced parallel to the track with a receiver on the other side of the track. These would work by signaling when the cart has passed between an emitter and receiver breaking the signal.

• **Pro**

a) Simple Concept.

• **Cons**

a) Accuracy is limited to the spacing of the sensors.
b) Many sensors necessary for an accurate reading.

Radar and sonar sensors are also possible. These would emit a light or sound, respectively, and wait for the reflection. The time it takes for a reflection would be used to then calculate the distance. Unfortunately these sensors are more expensive and could take up a great deal of processing time which would slow down the reaction of the system. Thus they will not be considered for this project.

3.5.5 Angle Sensor

One of the easiest solutions would be to mount the pendulum on a circular potentiometer. Ideally, the potentiometer would have little friction. Though practical potentiometers will have some friction, which could influence the dynamics of the pendulum falling. More friction would slow down the reaction of the pendulum to any of the forces exerted on it, making it easier to balance.

• **Pros**

a) Easy Implementation.

• **Cons**

a) potentiometer output voltage may be nonlinear.
b) Friction of potentiometer may influence the dynamics of the pendulum falling.
c) Sampling frequency may be limited by ADC conversion time thus reaction time by the controller is limited.
d) Accuracy is limited by two factors.

i) Potentiometer is accurate to within a tolerance.

ii) ADC is accurate to within a tolerance.

A hub encoder could be another practical solution. This sensor reads the angle and then returns the digital representation . Unfortunately this sensor is expensive and will not be considered.

3.5.6 Controller Solutions

Once a mechanical system is developed with an accurate feedback network and an easy interface for controlling the cart, a controller can be designed. As this section will show, there are many ways to implement a working controller. The largest constraint of designing a working controller, is how well the system is modeled. If the system has mismodeled or unmodeled dynamics, it is quite likely the controller designed from this model will not work. This section will discuss the modeling of the system and the necessary assumptions that are required to design and implement a working controller. Once the modeling techniques have been discussed, possible controller design approaches will be presented.

3.5.7 Modeling the System Dynamics

In general, an accurate model of the system is desired. Unfortunately, modeling a system accurately may make it very difficult to design a controller. Most control techniques hinge on the assumption that the system is linear. Thus simplifications will need to be made in hopes that the resulting model is still a relatively accurate model of the real system.

3.5.7.1 Modeling Assumptions

A first approach to modeling would be to assume a frictionless system with all motion limited to the desired degrees of freedom. This may not seem like a fair approach since the system indeed has friction.. If friction was included in the model, the system may become very complex, thus making it difficult to design for. It is also quite possible to mismodel the friction given a mechanical system that is relatively complex. One way to compensate for not modeling the friction is to account for it while measuring certain parameters of the system.

3.5.7.2 Linearity

The inverted pendulum is adherently a nonlinear system due to the cosine and sine terms generated while modeling the pendulum. Assuming that the pendulum's angle will remain small, since there should be ample control to keep it balanced, it may be possible to approximate Sin $\theta = \theta$ and cos $\theta = 1$. Thus this will linearize the system, which will allow for controller design methods to use linear system techniques.

3.5.7.3 Complexity

Another assumption that may be made which will greatly reduce the complexity of the system is that all of the mass of the pendulum is located at the tip. This will allow for the moment of inertia of the pendulum to be zero, which simplifies the system dynamics.

3.5.7.4 Controller Design

Control Theory has evolved over the past 75 years. There has been the classic control era, modern control era, robust control era, and now the model predictive era. Thus there are many viable solutions for designing a controller specific to this system. Though when implementing these solutions, the Mega32 may impose constraints in timing and calculation ability which need to be considered.

3.5.7.5 Classical Control

One possible way to design a controller for this problem would be to use classical control techniques. In general classical control is used on Single Input Single Output (SISO) systems, but can be applied to Multiple Input Multiple Out (MIMO) systems using clever manipulations of the dynamics. One possible manipulation to change a MIMO system into a SISO system would be to combine the measurements into a “virtual” measurement. This would only work in certain situations where the measurements can be correlated. In this case, the displacement can be related to the angle.

- **Pros**
  - a) Many design tools.
    - i) Bode Plots.
    - ii) Root Locus.
    - iii) Nichols/Nyquist Plots.
  - b) Simple Implementation.
- **Cons**
  - a) Better suited for SISO systems and the “virtual” measurement may be inaccurate.
  - b) Many design variables could make for a long iterative process.

3.5.7.6 Modern Control

Modern Control may seem like the most logical choice for this design since this technique is focuses primarily on Multiple Input Multiple Output (MIMO) systems. The main design objective is to minimize the cost equation which is an integral over time of the weighted inputs and outputs. There are two cost equations that are typically used for this design technique. There Is the Linear Quadratic Regulator (LQR) cost equation for determining the control law which applies the control effort onto the system,

$$J_{LQR} = \lim_{T\to\infty} \frac{1}{T} \cdot \int_0^{t_f} \left[z^T(t) R_{zz} z(t) + u^T(t) R_{uu} u(t) \right] \cdot dt \qquad (1)$$

Where z is the performance, u is the input, and Rzz and Ruu are the corresponding weighting matrices. There is also the Kalman Filter cost equation used for the observer that estimates the state vector from the measure outputs,

$$J_{K.F.} = \lim_{T\to\infty} \frac{1}{T} \cdot \int_0^{T} \left[\left(x(t) - \hat{x}(t) \right)^T Q \left(x(t) - \hat{x}(t) \right) \right] \cdot dt \qquad (2)$$

Where x is the real state vector, ^X is the estimated state vector and Q is the weighting matrix that determines how noisy the measurements are. In general there is another term that incorporates the correlation between the noise of the sensors and the disturbance. For simplicity the assumption will be made that there is no correlation between the two noise measurements.

$$R_{zz} = \begin{bmatrix} \frac{1}{(\max z_1)^2} & 0 \\ 0 & \frac{1}{(\max z_2)^2} \end{bmatrix}, \qquad R_{uu} = \begin{bmatrix} \frac{1}{(\max u_1)^2} & 0 \\ 0 & \frac{1}{(\max u_2)^2} \end{bmatrix}. \qquad (3)$$

These two costs can be implemented and combined to form a controller known as the Linear quadratic Gaussian (LQG) controller. Implementing this type of controller would be quite involved since it would be necessary to keep track of the state variables internally. These internal representations would be used to calculate the errors between the measurements and expected state variables. The error would then be minimized by the Kalman filter and the LQR control law would be applied using the estimated state variables. For a continuous system, this could involve many matrix operations including inversions which are costly in terms of calculations, increasing the time needed for controlling one sample, hurting the actual control performance.

• Pros

a) Easy design procedure.
b) Increases the accuracy of the state variables by estimating the state.

• Cons

a) Tough to implement.
b) Lengthy calculations.
c) Works best for white noise disturbances (or filtered versions of).

3.5.7.7 Robust Control

This design strategy is used to help guarantee stability of a controller. Unfortunately certain criteria is required to use this method and it is typically good for systems that cannot model the disturbance as some sort of filtered white noise.

First off, for analyzing a controller, it is necessary to develop approximate uncertainties in their model parameters (not including unmodeled dynamics which could have a huge influence on the analysis). Many random system models can then be generated from the uncertainties by randomly choosing parameter values within their approximated ranges. With this information,loop transfer functions can be calculated and sensitivity and Nichols plots can be used to determine the stability of the controller. A system is considered robust if it is stable for a large sample of random system models with parameters that fall within these uncertainty ranges.

Synthesizing a robust controller may be as simple as designing an LQG controller and then testing it for stability. Though, LQG is only good for systems with a variation of a white noise disturbance. Many real systems tend to have something other than a white noise disturbance. Given that we may know the frequency range in which this disturbance will primarily occur, an extra term with parameter γ can be added into the cost equation as shown in Eq. (4) to incorporate the disturbance, d, when determining the controller.

$$J_{LQG} = \lim_{T \to \infty} \frac{1}{T} \cdot \int_{0}^{t_f} \left[z^T(t) R_{zz} z(t) + u^T(t) R_{uu} u(t) + \gamma^2 d^T d \right] \cdot dt \qquad (4)$$

Minimizing γ until the system is close to instability gives the H∞ control law. As γ approaches infinity we get the LQG controller and anywhere in between is considered H2 control.

• **Pros**

a) Handles non-white noise disturbances.
b) Better chance for stability assuming the system is not mismodeled.

• **Cons**

a) Tough to implement.
b) Lengthy calculations.
c) Set up for analysis requires some educated guesses about the.

3.5.7.8 Model Predictive Control

Model Predictive Control (MPC) is a good technique for problems that track a reference. This method is primarily used for discrete system models since the continuous system's cost equation cannot be solved in closed form. This technique can still be applied to continuous systems, since most practical control situations obtain their measurements by sampling values from the sensors, thus requiring that the system and/or the controller be discretized. The cost equation used for discrete systems is as follows,

$$J_{MPC} = \lim_{N\to\infty} \frac{1}{2} \cdot \sum_{k=1}^{N} \left[\left(y[k] - r_y[k] \right)^T R_{YY} \left(y[k] - r_y[k] \right) + u^T[k] R_{uu} u[k] \right]. \qquad (5)$$

The basic principle of this technique is to take a sample from the sensors and then estimate the system dynamics for the next N time steps called the horizon. From that, a control law is determined for every time step and only the first one is implemented using some sort of interpolation method. The process then repeats for the next time step. As the horizon increases,the tracking tends to become more accurate.

• **Pros**

a) Ideal for real time systems that are sampled.
b) Simple to design once the system has been discretized.

• **Cons**

a) Time consuming calculations to determine control for each sample.
b) Minimum sampling period for stability may be higher than maximum attainable sampling rate.
c) Requires knowledge of the referen.

3.6 The Final Design Choices and Reasoning

3.6.1 The Final Design Choices

For this project, the U-channel track was chosen with a DC motor that has a sprocket attached to its shaft to pull a chain. The control circuit is the H-bridge circuit designed for DC motors. The cart is a block of wood mounted on matchbox cars with horizontal wheels on the bottom and top to limit the degrees of freedom. The pendulum is mounted on the shaft on the angle measuring circular potentiometer which is mounted on top of the cart. The position sensor is also a circular potentiometer attached to the drive sprocket. The sensors are sampled at the rate of the internal ADC, but the values are only used to develop a control effort at a rate of 100Hz. This allows for the most recent measurements to be available when it is necessary to determine a new control effort. The actual controller was designed using LQR and is implemented using floating point arithmetic.

3.6.2 The Reasoning Behind the Choices

The mechanical system was the most important subsystem of the design, which everything else would need to be based off of. Thus the mechanical system needed to allow for easy compatibility with the other subsystems. The U-channel seemed to offer this while still maintaining the ability to actually work. It also would help restrict the degrees of freedom of the cart making the cart design simpler. The friction might be a problem, but it should be possible to model the friction and compensate for it using the controller. The U-channel also had the appeal that the motor was a separate component that was not embedded in the cart. Thus if the motor required any modifications, it could easily be removed and fixed. This system should be easier to build which would reduce the chance for error due to the machine shop constraints. The feedback network also was relatively easy to implement because of the U-channel. Circular potentiometers would be simple to mount on both the cart and on the drive wheel.

3.6.3 Design Process and Its Implementation

Now that the basic design has been chosen, this section will discuss the actual design process. Obviously there are many different components that need to be designed and built in order to get this whole system up and running so the question is where to start? What makes this more complicated is that most of the components are dependent upon each other. The answer to this question is the motor, which would probably be the limiting factor in the whole design (as well as most expensive). Thus it is necessary to determine what constitutes a “good” motor and then design the system around the motor such that the motor would be capable of controlling the cart to keep the pendulum balanced. Detailed in the following section is the design process followed for each subsystem and its final implementation.

3.6.4 Center Average Defuzzifier (or Weighted Average Method)

This method is only valid for symmetrical output membership functions. Since, the fuzzy set C′ is the union or intersection of M fuzzy sets, a good approximation of (13.15) is the weighted average (mean) of the centers of the M fuzzy sets, with the weights equal to the heights (maximum)of the corresponding fuzzy sets. Specifically, let zr (z bar r) be the center of the r-th fuzzy set and Wr (omega r)be Its height, the center average defuzzifier determines z* as

$$z^* = \frac{\sum_{r=1}^{m} \bar{z}_r \, w_r}{\sum_{r=1}^{m} w_r} \qquad (6)$$

Figure 13.5 illustrates this operation graphically for a simple example with M = 2, where z1(z1bar) = 3 and z2 (z2bar)= 5

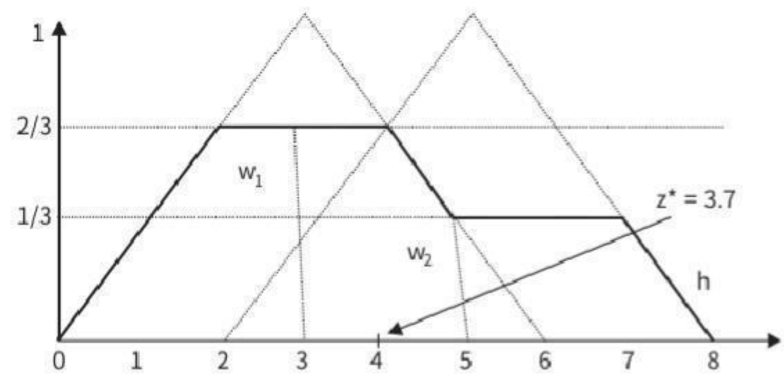


Fig 3: A graphical representation of the center average defuzzifier

The center average defuzzifier is the most commonly used defuzzifier in fuzzy systems and fuzzy control. It is computationally simple and intuitively plausible. Also, small changes in zr(z bar r) and wr (omega r) result in small changes in z*.

3.7 Fuzzy Control System Design: INVERTED PENDULUM

Figure 3 shows the classic inverted pendulum system.

Let us design a fuzzy controller for the simplified version of the inverted pendulum system shown in Fig. 4 The linearized differential equation describing the dynamics of the system is given by the equation which is reproduced below:

$$\frac{4l}{3}\frac{(4M+m)}{4m}\ddot{\theta} - \frac{(M+m)g}{m}\theta = -\frac{u}{m}$$

Where M = the mass of the pole assumed to be concentrated at the center of the pendulum, Kg

M = mass of the cart, Kg

2l = the length of the pendulum, meter

θ = the deviation angle from vertical in the clockwise direction, in radian

T(t) = the torque applied to the pole in the counterclockwise direction, Nm

U(t) = the control on the cart acting from left to the right producing the counterclockwise torque T(t), Nm

T = time in a sec, and

G = the gravitational acceleration constant

When θ is expressed in degrees, the coefficient of u is to be multiplied by 180/π. For convenience of hand calculation, we choose the following parameters for the system in equation (6) where θ is expressed in degrees and θ^.(θ dot) In degrees per second.

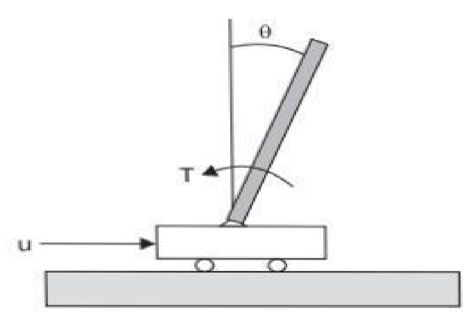
M=180/πg- m and l=3(M-m)g/4M+m **(8)**

With the above choices of the parameters, the equation (7) reduces to θ^..(θ double dot) = θ – u **(9)**

With x1 = θ and x2 = θ^.(θ dot) as state variables, the state-space representation for the linearized system defined by Equation (8) is given by

dx1/dt = x2 **(10)**

dx2/dt = x1 – u(dx2/dt = x1 – u(t) **(11)**



Inverted pendulum control problem

The discrete-time state-space equations can be represented as matrix difference equations, with sampling time T set to 1 sec as :

X1(k + 1) = x1(k) + Tx2(k) = x1(k) + x2(k) **(12)**

X2(k + 1) = Tx1(k) + x2(k) – Tu(k) = x1(k) + x2(k) – u(k) **(13)**

For this problem we assume the universe of discourse for the two variables to be -2° ≤ x1 ≤ 2° and – 4 dps ≤ x2 ≤ 4 dps [dps = degree per second].

Step 1: we decide to use 3 term sets to cover the universe of discourse for both the states x1 and x2. The term sets are designated as positive (P), zero (Z), and negative (N). The membership functions of the three-term set functions for x1 on its universe are as shown in Fig. 5. Similarly, the membership functions Of the three-term set functions for x2 on its universe are shown in Fig. 6.

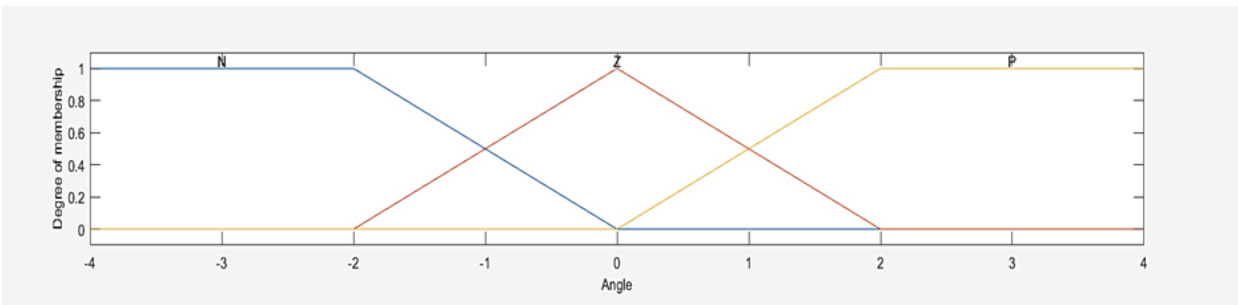


Fig. 5 Input x1 partitioned

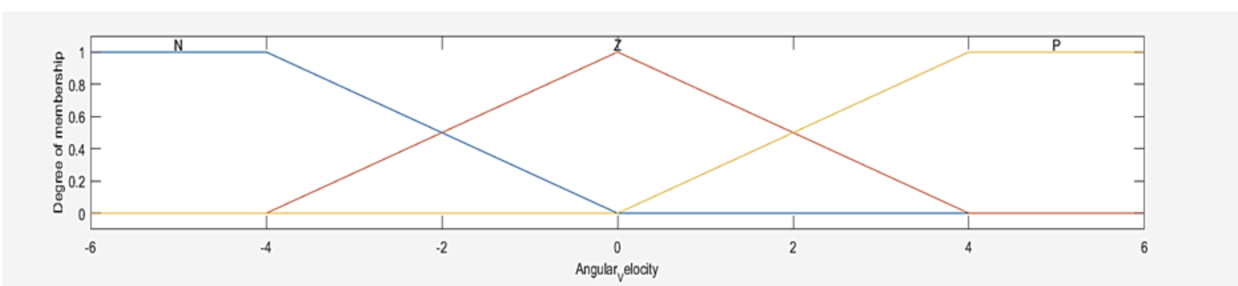


Fig. 6 Input x2 partitioned

Step 2: In contrast to 3 term sets for the inputs to the fuzzy controller (the states x1, x2),we decide to use 5 term sets for the output of the controller, u(k), designated as negative Big (NB), Negative (N), Zero (Z), Positive (P), Positive Big (PB) respectively on its universe, which is $-10 \leq u(k) \leq 10$. The membership functions of the 5 term sets are shown in Fig. 7

Step 3: We construct nine rules in a 3 × 3 Fuzzy Associated Memory (FAM) Table 1 system, which would involve θ and θ^. (θ dot) for balancing the inverted pendulum in the vertical position. The Rule numbers are indicated in the parenthesis in the table cells along with the control actions,

Step 4: We shall now prepare a simulation of the controller using the rules in the FAM table 1 starting with initial conditions:

$$X1(0) = 1° \text{ and } x2(0) = 0 \text{ dps.}$$

We shall employ graphical methods for computing controller outputs using the rules and the plant equation (12) and (13). The final controller outputs are summarized for Few discrete steps $0 \leq k \leq 12$ in Table 3.

In each discrete step of simulation, we will find the firing strengths of the rules, which yields the controller output by the rules in the FAM Table 1. The control action u(k) is computed by using Mamdani implication. The membership function is defuzzified for the control action using the center of area or centroid method. The recursive difference equation is then solved for new values of x1 and x2 to be used as the initial conditions to the next discrete step of the recursive solution.

With reference to Figs. 5 and 6, we note that with singleton initial conditions for X1(0) = 1° and x2(0) = 0 dps respectively, rules 1 and 2 in Table 1. are activated (see also Fig. 8)

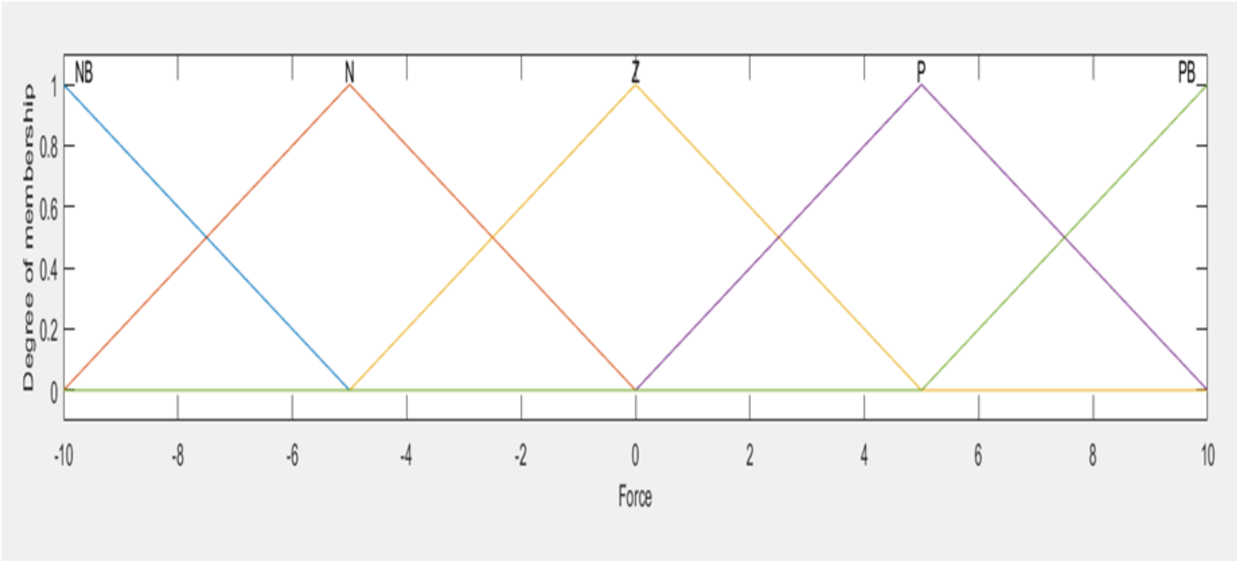
Rule 1 : If x1 is Z and x2 is Z then u = Z, So, the firing strength,

$$\alpha\,\text{^}1= \min (0.5, 1) = 0.5 \; ; \; u = 0.5(Z)$$

Rule 2: If x1 is P and x2 is Z then u = P, with firing strength,

$$\alpha\,\text{^}2=\min (0.5, 1) = 0.5 \; ; \; u = 0.5(P)$$

The firing of the rules are graphically depicted in Fig. 8 In part (c) of Fig. 8, the union of the clipped fuzzy consequences are shown for the control variable, u. The final form with the defuzzified control value, is shown at the bottom of Fig. 8 (c) The defuzzified value of u may be computed by using Equation (6) after quantization of the domain u from -10 to + 10 , which yields u* = 2.50.



Output Membership Functions

Fig. 7: Output variable u, partitioned

Table 1. FAM table

x_1 \ x_2	P	Z	N
P	PB (3)	P (6)	Z (9)
Z	P (2)	Z (5)	N (8)
N	Z (1)	N (4)	NB (7)

This completes the first discrete step of the simulation. Now taking the value of the defuzzified control variable, u* = 2.50. The initial conditions for the next iteration are found, with the help of state equations (12)and (13) as:

$$X1(1) = x1(0) + x2(0) = 1 + 0 = 1$$

$$X2(1) = x1(0) + x2(0) – u(0) = 1 + 0 – 2.5 = – 1.50$$

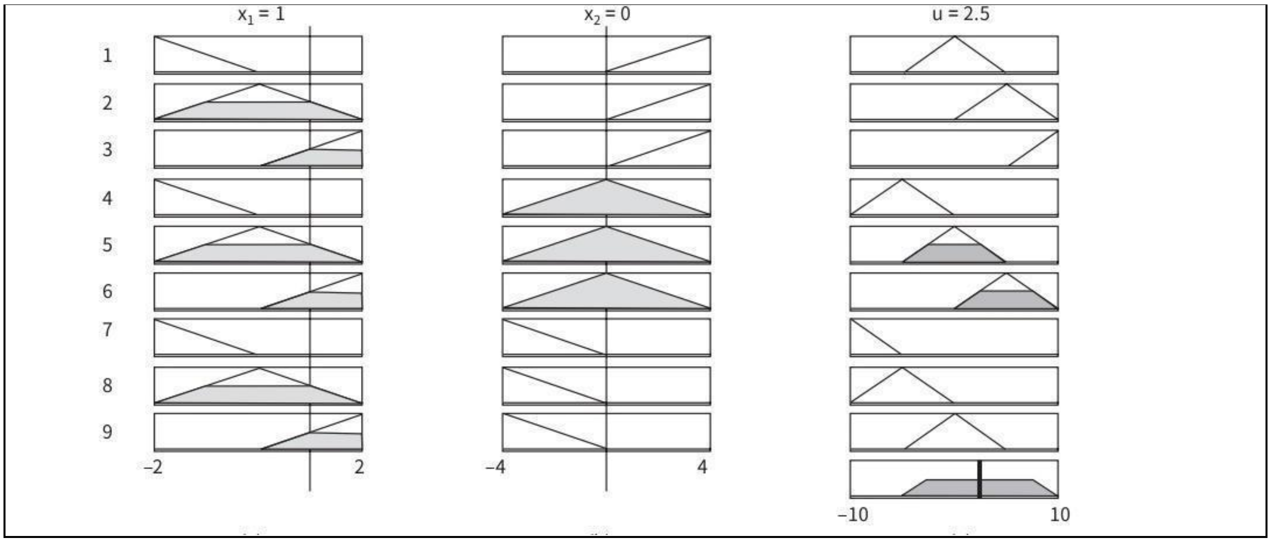
So, the initial conditions for the second discrete step are x1(1) = 1 and x2(1) = – 1.50,respectively and are shown graphically in Fig. 9(a) and 9(b). From Table 9, we note that for these initial conditions, Rule 1, Rule 2,Rule 3 and Rule 4 are fired. The Rules along with their firing strengths α's are shown below:

Rule 1: If x1 is Z and x2 is Z then u is Z, α1 = min (0.5, 0.625) = 0.5, u = 0.5(Z)

Rule 2: If x1 is P and x2 is Z then u is P, α2 = min (0.5, 0.625) = 0.5, u = 0.5(P)

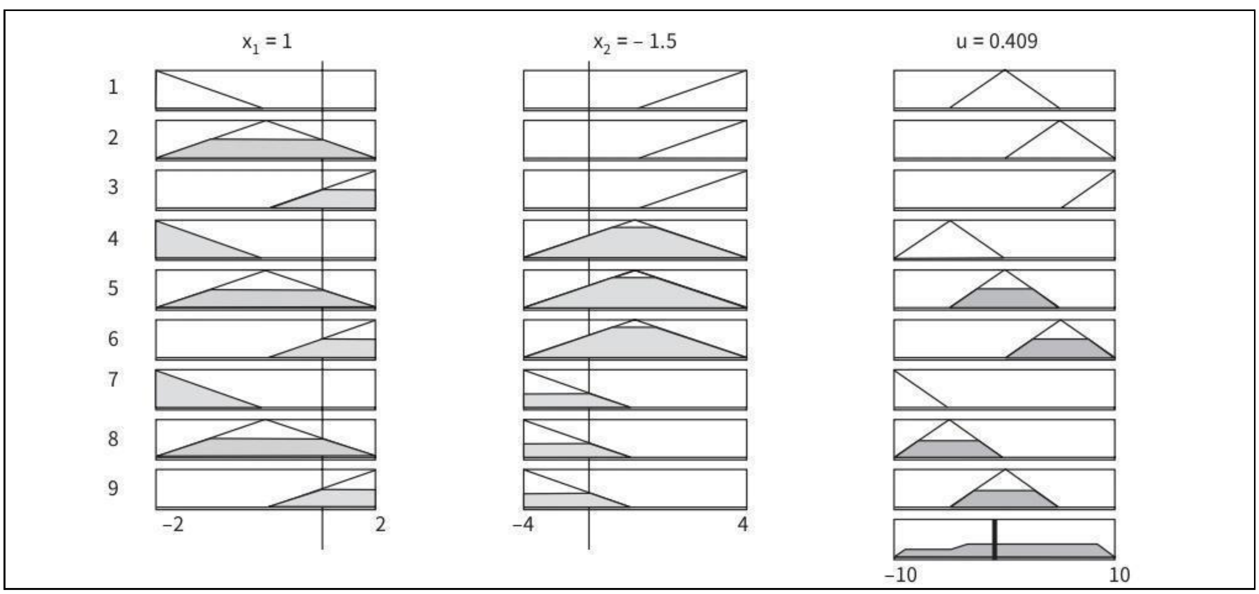
Rule 3: If x1 is Z and x2 is N then u is N, α3 = min (0.5, 0.375) = 0.375, u = 0.375(N)

Rule 4: If x1 is P and x2 is N then u is Z, α4 = min (0.5, 0.375) = 0.375, u = 0.375(Z)



(a) (b) (c)

Fig.8. Firing of rules with initial values of x1(0)=1 and X2(0) = 0 and the defuzzified output u = 2.5



(a) (b) (c).

Fig. 9. Firing of rules with initial values of x1(1) =1 and x2(1) = – 1.5 and the defuzzified output u = 0.409

The union of the fuzzy consequents and the resulting defuzzified output are shown at the bottom of Fig. 9(c).The defuzzified value (using Equation 6) is u* = 0.409

We now use u = 0.409 to find the initial conditions for the third discrete iterative step.

X1(2) = x1(1) + x2(1) = 1 – 1.5 = – 0.50

$X2(2) = x1(1) + x2(1) – u(t) = 1 – 1.5 – 0.409 = – 0.909$

Thus, with initial conditions x1(2) = – 0.50 and x2(2) = – 0.909, we can proceed as before using Table 1. and graphically compute the defuzzified controller output as u* = – 1.65. The results up to 12 cycles are shown in tabular form in Table

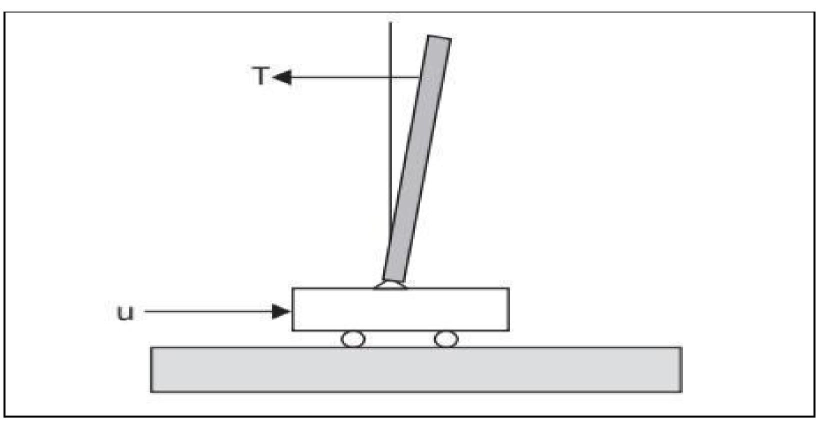
Table 2: FAM table

k	$x_1(k)$	$x_2(k)$	$u(k)$
0	1	0	2.500
1	1	– 1.500	0.409
2	– 0.500	– 0.909	– 1.650
3	– 1.409	0.241	– 2.840
4	– 1.168	1.672	– 0.501
5	0.504	1.005	1.700
6	1.509	– 0.191	3.150
7	1.318	– 1.832	0.670
8	– 0.514	– 1.184	– 1.900
9	– 1.698	0.202	– 3.560
10	– 1.496	2.064	– 0.875
11	0.568	1.443	2.210
12	2.011	– 0.199	4.650

Fig. 10.Initial conditions X1(0) = 1 0, x2(0) = 0 dps, U* = 2.5

3.8 Feedback Network

The angle sensor design proved to be relatively simple. A single-turn potentiometer was deemed adequate since the pendulum should never exceed more than 15° in either direction. Due to this decision, the possible extension of having the pendulum start in the down position was scrapped. Mounting the angle sensor was accounted for in the mechanical system design.



The position sensor required more thought. A 10-turn potentiometer was used and mounted onto a block of wood such that the shaft of the potentiometer was inserted into the hollow cylinder used to mount the sprocket onto the motor. The shaft of the potentiometer was smaller than the hole, so duct tape was wrapped around the shaft to make it the correct size. A part could have been machined for this, but the duct tape prevented possible damage to the potentiometer during the testing period when the sprocket could turn more times than allowed by the potentiometer. Thus, the duct tape was essentially a clutch that would release when the potentiometer was at its maximum number of turns.

Once the sensors were mounted in place, software was necessary to sample the voltage levels of the potentiometers and then change them into the correct values given conversion factors. These conversion factors were calculated to determine the relationship between the number of turns relative to either the angle or the displacement. Ideally this software would be able to sample at a very high speed to give the most accurate measurements.

3.8.1 The System Dynamics

Now that the design process is outlined, the final system model, state space representation and controller design will now be described in detail. The actual design of the controller took a few steps. First the system will need to be modeled with the desired assumptions such that the system is linear and simple enough to design a controller for. From that, the model can then be converted into state space form, which can then be used to design a controller using the LQR cost equation.

Beginning with the simplified free-body diagram of the cart and pendulum controlled by a generic control effort as shown in Figure 6 (which is Figure 1 redrawn for reader convenience), the dynamics of the system can be modeled using Newton's Laws of motion

The dynamics of the pendulum will be described first. The sum of the forces around the rotational point is a follows,

$$I\ddot{\theta} = \left(\sum Fy_{pendulum}\right)\cdot\frac{L}{2}\cdot\sin\theta - \left(\sum Fx_{pendulum}\right)\cdot\frac{L}{2}\cdot\cos\theta \qquad (14)$$

With

$$\sum Fx_{pendulum} = M_{pendulum}\cdot\ddot{q}_P \qquad (15)$$

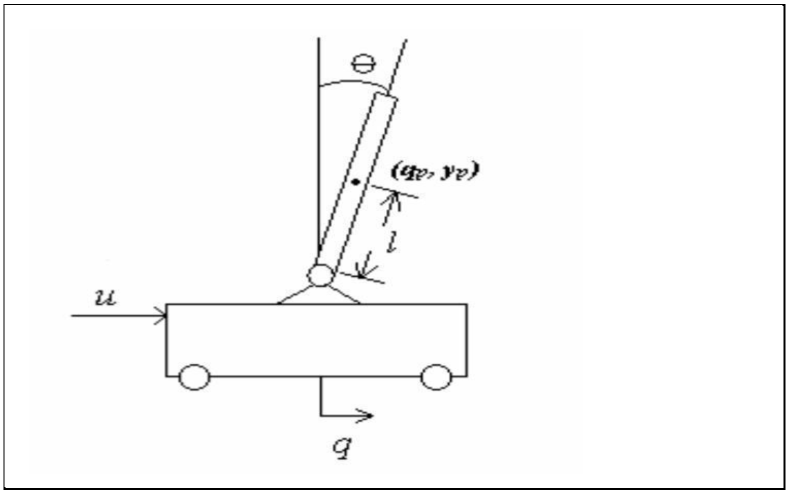


Fig.11. Basic Diagram of a cart with a pendulum and a generic force being applied to the system.

And

$$\sum Fy_{pendulum} = M_{pendulum} \cdot g + M_{pendulum} \cdot \ddot{y}_P . \qquad (16)$$

Expanding out the sum of the forces in the x and y directions of the pendulum by substituting in for qP and yP yields,

$$\sum Fx_{pendulum} = M_{pendulum} \cdot \frac{\partial^2}{\partial t^2}\left(q + \frac{L}{2} \cdot \sin\ \theta \right) \qquad (17)$$

And

$$\sum Fy_{pendulum} = M_{pendulum} \cdot g + M_{pendulum} \cdot \frac{\partial^2}{\partial t^2}\left(\frac{L}{2} \cdot \cos\ \theta \right). \qquad (18)$$

Simplifying these equations through linearization based on the assumption that θ will remain small thus sin θ = θ and cos θ = 1 yields,

$$\sum Fx_{pendulum} = M_{pendulum} \cdot \frac{\partial^2}{\partial t^2}\left(q + \frac{L}{2} \cdot \theta \right) = M_{pendulum} \cdot \ddot{q} + M_{pendulum} \cdot \frac{L}{2} \cdot \ddot{\theta} \qquad (19)$$

And

$$\sum Fy_{pendulum} = M_{pendulum} \cdot g + M_{pendulum} \cdot \frac{\partial^2}{\partial t^2}\left(\frac{L}{2}\right) = M_{pendulum} \cdot g \tag{20}$$

And

$$I\ddot{\theta} = \left(\sum Fy_{pendulum}\right)\cdot\frac{L}{2}\cdot\theta - \left(\sum Fx_{pendulum}\right)\cdot\frac{L}{2}. \tag{21}$$

With these simplifications, Eq. 19 and Eq.20 can be substituted into Eq. 21 as follows,

$$I\ddot{\theta} = \left(M_{pendulum} \cdot g\right)\cdot\frac{L}{2}\cdot\theta - \left(M_{pendulum} \cdot \ddot{q} + M_{pendulum} \cdot \frac{L}{2}\cdot\ddot{\theta}\right)\cdot\frac{L}{2}. \tag{22}$$

Reduction yields

$$I\ddot{\theta} = M_{pendulum} \cdot g \cdot \frac{L}{2}\cdot\theta - M_{pendulum} \cdot \frac{L}{2}\cdot\ddot{q} - M_{pendulum} \cdot \frac{L^2}{4}\cdot\ddot{\theta}. \tag{23}$$

Eq.(23) can be used to describe the dynamics of the pendulum. Since the cart dynamics are simple, the system dynamics will be developed at the same time. Writing down the sum of the forces in x and y directions yields,

$$\sum Fx_{cart} = M_{chain} \cdot \ddot{q} + M_{cart} \cdot \ddot{q} - u + \sum Fx_{pendulum} \tag{24}$$

And

$$\sum Fy_{cart} = 0. \tag{25}$$

Substituting in the dynamics of the pendulum gives,

$$\sum Fx_{cart} = M_{chain} \cdot \ddot{q} + M_{cart} \cdot \ddot{q} - u + M_{pendulum} \cdot \ddot{q} + M_{pendulum} \cdot \frac{L}{2}\cdot\ddot{\theta} = 0 \tag{26}$$

And

$$\sum Fx_{cart} = \left(M_{chain} + M_{cart} + M_{pendulum}\right)\cdot \ddot{q} + M_{pendulum} \cdot \frac{L}{2}\cdot \ddot{\theta} - u = 0\,. \qquad (27)$$

Let M = (M chain + M cart + M pendulum) in order to make the equations easier to read. Substituting this in gives,

$$\sum Fx_{cart} = M \cdot \ddot{q} + M_{pendulum} \cdot \frac{L}{2}\cdot \ddot{\theta} - u = 0\,. \qquad (28)$$

Rearranging terms shows that

$$M \cdot \ddot{q} + M_{pendulum} \cdot \frac{L}{2}\cdot \ddot{\theta} = u\,. \qquad (29)$$

This simple equation can be used to solve for q^..(Q double dot) and θ ^..(θ double dot) before substituting back into the main equation for the dynamics of the pendulum. Two equations can be developed from this: one for q^..(q double dot)and one for θ ^..(θ double dot)which are in terms of q and θ. This will allow for an easy conversion into state Space. Now substituting in for q yields,

$$\left(I + M_{pendulum} \cdot \frac{L^2}{4}\right)\cdot \ddot{\theta} = M_{pendulum}\cdot g \cdot \frac{L}{2}\cdot \theta - M_{pendulum}\cdot \frac{L}{2}\cdot \ddot{q} \qquad (30)$$

And

$$\left(I + M_{pendulum} \cdot \frac{L^2}{4} - \frac{M_{pendulum}^{\ 2}}{M}\cdot \frac{L^2}{4}\right)\cdot \ddot{\theta} = M_{pendulum}\cdot g \cdot \frac{L}{2}\cdot \theta - M_{pendulum}\cdot \frac{L}{2}\cdot \frac{1}{M}\cdot u\,. \qquad (31)$$

Substituting for θ ^..(θ double dot) produces

$$\left(I + M_{pendulum} \cdot \frac{L^2}{4} \right) \cdot \left(u \cdot \frac{2}{L} \cdot \frac{1}{M_{pendulum}} - M \cdot \frac{2}{L} \cdot \frac{1}{M_{pendulum}} \cdot \ddot{q} \right) =$$
$$M_{pendulum} \cdot g \cdot \frac{L}{2} \cdot \theta - M_{pendulum} \cdot \frac{L}{2} \cdot \ddot{q} \,. \qquad (32)$$

Simplifying,

$$\left[\left(I + M_{pendulum} \cdot \frac{L^2}{4} \right) \cdot \left(M \cdot \frac{2}{L} \cdot \frac{1}{M_{pendulum}} \right) + M_{pendulum} \cdot \frac{L}{2} \right] \cdot \ddot{q} =$$
$$-\left(I + M_{pendulum} \cdot \frac{L^2}{4} \right) \cdot u \cdot \frac{2}{L} \cdot \frac{1}{M_{pendulum}} + M_{pendulum} \cdot g \cdot \frac{L}{2} \cdot \theta \quad . \qquad (33)$$

These last two equations can then be used for describing the dynamics of the system.

4. SOFTWARE REQUIREMENT

4.1 MATLAB

Cleve Moler, the chairman of the computer science department at the University of New Mexico, started developing MATLAB in the late 1970s. He designed it to give his students access to LINPACK and EISPACK without them having to learn Fortran. It soon spread to other universities and found a strong audience within the applied mathematics community. Jack Little, an engineer, was exposed to it during a visit Moler made to Stanford University in 1983. Recognizing its commercial potential, he joined with Moler and Steve Bangert. They rewrote MATLAB in C and founded MathWorks in 1984 to continue its development.

MATLAB (matrix laboratory) is a multi-paradigm numerical computing environment and proprietary programming language developed by MathWorks. MATLAB allows matrix manipulations, plotting of functions and data, implementation of algorithms, creation of user interfaces, and interfacing with programs written in other languages, including C, C++, C#, Java, Fortran, and Python. Although MATLAB is intended primarily for numerical computing, an optional toolbox uses the MuPAD symbolic engine, allowing access to symbolic computing abilities. An additional package, Simulink, adds graphical multi-domain simulation and model-based design for dynamic and embedded systems. As of 2018, MATLAB has more than 3 million users worldwide.MATLAB users come from various backgrounds of engineering, science, and economics.

4.2 Get Started with Fuzzy Logic Toolbox

Fuzzy Logic Toolbox™ provides MATLAB® functions, apps, and a Simulink® block for analyzing, designing, and simulating systems based on fuzzy logic. The product guides you through the steps of designing fuzzy inference systems. Functions are provided for many common methods, including fuzzy clustering and adaptive neuro-fuzzy learning.

The toolbox lets you model complex system behaviors using simple logic rules, and then implement these rules in a fuzzy inference system. You can use it as a stand-alone fuzzy inference engine. Alternatively, you can use fuzzy inference blocks in Simulink and simulate the fuzzy systems within a comprehensive model of the entire dynamic system

4.3 Fuzzy Inference System Modeling

Fuzzy inference is the process of formulating input/output mappings using fuzzy logic. Fuzzy Logic Toolbox™ software provides tools for creating:

- Type-1 or interval type-2 Mamdani fuzzy inference systems
- Type-1 or interval type-2 Sugeno fuzzy inference systems

Trees of interconnected fuzzy inference systems

5. Matlab Code

5.1 Fuzzy System Design

```
clc;
clearvars;
A = newfis('Inverted Pendulum','mamdani','min','max','min','max','bisector');

%%defining input 1
A = addvar(A,'input','Angle',[-4 4]);
A = addmf(A,'input',1,'N','trapmf',[-4 -4 -2 0]);
A = addmf(A,'input',1,'Z','trimf',[-2 0 2]);
A = addmf(A,'input',1,'P','trapmf',[0 2 4 4]);
subplot(311); plotmf(A,'input',1)

%%defining input 2
A = addvar(A,'input','Angular_Velocity',[-6 6]);
A = addmf(A,'input',2,'N','trapmf',[-6 -6 -4 0]);
A = addmf(A,'input',2,'Z','trimf',[-4 0 4]);
A = addmf(A,'input',2,'P','trapmf',[0 4 6 6]);
subplot(312);
plotmf(A,'input',2)

%%defining output
A = addvar(A,'output','Force',[-10 10]);
A = addmf(A,'output',1,'NB','trimf',[-15 -10 -5]);
A = addmf(A,'output',1,'N','trimf',[-10 -5 0]);
A = addmf(A,'output',1,'Z','trimf',[-5 0 5]);
A = addmf(A,'output',1,'P','trimf',[0 5 10]);
A = addmf(A,'output',1,'PB','trimf',[5 10 15]);
subplot(313);
plotmf(A,'output',1)
```

```
%%defining the rules
A.rule = [];
rule1 = [1 3 3 1 1];
rule2 = [2 3 4 1 1];
rule3 = [3 3 5 1 1];
rule4 = [1 2 2 1 1];
rule5 = [2 2 3 1 1];
rule6 = [3 2 4 1 1];
rule7 = [1 1 1 1 1];
rule8 = [2 1 2 1 1];
rule9 = [3 1 3 1 1];

ruleList = [rule1;rule2;rule3;rule4;rule5;rule6;rule7;rule8;rule9];

A = addrule(A,ruleList);

writefis(A,'Inverted_Pendulam_Fuzzy');

A = showrule(A,1:9);
```

5.2 Evaluating and Checking Iterations:

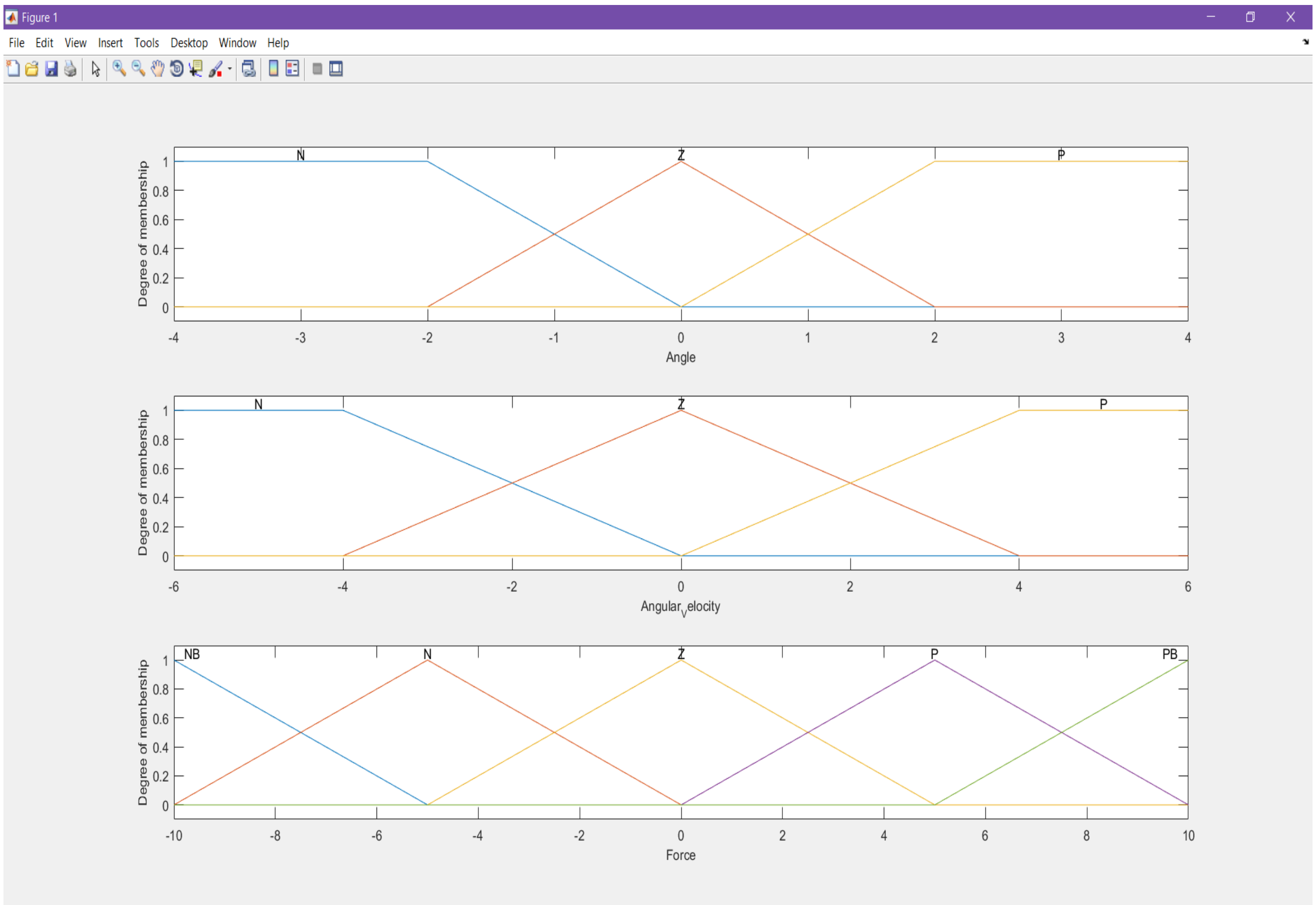
```
ruleview('Inverted_Pendulam_Fuzzy');
clc;
clearvars;
fis = readfis('Inverted_Pendulam_Fuzzy');
x1 = 1;
x2 = 0;

out1 = [];
for k = 0:10000
    u = evalfis([x1 x2],fis);
    out1 = [out1;k x1 x2 u];
    x1 = x1 + x2;
    x2 = x1 - u;
    if x1 >= -0.0005 && x1 <= 0.0005
        break
    end

end
```

5.3 Plotting Comparison Graph:

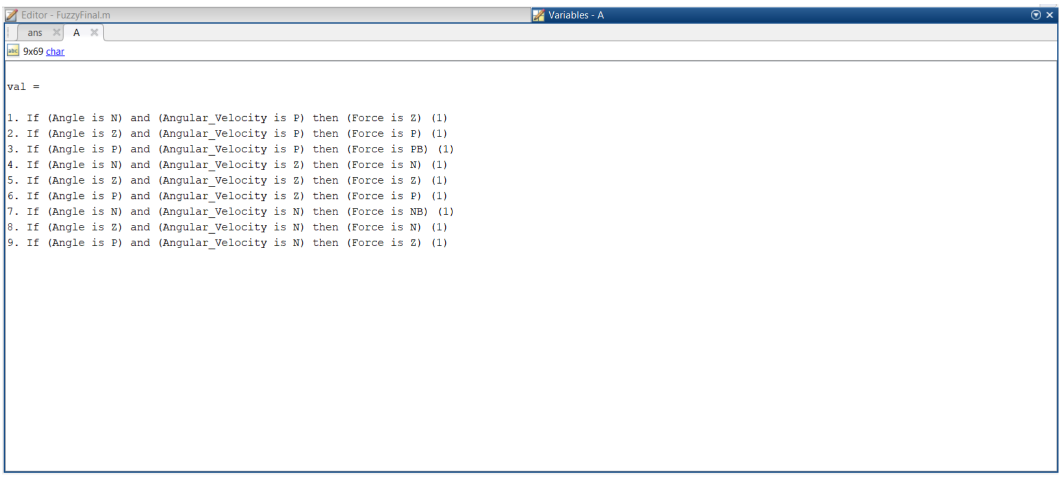
```
clc;
clearvars;
DefuzzMethod = {'Centroid';'MOM';'Bisector';'lom';'som'};
y = [156 122 6325 10000 10000]';
bar(y);
set(gca,'xticklabel',DefuzzMethod);
xlabel('Defuzzification methods');ylabel('No. of iterations(K)')
title('Comparison between Defuzzification Method')
```

6. OUTPUT

6.1 INPUT AND OUTPUT MEMBERSHIP FUNCTION



6.2 RULE VIEWER



Editor - FuzzyFinal.m | Variables - A

ans | A

9x69 char

```
val =

1. If (Angle is N) and (Angular_Velocity is P) then (Force is Z) (1)
2. If (Angle is Z) and (Angular_Velocity is P) then (Force is P) (1)
3. If (Angle is P) and (Angular_Velocity is P) then (Force is PB) (1)
4. If (Angle is N) and (Angular_Velocity is Z) then (Force is N) (1)
5. If (Angle is Z) and (Angular_Velocity is Z) then (Force is Z) (1)
6. If (Angle is P) and (Angular_Velocity is Z) then (Force is P) (1)
7. If (Angle is N) and (Angular_Velocity is N) then (Force is NB) (1)
8. If (Angle is Z) and (Angular_Velocity is N) then (Force is N) (1)
9. If (Angle is P) and (Angular_Velocity is N) then (Force is Z) (1)
```

FIG. (i)

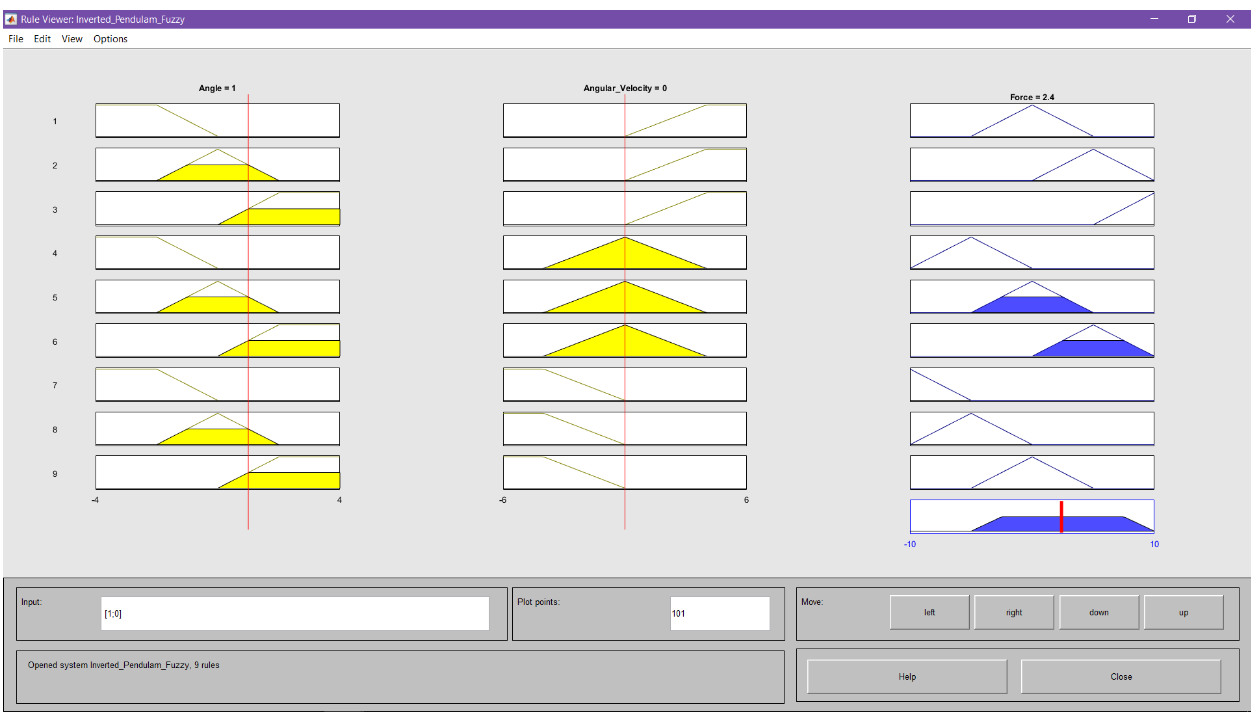
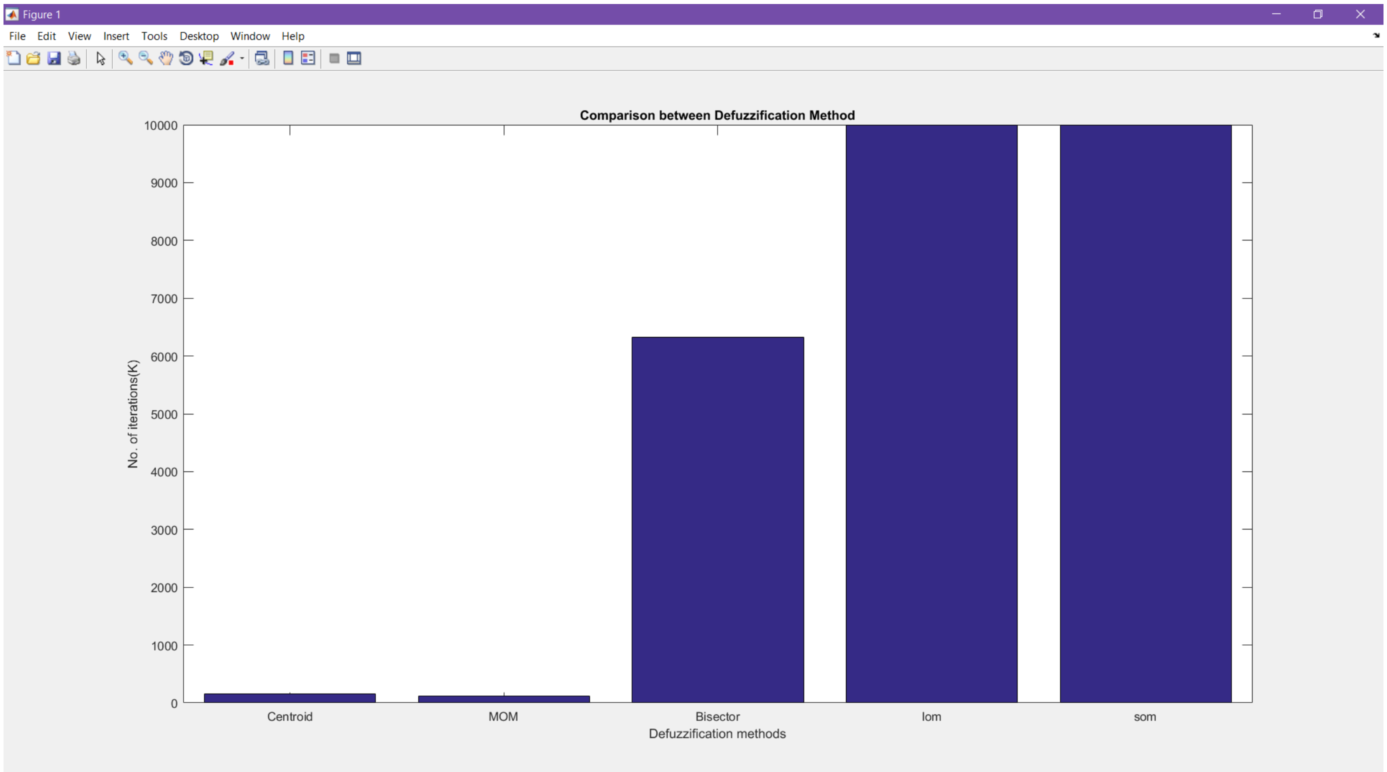


FIG (ii)

6. 3 COMPARISON BETWEEN DEFUZZIFICATION METHOD



6.4 ITERATION FOR DIFFERENT INITIAL CONDITIONS

	Initial Conditions		No. of iterations				
S no.	**Angle**	**Angular velocity**	**Centroid**	**MOM**	**LOM**	**SOM**	**Bisector**
1	1	0	155	121	>10,000	>10,000	6324
2	2	1	8976	122	>10,000	>10,000	6124
3	-1.2	0	>10,000	321	>10,000	>10,000	2040
4	2	-3.2	>10,000	190	>10,000	>10,000	5054
5	-1.5	-0.5	>10,000	118	>10,000	12	384
6	-2.1	-0.7	8892	62	>10,000	10	>10,000
7	-0.6	2.3	>10,000	812	>10,000	>10,000	>10,000
8	0	0	0	0	0	0	0
9	0	1.3	>10,000	237	>10,000	>10,000	540
10	0	-2.7	>10,000	235	>10,000	10	177

7. CONCLUSION

Overall the project can be considered a success. Despite the fact that the main goal of the project was not reached- the cart was unable to balance the pendulum for an extended period of time- the foundation is laid for future research. Many requirements were met such that a working mechanical system was developed along with a control circuit and an accurate feedback network. From this, it seems that with the right modifications, it would be quite feasible to balance a pendulum for an extended period of time. The system would need to be modified to reduce some of the imperfections and increase the torque and speed of the motor. A variety of control designs could be implemented using some of the other suggested methods in this report.

So we conclude that **MOM(Mean of Maximum)** is the most efficient defuzzification method with minimum no. of iterations in this case.

We also conclude that when **X2(Angular Velocity) < 0** then **SOM(Smallest of Maximum**) is more efficient than any other defuzzification method.

The most beneficial aspect of this project was that it gave exposure to a full system design. The experience gained from developing each of the subsystems given the constraints they imposed on each other and then integrating them together proved to be invaluable

8. ACKNOWLEDGEMENT

We take this opportunity to thank our mentor and guide Prof Anup Kumar Mallik ,who has enlightened us with such an amazing concept of fuzzy control systems ,he clarified all our doubts and supplied us with all the necessary study materials we would require in order to gain a considerable amount of knowledge in this field so that we are able to do further analysis on our subject .

9. REFERENCES

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