# Making Our Own Types and Typeclasses

In the previous chapters, we covered some existing Haskell types and typeclasses. In this chapter, we'll learn how to make our own and how to put them to work!

## Algebraic data types intro

So far, we've run into a lot of data types. Bool, Int, Char, Maybe, etc. But how do we make our own? Well, one way is to use the **data** keyword to define a type. Let's see how the **Bool** type is defined in the standard library.

data Bool = False | True

data means that we're defining a new data type. The part before the = denotes the type, which is **Bool**. The parts after the = are value constructors. They specify the different values that this type can have. The | is read as or. So we can read

this as: the **Bool** type can have a value of **True** or **False**. Both the type name and the value constructors have to be capital cased.

In a similar fashion, we can think of the Int type as being defined like this:

#### data Int = -2147483648 | -2147483647 | ... | -1 | 0 | 1 | 2 | ... | 2147483647



The first and last value constructors are the minimum and maximum possible values of Int. It's not actually defined like this, the ellipses are here because we omitted a heapload of numbers, so this is just for illustrative purposes.

Now, let's think about how we would represent a shape in Haskell. One way would be to use tuples. A circle could be denoted as (43.1, 55.0, 10.4) where the first and second fields are the coordinates of the circle's center and the third field is the radius. Sounds OK, but those could also represent a 3D vector or anything else. A better solution would be to make our own type to represent a shape. Let's say that a shape can be a circle or a rectangle. Here it is:

data Shape = Circle Float Float Float | Rectangle Float Float Float Float

Now what's this? Think of it like this. The Circle value constructor has three fields, which take floats. So when we write a

value constructor, we can optionally add some types after it and those types define the values it will contain. Here, the first two fields are the coordinates of its center, the third one its radius. The **Rectangle** value constructor has four fields which accept floats. The first two are the coordinates to its upper left corner and the second two are coordinates to its lower right one.

Now when I say fields, I actually mean parameters. Value constructors are actually functions that ultimately return a value of a data type. Let's take a look at the type signatures for these two value constructors.

```
ghci> :t Circle
Circle :: Float -> Float -> Shape
ghci> :t Rectangle
Rectangle :: Float -> Float -> Float -> Float -> Shape
```

Cool, so value constructors are functions like everything else. Who would have thought? Let's make a function that takes a shape and returns its surface.

```
surface :: Shape -> Float
surface (Circle _ _ r) = pi * r ^ 2
surface (Rectangle x1 y1 x2 y2) = (abs $ x2 - x1) * (abs $ y2 - y1)
```

The first notable thing here is the type declaration. It says that the function takes a shape and returns a float. We couldn't write a type declaration of Circle -> Float because Circle is not a type, Shape is. Just like we can't write a function with a type declaration of True -> Int. The next thing we notice here is that we can pattern match against constructors. We pattern matched against constructors before (all the time actually) when we pattern matched against values like [] or False or 5,

only those values didn't have any fields. We just write a constructor and then bind its fields to names. Because we're interested in the radius, we don't actually care about the first two fields, which tell us where the circle is.

```
ghci> surface $ Circle 10 20 10
314.15927
ghci> surface $ Rectangle 0 0 100 100
10000.0
```

Yay, it works! But if we try to just print out Circle 10 20 5 in the prompt, we'll get an error. That's because Haskell doesn't know how to display our data type as a string (yet). Remember, when we try to print a value out in the prompt, Haskell first runs the show function to get the string representation of our value and then it prints that out to the terminal. To make our Shape type part of the **Show** typeclass, we modify it like this:

```
data Shape = Circle Float Float Float | Rectangle Float Float Float Float deriving (Show)
```

We won't concern ourselves with deriving too much for now. Let's just say that if we add deriving (Show) at the end of a data declaration, Haskell automagically makes that type part of the **Show** typeclass. So now, we can do this:

```
ghci> Circle 10 20 5
Circle 10.0 20.0 5.0
ghci> Rectangle 50 230 60 90
Rectangle 50.0 230.0 60.0 90.0
```

Value constructors are functions, so we can map them and partially apply them and everything. If we want a list of concentric circles with different radii, we can do this.

```
ghci> map (Circle 10 20) [4,5,6,6]
[Circle 10.0 20.0 4.0, Circle 10.0 20.0 5.0, Circle 10.0 20.0 6.0, Circle 10.0 20.0 6.0]
```

Our data type is good, although it could be better. Let's make an intermediate data type that defines a point in two-dimensional space. Then we can use that to make our shapes more understandable.

```
data Point = Point Float Float deriving (Show)
data Shape = Circle Point Float | Rectangle Point Point deriving (Show)
```

Notice that when defining a point, we used the same name for the data type and the value constructor. This has no special meaning, although it's common to use the same name as the type if there's only one value constructor. So now the Circle has two fields, one is of type Point and the other of type Float. This makes it easier to understand what's what. Same goes for the rectangle. We have to adjust our **surface** function to reflect these changes.

```
surface :: Shape -> Float
surface (Circle r) = pi * r \wedge 2
surface (Rectangle (Point x1 y1) (Point x2 y2)) = (abs $x2 - x1) * (abs $y2 - y1)
```

The only thing we had to change were the patterns. We disregarded the whole point in the circle pattern. In the rectangle pattern, we just used a nested pattern matching to get the fields of the points. If we wanted to reference the points themselves for some reason, we could have used as-patterns.

```
ghci> surface (Rectangle (Point 0 0) (Point 100 100))
10000.0
ghci> surface (Circle (Point 0 0) 24)
1809.5574
```

How about a function that nudges a shape? It takes a shape, the amount to move it on the x axis and the amount to move it on the y axis and then returns a new shape that has the same dimensions, only it's located somewhere else.

```
nudge :: Shape -> Float -> Float -> Shape
nudge (Circle (Point x y) r) a b = Circle (Point (x+a) (y+b)) r
nudge (Rectangle (Point x1 y1) (Point x2 y2)) a b = Rectangle (Point (x1+a) (y1+b)) (Point (x2+a)
```

Pretty straightforward. We add the nudge amounts to the points that denote the position of the shape.

```
ghci> nudge (Circle (Point 34 34) 10) 5 10
Circle (Point 39.0 44.0) 10.0
```

If we don't want to deal directly with points, we can make some auxilliary functions that create shapes of some size at the zero coordinates and then nudge those.

```
baseCircle :: Float -> Shape
baseCircle r = Circle (Point 0 0) r
baseRect :: Float -> Float -> Shape
baseRect width height = Rectangle (Point 0 0) (Point width height)
```

```
ghci> nudge (baseRect 40 100) 60 23
Rectangle (Point 60.0 23.0) (Point 100.0 123.0)
```

You can, of course, export your data types in your modules. To do that, just write your type along with the functions you are exporting and then add some parentheses and in them specify the value constructors that you want to export for it, separated by commas. If you want to export all the value constructors for a given type, just write . . .

If we wanted to export the functions and types that we defined here in a module, we could start it off like this:

```
module Shapes
( Point(..)
 Shape(..)
  surface
 nudge
 baseCircle
  baseRect
 where
```

By doing Shape (...), we exported all the value constructors for Shape, so that means that whoever imports our module can make shapes by using the Rectangle and Circle value constructors. It's the same as writing Shape (Rectangle, Circle).

We could also opt not to export any value constructors for **Shape** by just writing **Shape** in the export statement. That way, someone importing our module could only make shapes by using the auxilliary functions baseCircle and baseRect. Data. Map uses that approach. You can't create a map by doing Map. Map [(1,2), (3,4)] because it doesn't export that value constructor. However, you can make a mapping by using one of the auxilliary functions like Map.fromList. Remember, value constructors are just functions that take the fields as parameters and return a value of some type (like Shape) as a result. So when we choose not to export them, we just prevent the person importing our module from using those functions, but if some other functions that are exported return a type, we can use them to make values of our custom data types.

Not exporting the value constructors of a data types makes them more abstract in such a way that we hide their implementation. Also, whoever uses our module can't pattern match against the value constructors.

## **Record syntax**

OK, we've been tasked with creating a data type that describes a person. The info that we want to store about that person is: first name, last name, age, height, phone number, and favorite ice-cream flavor. I don't know about you, but that's all I ever want to know about a person. Let's give it a go!



```
data Person = Person String String Int Float String String deriving (Show)
```

O-kay. The first field is the first name, the second is the last name, the third is the age and so on. Let's make a person.

```
ghci> let guy = Person "Buddy" "Finklestein" 43 184.2 "526-2928" "Chocolate"
ghci> guy
Person "Buddy" "Finklestein" 43 184.2 "526-2928" "Chocolate"
```

That's kind of cool, although slightly unreadable. What if we want to create a function to get seperate info from a person? A function that gets some person's first name, a function that gets some person's last name, etc. Well, we'd have to define them kind of like this.

```
firstName :: Person -> String
firstName (Person firstname _ _ _ _ ) = firstname
lastName :: Person -> String
lastName (Person _ lastname _ _ _ _ ) = lastname
age :: Person -> Int
age (Person _ _ age _ _ _) = age
height :: Person -> Float
height (Person _ _ _ height _ _) = height
phoneNumber :: Person -> String
phoneNumber (Person _ _ _ number _) = number
```

```
flavor :: Person -> String
flavor (Person _ _ _ _ flavor) = flavor
```

Whew! I certainly did not enjoy writing that! Despite being very cumbersome and BORING to write, this method works.

```
ghci> let guy = Person "Buddy" "Finklestein" 43 184.2 "526-2928" "Chocolate"
ghci> firstName guy
"Buddy"
ghci> height guy
184.2
ghci> flavor guy
"Chocolate"
```

There must be a better way, you say! Well no, there isn't, sorry.

Just kidding, there is. Hahaha! The makers of Haskell were very smart and anticipated this scenario. They included an alternative way to write data types. Here's how we could achieve the above functionality with record syntax.

```
data Person = Person { firstName :: String
                     , lastName :: String
                     , age :: Int
                     , height :: Float
                     , phoneNumber :: String
                       flavor :: String
```

#### deriving (Show)

So instead of just naming the field types one after another and separating them with spaces, we use curly brackets. First we write the name of the field, for instance, **firstName** and then we write a double colon :: (also called Paamayim Nekudotayim, haha) and then we specify the type. The resulting data type is exactly the same. The main benefit of this is that it creates functions that lookup fields in the data type. By using record syntax to create this data type, Haskell automatically made these functions: **firstName**, **lastName**, **age**, **height**, **phoneNumber** and **flavor**.

```
ghci> :t flavor
flavor :: Person -> String
ghci> :t firstName
firstName :: Person -> String
```

There's another benefit to using record syntax. When we derive **Show** for the type, it displays it differently if we use record syntax to define and instantiate the type. Say we have a type that represents a car. We want to keep track of the company that made it, the model name and its year of production. Watch.

```
data Car = Car String String Int deriving (Show)
```

```
ghci> Car "Ford" "Mustang" 1967
Car "Ford" "Mustang" 1967
```

If we define it using record syntax, we can make a new car like this.

```
data Car = Car {company :: String, model :: String, year :: Int} deriving (Show)
```

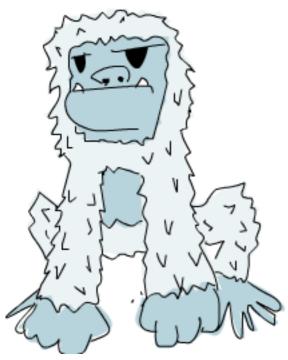
```
ghci> Car {company="Ford", model="Mustang", year=1967}
Car {company = "Ford", model = "Mustang", year = 1967}
```

When making a new car, we don't have to necessarily put the fields in the proper order, as long as we list all of them. But if we don't use record syntax, we have to specify them in order.

Use record syntax when a constructor has several fields and it's not obvious which field is which. If we make a 3D vector data type by doing data Vector = Vector Int Int Int, it's pretty obvious that the fields are the components of a vector. However, in our **Person** and **Car** types, it wasn't so obvious and we greatly benefited from using record syntax.

## Type parameters

A value constructor can take some values parameters and then produce a new value. For instance, the car constructor takes three values and produces a car value. In a similar manner, type constructors can take types as parameters to produce new types. This might sound a bit too meta at first, but it's not that complicated. If you're familiar with templates in C++, you'll see some parallels. To get a clear picture of what type parameters work like in action, let's take a look at how a type we've already met is implemented.



The a here is the type parameter. And because there's a type parameter involved, we call Maybe a type constructor. Depending on what we want this data type to hold when it's not Nothing, this type constructor can end up producing a type of Maybe Int, Maybe Car, Maybe String, etc. No value can have a type of just Maybe, because that's not a type per se, it's a type constructor. In order for this to be a real type that a value can be part of, it has to have all its type parameters filled up.

So if we pass Char as the type parameter to Maybe, we get a type of Maybe Char. The value Just 'a' has a type of Maybe Char, for example.

You might not know it, but we used a type that has a type parameter before we used

Maybe. That type is the list type. Although there's some syntactic sugar in play, the list type takes a parameter to produce a concrete type. Values can have an [Int] type, a [Char] type, a [[String]] type, but you can't have a value that just has a type of [].

Let's play around with the Maybe type.

```
ghci> Just "Haha"
Just "Haha"
ghci> Just 84
```

```
Just 84
ghci> :t Just "Haha"
Just "Haha" :: Maybe [Char]
ghci> :t Just 84
Just 84 :: (Num t) \Rightarrow Maybe t
ghci> :t Nothing
Nothing :: Maybe a
ghci> Just 10 :: Maybe Double
Just 10.0
```

Type parameters are useful because we can make different types with them depending on what kind of types we want contained in our data type. When we do :t Just "Haha", the type inference engine figures it out to be of the type Maybe [Char], because if the a in the Just a is a string, then the a in Maybe a must also be a string.

Notice that the type of **Nothing** is **Maybe a**. Its type is polymorphic. If some function requires a **Maybe Int** as a parameter, we can give it a **Nothing**, because a **Nothing** doesn't contain a value anyway and so it doesn't matter. The **Maybe** a type can act like a Maybe Int if it has to, just like 5 can act like an Int or a Double. Similarly, the type of the empty list is [a]. An empty list can act like a list of anything. That's why we can do [1,2,3] ++ [] and ["ha", "ha", "ha"] ++ [].

Using type parameters is very beneficial, but only when using them makes sense. Usually we use them when our data type would work regardless of the type of the value it then holds inside it, like with our Maybe a type. If our type acts as some kind of box, it's good to use them. We could change our **car** data type from this:

```
data Car = Car { company :: String
```

```
, model :: String
, year :: Int
} deriving (Show)
```

To this:

```
data Car a b c = Car { company :: a
    , model :: b
    , year :: c
    } deriving (Show)
```

But would we really benefit? The answer is: probably no, because we'd just end up defining functions that only work on the Car String Int type. For instance, given our first definition of Car, we could make a function that displays the car's properties in a nice little text.

```
tellCar :: Car -> String
tellCar (Car {company = c, model = m, year = y}) = "This " ++ c ++ " " ++ m ++ " was made in " ++ s
```

```
ghci> let stang = Car {company="Ford", model="Mustang", year=1967}
ghci> tellCar stang
"This Ford Mustang was made in 1967"
```

A cute little function! The type declaration is cute and it works nicely. Now what if Car was Car a b c?

```
tellCar :: (Show a) => Car String String a -> String tellCar (Car {company = c, model = m, year = y}) = "This " ++ c ++ " " ++ m ++ " was made in " ++ s
```

We'd have to force this function to take a Car type of (Show a) => Car String String a. You can see that the type signature is more complicated and the only benefit we'd actually get would be that we can use any type that's an instance of the Show typeclass as the type for c.

```
ghci> tellCar (Car "Ford" "Mustang" 1967)
"This Ford Mustang was made in 1967"
ghci> tellCar (Car "Ford" "Mustang" "nineteen sixty seven")
"This Ford Mustang was made in \"nineteen sixty seven\""
ghci> :t Car "Ford" "Mustang" 1967
Car "Ford" "Mustang" 1967 :: (Num t) => Car [Char] [Char] t
ghci> :t Car "Ford" "Mustang" "nineteen sixty seven"
Car "Ford" "Mustang" "nineteen sixty seven" :: Car [Char] [Char] [Char]
```

In real life though, we'd end up using <code>Car String String Int</code> most of the time and so it would seem that parameterizing the <code>Car</code> type isn't really worth it. We usually use type parameters when the type that's contained inside the data type's various value constructors isn't really that important for the type to work. A list of stuff is a list of stuff and it doesn't matter what the type of that stuff is, it can still work. If we want to sum a list of numbers, we can specify later in the summing function that we specifically want a list of numbers. Same goes for <code>Maybe</code>. <code>Maybe</code> represents an option of either

having nothing or having one of something. It doesn't matter what the type of that something is.



Another example of a parameterized type that we've already met is Map k v from Data. Map. The k is the type of the keys in a map and the v is the type of the values. This is a good example of where type parameters are very useful. Having maps parameterized enables us to have mappings from any type to any other type, as long as the type of the key is part of the **ord** typeclass. If we were defining a mapping type, we could add a typeclass constraint in the *data* declaration:



data (Ord k) 
$$\Rightarrow$$
 Map k v  $\Rightarrow$  ...

However, it's a very strong convention in Haskell to never add typeclass constraints in data declarations. Why? Well, because we don't benefit a lot, but we end up writing more class constraints, even when we don't need them. If we put or don't put the Ord k constraint in the data declaration for Map k v, we're going to have to put the constraint into functions that assume the keys in a map can be ordered. But if we don't put the constraint in the data declaration, we don't have to put (Ord k) => in the type declarations of functions that don't care whether the keys can be ordered or not. An example of such a function is toList, that just takes a mapping and converts it to an associative list. Its type signature is toList :: Map k a -> [(k, a)]. If Map k v had a type constraint in its data declaration, the type for toList would have to be toList :: (Ord k) => Map k a -> [(k, a)], even though the function doesn't do any comparing of keys by order.

So don't put type constraints into data declarations even if it seems to make sense, because you'll have to put them into the function type declarations either way.

Let's implement a 3D vector type and add some operations for it. We'll be using a parameterized type because even though it will usually contain numeric types, it will still support several of them.

```
data Vector a = Vector a a a deriving (Show)

vplus :: (Num t) => Vector t -> Vector t
(Vector i j k) `vplus` (Vector l m n) = Vector (i+l) (j+m) (k+n)

vectMult :: (Num t) => Vector t -> t -> Vector t
(Vector i j k) `vectMult` m = Vector (i*m) (j*m) (k*m)

scalarMult :: (Num t) => Vector t -> Vector t -> t
(Vector i j k) `scalarMult` (Vector l m n) = i*l + j*m + k*n
```

vplus is for adding two vectors together. Two vectors are added just by adding their corresponding components.

scalarMult is for the scalar product of two vectors and vectMult is for multiplying a vector with a scalar. These functions can operate on types of Vector Int, Vector Integer, Vector Float, whatever, as long as the a from Vector a is from the Num typeclass. Also, if you examine the type declaration for these functions, you'll see that they can operate only on vectors of the same type and the numbers involved must also be of the type that is contained in the vectors. Notice that we didn't put a Num class constraint in the data declaration, because we'd have to repeat it in the functions anyway.

Once again, it's very important to distinguish between the type constructor and the value constructor. When declaring a data type, the part before the = is the type constructor and the constructors after it (possibly separated by | 's) are value constructors. Giving a function a type of Vector t t t -> Vector t t t -> t would be wrong, because we have to put

types in type declaration and the vector **type** constructor takes only one parameter, whereas the value constructor takes three. Let's play around with our vectors.

```
ghci> Vector 3 5 8 `vplus` Vector 9 2 8

Vector 12 7 16

ghci> Vector 3 5 8 `vplus` Vector 9 2 8 `vplus` Vector 0 2 3

Vector 12 9 19

ghci> Vector 3 9 7 `vectMult` 10

Vector 30 90 70

ghci> Vector 4 9 5 `scalarMult` Vector 9.0 2.0 4.0

74.0

ghci> Vector 2 9 3 `vectMult` (Vector 4 9 5 `scalarMult` Vector 9 2 4)

Vector 148 666 222
```

#### **Derived instances**

In the <u>Typeclasses 101</u> section, we explained the basics of typeclasses. We explained that a typeclass is a sort of an interface that defines some behavior. A type can be made an **instance** of a typeclass if it supports that behavior. Example: the <u>Int</u> type is an instance of the <u>Eq</u> typeclass because the <u>Eq</u> typeclass defines behavior for stuff that can be equated. And because integers can be equated, <u>Int</u> is a part of the <u>Eq</u> typeclass. The real usefulness comes with the functions that act as the interface for <u>Eq</u>, namely <u>— and /=.</u> If a type is a part of the <u>Eq</u> typeclass, we can use the <u>— functions with values of that type.</u> That's why expressions like <u>4 — 4 and "foo" /= "bar"</u> typecheck.

We also mentioned that they're often confused with classes in languages like Java, Python, C++ and the



like, which then baffles a lot of people. In those languages, classes are a blueprint from which we then create objects that contain state and can do some actions. Typeclasses are more like interfaces. We don't make data from typeclasses. Instead, we first make our data type and then we think about what it can act like. If it can act like something that can be equated, we make it an instance of the Eq typeclass. If it can act like something that can be ordered, we make it an instance of the ord typeclass.



In the next section, we'll take a look at how we can manually make our types instances of typeclasses by implementing the functions defined by the typeclasses. But right now, let's see how Haskell can automatically make our type an instance of any of the following typeclasses: Eq. Ord, Enum, Bounded, Show, Read. Haskell can derive the behavior of our types in these contexts if we use the *deriving* keyword when making our data type.

Consider this data type:

```
data Person = Person { firstName :: String
                       lastName :: String
                       age :: Int
```

It describes a person. Let's assume that no two people have the same combination of first name, last name and age. Now, if we have records for two people, does it make sense to see if they represent the same person? Sure it does. We can try to equate them and see if they're equal or not. That's why it would make sense for this type to be part of the Eq typeclass. We'll derive the instance.

```
data Person = Person { firstName :: String
                     , lastName :: String
                       age :: Int
                     } deriving (Eq)
```

When we derive the Eq instance for a type and then try to compare two values of that type with == or /=, Haskell will see if the value constructors match (there's only one value constructor here though) and then it will check if all the data contained inside matches by testing each pair of fields with ==. There's only one catch though, the types of all the fields also have to be part of the Eq typeclass. But since both String and Int are, we're OK. Let's test our Eq instance.

```
ghci> let mikeD = Person {firstName = "Michael", lastName = "Diamond", age = 43}
ghci> let adRock = Person {firstName = "Adam", lastName = "Horovitz", age = 41}
ghci> let mca = Person {firstName = "Adam", lastName = "Yauch", age = 44}
ghci> mca == adRock
False
ghci> mikeD == adRock
False
ghci> mikeD == mikeD
True
ghci> mikeD == Person {firstName = "Michael", lastName = "Diamond", age = 43}
True
```

Of course, since Person is now in Eq, we can use it as the a for all functions that have a class constraint of Eq a in their type signature, such as elem.

```
ghci> let beastieBoys = [mca, adRock, mikeD]
ghci> mikeD `elem` beastieBoys
True
```

The **Show** and **Read** typeclasses are for things that can be converted to or from strings, respectively. Like with **Eq**, if a type's constructors have fields, their type has to be a part of **Show** or **Read** if we want to make our type an instance of them. Let's make our **Person** data type a part of **Show** and **Read** as well.

```
data Person = Person { firstName :: String
                     , lastName :: String
                     , age :: Int
                     } deriving (Eq, Show, Read)
```

Now we can print a person out to the terminal.

```
ghci> let mikeD = Person {firstName = "Michael", lastName = "Diamond", age = 43}
ghci> mikeD
Person {firstName = "Michael", lastName = "Diamond", age = 43}
ghci> "mikeD is: " ++ show mikeD
"mikeD is: Person {firstName = \"Michael\", lastName = \"Diamond\", age = 43}"
```

Had we tried to print a person on the terminal before making the **Person** data type part of **Show**, Haskell would have complained at us, claiming it doesn't know how to represent a person as a string. But now that we've derived a **show** instance for it, it does know.

Read is pretty much the inverse typeclass of Show. Show is for converting values of our a type to a string, Read is for converting strings to values of our type. Remember though, when we use the read function, we have to use an explicit type annotation to tell Haskell which type we want to get as a result. If we don't make the type we want as a result explicit, Haskell doesn't know which type we want.

```
ghci> read "Person {firstName =\"Michael\", lastName =\"Diamond\", age = 43}" :: Person
Person {firstName = "Michael", lastName = "Diamond", age = 43}
```

If we use the result of our read later on in a way that Haskell can infer that it should read it as a person, we don't have to use type annotation.

```
ghci> read "Person {firstName =\"Michael\", lastName =\"Diamond\", age = 43}" == mikeD
True
```

We can also read parameterized types, but we have to fill in the type parameters. So we can't do read "Just 't'" :: Maybe a, but we can do read "Just 't'" :: Maybe Char.

We can derive instances for the <code>Ord</code> type class, which is for types that have values that can be ordered. If we compare two values of the same type that were made using different constructors, the value which was made with a constructor that's defined first is considered smaller. For instance, consider the <code>Bool</code> type, which can have a value of either <code>False</code> or <code>True</code>.

For the purpose of seeing how it behaves when compared, we can think of it as being implemented like this:

```
data Bool = False | True deriving (Ord)
```

Because the False value constructor is specified first and the True value constructor is specified after it, we can consider True as greater than False.

```
ghci> True `compare` False
GT
ghci> True > False
True
ghci> True < False
False
```

In the Maybe a data type, the Nothing value constructor is specified before the Just value constructor, so a value of Nothing is always smaller than a value of Just something, even if that something is minus one billion trillion. But if we compare two Just values, then it goes to compare what's inside them.

```
ghci> Nothing < Just 100</pre>
True
ghci> Nothing > Just (-49999)
False
ghci> Just 3 `compare` Just 2
GT
```

```
ghci> Just 100 > Just 50
True
```

But we can't do something like Just (\*3) > Just (\*2), because (\*3) and (\*2) are functions, which aren't instances of Ord.

We can easily use algebraic data types to make enumerations and the **Enum** and **Bounded** typeclasses help us with that. Consider the following data type:

```
data Day = Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
                                                                         Sunday
```

Because all the value constructors are nullary (take no parameters, i.e. fields), we can make it part of the **Enum** typeclass. The Enum typeclass is for things that have predecessors and successors. We can also make it part of the **Bounded** typeclass, which is for things that have a lowest possible value and highest possible value. And while we're at it, let's also make it an instance of all the other derivable typeclasses and see what we can do with it.

```
data Day = Monday | Tuesday | Wednesday | Thursday | Friday |
                                                              Saturday |
                                                                         Sunday
           deriving (Eq. Ord, Show, Read, Bounded, Enum)
```

Because it's part of the **Show** and **Read** typeclasses, we can convert values of this type to and from strings.

```
ghci> Wednesday
Wednesday
ghci> show Wednesday
"Wednesday"
ghci> read "Saturday" :: Day
Saturday
```

Because it's part of the Eq and Ord typeclasses, we can compare or equate days.

```
ghci> Saturday == Sunday
False
ghci> Saturday == Saturday
True
ghci> Saturday > Friday
True
ghci> Monday `compare` Wednesday
LT
```

It's also part of **Bounded**, so we can get the lowest and highest day.

```
ghci> minBound :: Day
Monday
ghci> maxBound :: Day
Sunday
```

It's also an instance of Enum. We can get predecessors and successors of days and we can make list ranges from them!

```
ghci> succ Monday
Tuesday
ghci> pred Saturday
Friday
ghci> [Thursday .. Sunday]
[Thursday, Friday, Saturday, Sunday]
ghci> [minBound .. maxBound] :: [Day]
[Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday]
```

That's pretty awesome.

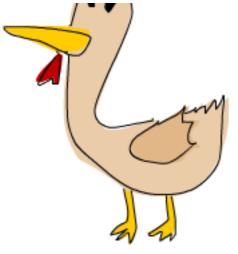
## Type synonyms

Previously, we mentioned that when writing types, the [Char] and String types are equivalent and interchangeable. That's implemented with type synonyms. Type synonyms don't really do anything per se, they're just about giving some types different names so that they make more sense to someone reading our code and documentation. Here's how the standard library defines String as a synonym for [Char].

```
type String = [Char]
```



We've introduced the *type* keyword. The keyword might be misleading to some, because we're not actually making anything new (we did that with the *data* keyword), but we're just making a



synonym for an already existing type.

If we make a function that converts a string to uppercase and call it toUpperString or something, we can give it a type declaration of toUpperString :: [Char] -> [Char] or toUpperString :: String -> String. Both of these are essentially the same, only the latter is nicer to read.

When we were dealing with the Data. Map module, we first represented a phonebook with an association list before converting it into a map. As we've already found out, an association list is a list of key-value pairs. Let's look at a phonebook that we had.

```
phoneBook :: [(String, String)]
phoneBook =
    [("betty", "555-2938")
    ,("bonnie","452-2928")
    ,("patsy","493-2928")
    ,("lucille","205-2928")
    , ("wendy", "939-8282")
    , ("penny", "853-2492")
```

We see that the type of phoneBook is [(String, String)]. That tells us that it's an association list that maps from strings to strings, but not much else. Let's make a type synonym to convey some more information in the type declaration.

```
type PhoneBook = [(String, String)]
```

Now the type declaration for our phonebook can be **phoneBook**:: **PhoneBook**. Let's make a type synonym for **String** as well.

```
type PhoneNumber = String
type Name = String
type PhoneBook = [(Name, PhoneNumber)]
```

Giving the String type synonyms is something that Haskell programmers do when they want to convey more information about what strings in their functions should be used as and what they represent.

So now, when we implement a function that takes a name and a number and sees if that name and number combination is in our phonebook, we can give it a very pretty and descriptive type declaration.

```
inPhoneBook :: Name -> PhoneNumber -> PhoneBook -> Bool
inPhoneBook name pnumber pbook = (name, pnumber) `elem` pbook
```

If we decided not to use type synonyms, our function would have a type of

String -> String -> [(String, String)] -> Bool. In this case, the type declaration that took advantage of type synonyms is easier to understand. However, you shouldn't go overboard with them. We introduce type synonyms either to describe what some existing type represents in our functions (and thus our type declarations become better documentation) or when something has a long-ish type that's repeated a lot (like [(String, String)]) but represents something more specific in the context of our functions.

Type synonyms can also be parameterized. If we want a type that represents an association list type but still want it to be general so it can use any type as the keys and values, we can do this:

### type AssocList k v = [(k, v)]

Now, a function that gets the value by a key in an association list can have a type of

(Eq k) = k - AssocList k v - Maybe v. AssocList is a type constructor that takes two types and produces a concrete type, like AssocList Int String, for instance.

Fonzie says: Aaay! When I talk about concrete types I mean like fully applied types like Map Int String or if we're dealin' with one of them polymorphic functions, [a] or (Ord a) => Maybe a and stuff. And like, sometimes me and the boys say that Maybe is a type, but we don't mean that, cause every idiot knows Maybe is a type constructor. When I apply an extra type to Maybe, like Maybe String, then I have a concrete type. You know, values can only have types that are concrete types! So in conclusion, live fast, love hard and don't let anybody else use your comb!

Just like we can partially apply functions to get new functions, we can partially apply type parameters and get new type constructors from them. Just like we call a function with too few parameters to get back a new function, we can specify a type constructor with too few type parameters and get back a partially applied type constructor. If we wanted a type that represents a map (from Data.Map) from integers to something, we could either do this:

**type** IntMap 
$$v = Map$$
 Int  $v$ 

Or we could do it like this:

Either way, the IntMap type constructor takes one parameter and that is the type of what the integers will point to.

Oh yeah. If you're going to try and implement this, you'll probably going to do a qualified import of Data. Map. When you do a qualified import, type constructors also have to be preceded with a module name. So you'd write type IntMap = Map.Map Int.

Make sure that you really understand the distinction between type constructors and value constructors. Just because we made a type synonym called IntMap or AssocList doesn't mean that we can do stuff like AssocList [(1,2),(4,5),(7,9)]. All it means is that we can refer to its type by using different names. We can do

[(1,2),(3,5),(8,9)] :: AssocList Int Int, which will make the numbers inside assume a type of Int, but we can still use that list as we would any normal list that has pairs of integers inside. Type synonyms (and types generally) can only be used in the type portion of Haskell. We're in Haskell's type portion whenever we're defining new types (so in data and type

declarations) or when we're located after a :: . The :: is in type declarations or in type annotations.

Another cool data type that takes two types as its parameters is the **Either a b** type. This is roughly how it's defined:

```
data Either a b = Left a | Right b deriving (Eq, Ord, Read, Show)
```

It has two value constructors. If the Left is used, then its contents are of type a and if Right is used, then its contents are of type b. So we can use this type to encapsulate a value of one type or another and then when we get a value of type Either a b, we usually pattern match on both Left and Right and we different stuff based on which one of them it was.

```
ghci> Right 20
Right 20
ghci> Left "w00t"
Left "woot"
ghci> :t Right 'a'
Right 'a' :: Either a Char
ghci> :t Left True
Left True :: Either Bool b
```

So far, we've seen that Maybe a was mostly used to represent the results of computations that could have either failed or not. But somtimes, Maybe a isn't good enough because Nothing doesn't really convey much information other than that something has failed. That's cool for functions that can fail in only one way or if we're just not interested in how and why they failed. A Data. Map lookup fails only if the key we were looking for wasn't in the map, so we know exactly what happened.

However, when we're interested in how some function failed or why, we usually use the result type of **Either a b**, where **a** is some sort of type that can tell us something about the possible failure and **b** is the type of a successful computation. Hence, errors use the Left value constructor while results use Right.

An example: a high-school has lockers so that students have some place to put their Guns'n'Roses posters. Each locker has a code combination. When a student wants a new locker, they tell the locker supervisor which locker number they want and he gives them the code. However, if someone is already using that locker, he can't tell them the code for the locker and they have to pick a different one. We'll use a map from Data. Map to represent the lockers. It'll map from locker numbers to a pair of whether the locker is in use or not and the locker code.

```
import qualified Data.Map as Map
data LockerState = Taken | Free deriving (Show, Eg)
type Code = String
type LockerMap = Map.Map Int (LockerState, Code)
```

Simple stuff. We introduce a new data type to represent whether a locker is taken or free and we make a type synonym for the locker code. We also make a type synonym for the type that maps from integers to pairs of locker state and code. And now, we're going to make a function that searches for the code in a locker map. We're going to use an **Either String Code** type to represent our result, because our lookup can fail in two ways — the locker can be taken, in which case we can't tell the code or the locker number might not exist at all. If the lookup fails, we're just going to use a **string** to tell what's happened.

```
lockerLookup :: Int -> LockerMap -> Either String Code
lockerLookup lockerNumber map =
    case Map.lookup lockerNumber map of
        Nothing -> Left $ "Locker number " ++ show lockerNumber ++ " doesn't exist!"
        Just (state, code) -> if state /= Taken
                                then Right code
                                else Left $ "Locker " ++ show lockerNumber ++ " is already taken!"
```

We do a normal lookup in the map. If we get a **Nothing**, we return a value of type **Left String**, saying that the locker doesn't exist at all. If we do find it, then we do an additional check to see if the locker is taken. If it is, return a Left saying that it's already taken. If it isn't, then return a value of type Right Code, in which we give the student the correct code for the locker. It's actually a Right String, but we introduced that type synonym to introduce some additional documentation into the type declaration. Here's an example map:

```
lockers :: LockerMap
lockers = Map.fromList
    [(100, (Taken, "ZD39I"))
    ,(101,(Free, "JAH3I"))
    ,(103,(Free,"IQSA9"))
    ,(105,(Free,"QOTSA"))
    , (109, (Taken, "893JJ"))
    , (110, (Taken, "99292"))
```

Now let's try looking up some locker codes.

```
ghci> lockerLookup 101 lockers
Right "JAH3I"
ghci> lockerLookup 100 lockers
Left "Locker 100 is already taken!"
ghci> lockerLookup 102 lockers
Left "Locker number 102 doesn't exist!"
ghci> lockerLookup 110 lockers
Left "Locker 110 is already taken!"
ghci> lockerLookup 105 lockers
Right "QOTSA"
```

We could have used a Maybe a to represent the result but then we wouldn't know why we couldn't get the code. But now, we have information about the failure in our result type.

#### Recursive data structures

As we've seen, a constructor in an algebraic data type can have several (or none at all) fields and each field must be of some concrete type. With that in mind, we can make types whose constructors have fields that are of the same type! Using that, we can create recursive data types, where one value of some type contains values of that type, which in turn contain more values of the same type and so on.

Think about this list: [5]. That's just syntactic sugar for 5:[]. On the left side of the :, there's a value and on the right side, there's a list. And in this case, it's an empty list. Now how about the



list [4,5]? Well, that desugars to 4: (5:[]). Looking at the first:, we see that it also has an element on its left side and a list (5:[]) on its right side. Same goes for a list like 3: (4: (5:6:[])), which could be written either like that or like 3:4:5:6:[] (because: is right-associative) or [3,4,5,6].



We could say that a list can be an empty list or it can be an element joined together with a : with another list (that can be either the empty list or not).

Let's use algebraic data types to implement our own list then!

```
data List a = Empty | Cons a (List a) deriving (Show, Read, Eq, Ord)
```

This reads just like our definition of lists from one of the previous paragraphs. It's either an empty list or a combination of a head with some value and a list. If you're confused about this, you might find it easier to understand in record syntax.

```
data List a = Empty | Cons { listHead :: a, listTail :: List a} deriving (Show, Read, Eq, Ord)
```

You might also be confused about the **Cons** constructor here. cons is another word for :. You see, in lists, : is actually a constructor that takes a value and another list and returns a list. We can already use our new list type! In other words, it has two fields. One field is of the type of a and the other is of the type [a].

```
ghci> Empty
Empty
ghci> 5 `Cons` Empty
Cons 5 Empty
ghci> 4 `Cons` (5 `Cons` Empty)
Cons 4 (Cons 5 Empty)
ghci> 3 `Cons` (4 `Cons` (5 `Cons` Empty))
Cons 3 (Cons 4 (Cons 5 Empty))
```

We called our Cons constructor in an infix manner so you can see how it's just like : . Empty is like [] and 4 `Cons` (5 `Cons` Empty) is like 4: (5:[]).

We can define functions to be automatically infix by making them comprised of only special characters. We can also do the same with constructors, since they're just functions that return a data type. So check this out.

```
infixr 5 :-:
data List a = Empty | a :-: (List a) deriving (Show, Read, Eq, Ord)
```

First off, we notice a new syntactic construct, the fixity declarations. When we define functions as operators, we can use that to give them a fixity (but we don't have to). A fixity states how tightly the operator binds and whether it's left-associative or right-associative. For instance, \*'s fixity is infix1 7 \* and +'s fixity is infix1 6. That means that they're both left-associative (4 \* 3 \* 2 is (4 \* 3) \* 2) but \* binds tighter than +, because it has a greater fixity, so 5 \* 4 + 3 is (5 \* 4) + 3.

Otherwise, we just wrote a :-: (List a) instead of Cons a (List a). Now, we can write out lists in our list type like so:

```
ghci> 3 :-: 4 :-: 5 :-: Empty
(:-:) 3 ((:-:) 4 ((:-:) 5 Empty))
\frac{1}{2} qhci> \frac{1}{2} a = \frac{3}{2} :-: \frac{4}{2} :-: Empty
ghci> 100 :-: a
(:-:) 100 ((:-:) 3 ((:-:) 4 ((:-:) 5 Empty)))
```

When deriving **Show** for our type, Haskell will still display it as if the constructor was a prefix function, hence the parentheses around the operator (remember, 4 + 3 is (+) 4 3).

Let's make a function that adds two of our lists together. This is how ++ is defined for normal lists:

```
infixr 5 ++
(++) :: [a] -> [a] -> [a]
  ++ ys = ys
П
(x:xs) ++ ys = x : (xs ++ ys)
```

So we'll just steal that for our own list. We'll name the function .++.

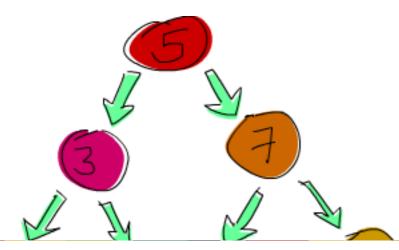
```
infixr 5 .++
(.++) :: List a -> List a -> List a
Empty .++ ys = ys
(x : -: xs) .++ ys = x : -: (xs .++ ys)
```

And let's see if it works ...

```
ghci> let a = 3 :-: 4 :-: 5 :-: Empty
ghci> let b = 6 :-: 7 :-: Empty
ghci> a .++ b
(:-:) 3 ((:-:) 4 ((:-:) 5 ((:-:) 6 ((:-:) 7 Empty))))
```

Nice. Is nice. If we wanted, we could implement all of the functions that operate on lists on our own list type.

Notice how we pattern matched on (x:-: xs). That works because pattern matching is actually about matching constructors. We can match on:-: because it is a constructor for our own list type and we can also match on: because it is a constructor for the built-in list type. Same goes for []. Because pattern matching works (only) on constructors, we can match for stuff like that, normal prefix constructors or stuff like 8 or 'a', which are basically constructors for the numeric and character types, respectively.



Now, we're going to implement a **binary search tree**. If you're not familiar with binary search trees from languages like C, here's what they are: an element points to two elements, one on its left and one on its right. The element to the left is smaller, the element to the right is bigger. Each of those elements can also point to two elements (or one, or none). In effect, each element has up to two sub-trees. And a cool thing about binary search trees is that we know that all the elements at the left







sub-tree of, say, 5 are going to be smaller than 5. Elements in its right sub-tree are going to be bigger. So if we need to find if 8 is in our tree, we'd start at 5 and then because 8 is greater than 5, we'd go right.

We're now at 7 and because 8 is greater than 7, we go right again. And we've found our element in three hops! Now if this were a normal list (or a tree, but really unbalanced), it would take us seven hops instead of three to see if 8 is in there.

Sets and maps from Data. Set and Data. Map are implemented using trees, only instead of normal binary search trees, they use balanced binary search trees, which are always balanced. But right now, we'll just be implementing normal binary search trees.

Here's what we're going to say: a tree is either an empty tree or it's an element that contains some value and two trees. Sounds like a perfect fit for an algebraic data type!

#### data Tree a = EmptyTree | Node a (Tree a) (Tree a) deriving (Show, Read, Eq)

Okay, good, this is good. Instead of manually building a tree, we're going to make a function that takes a tree and an element and inserts an element. We do this by comparing the value we want to insert to the root node and then if it's smaller, we go left, if it's larger, we go right. We do the same for every subsequent node until we reach an empty tree. Once we've reached an empty tree, we just insert a node with that value instead of the empty tree.

In languages like C, we'd do this by modifying the pointers and values inside the tree. In Haskell, we can't really modify our tree, so we have to make a new sub-tree each time we decide to go left or right and in the end the insertion function returns a

completely new tree, because Haskell doesn't really have a concept of pointer, just values. Hence, the type for our insertion function is going to be something like a -> Tree a - > Tree a. It takes an element and a tree and returns a new tree that has that element inside. This might seem like it's inefficient but laziness takes care of that problem.

So, here are two functions. One is a utility function for making a singleton tree (a tree with just one node) and a function to insert an element into a tree.

```
singleton :: a -> Tree a
singleton x = Node \times EmptyTree EmptyTree
treeInsert :: (Ord a) => a -> Tree a -> Tree a
treeInsert x \in T EmptyTree = singleton x
treeInsert x (Node a left right)
     x == a = Node \times left right
     x < a = Node a (treeInsert x left) right
     x > a = Node a left (treeInsert x right)
```

The singleton function is just a shortcut for making a node that has something and then two empty sub-trees. In the insertion function, we first have the edge condition as a pattern. If we've reached an empty sub-tree, that means we're where we want and instead of the empty tree, we put a singleton tree with our element. If we're not inserting into an empty tree, then we have to check some things. First off, if the element we're inserting is equal to the root element, just return a tree that's the same. If it's smaller, return a tree that has the same root value, the same right sub-tree but instead of its left sub-tree, put a tree that has our value inserted into it. Same (but the other way around) goes if our value is bigger than the root element.

Next up, we're going to make a function that checks if some element is in the tree. First, let's define the edge condition. If we're looking for an element in an empty tree, then it's certainly not there. Okay. Notice how this is the same as the edge condition when searching for elements in lists. If we're looking for an element in an empty list, it's not there. Anyway, if we're not looking for an element in an empty tree, then we check some things. If the element in the root node is what we're looking for, great! If it's not, what then? Well, we can take advantage of knowing that all the left elements are smaller than the root node. So if the element we're looking for is smaller than the root node, check to see if it's in the left sub-tree. If it's bigger, check to see if it's in the right sub-tree.

```
treeElem :: (Ord a) => a -> Tree a -> Bool
treeElem x EmptyTree = False
treeElem x (Node a left right)
     x == a = True
     x < a = treeElem x left
     x > a = treeElem x right
```

All we had to do was write up the previous paragraph in code. Let's have some fun with our trees! Instead of manually building one (although we could), we'll use a fold to build up a tree from a list. Remember, pretty much everything that traverses a list one by one and then returns some sort of value can be implemented with a fold! We're going to start with the empty tree and then approach a list from the right and just insert element after element into our accumulator tree.

```
ghci> let nums = [8, 6, 4, 1, 7, 3, 5]
ghci> let numsTree = foldr treeInsert EmptyTree nums
ghci> numsTree
Node 5 (Node 3 (Node 1 EmptyTree EmptyTree) (Node 4 EmptyTree EmptyTree)) (Node 7 (Node 6 EmptyTree
```

In that foldr, treeInsert was the folding function (it takes a tree and a list element and produces a new tree) and EmptyTree was the starting accumulator. nums, of course, was the list we were folding over.

When we print our tree to the console, it's not very readable, but if we try, we can make out its structure. We see that the root node is 5 and then it has two sub-trees, one of which has the root node of 3 and the other a 7, etc.

```
ghci> 8 `treeElem` numsTree
True
ghci> 100 `treeElem` numsTree
False
ghci> 1 `treeElem` numsTree
True
ghci> 10 `treeElem` numsTree
False
```

Checking for membership also works nicely. Cool.

So as you can see, algebraic data structures are a really cool and powerful concept in Haskell. We can use them to make anything from boolean values and weekday enumerations to binary search trees and more!

## **Typeclasses 102**

So far, we've learned about some of the standard Haskell typeclasses and we've seen which



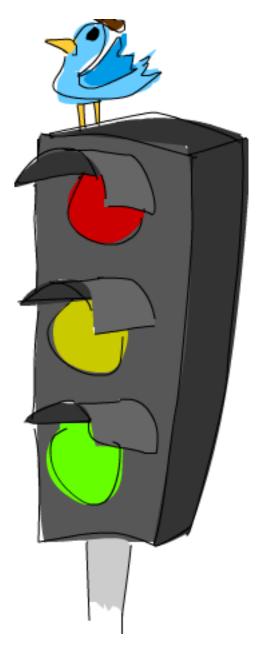
types are in them. We've also learned how to automatically make our own types instances of the standard typeclasses by asking Haskell to derive the instances for us. In this section, we're going to learn how to make our own typeclasses and how to make types instances of them by hand.

A quick recap on typeclasses: typeclasses are like interfaces. A typeclass defines some behavior (like comparing for equality, comparing for ordering, enumeration) and then types that can behave in that way are made instances of that typeclass. The behavior of typeclasses is achieved by defining functions or just type declarations that we then implement. So when we say that a type is an instance of a typeclass, we mean that we can use the functions that the typeclass defines with that type.

Typeclasses have pretty much nothing to do with classes in languages like Java or Python. This confuses many people, so I want you to forget everything you know about classes in imperative languages right now.

For example, the Eq typeclass is for stuff that can be equated. It defines the functions == and /= . If we have a type (say, Car) and comparing two cars with the equality function == makes sense, then it makes sense for Car to be an instance of Eq.

This is how the **Eq** class is defined in the standard prelude:



```
class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool
  x == y = not (x /= y)
  x /= y = not (x == y)
```

Woah, woah! Some new strange syntax and keywords there! Don't worry, this will all be clear in a second. First off, when we write class Eq a where, this means that we're defining a new typeclass and that's called Eq. The a is the type variable and it means that a will play the role of the type that we will soon be making an instance of Eq. It doesn't have to be called a, it doesn't even have to be one letter, it just has to be a lowercase word. Then, we define several functions. It's not mandatory to implement the function bodies themselves, we just have to specify the type declarations for the functions.

Some people might understand this better if we wrote class Eq equatable where and then specified the type declarations like (==) :: equatable -> equatable -> Bool.

Anyway, we *did* implement the function bodies for the functions that **Eq** defines, only we defined them in terms of mutual recursion. We said that two instances of **Eq** are equal if they are not different and they are different if they are not equal. We didn't have to do this, really, but we did and we'll see how this helps us soon.

If we have say class Eq a where and then define a type declaration within that class like (==) :: a -> -a -> Bool, then when we examine the type of that function later on, it will have the type of (Eq a) => a -> a -> Bool.

So once we have a class, what can we do with it? Well, not much, really. But once we start making types instances of that class, we start getting some nice functionality. So check out this type:

```
data TrafficLight = Red | Yellow | Green
```

It defines the states of a traffic light. Notice how we didn't derive any class instances for it. That's because we're going to write up some instances by hand, even though we could derive them for types like **Eg** and **Show**. Here's how we make it an instance of Eq.

```
instance Eq TrafficLight where
    Red == Red = True
   Green == Green = True
   Yellow == Yellow = True
    _ == _ = False
```

We did it by using the *instance* keyword. So *class* is for defining new typeclasses and *instance* is for making our types instances of typeclasses. When we were defining Eq, we wrote class Eq a where and we said that a plays the role of whichever type will be made an instance later on. We can see that clearly here, because when we're making an instance, we write instance Eq TrafficLight where. We replace the a with the actual type.

Because == was defined in terms of /= and vice versa in the class declaration, we only had to overwrite one of them in the instance declaration. That's called the minimal complete definition for the typeclass — the minimum of functions that we have to implement so that our type can behave like the class advertises. To fulfill the minimal complete definition for Eq, we have to overwrite either one of == or /= . If Eq was defined simply like this:

```
class Eq a where
    (==) :: a -> a -> Bool
    (/=) :: a -> a -> Bool
```

we'd have to implement both of these functions when making a type an instance of it, because Haskell wouldn't know how these two functions are related. The minimal complete definition would then be: both == and /=.

You can see that we implemented == simply by doing pattern matching. Since there are many more cases where two lights aren't equal, we specified the ones that are equal and then just did a catch-all pattern saying that if it's none of the previous combinations, then two lights aren't equal.

Let's make this an instance of **Show** by hand, too. To satisfy the minimal complete definition for **Show**, we just have to implement its **show** function, which takes a value and turns it into a string.

```
instance Show TrafficLight where
    show Red = "Red light"
    show Yellow = "Yellow light"
    show Green = "Green light"
```

Once again, we used pattern matching to achieve our goals. Let's see how it works in action:

```
ghci> Red == Red
True
ghci> Red == Yellow
False
ghci> Red `elem` [Red, Yellow, Green]
True
ghci> [Red, Yellow, Green]
[Red light, Yellow light, Green light]
```

Nice. We could have just derived Eq and it would have had the same effect (but we didn't for educational purposes). However, deriving **show** would have just directly translated the value constructors to strings. But if we want lights to appear like "Red light", then we have to make the instance declaration by hand.

You can also make typeclasses that are subclasses of other typeclasses. The *class* declaration for **Num** is a bit long, but here's the first part:

```
class (Eq a) => Num a where
```

As we mentioned previously, there are a lot of places where we can cram in class constraints. So this is just like writing class Num a where, only we state that our type a must be an instance of Eq. We're essentially saying that we have to make a type an instance of Eq before we can make it an instance of Num. Before some type can be considered a number, it makes sense that we can determine whether values of that type can be equated or not. That's all there is to subclassing really, it's just a class constraint on a *class* declaration! When defining function bodies in the *class* declaration or when defining them in *instance* declarations, we can assume that **a** is a part of **Eq** and so we can use **==** on values of that type.

But how are the Maybe or list types made as instances of typeclasses? What makes Maybe different from, say,

TrafficLight is that Maybe in itself isn't a concrete type, it's a type constructor that takes one type parameter (like Char or something) to produce a concrete type (like Maybe Char). Let's take a look at the Eq typeclass again:

```
class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool
  x == y = not (x /= y)
  x /= y = not (x == y)
```

From the type declarations, we see that the a is used as a concrete type because all the types in functions have to be concrete (remember, you can't have a function of the type a -> Maybe but you can have a function of a -> Maybe a or Maybe Int -> Maybe String). That's why we can't do something like

```
instance Eq Maybe where
...
```

Because like we've seen, the a has to be a concrete type but Maybe isn't a concrete type. It's a type constructor that takes one parameter and then produces a concrete type. It would also be tedious to write instance Eq (Maybe Int) where,

instance Eq (Maybe Char) where, etc. for every type ever. So we could write it out like so:

```
instance Eq (Maybe m) where
   Just x == Just y = x == y
   Nothing == Nothing = True
   == = False
```

This is like saying that we want to make all types of the form Maybe something an instance of Eq. We actually could have written (Maybe something), but we usually opt for single letters to be true to the Haskell style. The (Maybe m) here plays the role of the a from class Eq a where. While Maybe isn't a concrete type, Maybe m is. By specifying a type parameter (m, which is in lowercase), we said that we want all types that are in the form of Maybe m, where m is any type, to be an instance of Eq.

There's one problem with this though. Can you spot it? We use == on the contents of the Maybe but we have no assurance that what the **Maybe** contains can be used with **Eq!** That's why we have to modify our *instance* declaration like this:

```
instance (Eq m) => Eq (Maybe m) where
    Just x == Just y = x == y
    Nothing == Nothing = True
    _ == _ = False
```

We had to add a class constraint! With this *instance* declaration, we say this: we want all types of the form Maybe m to be part of the Eq typeclass, but only those types where the m (so what's contained inside the Maybe) is also a part of Eq. This is actually how Haskell would derive the instance too.

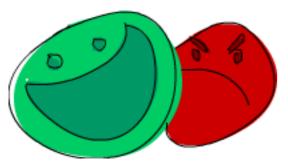
Most of the times, class constraints in *class* declarations are used for making a typeclass a subclass of another typeclass and class constraints in *instance* declarations are used to express requirements about the contents of some type. For instance, here we required the contents of the Maybe to also be part of the Eq typeclass.

When making instances, if you see that a type is used as a concrete type in the type declarations (like the a in a -> a -> Bool), you have to supply type parameters and add parentheses so that you end up with a concrete type.

Take into account that the type you're trying to make an instance of will replace the parameter in the class declaration. The a from class Eq a where will be replaced with a real type when you make an instance, so try mentally putting your type into the function type declarations as well. (==) :: Maybe -> Maybe -> Bool doesn't make much sense but (==) :: (Eq m) => Maybe m -> Maybe m -> Bool does. But this is just something to think about, because == will always have a type of (==) :: (Eq a) => a -> a -> Bool, no matter what instances we make.

Ooh, one more thing, check this out! If you want to see what the instances of a typeclass are, just do :info YourTypeClass in GHCI. So typing :info Num will show which functions the typeclass defines and it will give you a list of the types in the typeclass. :info works for types and type constructors too. If you do :info Maybe, it will show you all the typeclasses that **Maybe** is an instance of. Also :info can show you the type declaration of a function. I think that's pretty cool.

### A yes-no typeclass



In JavaScript and some other weakly typed languages, you can put almost anything inside an if expression. For example, you can do all of the following:

```
if (0) alert("YEAH!") else alert("NO!"),
if ("") alert ("YEAH!") else alert("NO!"),
if (false) alert("YEAH") else alert("NO!), etc. and all of these will throw an alert
of NO! . If you do if ("WHAT") alert ("YEAH") else alert("NO!") , it will alert a
```

"YEAH!" because JavaScript considers non-empty strings to be a sort of true-ish value.

Even though strictly using **Bool** for boolean semantics works better in Haskell, let's try and implement that JavaScript-ish behavior anyway. For fun! Let's start out with a *class* declaration.

```
class YesNo a where
    yesno :: a -> Bool
```

Pretty simple. The YesNo typeclass defines one function. That function takes one value of a type that can be considered to hold some concept of true-ness and tells us for sure if it's true or not. Notice that from the way we use the a in the function, a has to be a concrete type.

Next up, let's define some instances. For numbers, we'll assume that (like in JavaScript) any number that isn't 0 is true-ish and 0 is false-ish.

```
instance YesNo Int where
    yesno 0 = False
    yesno _{-} = True
```

Empty lists (and by extensions, strings) are a no-ish value, while non-empty lists are a yes-ish value.

```
instance YesNo [a] where
    yesno [] = False
    yesno _{-} = True
```

Notice how we just put in a type parameter a in there to make the list a concrete type, even though we don't make any assumptions about the type that's contained in the list. What else, hmm ... I know, Bool itself also holds true-ness and falseness and it's pretty obvious which is which.

```
instance YesNo Bool where
    yesno = id
```

Huh? What's id? It's just a standard library function that takes a parameter and returns the same thing, which is what we would be writing here anyway.

Let's make Maybe a an instance too.

```
instance YesNo (Maybe a) where
   yesno (Just _) = True
   yesno Nothing = False
```

We didn't need a class constraint because we made no assumptions about the contents of the Maybe. We just said that it's true-ish if it's a Just value and false-ish if it's a Nothing. We still had to write out (Maybe a) instead of just Maybe because if you think about it, a Maybe -> Bool function can't exist (because Maybe isn't a concrete type), whereas a Maybe a -> Bool is fine and dandy. Still, this is really cool because now, any type of the form Maybe something is part of YesNo and it doesn't matter what that something is.

Previously, we defined a **Tree** a type, that represented a binary search tree. We can say an empty tree is false-ish and anything that's not an empty tree is true-ish.

```
instance YesNo (Tree a) where
    yesno EmptyTree = False
    yesno _{-} = True
```

Can a traffic light be a yes or no value? Sure. If it's red, you stop. If it's green, you go. If it's yellow? Eh, I usually run the yellows because I live for adrenaline.

```
instance YesNo TrafficLight where
    yesno Red = False
```

```
yesno _ = True
```

Cool, now that we have some instances, let's go play!

```
ghci> yesno $ length []
False
ghci> yesno "haha"
True
ghci> yesno ""
False
ghci> yesno $ Just 0
True
ghci> yesno True
True
ghci> yesno EmptyTree
False
ghci> yesno []
False
ghci> yesno [0,0,0]
True
ghci> :t yesno
yesno :: (YesNo a) \Rightarrow a \rightarrow Bool
```

Right, it works! Let's make a function that mimics the if statement, but it works with YesNo values.

```
yesnoIf :: (YesNo y) \Rightarrow y \rightarrow a \rightarrow a \rightarrow a
yesnoIf yesnoVal yesResult noResult = if yesno yesnoVal then yesResult else noResult
```

Pretty straightforward. It takes a yes-no-ish value and two things. If the yes-no-ish value is more of a yes, it returns the first of the two things, otherwise it returns the second of them.

```
ghci> yesnoIf [] "YEAH!" "NO!"
"NO!"
ghci> yesnoIf [2,3,4] "YEAH!" "NO!"
"YEAH!"
ghci> yesnoIf True "YEAH!" "NO!"
"YEAH!"
ghci> yesnoIf (Just 500) "YEAH!" "NO!"
"YEAH!"
ghci> yesnoIf Nothing "YEAH!" "NO!"
"NO!"
```

## The Functor typeclass

So far, we've encountered a lot of the typeclasses in the standard library. We've played with ord, which is for stuff that can be ordered. We've palled around with Eq, which is for things that can be equated. We've seen Show, which presents an interface for types whose values can be displayed as strings. Our good friend **Read** is there whenever we need to convert a string to a value of some type. And now, we're going to take a look at the **Functor** typeclass, which is basically for things that can be mapped over. You're probably thinking about lists now, since mapping over lists is such a dominant idiom in Haskell. And you're right, the list type is part of the **Functor** typeclass.

What better way to get to know the **Functor** typeclass than to see how it's implemented? Let's take a peek.

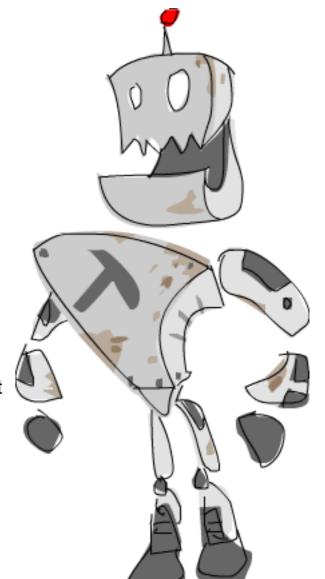
```
class Functor f where

fmap :: (a \rightarrow b) \rightarrow f a \rightarrow f b
```

Alright. We see that it defines one function, <code>fmap</code>, and doesn't provide any default implementation for it. The type of <code>fmap</code> is interesting. In the definitions of typeclasses so far, the type variable that played the role of the type in the typeclass was a concrete type, like the <code>a</code> in <code>(==) :: (Eq a) => a -> a -> Bool</code>. But now, the <code>f</code> is not a concrete type (a type that a value can hold, like <code>Int</code>, <code>Bool</code> or <code>Maybe String</code>), but a type constructor that takes one type parameter. A quick refresher example: <code>Maybe Int</code> is a concrete type, but <code>Maybe</code> is a type constructor that takes one type as the parameter. Anyway, we see that <code>fmap</code> takes a function from one type to another and a functor applied with one type and returns a functor applied with another type.

If this sounds a bit confusing, don't worry. All will be revealed soon when we check out a few examples. Hmm, this type declaration for fmap reminds me of something. If you don't know what the type signature of map is, it's: map :: (a -> b) -> [a] -> [b].

Ah, interesting! It takes a function from one type to another and a list of one type and returns a list of another type. My friends, I think we have ourselves a functor! In fact, map is just a fmap that works only on lists. Here's how the list is an instance of the



Functor typeclass.

```
instance Functor [] where
fmap = map
```

That's it! Notice how we didn't write instance Functor [a] where, because from fmap :: (a -> b) -> f a -> f b, we see that the f has to be a type constructor that takes one type. [a] is already a concrete type (of a list with any type inside it), while [] is a type constructor that takes one type and can produce types such as [Int], [String] or even [[String]].

Since for lists, fmap is just map, we get the same results when using them on lists.

```
map :: (a -> b) -> [a] -> [b]
ghci> fmap (*2) [1..3]
[2,4,6]
ghci> map (*2) [1..3]
[2,4,6]
```

What happens when we map or fmap over an empty list? Well, of course, we get an empty list. It just turns an empty list of type [a] into an empty list of type [b].

Types that can act like a box can be functors. You can think of a list as a box that has an infinite amount of little compartments and they can all be empty, one can be full and the others empty or a number of them can be full. So, what else has the

properties of being like a box? For one, the Maybe a type. In a way, it's like a box that can either hold nothing, in which case it has the value of Nothing, or it can hold one item, like "HAHA", in which case it has a value of Just "HAHA". Here's how Maybe is a functor.

```
instance Functor Maybe where
  fmap f (Just x) = Just (f x)
  fmap f Nothing = Nothing
```

Again, notice how we wrote instance Functor Maybe where instead of instance Functor (Maybe m) where, like we did when we were dealing with Maybe and YesNo. Functor wants a type constructor that takes one type and not a concrete type. If you mentally replace the fs with Maybes, fmap acts like a (a -> b) -> Maybe a -> Maybe b for this particular type, which looks OK. But if you replace f with (Maybe m), then it would seem to act like a

(a -> b) -> Maybe m a -> Maybe m b, which doesn't make any damn sense because Maybe takes just one type parameter.

Anyway, the fmap implementation is pretty simple. If it's an empty value of Nothing, then just return a Nothing. If we map over an empty box, we get an empty box. It makes sense. Just like if we map over an empty list, we get back an empty list. If it's not an empty value, but rather a single value packed up in a Just, then we apply the function on the contents of the Just.

```
ghci> fmap (++ " HEY GUYS IM INSIDE THE JUST") (Just "Something serious.")
Just "Something serious. HEY GUYS IM INSIDE THE JUST"
ghci> fmap (++ " HEY GUYS IM INSIDE THE JUST") Nothing
Nothing
```

```
ghci> fmap (*2) (Just 200)
Just 400
ghci> fmap (*2) Nothing
Nothing
```

Another thing that can be mapped over and made an instance of **Functor** is our **Tree a** type. It can be thought of as a box in a way (holds several or no values) and the **Tree** type constructor takes exactly one type parameter. If you look at **fmap** as if it were a function made only for Tree, its type signature would look like (a -> b) -> Tree a -> Tree b. We're going to use recursion on this one. Mapping over an empty tree will produce an empty tree. Mapping over a non-empty tree will be a tree consisting of our function applied to the root value and its left and right sub-trees will be the previous sub-trees, only our function will be mapped over them.

```
instance Functor Tree where
    fmap f EmptyTree = EmptyTree
    fmap f (Node x leftsub rightsub) = Node (f x) (fmap f leftsub) (fmap f rightsub)
```

```
ghci> fmap (*2) EmptyTree
EmptyTree
ghci> fmap (*4) (foldr treeInsert EmptyTree [5,7,3,2,1,7])
Node 28 (Node 4 EmptyTree (Node 8 EmptyTree (Node 12 EmptyTree (Node 20 EmptyTree EmptyTree)))) EmptyTree
```

Nice! Now how about **Either a b?** Can this be made a functor? The **Functor** typeclass wants a type constructor that takes only one type parameter but **Either** takes two. Hmmm! I know, we'll partially apply **Either** by feeding it only one parameter

so that it has one free parameter. Here's how **Either a** is a functor in the standard libraries:

```
instance Functor (Either a) where
  fmap f (Right x) = Right (f x)
  fmap f (Left x) = Left x
```

Well well, what did we do here? You can see how we made **Either a** an instance instead of just **Either**. That's because **Either a** is a type constructor that takes one parameter, whereas **Either** takes two. If **fmap** was specifically for **Either a**, the type signature would then be (b -> c) -> **Either a b -> Either a c** because that's the same as (b -> c) -> (**Either a**) b -> (**Either a**) c. In the implementation, we mapped in the case of a **Right** value constructor, but we didn't in the case of a **Left**. Why is that? Well, if we look back at how the **Either a b** type is defined, it's kind of like:

#### data Either a b = Left a | Right b

Well, if we wanted to map one function over both of them, **a** and **b** would have to be the same type. I mean, if we tried to map a function that takes a string and returns a string and the **b** was a string but the **a** was a number, that wouldn't really work out. Also, from seeing what **fmap**'s type would be if it operated only on **Either** values, we see that the first parameter has to remain the same while the second one can change and the first parameter is actualized by the **Left** value constructor.

This also goes nicely with our box analogy if we think of the Left part as sort of an empty box with an error message written on the side telling us why it's empty.

Maps from Data.Map can also be made a functor because they hold values (or not!). In the case of Map k v, fmap will map a function v -> v' over a map of type Map k v and return a map of type Map k v'.

Note, the has no special meaning in types just like it doesn't have special meaning when naming values. It's used to denote things that are similar, only slightly changed.

Try figuring out how Map k is made an instance of Functor by yourself!

With the **Functor** typeclass, we've seen how typeclasses can represent pretty cool higher-order concepts. We've also had some more practice with partially applying types and making instances. In one of the next chapters, we'll also take a look at some laws that apply for functors.

Just one more thing! Functors should obey some laws so that they may have some properties that we can depend on and not think about too much. If we use fmap (+1) over the list [1,2,3,4], we expect the result to be [2,3,4,5] and not its reverse, [5,4,3,2]. If we use fmap (\a -> a) (the identity function, which just returns its parameter) over some list, we expect to get back the same list as a result. For example, if we gave the wrong functor instance to our Tree type, using fmap over a tree where the left sub-tree of a node only has elements that are smaller than the node and the right sub-tree only has nodes that are larger than the node might produce a tree where that's not the case. We'll go over the functor laws in more detail in one of the next chapters.

# Kinds and some type-foo

Type constructors take other types as parameters to eventually produce concrete types. That kind of reminds me of functions, which take values as parameters to produce values. We've seen that type constructors can be partially applied (Either String is a type that takes one type and produces a concrete type, like Either String Int), just like functions can. This is all very interesting indeed. In this section, we'll take a look at formally defining how types are applied to type constructors, just like we took a look at formally defining how values are applied to functions by using type declarations. You don't really have to read this section to continue on your magical Haskell quest and if you don't understand it, don't worry about it. However, getting this will give you a very thorough understanding of the type system.

So, values like 3, "YEAH" or takeWhile (functions are also values, because we can pass them around and such) each have their own type. Types are little labels that values carry so that we can reason about the values. But types have their own little labels, called kinds. A kind is more or less the type of a type. This may sound a bit weird and confusing, but it's actually a really cool concept.



What are kinds and what are they good for? Well, let's examine the kind of a type by using the :k command in GHCI.

```
ghci> :k Int
Int :: *
```

A star? How quaint. What does that mean? A \* means that the type is a concrete type. A concrete type is a type that doesn't take any type parameters and values can only have types that are concrete types. If I had to read \* out loud (I haven't had to do that so far), I'd say star or just type.

Okay, now let's see what the kind of Maybe is.

```
ghci> :k Maybe
Maybe :: * -> *
```

The Maybe type constructor takes one concrete type (like Int) and then returns a concrete type like Maybe Int. And that's what this kind tells us. Just like Int -> Int means that a function takes an Int and returns an Int, \* -> \* means that the type constructor takes one concrete type and returns a concrete type. Let's apply the type parameter to Maybe and see what the kind of that type is.

```
ghci> :k Maybe Int
Maybe Int :: *
```

Just like I expected! We applied the type parameter to Maybe and got back a concrete type (that's what \* -> \* means. A

parallel (although not equivalent, types and kinds are two different things) to this is if we do :t isUpper and :t isUpper 'A' . isUpper has a type of Char -> Bool and isUpper 'A' has a type of Bool, because its value is basically **True**. Both those types, however, have a kind of \*.

We used :k on a type to get its kind, just like we can use :t on a value to get its type. Like we said, types are the labels of values and kinds are the labels of types and there are parallels between the two.

Let's look at another kind.

```
ghci> :k Either
Either :: * -> * -> *
```

Aha, this tells us that **Either** takes two concrete types as type parameters to produce a concrete type. It also looks kind of like a type declaration of a function that takes two values and returns something. Type constructors are curried (just like functions), so we can partially apply them.

```
ghci> :k Either String
Either String :: * -> *
ghci> :k Either String Int
Either String Int :: *
```

When we wanted to make **Either** a part of the **Functor** typeclass, we had to partially apply it because **Functor** wants

types that take only one parameter while **Either** takes two. In other words, **Functor** wants types of kind \* -> \* and so we had to partially apply **Either** to get a type of kind \* -> \* instead of its original kind \* -> \* . If we look at the definition of **Functor** again

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

we see that the f type variable is used as a type that takes one concrete type to produce a concrete type. We know it has to produce a concrete type because it's used as the type of a value in a function. And from that, we can deduce that types that want to be friends with Functor have to be of kind \* -> \*.

Now, let's do some type-foo. Take a look at this typeclass that I'm just going to make up right now:

```
class Tofu t where
tofu :: j a -> t a j
```

Man, that looks weird. How would we make a type that could be an instance of that strange typeclass? Well, let's look at what its kind would have to be. Because j a is used as the type of a value that the tofu function takes as its parameter, j a has to have a kind of \*. We assume \* for a and so we can infer that j has to have a kind of \* -> \*. We see that t has to produce a concrete value too and that it takes two types. And knowing that a has a kind of \* and j has a kind of \* -> \*, we infer that t has to have a kind of \* -> (\* -> \*) -> \*. So it takes a concrete type (a), a type constructor that takes one concrete type (j) and produces a concrete type. Wow.

OK, so let's make a type with a kind of  $* \rightarrow (* \rightarrow *) \rightarrow *$ . Here's one way of going about it.

```
data Frank a b = Frank {frankField :: b a} deriving (Show)
```

How do we know this type has a kind of  $* \rightarrow (* \rightarrow *) \rightarrow *$ ? Well, fields in ADTs are made to hold values, so they must be of kind \*, obviously. We assume \* for a, which means that b takes one type parameter and so its kind is \* -> \*. Now we know the kinds of both a and b and because they're parameters for Frank, we see that Frank has a kind of \* -> (\* -> \*) -> \* The first \* represents a and the (\* -> \*) represents b. Let's make some Frank values and check out their types.

```
ghci> :t Frank {frankField = Just "HAHA"}
Frank {frankField = Just "HAHA"} :: Frank [Char] Maybe
ghci> :t Frank {frankField = Node 'a' EmptyTree EmptyTree}
Frank {frankField = Node 'a' EmptyTree EmptyTree} :: Frank Char Tree
ghci> :t Frank {frankField = "YES"}
Frank {frankField = "YES"} :: Frank Char []
```

Hmm. Because **frankField** has a type of form **a b**, its values must have types that are of a similar form as well. So they can be Just "HAHA", which has a type of Maybe [Char] or it can have a value of ['Y', 'E', 'S'], which has a type of [Char] (if we used our own list type for this, it would have a type of List Char). And we see that the types of the Frank values correspond with the kind for Frank. [Char] has a kind of \* and Maybe has a kind of \* -> \*. Because in order to

have a value, it has to be a concrete type and thus has to be fully applied, every value of **Frank blah blaah** has a kind of \*.

Making Frank an instance of Tofu is pretty simple. We see that tofu takes a j a (so an example type of that form would be Maybe Int) and returns a t a j. So if we replace Frank with j, the result type would be Frank Int Maybe.

```
instance Tofu Frank where
tofu x = Frank x
```

```
ghci> tofu (Just 'a') :: Frank Char Maybe
Frank {frankField = Just 'a'}
ghci> tofu ["HELLO"] :: Frank [Char] []
Frank {frankField = ["HELLO"]}
```

Not very useful, but we did flex our type muscles. Let's do some more type-foo. We have this data type:

```
data Barry t k p = Barry { yabba :: p, dabba :: t k }
```

And now we want to make it an instance of Functor. Functor wants types of kind \* -> \* but Barry doesn't look like it has that kind. What is the kind of Barry? Well, we see it takes three type parameters, so it's going to be something -> something -> \*. It's safe to say that p is a concrete type and thus has a kind of \*. For k, we assume \* and so by extension, t has a kind of \* -> \*. Now let's just replace those kinds with the somethings that we

used as placeholders and we see it has a kind of (\* -> \*) -> \* -> \*. Let's check that with GHCI.

```
ghci> :k Barry
Barry :: (* -> *) -> * -> *
```

Ah, we were right. How satisfying. Now, to make this type a part of Functor we have to partially apply the first two type parameters so that we're left with \* -> \*. That means that the start of the instance declaration will be:

instance Functor (Barry a b) where. If we look at fmap as if it was made specifically for Barry, it would have a type of fmap :: (a -> b) -> Barry c d a -> Barry c d b, because we just replace the Functor's f with Barry c d.

The third type parameter from Barry will have to change and we see that it's conviniently in its own field.

```
instance Functor (Barry a b) where
fmap f (Barry {yabba = x, dabba = y}) = Barry {yabba = f x, dabba = y}
```

There we go! We just mapped the **f** over the first field.

In this section, we took a good look at how type parameters work and kind of formalized them with kinds, just like we formalized function parameters with type declarations. We saw that there are interesting parallels between functions and type constructors. They are, however, two completely different things. When working on real Haskell, you usually won't have to mess with kinds and do kind inference by hand like we did now. Usually, you just have to partially apply your own type to \* -> \* or \* when making it an instance of one of the standard typeclasses, but it's good to know how and why that actually works. It's also interesting to see that types have little types of their own. Again, you don't really have to understand everything we did

here to read on, but if you understand how kinds work, chances are that you have a very solid grasp of Haskell's type system.



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