

# Assignment 2 - Alloy Corporation

## Introduction

The **Manufacturing INC** (MNC) makes four different metal alloys: A, B, C and D. The company is barely able to keep up with the increasing demand for these products. However, increasing raw material prices and foreign competition are forcing MNC to watch its margins to ensure it is operating in the most efficient manner possible.

## Marketing Demands

To meet marketing demands for the coming week, MNC needs to produce

- at least 1,200 pounds of the A product,
- between 450 and 650 pounds of the B product,
- no more than 120 pounds of the C product, and
- no more than 150 pounds of D product.

## Production Requirements

Each pound of the A, B, C, and D product contains, respectively, 55%, 45%, 25%, and 100% aluminum made up of copper.

The company has 1300 pounds of aluminum and 900 pounds of copper for use in the next week.

The various products are made using four different machines: forger, reamer, shaper, and planer. The following table summarizes the time required by each product on each machine. Each machine has 60 hours of time available in the coming week.

## Financial summary

The controller recently presented management with the following financial summary of MNC's average weekly operations over the past quarter. From this report, the controller is arguing that the company should cease producing its B and C products.

Machine	Minutes required per pound			
	A	B	C	D
Forger	1.20	1.00	1.00	1.5
Reamer	1.75	1.00	1.00	1.50
Shaper	1.30	0.60	0.40	0.00
Planer	2.50	1.20	2.00	1.50

Figure 1: Districts

	Product				
	A	B	C	D	Total
Sales Revenue	5769	2000	700	879	9348
Variables costs	3200	1550	300	350	5050
Fixed costs	600	500	250	300	1650
Net profit	1969	-50	150	229	2648
Pounds sold	1200	400	100	250	1950
Net profit per pound	1.64	-0.12	1.50	0.91	1.35

Figure 2: Financial summary

## Questions

- Do you agree with the controller's recommendation? Why or why not?
- Formulate an LP model for this problem.
- Create a executable model in R for this problem and solve it.
- What is the optimal solution?
- Perform the sensitivity analysis for this solution and answer the following questions.
- Is the solution degenerate?
- Is the solution unique?
- If MNC wanted to decrease the production on any product, which one would you recommend and why?
- If MNC wanted to increase the production of any product, which one would you recommend and why?
- Which resources are preventing MNS from making more money? If they could acquire more of this resource how much should they acquire & how much should they be willing to pay to acquire it?
- How much should MNC be willing to pay to acquire more copper?
- If the marketing department wanted to decrease the price of the A product by \$0.25, would the optimal solution change?

## Solutions

- We have that using the current combination of Products, the Product B has negative net profit (-0.12 per Pound) while the Product C has positive net profit (1.5 per Pound), thus we **don't agree on controller's recommendation**. He is right on the cessation of the production of the product B but not of the product C.
- Our objective is to maximize the profit, for that we define the *Margin Profit* as the *Sales Revenue - Variable Costs* and the Margin Profit per Pound as *Margin Profit / Pounds Sold*, we ignore the fixed costs because if we included them, since they are constant, they will go on the right hand side of the objective function and will not change the problem itself.

Product	A	B	C	D
Sales Revenue	5769	2000	700	879
Variable Costs	3200	1550	300	350
Margin	2569	450	400	529
Pounds Sold	1200	400	100	250
Margin per Pound	2.14	1.13	4.00	2.12

So, indicated with A the pound sold of Product A (same for B, C and D) , the objective function is:

$$2.14 \cdot A + 1.13 \cdot B + 4 \cdot C + 2.12 \cdot D$$

The domain of the variables are:

$$\begin{aligned} 1200 &\leq A < +\infty \\ 450 &\leq B \leq 650 \\ 0 &\leq C \leq 120 \\ 0 &\leq D \leq 150 \end{aligned}$$

Based on the composition of the material in each Product and the limited resources we also have that:

$$\begin{aligned} 0.55 \cdot A + 0.45 \cdot B + 0.25 \cdot C + 1.00 \cdot D &\leq 1300 && \rightarrow \text{Maximum Alminium} \\ 0.45 \cdot A + 0.55 \cdot B + 0.75 \cdot C + 0.00 \cdot D &\leq 900 && \rightarrow \text{Maximum Copper} \end{aligned}$$

We also have a time limit on each Machine, so the following constraint is also valid:

$$\begin{aligned} 1.20 \cdot A + 1.00 \cdot B + 1.00 \cdot C + 1.50 \cdot D &\leq 3600 && \rightarrow \text{Forger} \\ 1.75 \cdot A + 1.00 \cdot B + 1.00 \cdot C + 1.50 \cdot D &\leq 3600 && \rightarrow \text{Reamer} \\ 1.30 \cdot A + 0.60 \cdot B + 0.40 \cdot C + 0.00 \cdot D &\leq 3600 && \rightarrow \text{Shaper} \\ 2.50 \cdot A + 1.20 \cdot B + 2.00 \cdot C + 1.50 \cdot D &\leq 3600 && \rightarrow \text{Planer} \end{aligned}$$

c. Here is the problem definition in R:

```
if(require(lpSolveAPI)==FALSE) install.packages("lpSolveAPI")
model = make.lp(0,4) # 4 Variables (A,B,C,D)
lp.control(model, sense = "max") #Maximazing Problem
set.objfn(model,obj = c(2.14,1.13,4,2.12)) #Objective function coefficients

row.add.mode(model,"on")

add.constraint(model,
  xt = c(0.55,0.45,0.25,1),
  type = "<=",
  rhs = 1300,
  indices = c(1:4)) #Aluminium Constraint

add.constraint(model,
  xt = c(0.25,0.55,0.75,0),
  type = "<=",
  rhs = 900,
  indices = c(1:4)) #Copper Constraint

add.constraint(model,
  xt = c(1.2,1,1,1.5),
  type = "<=",
  rhs = 3600,
  indices = c(1:4)) #Forger Time Constraint

add.constraint(model,
```

```

        xt = c(1.75,1,1,1.5),
        type = "<=",
        rhs = 3600,
        indices = c(1:4)) #Reamer Time Constraint

add.constraint(model,
               xt = c(1.3,0.6,0.4,0),
               type = "<=",
               rhs = 3600,
               indices = c(1:4)) #Shaper Time Constraint

add.constraint(model,
               xt = c(2.5,1.2,2,1.5),
               type = "<=",
               rhs = 3600,
               indices = c(1:4)) #Planer Time Constraint

#lower and upper bounds
set.bounds(model,lower = c(1200,450,0,0), upper = c(Inf,650,120,150))

row.add.mode(model,"off")

#Give names to constraints and variables
dimnames(model)<- list(c("Aluminium","Copper","Forger","Reamer","Shaper","Planer"),
                      c("A","B","C","D"))

print(model) #To see to model

```

```

## Model name:
##           A      B      C      D
## Maximize  2.14  1.13    4  2.12
## Aluminium 0.55  0.45  0.25    1  <=  1300
## Copper    0.25  0.55  0.75    0  <=   900
## Forger     1.2    1    1    1.5  <=  3600
## Reamer     1.75    1    1    1.5  <=  3600
## Shaper     1.3    0.6  0.4    0  <=  3600
## Planer     2.5    1.2    2    1.5  <=  3600
## Kind       Std   Std   Std   Std
## Type       Real  Real  Real  Real
## Upper      Inf   650   120   150
## Lower      1200  450    0    0

```

d. We use the solve() function to solve the model:

```

solve(model) #result = 0 -> solved

## [1] 0

solution <- get.variables(model)
cat("The optimal variables values are:", "\n",
    "A <-", solution[1], ", B <-", solution[2],
    ", C <-", solution[3], ", D <-", solution[4])

## The optimal variables values are:
## A <- 1200 , B <- 450 , C <- 30 , D <- 0

```

```
cat("The optimal objective function value is: ",get.objective(model))
```

```
## The optimal objective function value is: 3196.5
```