Assigment 2 - Alloy Corporation

Pranav Kasela - 846965

Introduction

The **Manifacturing INC** (MNC) makes four different metal alloys: A, B, C and D. The company is barely able to keep up with the increasing demand for these products. However, increasing raw material prices and foreign competition are forcing MNC to watch its margins to ensure it is operating in the most efficient manner possible.

Marketing Demands

To meet marketing demands for the coming week, MNC needs to produce

- at least 1,200 pounds of the A product,
- between 450 and 650 pounds of the B product,
- no more than 120 pounds of the C product, and
- no more than 150 pounds of D product.

Production Requirements

Each pound of the A, B, C, and D product contains, respectively, 55%, 45%, 25%, and 100% aluminum made up of copper.

The company has 1300 pounds of aluminum and 900 pounds of copper copper for use in the next week.

The various products are made using four different machines: forger, reamer, shaper, and planer. The following table summarizes the time required by each product on each machine. Each machine has 60 hours of time available in the coming week.

Financial summary

The controller recently presented management with the following financial summary of MNC's average weekly operations over the past quarter. From this report, the controller is arguing that the company should cease producing its B and C products.

	Minutes required per pound			
Machine	**A**	**B**	**C**	**D**
Forger	1.20	1.00	1.00	1.5
Reamer	1.75	1.00	1.00	1.50
Shaper	1.30	0.60	0.40	0.00
Planer	2.50	1.20	2.00	1.50

Figure 1: Districts

	Product				
	A	B	C	D	Total
Sales Revenue	5769	2000	700	879	9348
Variables costs	3200	1550	300	350	5050
Fixed costs	600	500	250	300	1650
Net profit	1969	-50	150	229	2648
Pounds sold	1200	400	100	250	1950
Net profit per pound	1.64	-0.12	1.50	0.91	1.35

Figure 2: Financial summary

Questions

a. Do you agree with the controller's recommendation? Why or why not?
b. Formulate an LP model for this problem.
c. Create a executable model in R for this problem and solve it.
d. What is the optimal solution?
e. Perform the sensitivity analysis for this solution and answer the following questions.
f. Is the solution degenerate?
g. Is the solution unique?
h. If MNC wanted to decrease the production on any product, which one would you recommend and why?
i. If MNC wanted to increase the production of any product, which one would you recommend and why?
j. Which resources are preventing MNS from making more money? If they could acquire more of this resource how much should they acquire & how much should they be willing to pay to acquire it?
k. How much should MNC be willing to pay to acquire more copper?
l. If the marketing department wanted to decrease the price of the A product by $0.25, would the optimal solution change?

Solutions

a.

We don't agree with the controller's recommendation, we have that using the current combination of Products, the *Product B* has negative net profit (-0.12 per Pound) while the *Product C* has positive net profit (1.5 per Pound). If we used the net profit as a performance measure of the products,which implies that the demand of the products will not change in the future, the controller would right on the cessation of the production of the *Product B* but not of the *Product C*.
But as we can see that the demand of the *Product B* changed, using the current minimum amount for the product B we have that the net profit in this case would be $amount \cdot Margin\ per\ Pound$(From the table below)$-$ $Variable\ Costs = 1.125 * 450 - 500 = 6.25 > 0$ which is positive thus profitable.
The interruption in the production of the *Product B* is arguable, but in the case of the *Product C* we see that the margin profit per Pound is 4.00, and it has a fixed cost of only 250, lowest one in all four products, thus we need to produce only 63 pounds of Product C to have a profit, as long as this demand is satisfied there is no need to interrupt the production of the Product C.

b.

Our objective is to maximize the profit, for that we define the *Margin Profit* as the *Sales Revenue*-*Variable Costs* and the Margin Profit per Pound as *Margin Profit/Pounds Sold*, we ignore the fixed costs because if we included them, since they are constant, they will go on the right hand side of the objective function and will not change the problem itself.

Product	A	B	C	D
Sales Revenue	5769	2000	700	879
Variable Costs	3200	1550	300	350
Margin	2569	450	400	529
Pounds Sold	1200	400	100	250
Margin per Pound	2.14	1.13	4.00	2.12

So, indicated with A the pound sold of Product A (same for B, C and D) , the **objective function** is:

$$2.14 \cdot A + 1.13 \cdot B + 4 \cdot C + 2.12 \cdot D$$

The domain of the variables are:

$$\begin{aligned} 1200 \leq A < +\infty \\ 450 \leq B \leq 650 \\ 0 \leq C \leq 120 \\ 0 \leq D \leq 150 \end{aligned}$$

Based on the composition of the material in each Product and the limited resources we also have that:

$$\begin{aligned} &0.55 \cdot A + 0.45 \cdot B + 0.25 \cdot C + 1.00 \cdot D \leq 1300 && \rightarrow \text{Maximum Alminium} \\ &0.45 \cdot A + 0.55 \cdot B + 0.75 \cdot C + 0.00 \cdot D \leq 900 && \rightarrow \text{Maximum Copper} \end{aligned}$$

We also have a time limit on each Machine, so the following constraint is also valid:

$$\begin{aligned} &1.20 \cdot A + 1.00 \cdot B + 1.00 \cdot C + 1.50 \cdot D \leq 3600 && \rightarrow \text{Forger} \\ &1.75 \cdot A + 1.00 \cdot B + 1.00 \cdot C + 1.50 \cdot D \leq 3600 && \rightarrow \text{Reamer} \\ &1.30 \cdot A + 0.60 \cdot B + 0.40 \cdot C + 0.00 \cdot D \leq 3600 && \rightarrow \text{Shaper} \\ &2.50 \cdot A + 1.20 \cdot B + 2.00 \cdot C + 1.50 \cdot D \leq 3600 && \rightarrow \text{Planer} \end{aligned}$$

c.

Here is the implementation of problem defined above in point **b.** in R:

```
if(require(lpSolveAPI)==FALSE) install.packages("lpSolveAPI")
model = make.lp(0,4)                              # 4 Variables (A,B,C,D)
lp.control(model, sense = "max")                  #Maximazing Problem
set.objfn(model,obj = c(2.141,1.125,4,2.116))     #Objective function coefficients

row.add.mode(model,"on")
```

```r
add.constraint(model,
               xt = c(0.55,0.45,0.25,1),
               type = "<=",
               rhs = 1300,
               indices = c(1:4)) #Aluminium Constraint

add.constraint(model,
               xt = c(0.45,0.55,0.75,0),
               type = "<=",
               rhs = 900,
               indices = c(1:4)) #Copper Constraint

add.constraint(model,
               xt = c(1.2,1,1,1.5),
               type = "<=",
               rhs = 3600,
               indices = c(1:4)) #Forger Time Constraint

add.constraint(model,
               xt = c(1.75,1,1,1.5),
               type = "<=",
               rhs = 3600,
               indices = c(1:4)) #Reamer Time Constraint

add.constraint(model,
               xt = c(1.3,0.6,0.4,0),
               type = "<=",
               rhs = 3600,
               indices = c(1:4)) #Shaper Time Constraint

add.constraint(model,
               xt = c(2.5,1.2,2,1.5),
               type = "<=",
               rhs = 3600,
               indices = c(1:4)) #Planer Time Constraint

#Setting lower and upper bounds
set.bounds(model,lower = c(1200,450,0,0), upper = c(Inf,650,120,150))

row.add.mode(model,"off")

#Giving names to the contraints and variables
dimnames(model)<- list(c("Aluminium","Copper","Forger","Reamer","Shaper","Planer"),
                       c("A","B","C","D"))
name.lp(model, "Alloy Corporation")
```

```r
print(model) #To see to model
```

```
## Model name: Alloy Corporation
##                 A      B      C      D
## Maximize    2.141  1.125      4  2.116
## Aluminium    0.55   0.45   0.25      1  <=  1300
## Copper       0.45   0.55   0.75      0  <=   900
## Forger        1.2      1      1    1.5  <=  3600
```

```
## Reamer      1.75      1      1    1.5  <=  3600
## Shaper       1.3    0.6    0.4      0  <=  3600
## Planer       2.5    1.2      2    1.5  <=  3600
## Kind         Std    Std    Std    Std
## Type        Real   Real   Real   Real
## Upper        Inf    650    120    150
## Lower       1200    450      0      0
```

d.

We use the solve() function to solve the model:

```r
solve(model) #result = 0 -> solved
```

```
## [1] 0
```

```r
solution <- get.variables(model)
fixed_cost <- 1650
cat("The optimal variables values are:","\n",
    "A <-",solution[1],", B <-",solution[2],
    ", C <-",solution[3],", D <-",solution[4])
```

```
## The optimal variables values are:
##  A <- 1200 , B <- 450 , C <- 30 , D <- 0
```

```r
cat("The optimal objective function value is: ",get.objective(model), "\n",
    "Thus the optimal net profit value is",get.objective(model) - fixed_cost)
```

```
## The optimal objective function value is:  3195.45
##  Thus the optimal net profit value is 1545.45
```

e.

```r
printSensitivityObj(model)
```

```
##   Objs         Sensitivity
## 1   C1    -inf <= C1 <= 5
## 2   C2  -inf <= C2 <= 2.4
## 3   C3 2.821 <= C3 <= inf
## 4   C4    -inf <= C4 <= 3
```

We have that the base remains if the coeffiecient are in the following range:

$$
\begin{aligned}
-\infty < C_1 \leq 5 \\
-\infty < C_2 \leq 2.4 \\
2.821 \leq C_3 < +\infty \\
-\infty < C_4 < 3
\end{aligned}
$$

The fact that C_1 and C_2 can decrease all they want is due to the fact that the optimal solution satisfies the minimum constraint given for Product A and B, thus if we were to decrease the marginal profit per pound from how it is now, we will have that it still needs to satify the minimum quantity for Product A and B, so

the solution will not change.
A similar argument applies for the coefficient C_4 of Product D, if we decrease C_4 the solution will not change since we will continue to not produce the Product C with the given constraints.
While for the coefficient C_3 if we increase it, since we produce only the product C, because the most profitable while satifying the given contraints even if we increased the obtained profit we will have that the optimal solution will not change.

```
printSensitivityRHS(model)
```

```
##   Rhs duals        Sensitivity
## 1  B1     0  -inf <= B1 <= inf
## 2  B2     0  -inf <= B2 <= inf
## 3  B3     0  -inf <= B3 <= inf
## 4  B4     0  -inf <= B4 <= inf
## 5  B5     0  -inf <= B5 <= inf
## 6  B6     2 3540 <= B6 <= 3780
```

So we have the the dual variables are $y_i = 0, \forall i = 1, ..., 5$ and $y_6 = 2$ and the interval for the resources are:

$$
\begin{gathered}
-\infty < b_1 < +\infty \\
-\infty < b_2 < +\infty \\
-\infty < b_3 < +\infty \\
-\infty < b_4 < +\infty \\
-\infty < b_5 < +\infty \\
3540 \leq b_6 \leq 3780
\end{gathered}
$$

All the shadow price except for the last one are 0 thus a small change in these contraints will not affect the solution.
The last contraint (Planer Time) has a shadow price of 2, which means that every minute we add/subtract to the planer time (staying in the given interval [3540,3780]), the solution increases/decreases by 2.

f.

There are two was of checking if a solution is degenrate, the first one is using the theory we have that since no constraint has an allowable increase or decrease = 0, the solution is not degenerate.
Another way is using the value of the following function:

```
solve.lpExtPtr(model)
```

```
## [1] 0
```

If the function has a value of 4 it means that the solution is degenerate, while 0 means that the optimal solution is found. Since in this case the value of the function confirms that the solution is not degenerate.

g.