Assignment 3 - The free position facility location problem

Pranav Kasela 846965

The Problem

In this assignment we will look at the **Facility Location Problem**. The problem can be described in general terms as follows:

We must decide the placement of a facility that distributes a certain good to a group of consumers that need it.

The placement must to be chosen in order to **minimize** the total compound distance from the facility and the customers.

The following assumptions has to be taken into account:

1. The possible location for the facility is unknown, that is the problem is to find the right spot to build it.
2. The facility building costs are fixed and independent from the position of the building site.

Notice that in this scenario there is one possible decision to make:

- where to build the facility, that is find the position (χ, υ) that minimises the compound distance of the facility with respect to all custumers.

Data

A file with the locations of the consumers can be found in the `Data` folder.

Distance function

Given the position of the facility $f = (\chi, \upsilon)$ and of a consumer $p_i = (x_i, y_i)$ use the following formula to calculate the distance between them.

$$d(f, p_i) = log((\chi - x_i)^2 + 1) + log((\upsilon - y_i)^2 + 1)$$

The assignment and the solution

1. Formulate the objective function to minimize for the described problem.

```r
library(ggplot2)

customer_locations <- read.csv("consumer_locations.csv")

ggplot(customer_locations, aes(x=x,y=y)) +
  geom_point() +
  theme(axis.title = element_blank(),
        axis.ticks = element_blank(),
        axis.text  = element_blank(),
        panel.grid = element_blank())
```

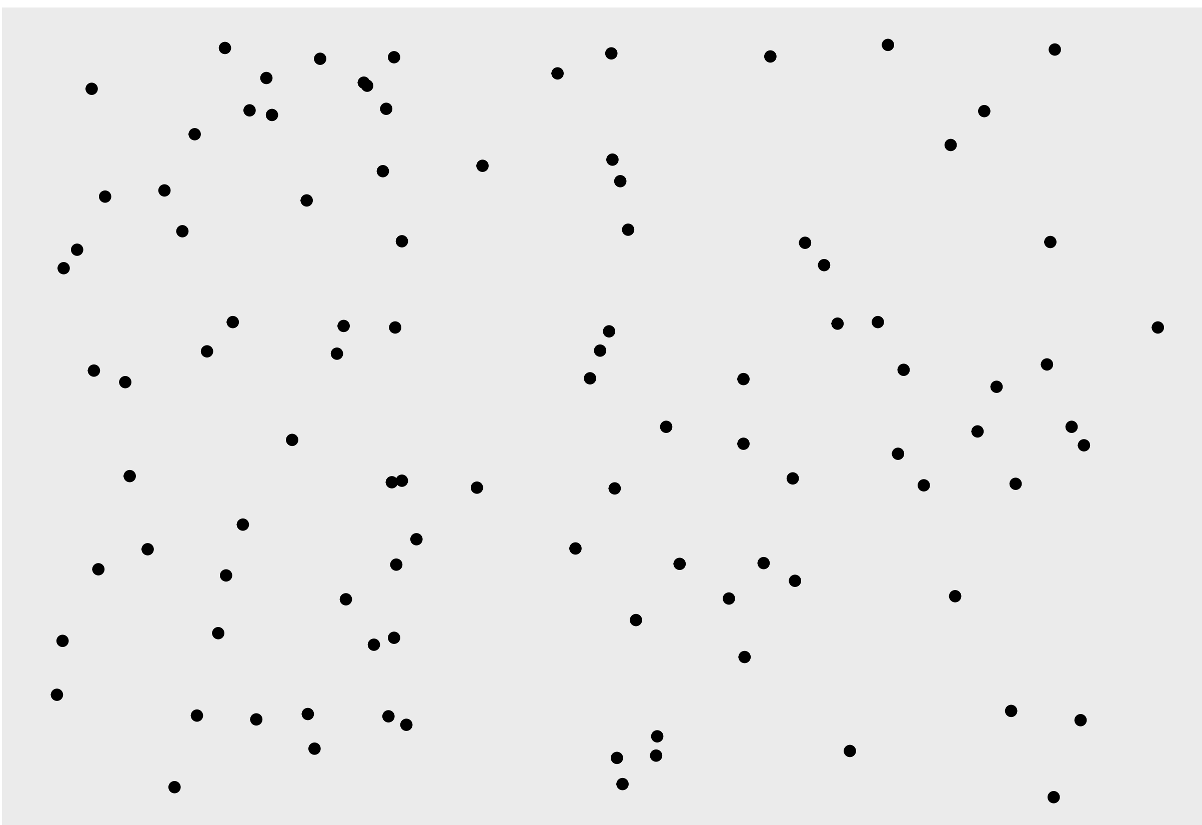

Figure 1: Customers Location Distribution

```
distance <- function(p1,p2){
  log((p1[,"x"]-p2[,"x"])^2+1) + log((p1[,"y"]-p2[,"y"])^2+1)
}
```

Our Problem in this case is to minimize the function:

$$\min \qquad \sum_{j=1}^{n} \text{distance}(P, p_j)$$

Wehre P is the location of the facility while p_j is the location of the j-th customer.

2. Express in analytical form the gradient for the objective to minimize.
3. Implement the `Gradiend Descent method` and solve the problem with it.
4. Solve the problem with a package provided by R (for instance, using the function `optimr` within the package `optimx`). Note that it is not required to use the `gradient descent` algorithm to solve the problem, other algorithms can be used as well.
5. Implement the `Stochastic gradient descent` algorithm with mini-batches and use it to solve the problem.