Computer Vision HW 1

1-1

a. (i)

$$f*g(n) = \sum_{k=0}^{N-1} f(n-k)g(n)$$

$$g*f(n) = \sum_{k'=0}^{N-1} g(n-k')f(k')$$

for $n=1$

$$f(n)g(0) + f(n-1)g(1) + \cdots + f(n-(N-1))g(N-1)$$

$$g(n)f(0) + g(n-1)f(1) + \cdots + g(n-(N-1))f(N-1)$$

$n=2$

$\vdots$

$n=N-1$ $$f(N-1)g(0) + f(N-1-1)g(1) + \cdots + f(0)g(N-1)$$

$$g(N-1)f(0) + g(N-1-1)f(1) + \cdots + g(0)f(N-1)$$

(ii)

$$(f*g)*h(n) = \sum_{k=0}^{n} (f*g)(k) \cdot h(n-k)$$

$$= \sum_{k=0}^{n}\sum_{l=0}^{k} f(l)g(k-l)h(n-k)$$

$$= \sum_{k=0}^{n}\sum_{k=l}^{n} f(l)g(k-l)h(n-k)$$

$$= \sum_{l=0}^{n}\sum_{k=0}^{n-l} f(l)g(k)h(n-k-l)$$

$$= \sum_{l=0}^{n} f(l)(g*h)(n-l) = f*(g*h)(n)$$

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(b)

$$f*g(0) = \sum_{k=0}^{N-1} f(0-k)g(k) = f(0)g(0) + f(-1)g(1) + \dots + f(-(N-1))g(N-1)$$

$$f*g(1) = \sum_{k=0}^{N-1} f(1-k)g(k) = f(1)g(0) + f(0)g(1) + \dots$$

$$f*g(N-1) = \sum_{k=0}^{N-1} f(N-1-k)\,g(k) = f(N-1)g(0) + f(N-2)g(0) + \dots + f(0)g(N-1)$$

$$f*g = \begin{bmatrix} f*g[0] \\ f*g[1] \\ \vdots \\ f*g[N-1] \end{bmatrix}_{N\times1} = \begin{bmatrix} f(0) & f(-1) & \cdots & f(-N+1) \\ f(1) & f(0) & & \\ \vdots & \vdots & \ddots & \\ f(N-1) & f(N-2) & \cdots & f(0) \end{bmatrix}_{N\times N} \begin{bmatrix} g[0] \\ g[1] \\ \vdots \\ g[N-1] \end{bmatrix}_{N\times1} = H\cdot g.$$

(C)

Since it is associative

$$y = f * g = (u\cdot v^T) * g = u * (v^T * g)$$
$$= v^T * (u * g)$$

ex:

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \text{ is seperable } = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

①

$$\begin{bmatrix} 10 & 20 & 30 \\ 40 & 50 & 60 \\ 70 & 80 & 90 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 1 \\ 1 & 2 & 1 \end{bmatrix} = 800$$

②

$$\begin{bmatrix} 10 & 20 & 30 \\ 40 & 50 & 60 \\ 70 & 80 & 90 \end{bmatrix} * \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 160 & 200 & 240 \end{bmatrix}, \quad \begin{bmatrix} 160 & 200 & 240 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} = 800$$

→ In that f is often got by SVD. $f = \sum \sigma u_i v_i^T$

$$y_{[m,n]} = f_{[m,n]} * g_{[m,n]} = \sum_{j=0} \sum_{j=0}^{N-1} f[i,j]\cdot g[m-i,n-j]$$

$$= \sum \sum u[i]\cdot v[j]\cdot x[m-i,n-j]$$

$$= \sum v[j] \left(\sum u[i]\cdot x[m-i,n-j] \right)$$

$(N-1)_{mod}$