Computer Vision HW 1

2.1

(a) (i)

$$(f*g)[n] = \sum_{k=0}^{N-1} f[n-k]g[k] = \sum_{l=n}^{n-(N-1)} f(n-l)g(l) = \sum_{0}^{N-1} g(l)f(n-l) = g*f$$

(ii)

$$((f*g)*h)(n) = \sum_{k=0}^{n} (f*g)(k)\cdot h(n-k) = \sum_{k=0}^{n}\sum_{l=0}$$

(b) Since $f*g = \underline{H}g$.

$$\begin{bmatrix} f[0] \\ f[1] \\ \vdots \\ f[M-1] \end{bmatrix}_{N\times 1} = \begin{bmatrix} h[0,0] & h[0,1] & \cdots & h[0,N-1] \\ h[1,0] & h[1,1] & \cdots & h[1,N-1] \\ \vdots & \vdots & \vdots & \vdots \\ h[M-1,0] & h[M-1,1] & \cdots & h[M-1,N-1] \end{bmatrix}_{N\times N} \begin{bmatrix} g[0] \\ g[1] \\ \vdots \\ g[N-1] \end{bmatrix}_{N\times 1}$$

(C)

Since it is associative

$$y = f*g = (u\cdot v^T)*g = u*(v^T*g)$$
$$= v^T*(u*g)$$

ex:

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \text{ is seperable } = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}[1\ 2\ 1]$$

①

$$\begin{bmatrix} 10 & 20 & 30 \\ 40 & 50 & 60 \\ 70 & 80 & 90 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 1 \\ 1 & 2 & 1 \end{bmatrix} = 800$$

②

$$\begin{bmatrix} 10 & 20 & 30 \\ 40 & 50 & 60 \\ 70 & 80 & 90 \end{bmatrix} * \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = [160\ 200\ 240], \quad [160\ 200\ 240]*[1\ 2\ 1] = 800$$

In that f is often got by SVD. $f = \sum \sigma u_i v_i^T$

$$y[m,n] = f[m,n] * g[m,n] = \sum_{j=0}^{N-1}\sum_{j=0} f[i,j]\cdot g[m-i,n-j]$$

$$= \sum\sum u[i]\cdot v_i[j]\cdot x[m-i,n-j]$$

$$= \sum v[j]\left(\sum u[i]\cdot x[m-i,n-j]\right)$$