EECS 568 Mobile Robotics HW2

PoKang Chen

Task 1

A. Extended Kalman Filter (EKF)

Mobile Robotics HW 2.

A. EKF.

$$\overset{\mu_1\ \mu_2\ \mu_3}{\mathbf{X} = x_k, y_k, \theta_k}, \quad \overset{u_1\ \ u_2}{u = (\hat{v}, \hat{\omega}, \hat{\gamma})}$$

For linearlization

$$f(u_t, x_{t-1}) \approx f(u_t, \mu_{t-1}) + \overset{F}{\left(\frac{\partial f(u_t, \mu_{t-1})}{\partial x_{t-1}}\right)}(x_{t-1} - \mu_{t-1})$$

$$h(x_t) \approx h(\bar{\mu}_t) + \overset{H}{\left(\frac{\partial h(\bar{\mu}_t)}{\partial x_t}\right)}(x_t - \bar{\mu}_t)$$

$$f = \begin{cases} x_{k+1} = x_k - \frac{\hat{v}}{\hat{\omega}}\sin(\theta_k) + \frac{\hat{v}}{\hat{\omega}}\sin(\theta_k + \hat{\omega}\Delta t) \\ y_{k+1} = y_k + \frac{\hat{v}}{\hat{\omega}}\cos(\theta_k) - \frac{\hat{v}}{\hat{\omega}}\cos(\theta_k + \hat{\omega}\Delta t) \\ \theta_{k+1} = \theta_k + \hat{\omega}\Delta t + \hat{\gamma}\Delta t \end{cases} \quad \text{gfun}$$

$$F = \left.\frac{\partial f}{\partial \mathbf{X}}\right|_{\mu_{k-1}} = \begin{bmatrix} 1 & 0 & \frac{\hat{v}}{\hat{\omega}}\cos(\theta + \hat{\omega}) - \frac{\hat{v}}{\hat{\omega}}\cos(\theta) \\ 0 & 1 & \frac{\hat{v}}{\hat{\omega}}\sin(\theta + \hat{\omega}\Delta t) - \frac{\hat{v}}{\hat{\omega}}\sin(\theta) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{cases} x_k = f(u_k, x_{k-1}) + w_k = x_{k-1} + w_k \\ z_k = h(x_k) + v_k = \begin{bmatrix} \sqrt{x_{k1}^2 + x_{k2}^2} \\ \mathrm{atan}\left(\frac{x_{k2}}{x_{k1}}\right) \end{bmatrix} + v_k \end{cases} \quad \text{hfun}$$

$$\therefore H = \left.\frac{\partial h}{\partial \mathbf{X}}\right|_{\bar{\mu}} = \begin{bmatrix} \frac{(\bar{y} - y_k)}{\hat{z}^2} & -\frac{(\bar{x} - x_k)}{\hat{z}^2} & -1 \\ -\frac{(\bar{x} - x_k)}{\hat{z}} & -\frac{(\bar{y} - y_k)}{\hat{z}} & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} \frac{1}{\hat{\omega}}\sin(\theta + \hat{\omega}) - \frac{1}{\hat{\omega}}\sin(\theta_k) & \frac{\hat{v}}{\hat{\omega}}\cos(\theta + \hat{\omega}) - \frac{\hat{v}}{\hat{\omega}^2}\sin(\theta_k + \hat{\omega}) + \frac{\hat{v}}{\hat{\omega}^2}\sin(\theta) & 0 \\ \frac{1}{\hat{\omega}}\cos(\theta) - \cos(\theta + \hat{\omega}) & \frac{\hat{v}}{\hat{\omega}^2}\cos(\theta + \hat{\omega}) + \frac{\hat{v}}{\hat{\omega}}\sin(\theta + \hat{\omega}) - \frac{\hat{v}}{\hat{\omega}^2}\cos(\theta) & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$\therefore$ Prediction :

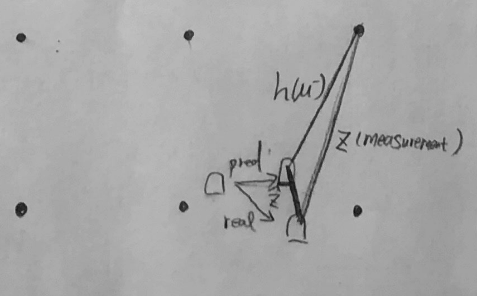
$$\begin{bmatrix} \bar{\mu}_k = f(u, \mu_{k-1}) \\ \bar{\Sigma}_k = F\Sigma_{k-1}F^T + W_k Q_k W_k^T \end{bmatrix}$$

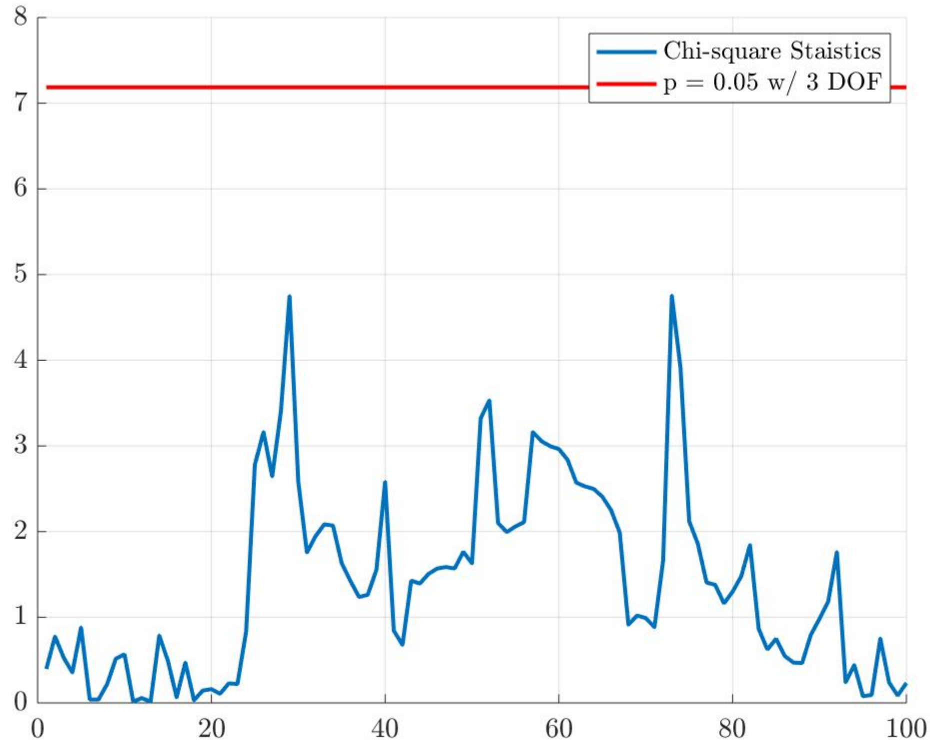
Innovation :

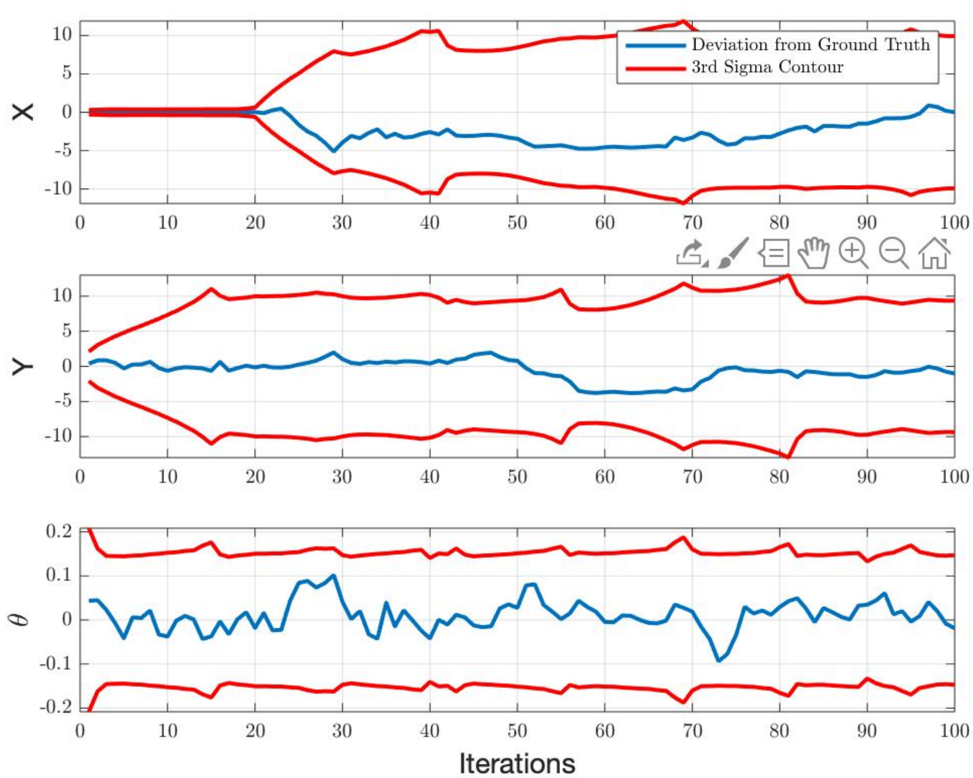
$$\begin{bmatrix} v_k = \underset{\text{real measurement}}{z_k} - h(\bar{\mu}_k) \\ S = H\bar{\Sigma}H^T + VRV^T \end{bmatrix} \quad K = \bar{\Sigma}H^T S^{-1}$$

Correction :

$$\begin{bmatrix} \mu_k = \bar{\mu}_k + K v \\ \Sigma_k = (I - KH)\Sigma \end{bmatrix}$$




Chi-square Staistics
p = 0.05 w/ 3 DOF


Deviation from Ground Truth
3rd Sigma Contour
X
Y
θ
Iterations

B. Unscented Kalman Filter (UKF)

B. UKF

Sigma Points.

$$\begin{cases} \chi^{[0]} = \mu \\ \chi^{[1]} = \mu + l_1' \\ \vdots \\ \chi^{[n]} = \mu + l_n' \end{cases} \qquad \begin{cases} \chi^{[n+1]} = \mu - l_1' \\ \vdots \\ \chi^{[2n]} = \mu - l_n' \end{cases}$$

$$\underset{n\times n}{\Sigma} = L L^T$$

$$\Sigma = VDV^{-1} = V\begin{bmatrix} \sqrt{d_{11}} & & \\ & \sqrt{d_{22}} & \\ & & \ddots \; \sqrt{d_{nn}} \end{bmatrix}\begin{bmatrix} \sqrt{d_{11}} & & \\ & \sqrt{d_{22}} & \\ & & \ddots \; \sqrt{d_{nn}} \end{bmatrix}V^{-1} = (\underline{V}D^{1/2}V^{-1})(VD^{1/2}\underline{V^{-1}}) = LL^T$$

$$\Rightarrow L' = \sqrt{\underset{\text{dim}}{n} + \underset{\text{scaling params}}{k}}\; L = \begin{bmatrix} | & | & & | \\ l_1' & l_2' & \cdots & l_n' \\ | & | & & | \end{bmatrix}$$

f, h is some as EKF

Weights :

$$\begin{cases} W_{mean}^{[0]} = \dfrac{k}{n+k} \\ W_{cov}^{[0]} = W_{mean}^{[0]} + (1-\alpha^2+\beta) \end{cases} \Rightarrow W_{mean}^{[i]} = W_{cov}^{[i]} = \frac{1}{2(n+k)} \quad i = 1, \dots, 2n$$

Predicted :

Calculated Sigma Points $\Rightarrow$ $\chi^{[0]} \sim \chi^{[2n]}$. (2n+1 points, using μ_{k-1} and Σ_{k-1})

weights $\Rightarrow$ W_k^- (2n+1 weights)

$$\mu_k^- = \sum_{i=0}^{2n} W_k^{[i]-} f(u_k, X_{k-1}^{[i]}) = W_k^{[0]} f(u_k, X_{k-1}^{[0]}) + \underbrace{W_k^{[1]} f(u_k, X_{k-1}^{[1]}) + \cdots + W_k^{[2n]} f(u_k, X_{k-1}^{[2n]})}_{2n}$$

$$\Sigma^- = \sum_{i=0}^{2n} W_k^{[i]-} \left(f(u_k, X_{k-1}^{[i]}) - \mu_k^- \right)\left(f(u_k, X_{k-1}^{[i]}) - \mu_k^- \right) + Q$$

Calculated Sigma Points $\Rightarrow$ $\bar{\chi}^{[0]} \sim \bar{\chi}^{[2n]}$ (2n+1 points, using $\bar{\mu}_{k-1}$ and $\bar{\Sigma}_{k-1}$)

weights $\Rightarrow$ W_k

Innovations :

$$\nu = z - \underline{\sum_{i=1}^{2n} W_k^{[i]} h(X_k^{[i]-})} = z - \hat{z}$$

$$S = \sum_{i=1}^{2n} W_k^{[i]} \left(h(X_k^{[i]-}) - \hat{z}\right)\left(h(X_k^{[i]-}) - \hat{z}\right)^T + R_k$$

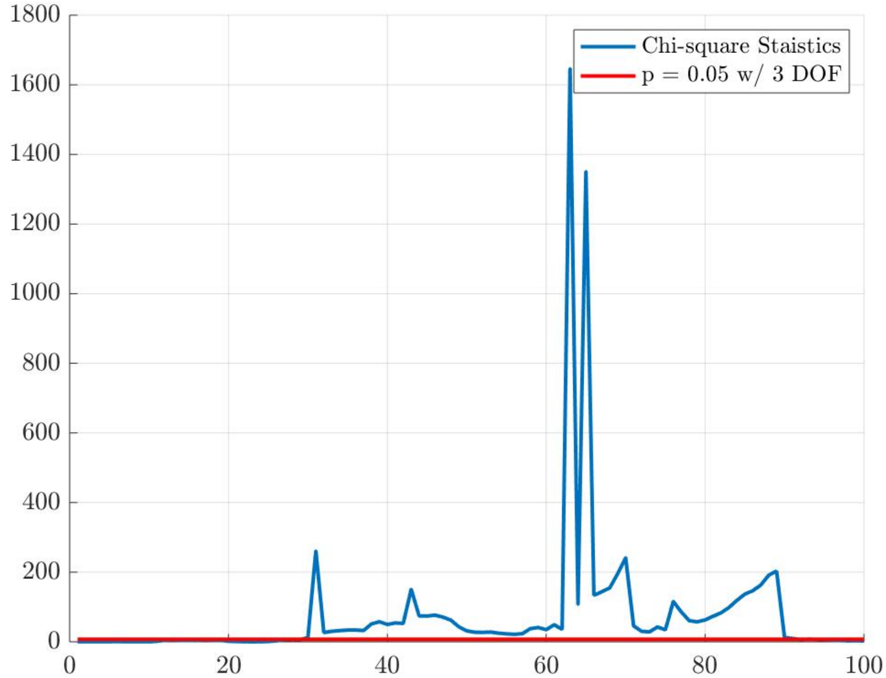
$$K = \Sigma_{xz} S^{-1}$$

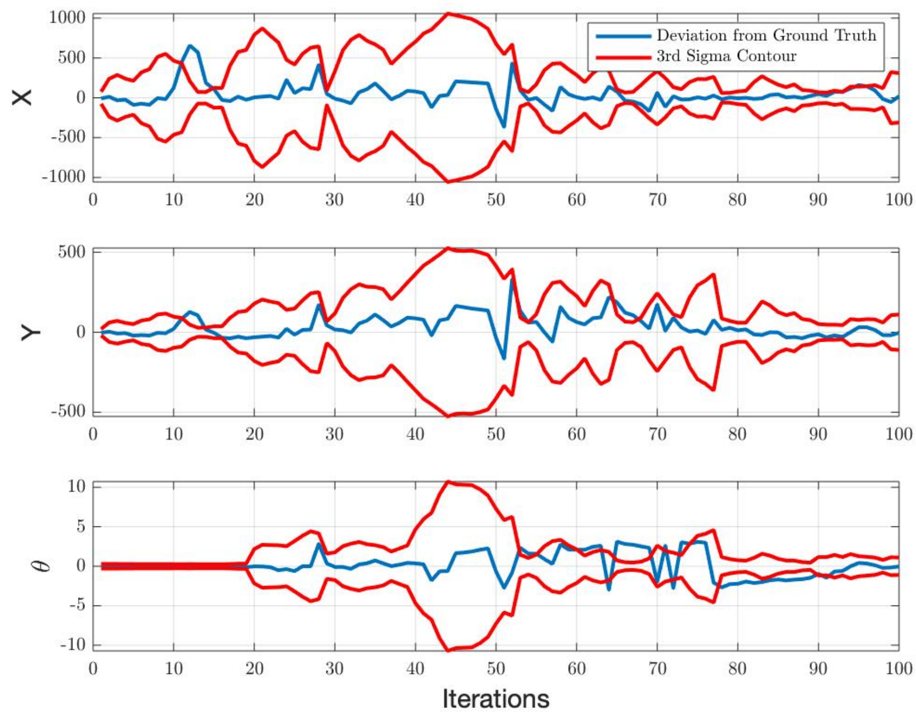
$$\Sigma_{xz} = \sum_{i=1}^{2n} W_k (X_k^{[i]-} - \mu_k^-)(h(X_k^{[i]-}) - \hat{z}_k)^T$$

Corrections :

$$\mu_k = \mu_k^- + K\nu$$

$$\Sigma_k = \Sigma_k^- - KSK^T$$


Chi-square Staistics
p = 0.05 w/ 3 DOF


Deviation from Ground Truth
3rd Sigma Contour
X
Y
θ
Iterations

C. Particle Filter (PF)

C. Particle Filter

Importance Sampling

$$\tilde{\omega} = \frac{p(x)}{q(x)} \quad \begin{matrix}\leftarrow \text{target} \\ \leftarrow \text{proposal}\end{matrix}$$

$$w_k^i \propto \frac{p(x_{0:k} \mid z_{1:k})}{q(x_{0:k} \mid z_{1:k})}$$

$$p(x_{0:k} \mid z_{1:k}) = \frac{p(z_k \mid x_{0:k}, z_{k-1}) \cdot p(x_k \mid x_{0:k-1}, z) \cdot p(x_{0:k-1} \mid z_{1:k-1})}{p(z_k \mid z_{1:k-1})}$$

$$\propto p(z_k \mid x_k)\, p(x_k \mid x_{k-1})\, \cancel{p(x_{0:k-1} \mid z_{1:k-1})}$$

$$q(x_{0:k} \mid z_{1:k}) \rightarrow q(x_k \mid x_{0:k-1}, z) \cdot \cancel{q(x_{0:k-1} \mid z_{1:k-1})}$$

$$\therefore \propto \frac{p(z_k \mid x_k^i) \cdot p(x_k^i \mid x_{k-1}^i)}{q(x_k^i \mid x_{0:k-1}^i, z_{1:k})} w_{k-1}^i \propto \frac{p(z_k \mid x_k^i) \cdot p(x_k^i \mid x_{k-1}^i)}{q(x_k^i \mid x_{k-1}^i, z_k)} w_{k-1}^i$$

Could ignore history $\left[\begin{matrix} x_{0:k-1} \rightarrow x_{k-1} \\ z_{1:k} \rightarrow z_k \end{matrix}\right.$

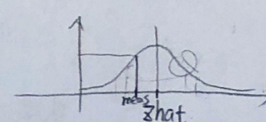
$$= \frac{\frac{p(z_k \mid x_k^i) \cdot p(x_k^i \mid x_{k-1}^i)}{p(z_k \mid x_k^i, x_{k-1}^i)\, p(x_k^i \mid x_{k-1}^i)}}{p(z_k \mid x_{k-1}^i)} w_{k-1}^i = \boxed{p(z_k \mid x_{k-1}^i)\, w_{k-1}^i}$$

suboptimal importance density

$\downarrow$

mvnpdf(measurement, zhat, Q)

$\downarrow$



$\therefore$ Algorithm:

$$x_k^i \sim p(x_k \mid x_{k-1}^i, u_k)$$

$$w_k^i = w_{k-1}^i \cdot p(z_k \mid x_{k-1}^i)\, w_{k-1}^i \longrightarrow$$

$$X_k = X_k \cup \{x_k^i, w_k^i / w_{total}\}_{i=1}^n, \quad w_{total} = \sum_{i=1}^n w_k^i$$

$$n_{eff} = 1 \Big/ \sum_{i=1}^n (w_k^i)^2$$

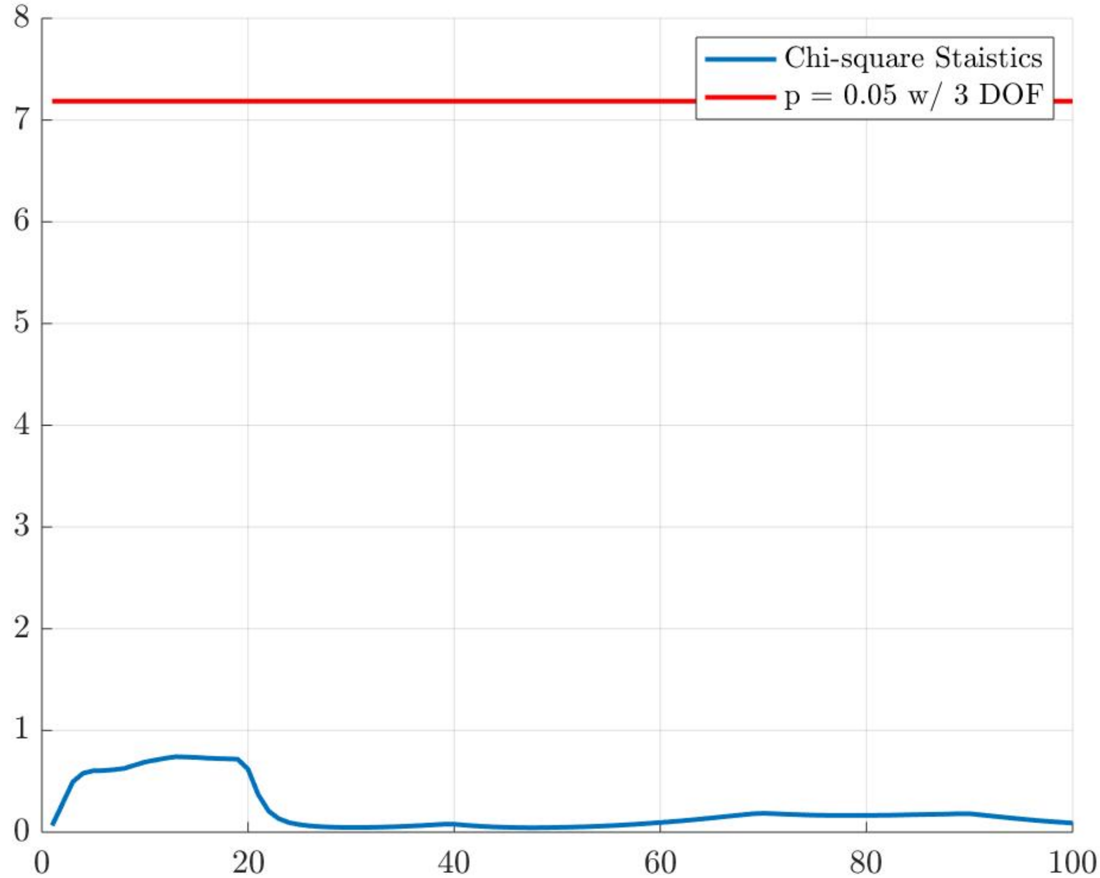
if $n_{eff} < n_t$

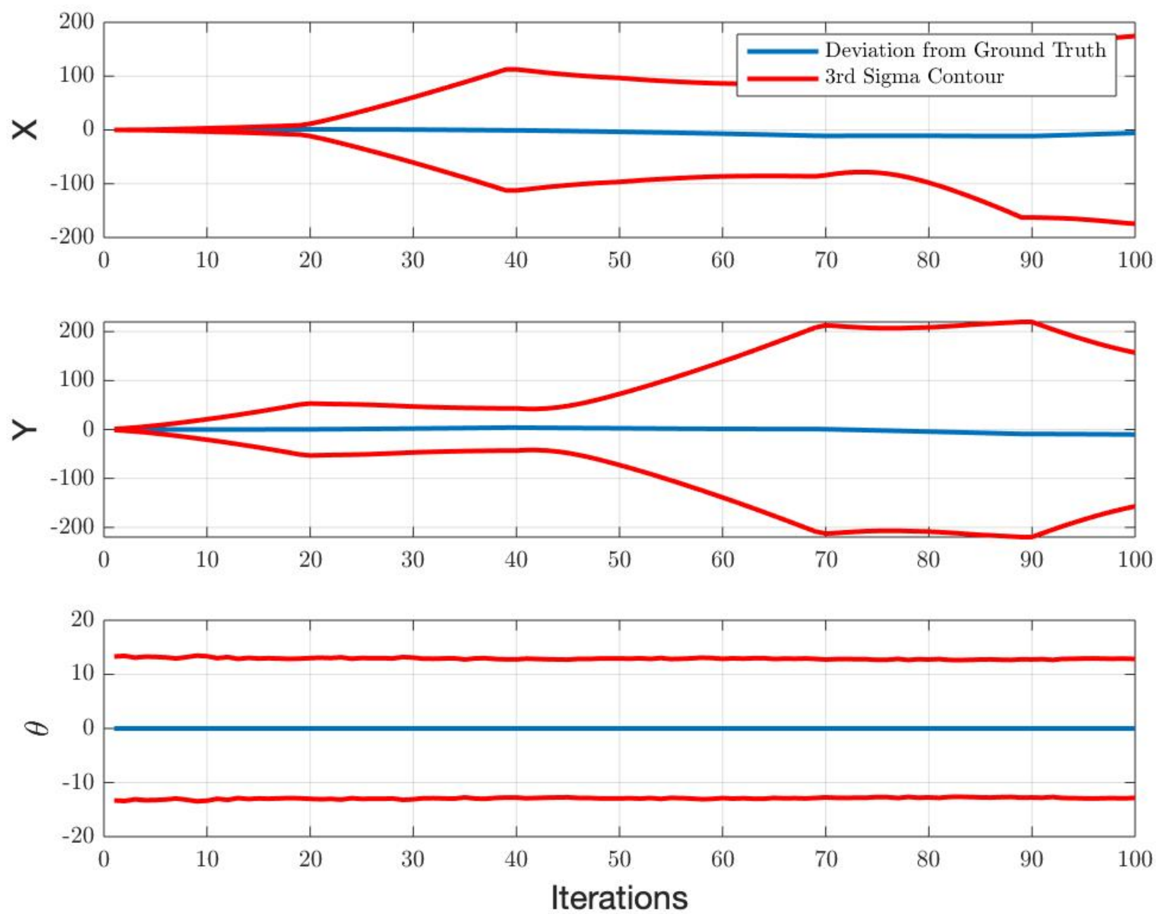
X_k = resample with X_k.

$$x_k = f(u_k, x_{k-1}) + w_k$$

$$z_k = h(x_k) + v_k = \begin{bmatrix} \sqrt{x_k^{[1]2} + x_k^{[2]2}} \\ \operatorname{atan2}(x_k^{[1]}, x_k^{[2]}) \end{bmatrix} + v_k$$

$\rightarrow$ Same as EKF, UKF


Chi-square Staistics
p = 0.05 w/ 3 DOF


Deviation from Ground Truth
3rd Sigma Contour
X
Y
θ
Iterations

D. Right-Invariant EKF (RI-EKF)

RI-EKF

Error :

$\eta_t^r = \bar{X}_t X_t^{-1} = (\bar{X}_t L)(X_t L)^{-1}$

$\uparrow$ estimate $\quad \uparrow$ true value

$g(e^{\xi_t}) = A\xi + O(\ )$,

$\frac{d(\eta_t^r)}{dt} = g(\eta_t^r)$, $\quad \frac{d}{dt}\xi_t = A_t \xi_t$,

$\eta_t = e^{(\xi_t)} \approx I + \xi_t^\wedge$

$\therefore \frac{d(\eta_t^r)}{dt} = \frac{d(e^{\xi_t})}{dt} = e^{(\xi_t)}\hat{w}_t - \hat{w}_t e^{(\xi_t)}$

$= \frac{d(I+\xi_t)}{dt} = (I + \hat{\xi}_t)\hat{w}_t - \hat{w}_t(I + \hat{\xi}_t)$

$\Rightarrow \frac{d\xi_t}{dt} = \hat{\xi}_t \hat{w}_t - \hat{w}_t \hat{\xi}_t$

$\mu_t = \mu^- + K\nu$

$\uparrow$

$(z - H_k\mu^-)$

Propagation :

$\frac{d}{dt}\bar{X}_t = f_{ut}(\bar{X}_t)$

$\frac{d\eta_t^r}{dt} = \underline{\quad\quad} \Rightarrow \frac{d}{dt}\xi_t^r = A_t^r \xi_t^r - Ad_{\bar{X}_t} w_t$

$\therefore \frac{d}{dt}P_t^r = A_t^r P_t^r + P_t^r A_t^{rT} + Ad\, Q_c\, Ad^T$

Update :

Kgain

$X_{t_k}^+ = e^{(L_{t_k}(\bar{X}_t Y - b))}\bar{X}_t$

$P_t^+ = (I - L_t H) P^- (I - LH)^T + LNL^T$

(R)

Kgain $\quad$ Measurement Jacobian

where

$L_t = P_t^r H^T S^{-1} \longrightarrow S = H\bar{P}H^T + \bar{N}_k$

$(\Sigma) \qquad\qquad\qquad (\Sigma) \quad (R)$

State : $x = (x, y, w)^T$

$so(2)$

$\uparrow$

$C = \begin{pmatrix} R & t \\ 0 & 1 \end{pmatrix} \in SE(2)$

$\xi = (\hat{v}, \hat{w})^T \in R^2$

$\hat{v}\, G_1 + \hat{w}\, G_2 + \theta G_3 \in se(2)$

generator of the algebra

Exp Map :

$\xi = (u\ \theta) \in se(2)$

$e^{(\xi)} = e\left(\begin{array}{c|c} \theta_\times & u \\ 0 & 0 \end{array}\right)$

$= I + \left(\begin{array}{c|c} \theta_\times & u \\ 0 & 0 \end{array}\right) + \frac{1}{2!}\left(\begin{array}{c|c} \theta_\times^2 & \theta_\times u \\ 0 & 0 \end{array}\right) + \frac{1}{3!}\left(\begin{array}{c|c} \theta_\times^3 & \theta_\times^2 u \\ 0 & 0 \end{array}\right) + \dots$

Adj

$\xi = (\hat{v}, \hat{w})^T$

$Adj_c = \left(\begin{array}{cc|c} R & & \begin{matrix} t_2 \\ -t_1 \end{matrix} \\ 0 & 0 & 1 \end{array}\right)$