Galaxy Rotation Curve

The rotation curve of any object is a plot showing how the orbital velocity (V) varies with the distance (R) from the center of the object.

A solid body(disk-like) will have a rotational velocity proportional to the distance from the center, i.e.

$$V \propto R$$

Consider the case shown in figure 1. The object shown here has a disk-like rotation, i.e., it rotates as a solid. As the disk rotates, the points A, B, C, and D, will remain in a straight line along a radius. If you plotted the orbital speed as a function of the distance from the center, the plot would be a straight line.

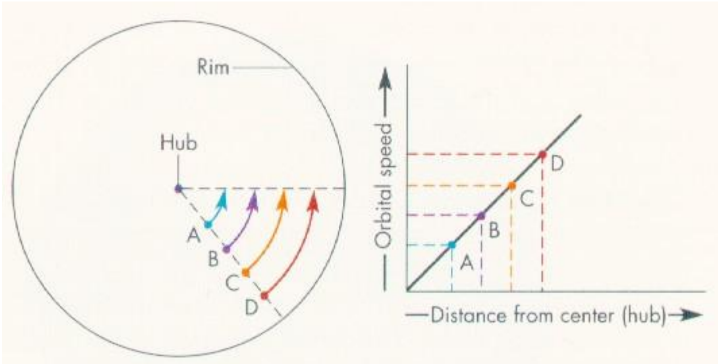

Figure. 1: Solid body rotation curve

(image credits: http://abyss.uoregon.edu/~js/cosmo/lectures/lec17.html)

We know our solar system has the Sun, eight planets, and various other bodies like moons, dwarf planets, comets, and asteroids. In figure. 2 we have the orbital velocity of each planet plotted against its distance from the Sun.

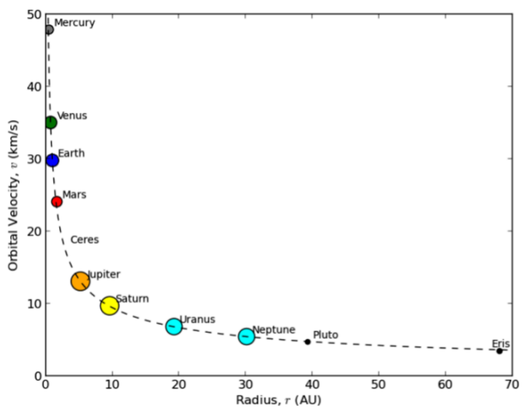

Figure. 2: Rotation curve of our solar system

(image credits: https://milkyway.cs.rpi.edu/milkyway/science.php)

This rotation curve is pretty different from a disk-like one. The planet-like rotation curve can be explained from Kepler's third law. This law states that, under the assumption of a circular orbit, the square of the period of a planet's orbit is proportional to the cube of its radius. The exact equation that explains the curve can be simply derived by equating the gravitational force to the centrifugal force of the planet-sun pair.

$$mv^2/R = GMm/R^2 \tag{1}$$

Here,

m = mass of planet

M = mass of star

R = Distance of planet from star

G = Gravitational Constant

v = orbital velocity of planet

Equation (1) simplifies to,

$$v^2 = GM/R$$

$$\Rightarrow v \propto 1/\sqrt{R}$$

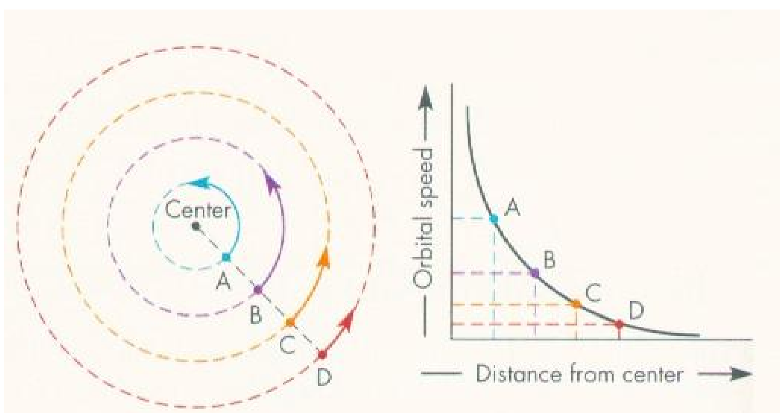

Figure. 3: Planet-like rotation curve

(image credits: http://abyss.uoregon.edu/~js/cosmo/lectures/lec17.html)

In contrast to the disk-like case shown in figure 1, planet-like rotation, which follows keplerian motion, is shown in figure 3. Unlike the case of figure 1, A, B, C, and D won't stay co-linear during rotation as the velocity profile follows Kepler's third law.

Coming to the concerned topic of the galaxy rotation curve, consider our own Milky Way Galaxy. Most of the stars in the Milky Way are concentrated near its center and as we move outwards the density of visible mass decreases. So, starting from the center till a certain distance the galaxy can be considered equivalent to a disk, and beyond that, we can expect that as we get further from the center the stars would revolve around the center of the galaxy more slowly in the same way that the planets orbit more slowly as we get further from the Star. Thus the rotation curve of a galaxy should like as shown in the figure. 4.

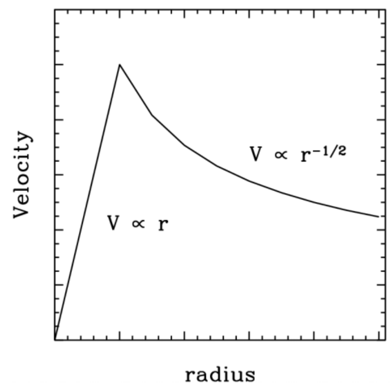

Figure. 4: Galaxy Rotation curve.
(image credits: https://ps.uci.edu/~observat/sites/default/files/COL_Activity_RotCurve.pdf)

But, unlike our expectations the observation of the galaxy rotation curve, looks quite different. A comparison can be seen in figure 5. Initially, the velocity is increasing up to a distance as expected, but beyond that, it doesn't decrease rather remains almost constant till large distances.

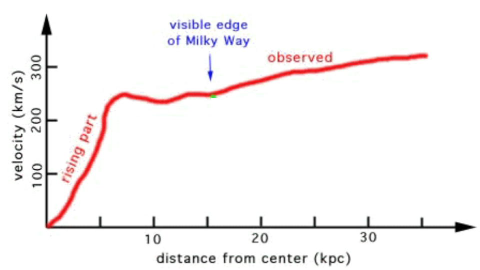

Figure. 5: Galaxy Rotation curve (observed)
(image credits: http://personal.psu.edu/mxe17/A020S/pages/darkmatter.html)

A flattened curve essentially means there is a large amount of unseen material, more widely spread than the stars and gas. These unseen materials are termed "dark matter". Thus similar to the near center region of the galaxy, the surrounding mass, felt by stars even at large distances increases due to the presence of this dark matter. They essentially trap the fast-moving stars and maintain a high average velocity of stars. So, theories involving the existence of dark matter are the main postulated solutions to account for the variance.

The observations made in the figure. 5 are mainly done for 21 cm wavelength. Thus using the data obtained from our horn we will try recreating this plot that contradicts the calculated value. In figure. 6 you can see a better comparison shown for M33.

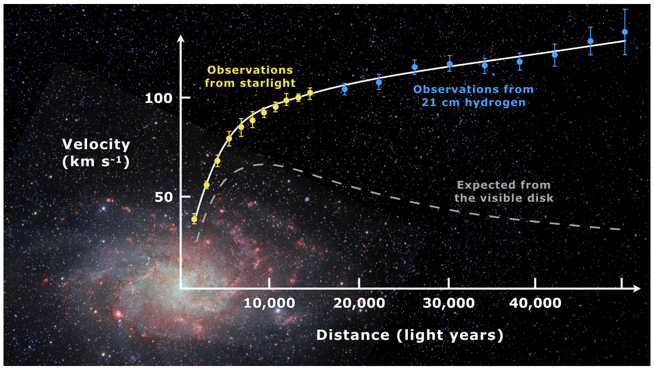

Figure. 6: Comparison of Expected and Observed Galaxy Rotation Curve for M33
(image credits: https://en.wikipedia.org/wiki/Galaxy_rotation_curve)

References:

1. "Rotation Curve of Galaxy" by James Schombert, University of Oregon
2. "Galaxy Masses and Dark Matter" by Richard Mushotzky, University of Maryland
3. M. Honma and Y. Sofue, "Rotation Curve of the Galaxy," in Publications of the Astronomical Society of Japan, vol. 49, no. 4, pp. 453-460, Aug. 1997, doi: 10.1093/pasj/49.4.453.