INF4490 Mandatory assignment 1: Travelling Salesman Problem

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Introduction

This assignement was solved with Python 3.7. The only package used that is not part of the Python Standard Library is matplotlib. This package can be installed with pip, i.e. `pip3 install matplotlib`. I have used version 2.2.3 of the matplotlib package.

You may run the algorithms presented in this report with the same parameters and on the same travelling salesman problem by running `python3 main.py`.

Exhaustive search

In the travelling salesman problem there are $n!$ different tours if we have n cities. Since it does not matter in which city we start a tour we only need to check $(n-1)!$ of these tours. Additionally it does not matter which direction we take a tour in so we can further divide the number of tours we need to check in half. We therefore end up having to check $(n-1)!/2$ tours in the exhaustive search.

Cities	Running time	Tours searched		Tours searched per second
6	0.0003	$5!/2 =$	60	
7	0.0020	$6!/2 =$	360	
8	0.0127	$7!/2 =$	2520	
9	0.1270	$8!/2 =$	20160	
10	1.1503	$9!/2 =$	181440	157732
11	12.144	$10!/2 =$	1814400	149407
12	141.22	$11!/2 =$	19958400	141328

Table 1: Running time for exhaustive search with different number of cities.

Assuming we can search 140000 tours per second and with 31557600 seconds in a year it will take about 3 billion years to do an exhaustive search for the best tour in our 24-city travelling salesman problem.

The shortest tour among the 10 first cities is of length 7486.

Hill climbing

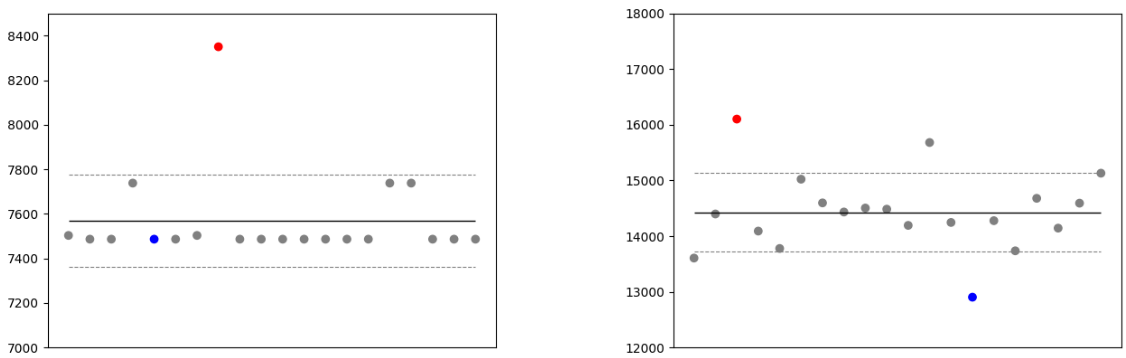
In figure 1a we see that our hill climber finds a tour very close in length to the best tour almost every time. In our 20 runs only once it is quite far off.

We see, as we would expect, that the standard deviation increases when we increase number of cities in our travelling salesman problem.

The results of 20 runs of the hill climber on the first 10 cities and all 24 cities are summarized in table 2.

	Cities	
	10	24
Best tour	7486.3	12903.6
Worst tour	8349.9	16101.5
Average tour	7568.9	14429.6
Standard deviation	205.2	709.0
Average running time	0.098	0.578

Table 2: Statistics for the hill climber on the first 10 cities and all 24 cities over 20 runs.



(a) 10 cities

(b) 24 cities

Figure 1: Plots showing tour lengths found with the hill climber.

Genetic algorithm

The genetic algorithm first inititalizes with a randomly generated population of tours. Then first an exploration phase is run before an exploitation phase. The default is to run the first 60% generations in a exploration phase.

In the exploration phase stochastic universal sampling, partially mapped crossover and scramble mutation is used for parent selection, as crossover operator and as mutation operator, respectively. Likewise, in the exploitation phase the operators as fitness proportional sampling, edge recombination and swap mutation. In both phases replacement is fitness based. The best 5% or a minimum of the best 5 individual in a population is kept from generation to generation.

10-city tours

We see that the genetic algorithm finds the shortest tour in the 10-city travelling salesman problem on all 20 runs. The results are summarized in table 3 and figure 2 shows a plot. The algorithm was run with a population size of 200 and for 500 generations.

Best tour	7486.31
Worst tour	7486.31
Average tour	7486.31
Standard deviation	0.00
Average running time	19.12

Table 3: Statistics for the genetic algorithm on the first 10 cities over 20 runs.

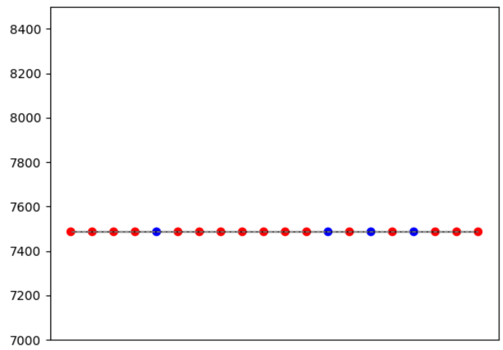


Figure 2: Plot showing tour lengths found with the genetic algorithm on a 10-city travelling salesman problem.

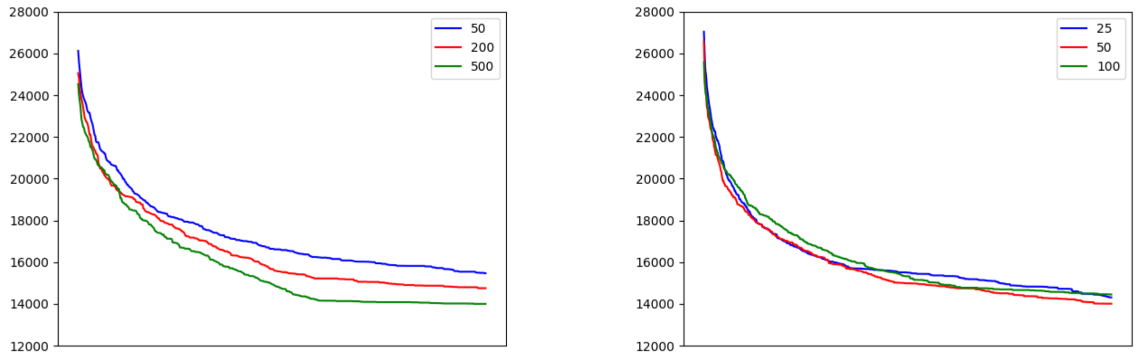
24-city tours

The 24-city travelling salesman problem was run with population sizes of 50, 200 and 500 for 500 generations, and with population population sizes of 25, 50 and 100 for 1000 generations. Each combinatoin was run 20 times.

The results are summarized in table 4. It seeems we get better results running smaller populations for more generations than larger populations for fewer generations.

Population size	Best tour	Worst tour	Average tour	Standard deviation	Average running time	Generations
50	13368.5	17470.1	15470.3	1000.0	10.1	500
200	13493.3	16214.6	14756.9	829.2	41.9	500
500	12684.6	15187.1	14004.8	682.3	109.3	500
25	13057.1	15547.4	14317.0	824.0	10.2	1000
50	12842.6	16870.0	14013.8	1005.4	20.8	1000
100	12737.8	16630.8	14454.7	1039.1	39.6	1000

Table 4: Statisics for the genetic algorithm on all 24 cities over 20 runs.



(a) 500 generations

(b) 1000 generations

Figure 3: Plot showing the average best tour in each generation over 20 runs with the genetic algorithm.

Hybrid algorithm

In the hybrid algorithm the selection of parents for the first generation of offspring is a fintess proportional selection based on the fitness value obtained from running the hill climber on the initial random population. In the Lamarckian version the parent is replaced with the hill-climbed version while in the Baldwinian the original parent is kept. After this inital the algorithm is the same as the genetic.

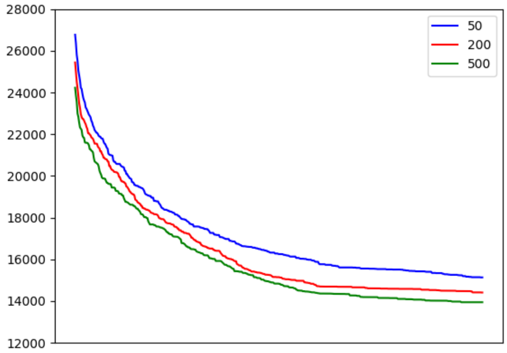
Baldwinian

We see that the Baldwinian version of the hybrid algorithm yields a result very similar to that of the genetic algorithm.

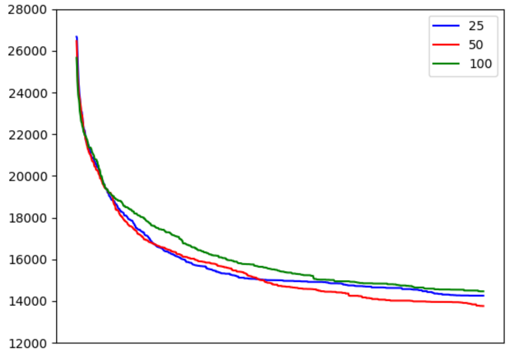
The results are summarized in table 5 and plots showing the average best in each generation is shown in figure 4.

Population size	Best tour	Worst tour	Average tour	Standard deviation	Average running time	Generations
50	13271.7	16463.1	15136.0	864.6	13.1	500
200	13079.3	16766.9	14411.7	967.1	52.9	500
500	12879.5	15845.6	13946.1	870.0	134.2	500
25	12807.4	15889.7	14260.2	939.2	11.0	1000
50	12423.0	15772.1	13764.1	887.9	22.0	1000
100	13078.0	15622.4	14467.6	776.4	47.0	1000

Table 5: Statistics for the Baldwinian version of the hybrid algorithm over 20 runs.



(a) 500 generations



(b) 1000 generations

Figure 4: Plots showing the average best tour in each generation over 20 runs with the Baldwinian version of the hybrid algorithm.

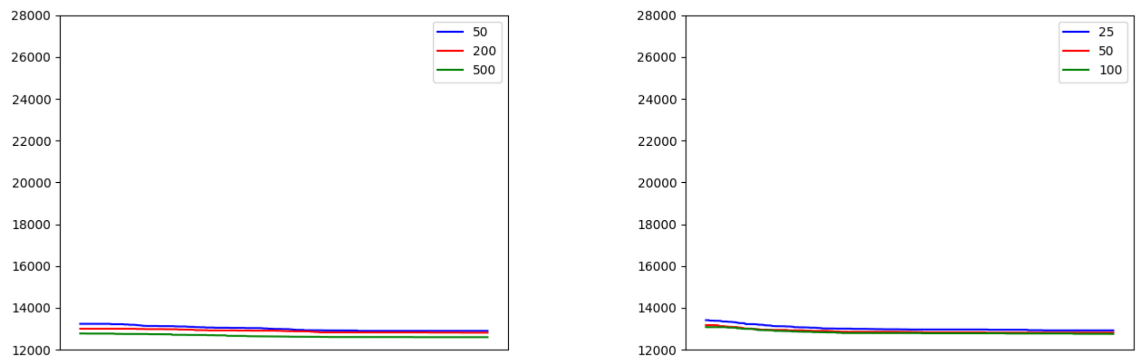
Lamarckian

With the Lamarckin version of the hybrid algorithm we start out with solutions with close to local minima. Then in subsequent generations there is little improvement. Because there is little room for improvement. This can been seen very clearly in the plots shown in figure 5.

The results are summarized in table 6.

Population size	Best tour	Worst tour	Average tour	Standard deviation	Average running time	Generations
50	12334.4	13753.0	12903.0	397.4	12.1	500
200	13057.1	15547.4	14317.0	824.0	10.2	500
500	12842.6	16870.0	14013.8	1005.4	20.8	500
25	12334.3	13977.6	12926.5	451.8	10.8	1000
50	12325.9	13351.0	12818.3	348.2	23.5	1000
100	12287.1	13424.0	12756.5	259.5	45.3	1000

Table 6: Statistics for the Lamarckian version of the hybrid algorithms over 20 runs.



(a) 500 generations

(b) 1000 generations

Figure 5: Plots showing the average best tour in each generaton over 20 runs with the Lamarckian version of the hybrid algorithm.