Lesson Report

Visualisation of Dimension: Development of Spatial Visualisation Skills

April 2018

undergraduate ambassadors scheme

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Introduction

Undergraduate Ambassador Scheme

The Undergraduate Ambassador Scheme (UAS) module is a module offered by the School of Mathematics & Statistics, University College Dublin (UCD). It is a core (compulsory) module for students in the Mathematics Education stream while also offering places to students who are not in Mathematics Education stream as an elective. The module requires the students to undertake the placement at an assigned post-primary school as a mathematics ambassador representing the school to connect the post-primary student with the studies of mathematics in third-level education while serving as a platform for the undergraduate student to experience teaching in practice.

School ethos

My placement took placed at Gonzaga College, a Catholic boys' secondary school with long-standing history tracing back to 1950. Gonzaga College is located at Ranelagh, Dublin, Ireland with currently around 600 students. The school aims to foster students with a selfless attitude to serve and contribute to the society. The school places attention on both academic and co-curriculum performance of their students, and the result was showed through the students' extraordinary achievement in academic, rugby, cricket, chess and more, placing itself among the top secondary schools in Ireland.

The two core objectives of the school are:

1. *Gonzaga will aim to develop a school community which is based on respect, love and service.*
2. *In the pursuit of excellence, the educational process will be collaborative and reflective with an openness to growth (The Gonzaga Record 2004).*

Observing teacher's classes and watching the interactions of students among their peers and teachers give strong indications of these ethos in action. The approach of taking an open stance toward teaching reminds me of Professor Jo Boaler's ideology for education, and demonstrate the commitment of the school to provide the highest standard and quality of education for their students. I am greatly honoured to be able to undertake my placement at Gonzaga College.

Placement and students' description

My placement required me to design and deliver an extra-curriculum mathematics course. There were two groups of students taking my mathematics course; a second-year group that consists of 13 students, and another group consisting of 7 third and transition year students. These students were selected with stronger mathematics background which creates a challenge for me to keep the course up to standard. The third-year students started the course two weeks behind the second-year group because of their mid-term exams and was made up by two additional classes. The two groups each have a 50 minutes session class every Tuesday morning for a duration of six classes. The table below shows the timetable and my trip to Gonzaga College.

Table 1: Timetable of the placement

	Date	1st period (0830 – 0920)	2nd period (0920 – 1010)
Class 0	23/01		Observation class
Class 1	06/02	2nd year	Extra class
Class 2	20/02	2nd year	Extra class
Class 3	27/02	3rd and transition year	2nd year
Class 4	06/03	2nd year	3rd and transition year
Class 5	13/03	3rd and transition year	2nd year
Class 6	20/03	2nd year	3rd and transition year
Class 7	10/04	3rd and transition year	Extra class
Class 8	17/04	3rd and transition year	Extra class

Observation class is the first time I visit Gonzaga College to meet with my mentor teacher Mr. Daniel Lynch, observe a Mathematics class of his and discuss the detail of the placement. During the extra classes, I went to some classroom to help students in other classes with their mathematics problems and talk about university life. Although these are not necessary for the placement, it does coincide with the objective of the UAS and I am certain it will contribute to my experience and learning.

Special Project: Visualisation of Dimension

Goal of the Module

The main goal of this module is to help students to develop spatial visualisation skills and therefore enhance the problem-solving ability of the students. The second goal is to introduce the concept of dimension in a mathematical context[1] and provide students with concrete examples of visualising mathematics ideas.

Motivation

Visualisation is one of the best ways to encourage deep mathematical understanding of science, technology, engineering and mathematics (STEM) subjects (Boaler, Chen, Williams, & Cordero, 2016; McGrath & Brown, 2005). Although effort has been made, visualisation is still yet to be fully integrated into the current mathematics curriculum and figure such as Professor Jo Boaler has constantly trying to improve it. One example is the way Pythagoras' theorem is taught in school. The Pythagoras' theorem is often stated as a mathematical fact and student were just required to learn off by heart without providing any mathematical insight to the theorem. This completely detaches from the fundamental notion of learning geometry – the study of shapes. Jeffes, et al. (2013) directly link the insufficient ability to visualise and construct diagrams to the lower achievement rate of Strand 2 (Geometry and Trigonometry) and Strand 5 (Functions) of leaving cert students. Indicating movement should be taken to act upon developing students visualisation skills.

Although some studies suggest that visual reasoning has less influence on problem solving ability (Lean & Clements, 1981), Hegarty and Kozhevnikov (1999) rightfully pointed out the difference between two types of visualisation: schematic representations (the type of visualisation that will be focusing in this report) and pictorial representations. In short, schematic representations extract the relevant mathematical information from the problem and pictorial representations have difficulties filtering the irrelevant details. While the former does correlate with mathematical performance and the later does not. This goes to show the importance of communicating the mathematician's way of thinking and the significance for teacher's guidance.

[1] Since It is not reasonable to define and explain abstract dimension in terms of vector spaces and bases to a post-primary student. For our purpose, only the Euclidean view of dimension (using the usual definition of orthogonality that can be measured with a protractor) will be discussed in this report.

Spatial visualisation ability refers to the ability to imagine two-dimensional (2D) and three-dimensional (3D), sometimes even higher dimensional objects. The skill has found useful in a lot of area especially in STEM fields and correlates with the success of STEM subjects among students (Uttal & Cohen, 2012; Mix & Cheng, 2012; Sorby & Baartmans, 2000). On the left (**Error! Not a valid bookmark self-reference.**) is a diagram extracted from the 2016 Higher-Level Mathematics Leaving Cert Exam paper 2. It was supposed to illustrate a pyramid in 3D space. It is, however, impossible to draw a 3D object on a 2D surface of a paper. The 3D pyramid perceived is an illusion based on intuition from our familiarity of the 3D surrounding. Students with lower spatial visualisation skills struggle with this kind of problems, often not able to imagine the diagram in 3D space hence withhold them from solving the problem. I therefore argue that developing spatial visualisation skills is crucial for successful mathematics education.

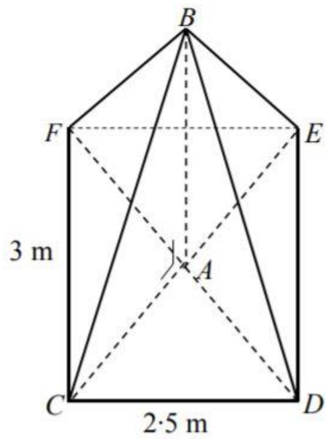

Figure 1: Diagram of a 3D pyramid

Rationale of approach

In search of the method for teaching spatial visualisation skills, a good place to start is by asking “How spatial visualisation ability could be assessed?”. It is challenging to measure spatial visualisation ability because of its grey boundary with problem solving ability (Bodner & Guay, 1997). A widely used spatial visualisation test known as the Purdue Spatial Visualisation Test: Rotations (PSVT:R) test developed by Professor Roland B. Guay from the Purdue University, Indiana (Guay, 1977) has been used across various field including Chemistry (Bodner & Guay, 1997), Engineering (Sorby & Baartmans, 2000) and Mathematics (Battista, Wheatley, & Talsma, 1982) to examine spatial visualisation ability. The test requires the participants to imagine the rotation of a 3D object (see Figure 2 below) and this is the task that a 3D modelling software able to demonstrate easily.

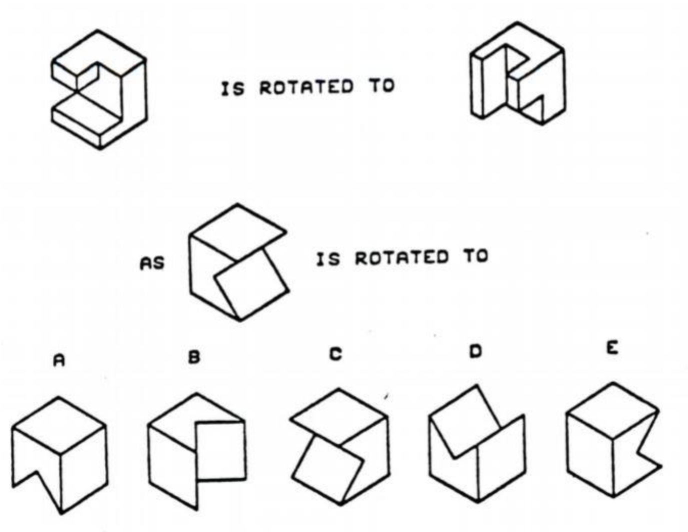

Figure 2: A sample question of the (PSVT:R) test

The content was structured around the usage of 3D modelling software, building an intuition of object in 3D space. The idea revolves around building familiarity by viewing and experiencing objects in 3D space. A good tool and arguably the best is by using actual models and object which students can touch and interact with, it is however constrained by the cost, resources, and the manipulability of the model. This is where 3D modelling software could serve as an alternative. Owing to the technology advancement and the boom in 3D modelling industry by the introduction of an affordable 3D printer, 3D modelling software is currently more affordable and accessible to everyone than ever. Although viewing through a screen is still a 2D experience, the built-in perspective simulates an actual 3D object while offering high manipulability and functionality such as transforming, rotating, x-ray, cross section, and plenty more, proving more insight into the understanding of 3D objects.

Lesson content

The content of the lesson aimed to serve five purposes:

1. To serve as a platform to showcase 3D models and demonstrate the manipulation of 3D objects.
2. To illustrate the importance of visualisation in mathematics.
3. To spark the interest, curiosity, and discussion of mathematics and science among students.
4. To introduce and stimulate the interest of 3D modelling and self-learning.
5. Finally, and most importantly, to be interesting, fun, and enjoyable.

This course consists of five sections which I will be referring as chapters, structured to be increasing in difficulties and slowly building up the knowledge from understanding the everyday 2D and 3D to eventually making sense of 4D. The content had been carefully chosen to be relevant with the same theme and plenty of ideas and concepts referencing to other chapters to stress their connections. A brief description and rationale for each section will be discussed in this section and the lesson slides will be included in the appendix.

Chapter 0: Introduction

I devoted the first lesson to introduce “hooks” to obtain the attention and the interest of the students and to introduce them to 3D model and 3D modelling software. In this lesson, I discussed how spatial visualisation ability can be useful in fields including, arts, mathematics, chemistry, engineering, and physics. Below (Figure 3) was an activity for the students to discuss while also a chance to introduce computer aid visualisation to them.

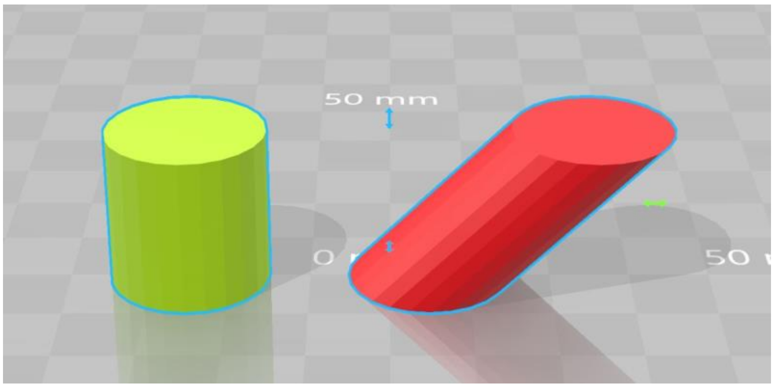

Figure 3: Two cylinders with identical volume

Chapter 1: Nets and polyhedral

In this chapter, act as a buffer to let students to get used to viewing objects through the 3D modelling software and to spot any difficulties they have with the software. Studying nets bridge the connections between 2D and 3D, giving them an alternative perspective and develop a deeper understanding of the internal structure of 3D objects. Discussing Durer's conjecture and the method of finding the shortest path on the surface of a polyhedron (Figure 4) aspire to show students how multiple facts in mathematics can be combined to produce a different fact, giving them a taste of university mathematics studies.

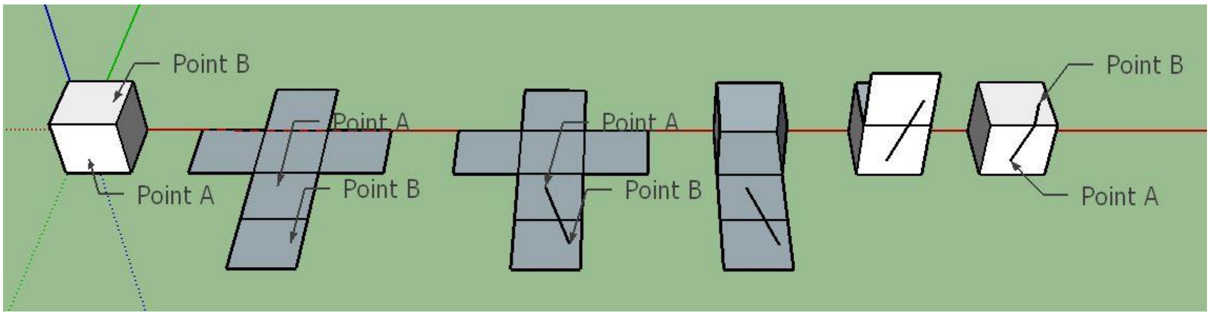

Figure 4: Constructing the shortest path between two points on the surface of a cube

Chapter 2: Platonic solids

The Platonic solids (Figure 5) are a subset of polyhedral and was one of the earliest mathematical concept ever been abstractly investigated and proven mathematically. The ability to reason is one of the most fundamental tools in mathematics. The proof that there are only five Platonic solids, featured in Euclid's book "The Elements", require the students to ask and reason the credibility of the statement, not simply believe in the statement, while using the concept of nets they had learned from Chapter 1.

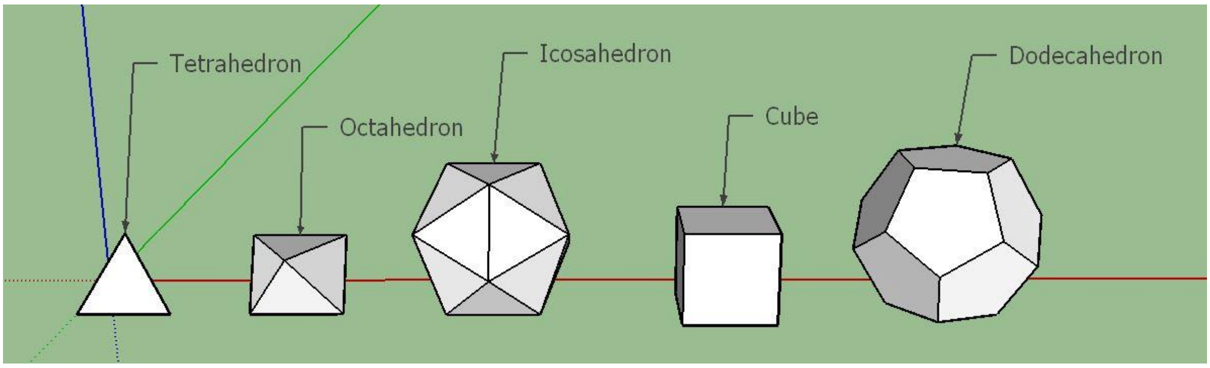

Figure 5: The five Platonic solids

Chapter 3: Dimension and parameters

To be able to discuss higher dimension, it is necessary to introduce the concept of dimension properly. The key idea in this section is to show the students that a dimension need not be spatial, other parameters such as temperature, population, and time could also represent a dimension. Using this idea, the complexity of higher dimensional space could be simplified and viewed intuitively. For instance, the temporal dimension of a 2D pendulum motion can be encoded as a third spatial dimension (Figure 6), with similar reasoning, spatial can be encoded as a temporal dimension. The knowledge of parameters could also potentially improve students' understanding of functions and relations.

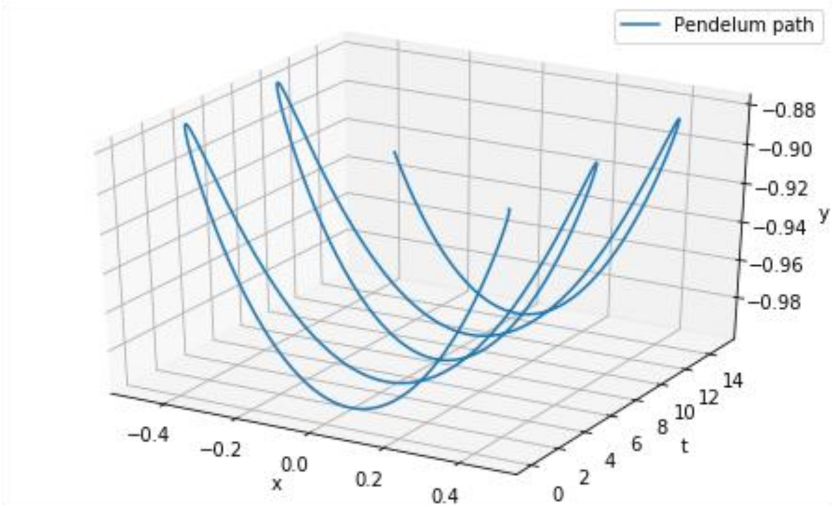

Figure 6: Space-time plot of the path of a simple pendulum motion

Chapter 4: Fourth dimension

This section dive into the deeper realm of mathematics. The purpose of the section is to introduce ideas that could only exist in mathematics, hence sparking their interest in mathematics. In this chapter, I gave multiple explanations and demonstration of how to make sense of a 4D object, using the ideas of nets and temporal dimension along with animations and 3D models. I also discussed the construction of a regular 4D polytopes using the Platonic solids and proved that there are only six possible regular 4D polytopes. Combining ideas from all previous chapters to a conclusion.

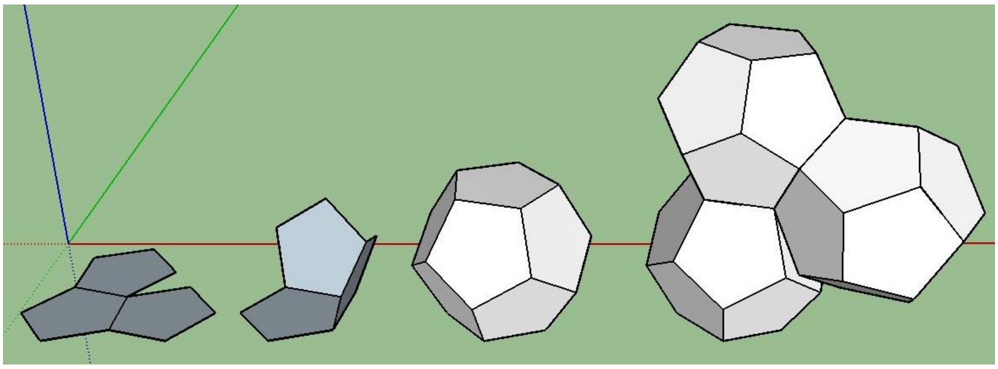

Figure 7: Construction of the net of the hyper dodecahedron and its connection to the 2D net

Learning objectives

Upon completing this course, students should be able to:

1. Differentiate 3D polyhedral from any non-polyhedral.
2. Group and classify 2D and 3D convex polytopes and concave polytopes.
3. Understand Durer's conjecture and its converse.
4. Construct the shortest path between two points on the surface of a simple polyhedral.
5. Distinguish regular convex polyhedral form non-regular polyhedral.
6. Reason and prove that there are only five Platonic solids in 3D space.
7. Construct all five Platonic solids and their nets.
8. Understand the concept of dimension as a coordinate system.
9. Visualise temporal dimension and understand its significance in modern physics.
10. Analyse and read 2D and 3D plot of spatial or any parameters.
11. Understand and reason the structure and the net of 4D regular polytopes.
12. Reason and proof that there are only six regular convex 4D polytopes in 4D space.

One might argue that most of these learning objectives are at the lower spectrum of the six level of intellectual behaviour categorised by Bloom (1956). However, the content as a whole has a clear and coherent objective to encourage long-term improvement. This includes spatial visualisation skills, logical thinking skills, and the ability to reason, simplify and break down complex problems. These are the necessary skills set that will allow students to better appreciate mathematics, which justify the content to be requisite to student's development. In most cases, it is difficult to promote understanding if students are not motivated and were only studying mathematics for the sake of exams.

Reflection

Course analysis

The Teaching for Robust Understanding (TRU) framework developed by Alan Schoenfeld (2013) is some well-studied guidelines for developing effective lessons that encourage engagement and enforce understanding. In this section, the suitability of the course and the execution of the lesson will be discussed in terms of five aspects of the TRU framework.

The mathematics

The course was structured to link ideas from different sections of the course, congregating different concepts to understand the contents and deduce new ideas. Examples of this discipline in practices included the understanding of nets and Durer’s conjecture to deduce the shortest path between two points on the surface of a polyhedron as well as using similar reasoning for the construction of the Platonic solids to construct regular 4D polytopes and showing that there are only six possible regular 4D polytopes.

Students had engaged deeper and showed greater appreciation to the mathematics knowledge they acquired, even if they find some contents were less interesting. After seeing previous knowledge was kept expanded and built upon in lessons, said one of the second-year students:

> *“I enjoyed learning about the 4th dimension, and also about space and time and speed, Einstein’s theory of relativity etc. My least favourite part (chapter) is maybe nets and geodesic, but they were necessary and I still enjoy them.”*

The constant referring to the concept of nets in later chapters had been successfully justified its importance as the foundation to acquiring more interesting ideas in later chapters and highlighting the core ideas and practices of problem solving.

Cognitive demand

The heavy usage of 3D geometry content served its purpose to create as many opportunities as possible for students to constantly engage in visual reasoning and thus, developing their spatial visualisation ability. This included the folding of 3D nets, the construction of the platonic solids and the 3D plot of different parameters. The content was observed to impose a good amount of cognitive demand for the students in terms of visualising different concept and ideas. For example, during the discussion of the 3D graph of the heat diffusion on a metal rod (see Figure 8), the students were observed to struggle to understand this concept at the beginning. With explanations and addresses to students' queries, development was seen.

The graph of the heat diffusion uses three different parameters, space (x), time (t), and temperature (u) on each of the axis. Some students had troubles to interpret the graph because they could not visualise the time and temperature dimension which we experience and feel. Thus, imagining the physical dimension was challenging for them. Although explanations were given, students were required to make connections to the evolution of temperature on a metal rod with the feature of the graph. This had to be done by their own interpretation to understand the graph. One of the less successful aspects of the lesson was its difficulty to assess students' thought process. This would be discussed in the following subsection.

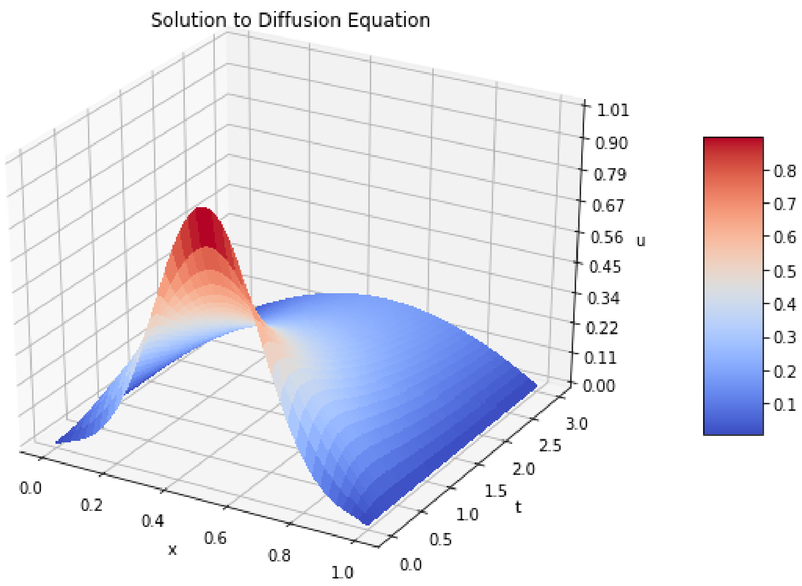

Figure 8: 3D graph of the heat diffusion on a metal rod

Formative assessment

As discussed by Schoenfeld, it is not always easy to grasp students' thought process. The difficulty of assessment in this course partly came from the content being heavily focused on visual reasoning, instead of tangible mathematical expression or formulae that could put down on paper. The challenging aspect of constructing tasks to assess student's internal thoughts was expected beforehand and two tests were planned since the beginning of the course. Although some of the students' strategies for problem solving such as dividing polyhedron into sections while drawing the net were revealed through students sketches (see Figure 9). Nevertheless, the tests were difficult to measure their performance concretely or give insight to students' misinterpretation that causes their mistakes. The result of the two tests and its flaws would be discussed in greater detail in the next section.

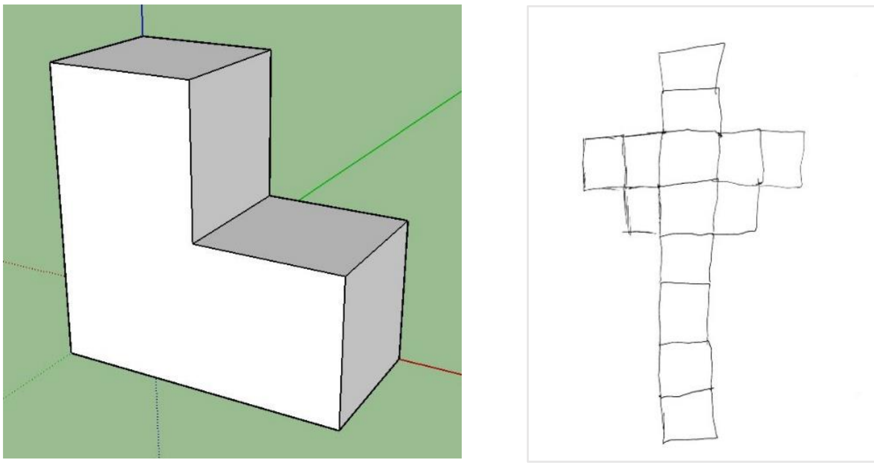

Figure 9: On the right shows a student's sketch of the net of the model on the left. The student breaks down the L-shape surface into squares demonstrating a certain degree of problem solving skills.

Similar difficulty where students' thought process was not fully understood was also found during the lessons. This was also partly due to the insufficient experience to follow up and further investigation of students' struggle during the lesson. This was observed specially when the course progressed towards the more abstract notion of higher dimension and it became more difficult of grasp students' though process. When students answered incorrectly or having troubles to understand, an explanation was provided for them to understand better but misunderstandings were not built on For example, when students were unable to understand the two 3D graphs above (Figure 6 and Figure 8), the explanation was a simple repentance in a greater detail without building up their misconception since their visual reasoning processes were not fully grasped. Overall, more effort had to be put on devising better fitting tasks and follow up questions to assess the students understanding and to gain insight of students' thought process.

Agency, ownership, and identity

The content supported discussions in 3D geometry and visual reasoning and required students to explain their reasoning. One such discussion could be seen in the introduction of the polygon – stellated octahedron. (see Figure 10a). Counting the number of faces is not an easy task since it requires a certain degree of spatial visualisation skills. This enabled to imagine the faces at the back of the model. Students were asked to count the number of faces of the stellated octahedron and explain their reasoning. A student noticed that the stellated octahedron could be constructed by overlapping two tetrahedrons (see Figure 10b, c). Since each tetrahedron has four vertices and each of the vertex is connected to three faces. The total number of faces of the stellated octahedron is, therefore, $2 \times 4 \times 3 = 24$.

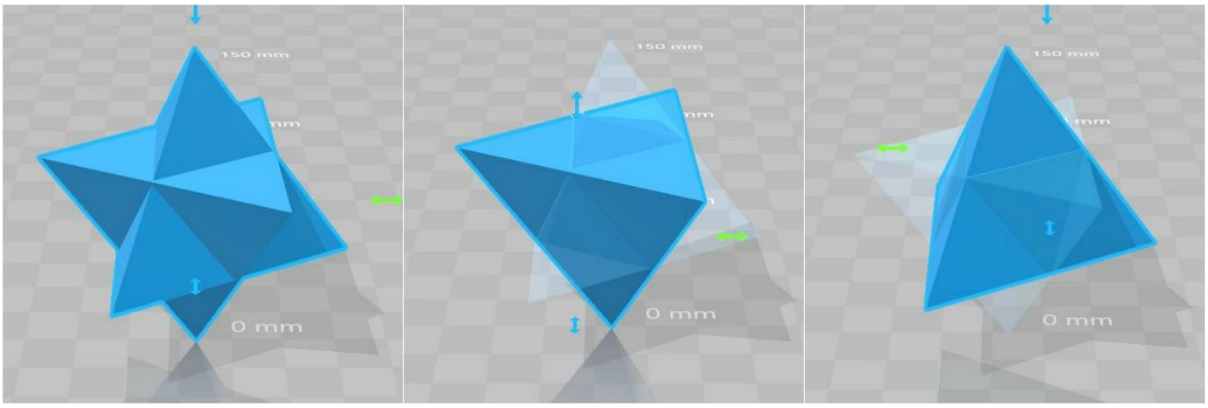

Figure 10: Stellated octahedron as two overlapping tetrahedron

Another student suggested that the stellated octahedron could also be viewed as eight smaller tetrahedron connected along their edges (see Figure 11). As each tetrahedron has four faces and one of them was hidden, the total number of faces is $(4 - 1) \times 8 = 24$, as explained by the student. These discussions demonstrated to the students that there are multiple ways to visualise a mathematical concept. Some students might find the first technique is easier to understand and others might think the second explanation is more intuitive. This was one of the key values, stressed throughout the course for encouraging students to engage in the discussion and explain their ideas.

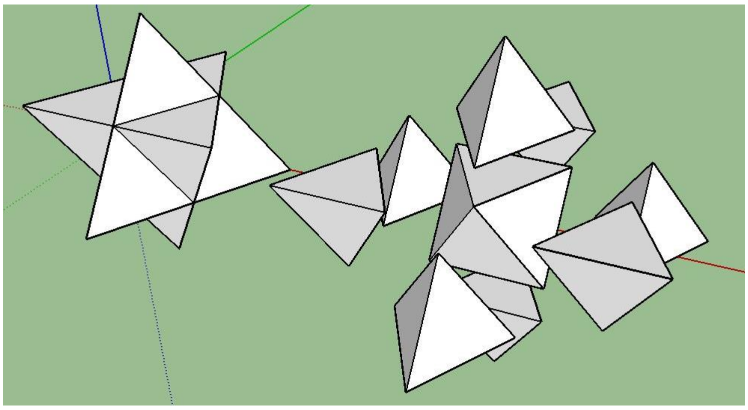

Figure 11: Stellated octahedron as eight edge-joint tetrahedron

The less favourable aspect was the insufficient opportunity for students to respond to each other's ideas. Although the students' ideas were enforced by visual aids such as ones above, it had not been fully put into advantage for students to build their ideas upon it. This was partly due the lack of initiation for discussion around students' own ideas and lack of implementation of small group discussion.

Equitable access to content

Since not all students share the same level of spatial visualisation skills, this created a challenge to decide the suitable content accessible to everyone. Effort was made to support and assist students who had troubles to visualise concepts by using visual aid, thus allowing greater portion of students participating in the discussion. For example, the discussion revolving the structure of the stellated octahedron discussed above along with use of visual interpretations served to bring the whole class onto the same page since students who had troubles to visualise were given sound interpretations from students who could do. The discussions had a purpose to balance the accessibility of mathematical content among students, enhancing students' ability to articulate their thinking while explaining to others who might find difficulty in visualisation. The development was also supported by the fact that these interpretations came from students themselves, allowing exchanges of ideas among peers. Henceforth presented different methods of solving a problem from multiple perspectives.

Although majority of the students participated in the discussion and contributed ideas, some of the students were observed to be less active during the lessons. In addition, lack of participation was observed especially within the third-year group. Most tasks required a certain degree of spatial visualisation skill or spatial reasoning, students who find difficulty in these tasks were less likely to participate and hesitant to propose their idea. As mentioned above, there were not enough small group discussions that would likely to improve students' participation and encourage them to support each other to come up with ideas as a team.

Conclusion

In conclusion, the contents demonstrated the use of visual approach to solve problems. Meanwhile it also gave a good amount of cognitive demand for visual reasoning. Students' improvements were shown by the fact that they could link previous concepts to connect with more advanced ideas. This includes the usage of angle defection within the net of the 3D Platonic solids and applying it to reason the existence and uniqueness of regular 4D polytopes. The execution of the lesson is, however, not so powerful in developing students' ideas in depth. More effort should be put on devising better fitting tasks to assess the students understanding and implementing group discussion to further develop students' ideas in depth.

Assessment and development

Two tests were placed to assess the development of spatial visualisation ability of the students, one at the start of the placement and another at the end. In this section, the method and result of the study will be discussed.

At the start of the placement, the students were asked to sketch the net of an L-shaped polyhedron (see Figure 13). The sketch was given a score of 0 to 5 based on the following criteria. Students' sketches are included in the appendix.

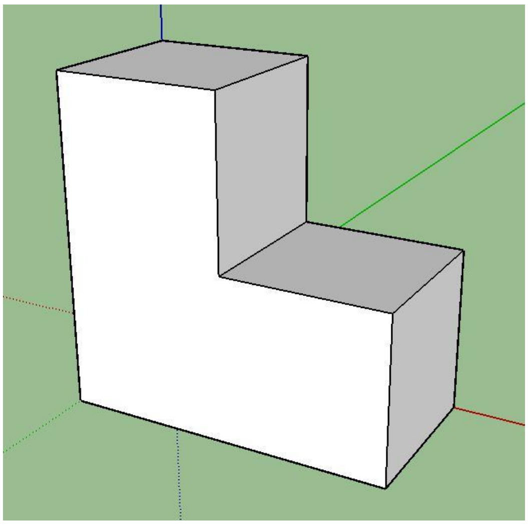

Figure 12: L-shaped model

Table 2: Score descriptor for first test

Category	Criteria	Points awarded
Accuracy (2 points)	The sketch is not a net of the model	0
	The sketch shows a rough structure of the model but contains some minor error	1
	The sketch is a correct net of the model	2
Structural representation (2 points)	The sketch does not represent any feature of the model	0
	Squares or rectangles were drawn demonstrating a basic intuition of perspective	1
	L-shape structure was drawn with edges connected to rectangles	2
Clarity (1 points)	The sketch has difficulty displaying the structure of the model	0
	The sketch clearly represents the model demonstrating the ease of visualisation	1

At the end of the course, a questionnaire consists of five (PSVT:R) questions was given to the students. Each correct answer contributes 1 point with a maximum score of 5 points. A sample of the questionnaire is included in the appendix. By subtracting the score of the second test form the score of the first test, a spatial visualisation improvement score ranging from -5 to 5 was investigated using a paired sample t-test.

Before discussing the result, there are a few things to be noted. Firstly, this study is far from a proper research. It is just a way to gauge the effectiveness of the content in developing students' spatial visualisation ability. Secondly, two different tests were used for obtaining the score and both tests do not measure every aspect of spatial visualisation ability. It should also be noted that the students taking the course have higher achievements in mathematics which could affect their prior spatial visualisation strength and development of the skills. The result does not represent the average post-primary students.

Table 3: Students result of first and second test

Group	Student	Score of first test	Score of second test	Difference
2nd year	1	4	5	1
	2	5	5	0
	3	4	5	1
	4	4	5	1
	5	5	4	-1
	6	4	0	-4
	7	5	3	-2
	8	5	5	0
	9	5	5	0
	10	3	5	2
	11	3	4	1
	12	3	5	2
	13	3	5	2
3rd and transition year	14	5	5	0
	15	3	5	2
	16	1	4	3
	17	1	5	4
	18	4	5	1
	19	4	5	1
	20	5	5	0

The improvement score has a mean of $\bar{x} = 0.7$, sample standard deviation $s = 1.75$, giving a sample t-value of $t_{test} = 1.789$. While the critical value for the upper tail t-test with $\alpha = 0.05$, degree of freedom $\nu = 19$ is $t_{crit} = 1.729$. The test shows a statistically *significant* result ($p \leq 0.044$) suggesting that there is a positive improvement in student's score.

Although the test shows a significant improvement, it is by no means conclusive. As discussed above, the two tests were structurally different, the improvement in score could be caused by students finding the second test easier than the first test. There is also an outlier (student 6) with a relatively high score for the first test and low score for the second test, hinting that two tests might be assessing a different aspect of spatial visualisation skills. Another part of the failure of the study is that the tests and the scoring mechanism have difficulty measuring the spatial visualisation ability on a comparable spectrum since most students are getting a 5 and the tests won't be able to measure their improvement accurately. On the other hand, it indicates that most students have impressive spatial visualisation skills considering the difficulty of the two tests.

One interesting aspect to note from the result of the first test is that students either had little trouble sketching the net or were completely unable to sketch the net of the model (see Figure 14). The first sketch shows a student who can visualise the model, although incomplete, the sketch displays a correct structure and features of the model. The second and third sketches show that the students have difficulty visualise the model and struggle to complete the task. This two-sided result signifies the impact of spatial visualisation in problem solving since students will not be able to attend the problem if they were not able to visualise the question being asked.

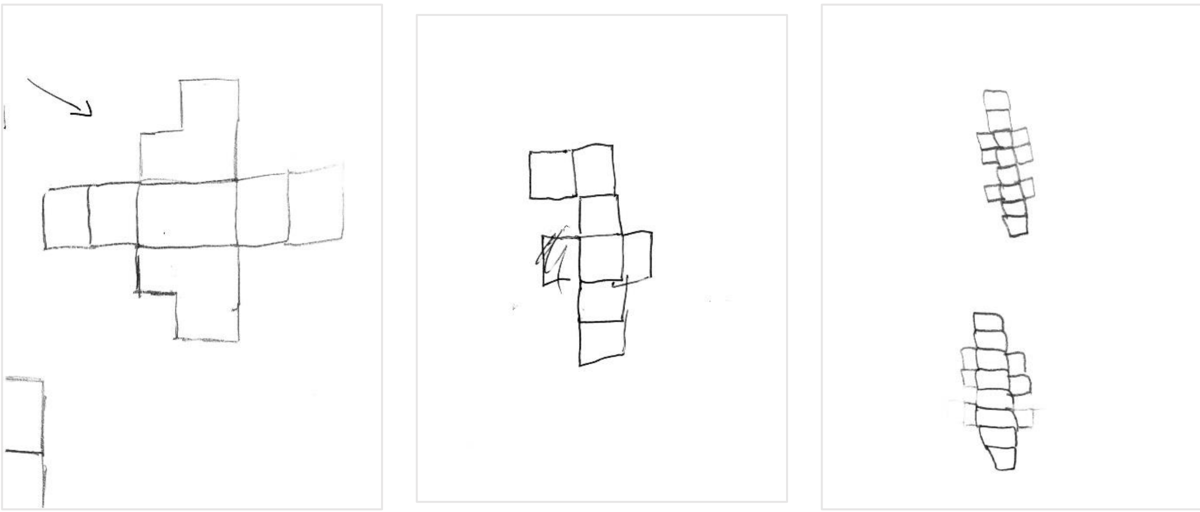

Figure 13: Three sketches of three students. Students' sketches seem to be able to categorise into able to visualise and not able to visualise.

Discussion and Suggestions

Based on the survey result, a significant portion of students thinks the difficulty is above average (see Figure 15). This is expected as the concepts are very foreign and there is a large amount of information in each lesson compared to their usual Mathematics class. The difficulty of the lesson could be adjusted by stretching the lesson to a longer period of time, this will encourage more discussions while providing the students with more time to absorb and make sense of the concepts. Although developing spatial visualisation skills at a later stage could also reduce the demand off the students, early stage development could benefit students the most by supporting their mathematical learning and thinking, thus early introduction is favourable.

Another aspect that is worthy of discussion is that most students find the course interesting despite the difficulty of the course. Based on the survey result, students like the more difficult chapters with two-third of the students listed Chapter 4: Spatial and Temporal Dimensions as their most favourite chapter of the course. This support what I think is the most important idea about learning mathematics, that mathematics lesson does not have to be easy to be enjoyable. Plenty of students also like the discussions on Einstein's Special Relativity and fourth dimension where these sections show them the significance and importance of the mathematics they had just acquired hence increase their appreciation towards mathematics.

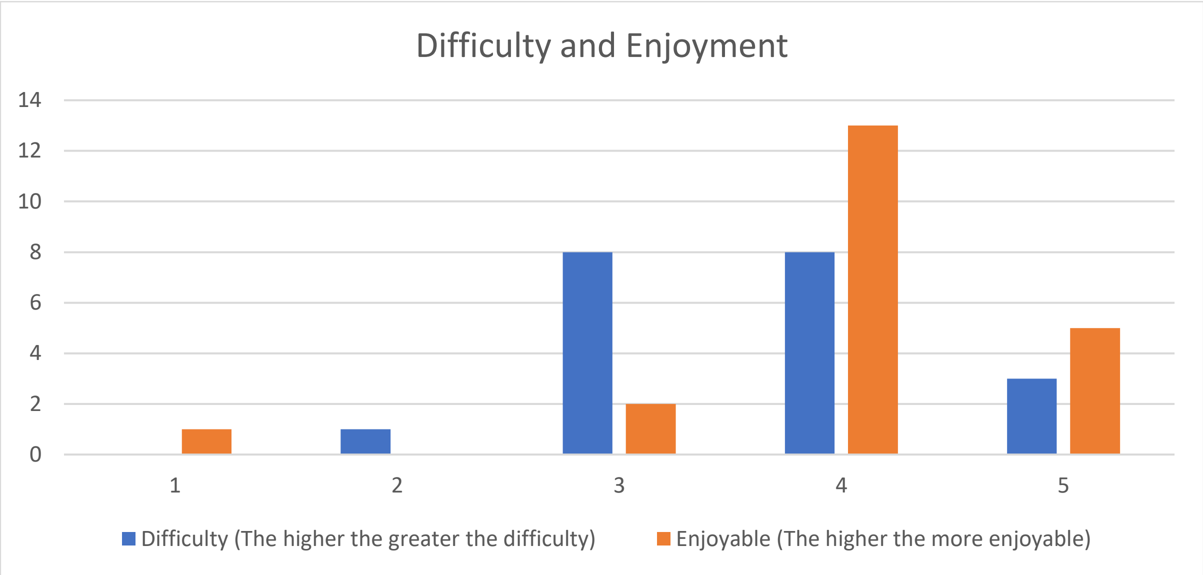

Figure 14: Distribution of students' impression on how difficult and interesting they find the course

Overall, the course had successfully improved students' spatial visualisation skills to a good extent. Moreover, the content had reached its goal, specifically, to engage and spark students interest in mathematics and science. The lesson is well-structured with no major gaps or disconnection between different sections of the content. It has its flaw such as difficulty stimulating group discussion and assessing students' knowledge. Better lesson activities and more systematic assessment should be further explored to better grasp students reasoning. In general, students demonstrated extraordinary spatial visualisation skills which were previously unexpected. Based on observations and the first test result, it shows a relatively large deviation of spatial visualisation ability across post-primary students compared to other academic ability usually with a small deviation within the same age group. This suggests that spatial visualisation development courses would be more beneficial for students with lower spatial visualisation ability as there will be a lesser gain for students with high spatial visualisation ability. Therefore, targeting this course to students with greater needs will likely to be more constructive.

(5177 words)

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Appendix

Computer software

SketchUp was used to construct the 3D models used in the course and support advance tools for manipulating the 3D models.

Microsoft 3D Builder provides a cleaner view of the 3D models and support some simple manipulation.

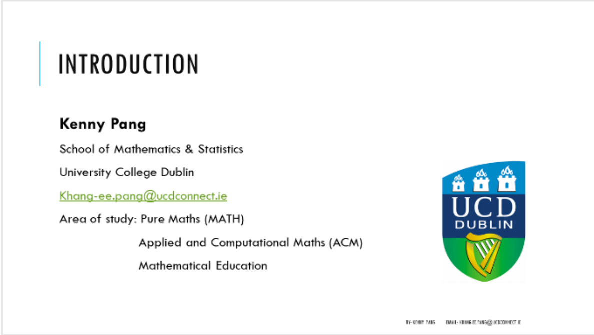
Desmos was used to generate animations for the simple pendulum motion and the small amplitude double pendulum motion.

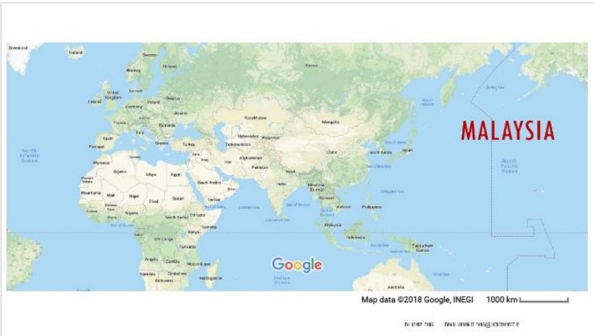
Python was used to generate the 3D plots of the simple pendulum motion and the solution of the diffusion equation.

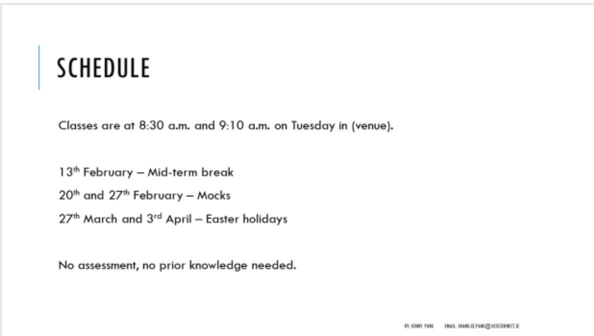
Stella4D was used to render the nets of 4D polytopes and demonstrate rotations in 4D space.

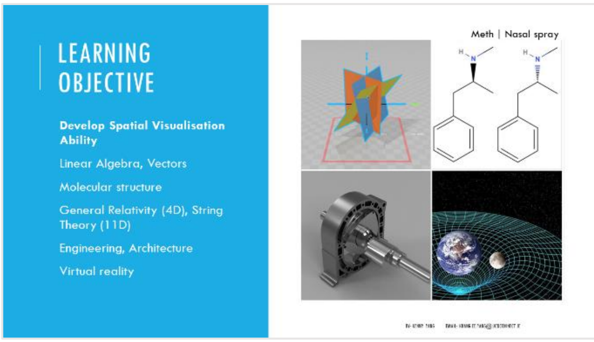
Student number: 14210030

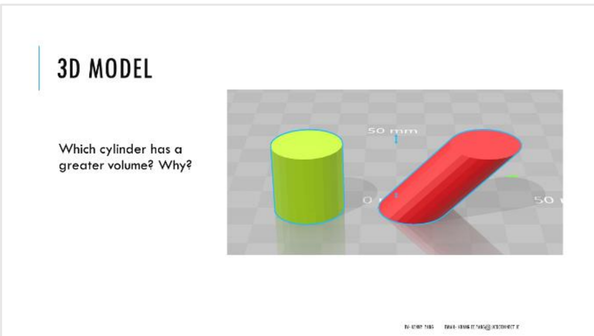
Lesson slides

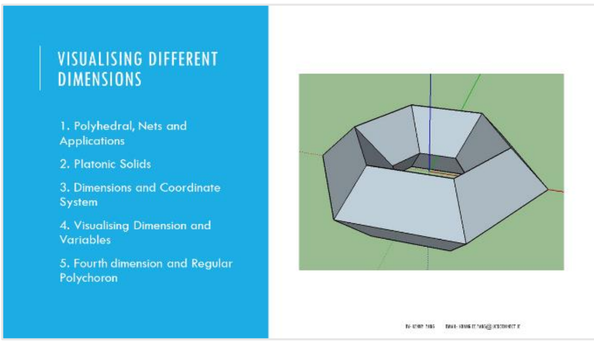




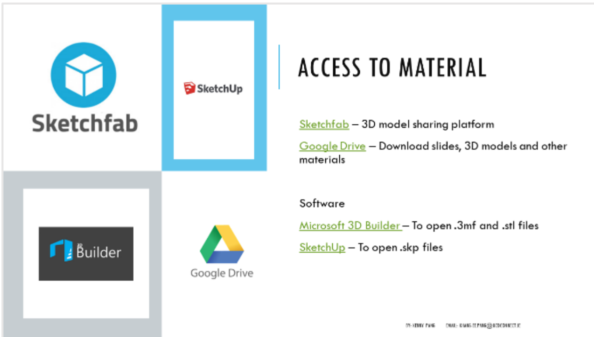


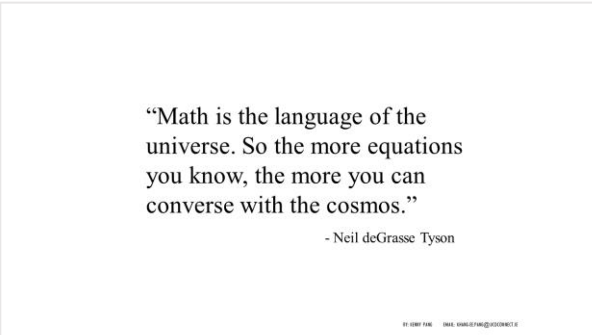


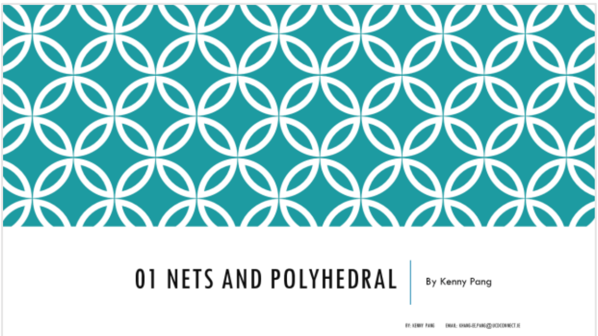


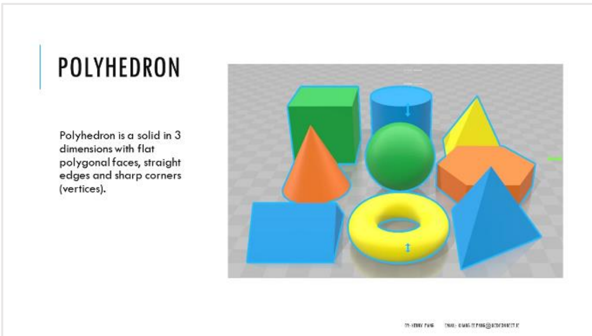


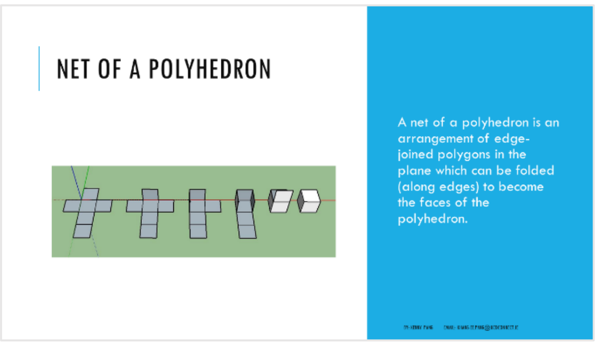

DOVER · THRIFT · EDITIONS
FLATLAND
A Romance of Many Dimensions
Edwin A. Abbott
THINGS TO MAKE AND DO in the FOURTH DIMENSION
MATT PARKER
READING SUGGESTION

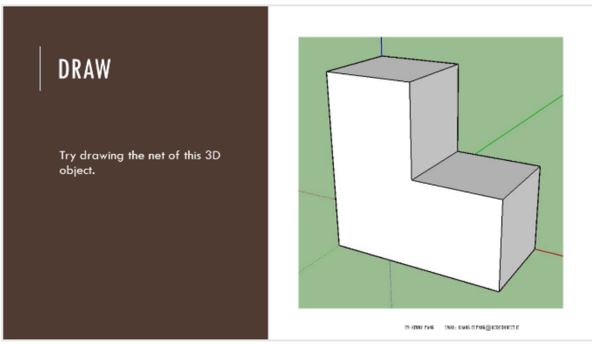

Sketchfab
SketchUp
Builder
Google Drive
ACCESS TO MATERIAL
Sketchfab – 3D model sharing platform
Google Drive – Download slides, 3D models and other materials
Software
Microsoft 3D Builder – To open .3mf and .stl files
SketchUp – To open .skp files

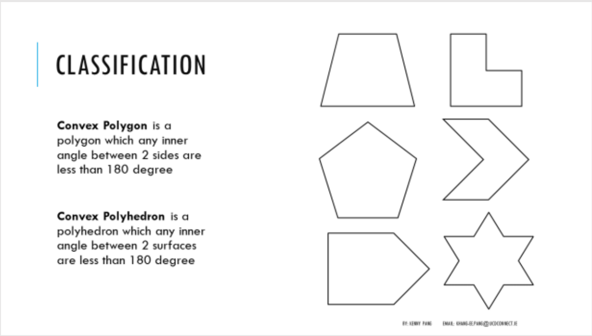

"Math is the language of the universe. So the more equations you know, the more you can converse with the cosmos."
- Neil deGrasse Tyson

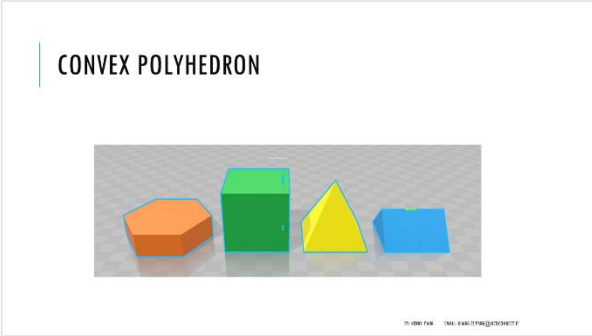

01 NETS AND POLYHEDRAL
By Kenny Pang


POLYHEDRON
Polyhedron is a solid in 3 dimensions with flat polygonal faces, straight edges and sharp corners (vertices).

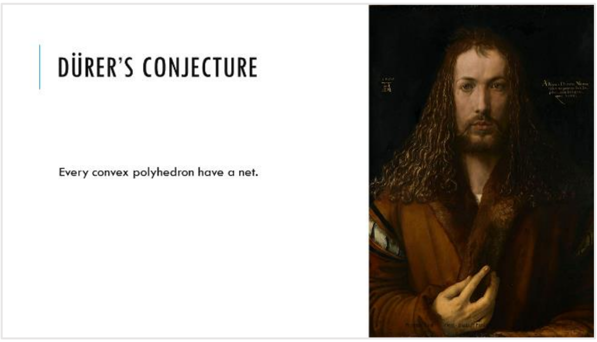

NET OF A POLYHEDRON
A net of a polyhedron is an arrangement of edge-joined polygons in the plane which can be folded (along edges) to become the faces of the polyhedron.

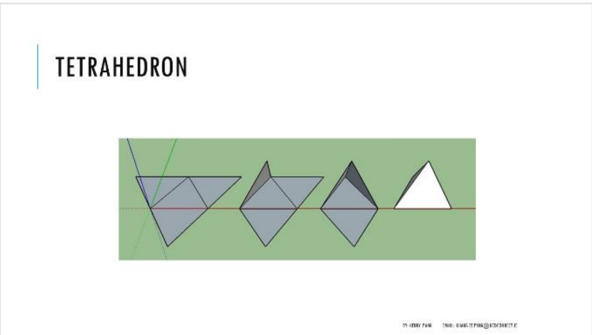

DRAW
Try drawing the net of this 3D object.

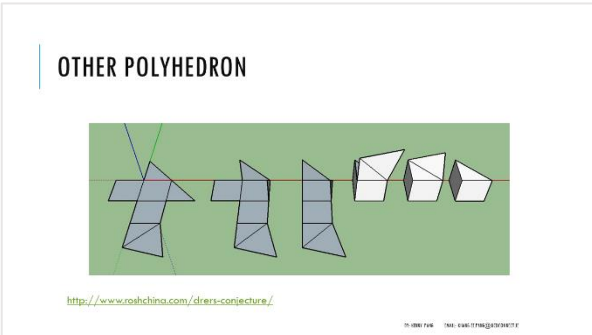

CLASSIFICATION
Convex Polygon is a polygon which any inner angle between 2 sides are less than 180 degree
Convex Polyhedron is a polyhedron which any inner angle between 2 surfaces are less than 180 degree

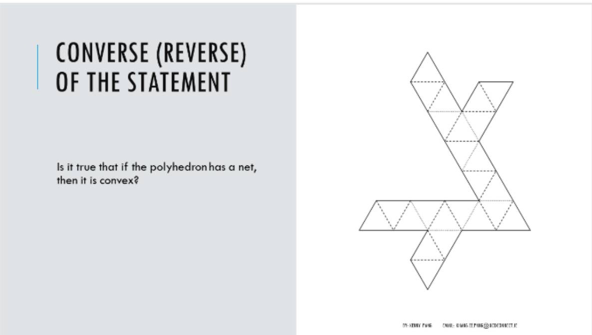

CONVEX POLYHEDRON

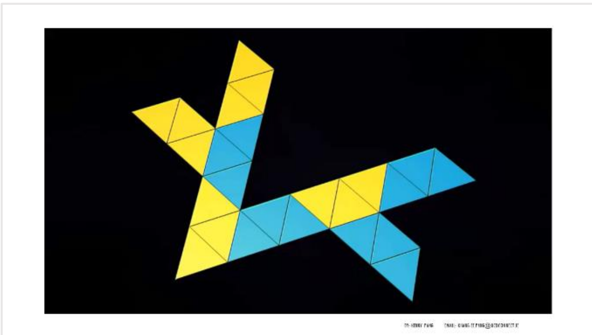

CONCAVE POLYHEDRON
Stella Octangula

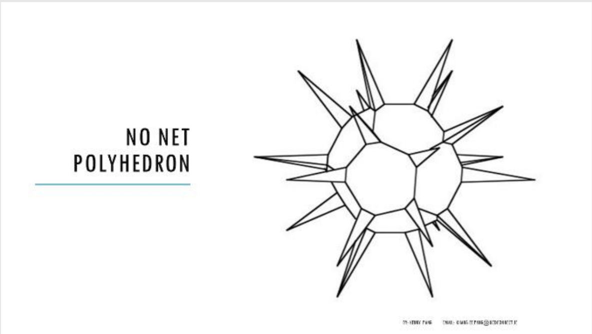

DÜRER'S CONJECTURE
Every convex polyhedron have a net.

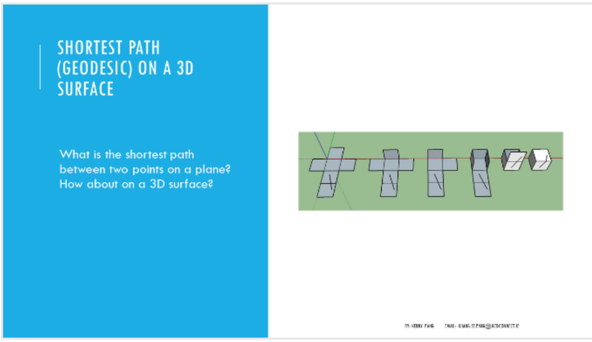

TETRAHEDRON

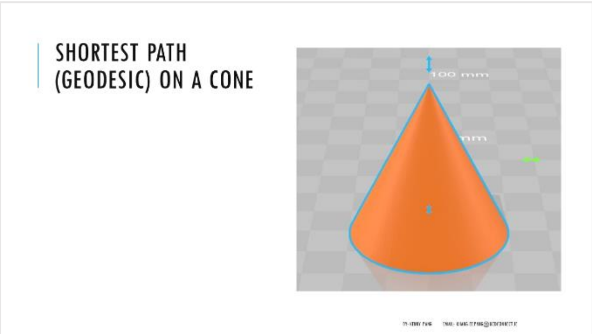

OTHER POLYHEDRON
http://www.roshchina.com/drers-conjecture/


CONVERSE (REVERSE) OF THE STATEMENT
Is it true that if the polyhedron has a net, then it is convex?

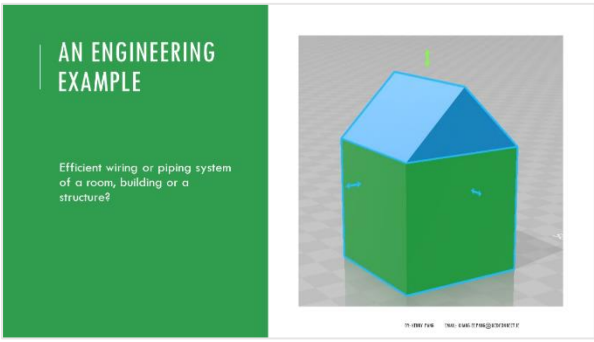


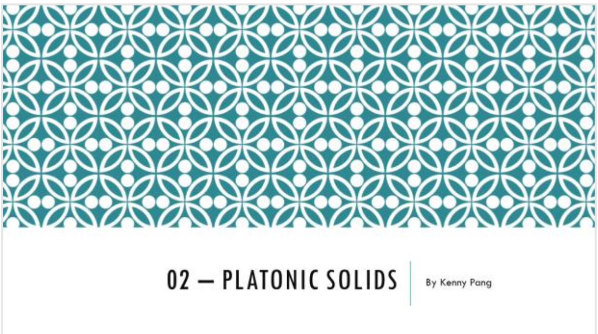

NO NET
POLYHEDRON

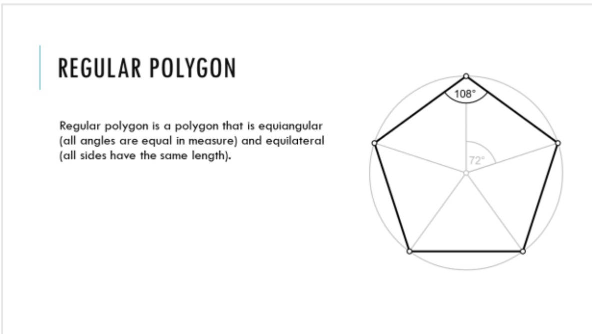

SHORTEST PATH
(GEODESIC) ON A 3D
SURFACE
What is the shortest path
between two points on a plane?
How about on a 3D surface?

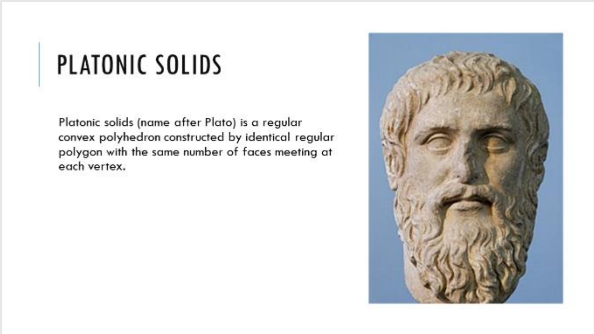

SHORTEST PATH
(GEODESIC) ON A CONE

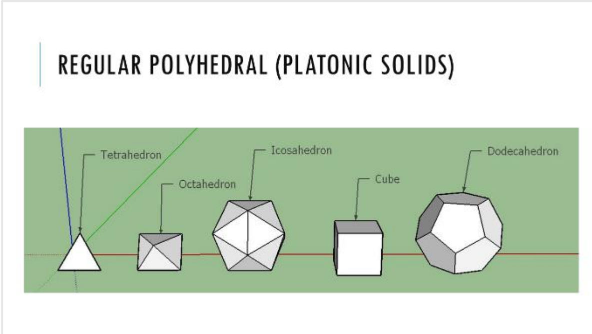

WHAT HAVE WE
LEARN SO FAR?
Dürer's conjecture: Every convex polyhedron
have a net.
Geodesic on a surface using net.

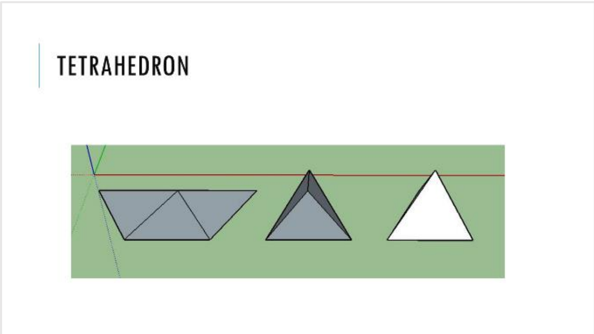

AN ENGINEERING
EXAMPLE
Efficient wiring or piping system
of a room, building or a
structure?

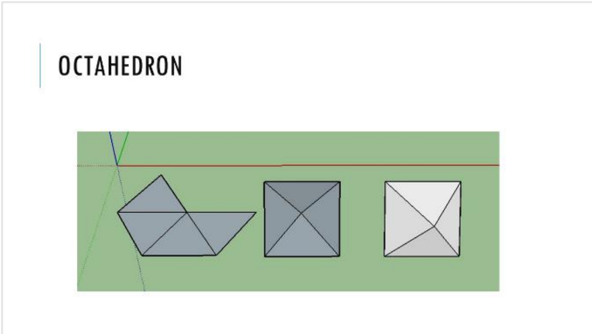

02 – PLATONIC SOLIDS
By Kenny Pang

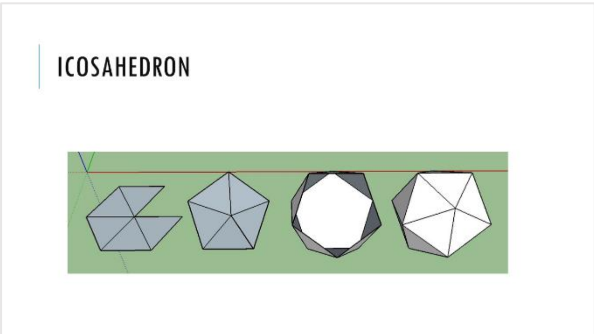

REGULAR POLYGON
Regular polygon is a polygon that is equiangular
(all angles are equal in measure) and equilateral
(all sides have the same length).
108°
72°

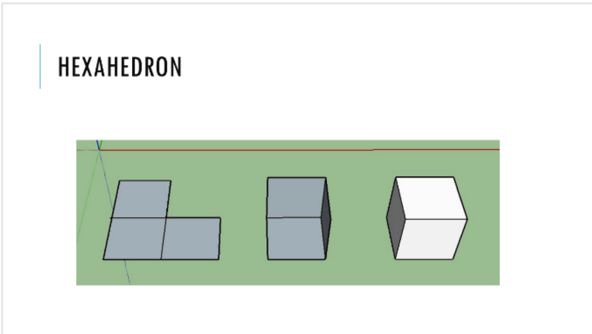

PLATONIC SOLIDS
Platonic solids (name after Plato) is a regular convex polyhedron constructed by identical regular polygon with the same number of faces meeting at each vertex.

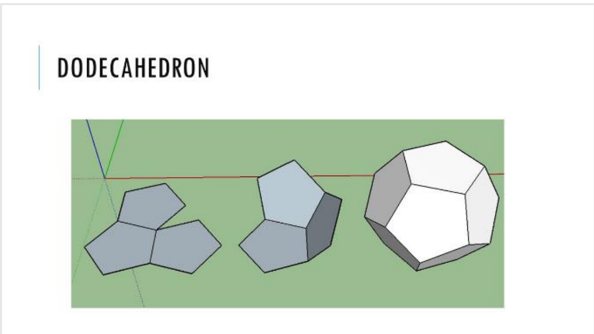

REGULAR POLYHEDRAL (PLATONIC SOLIDS)
Tetrahedron
Octahedron
Icosahedron
Cube
Dodecahedron

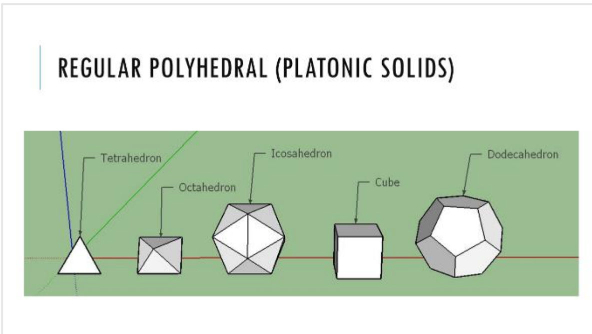

TETRAHEDRON

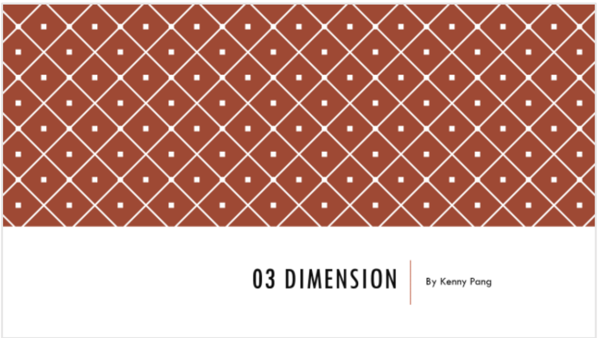

OCTAHEDRON

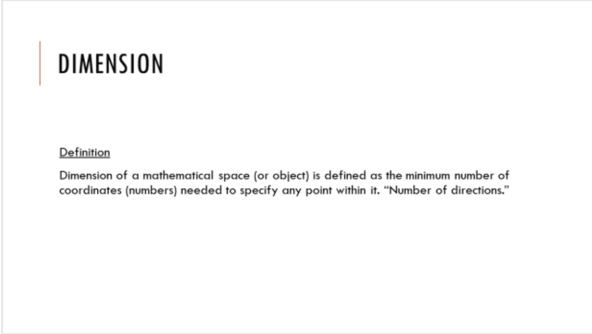

ICOSAHEDRON

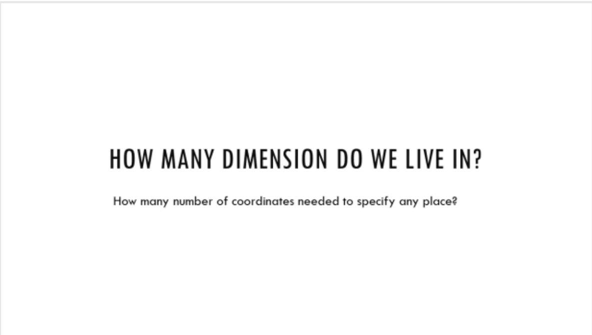

HEXAHEDRON


DODECAHEDRON

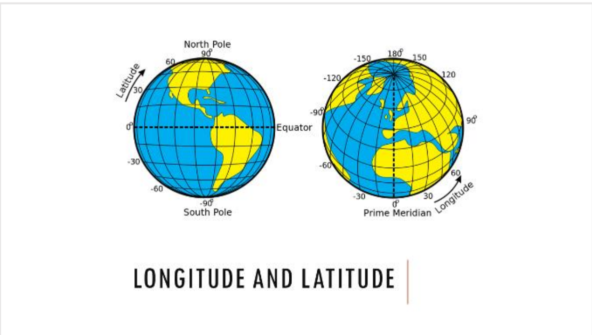

REGULAR POLYHEDRAL (PLATONIC SOLIDS)
Tetrahedron
Octahedron
Icosahedron
Cube
Dodecahedron

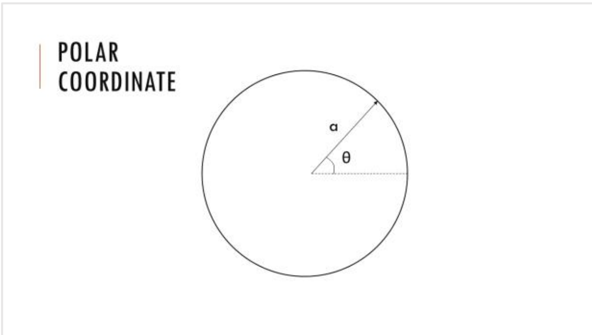

03 DIMENSION
By Kenny Pang


DIMENSION
Definition
Dimension of a mathematical space (or object) is defined as the minimum number of coordinates (numbers) needed to specify any point within it. "Number of directions."


HOW MANY DIMENSION DO WE LIVE IN?
How many number of coordinates needed to specify any place?

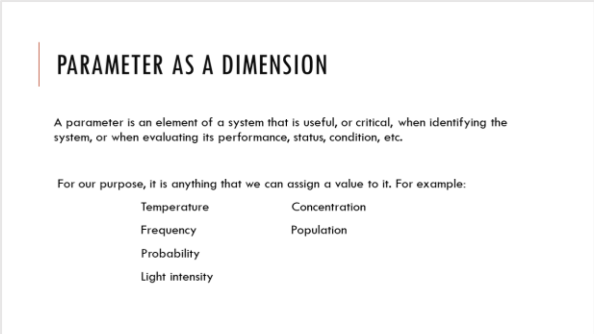

53.31674,
-6.249258000000054
Irish Christmas trees
Abbeyfield

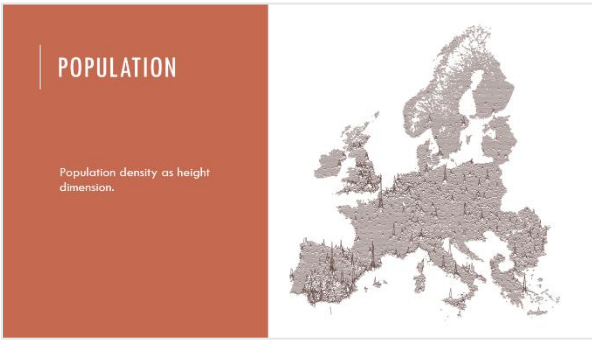

North Pole
Latitude
Equator
South Pole
Prime Meridian
Longitude
LONGITUDE AND LATITUDE


POLAR
COORDINATE
a
θ

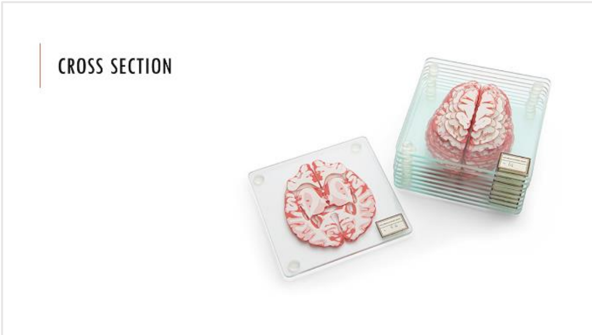

Google Maps
25°11'49.6"N 55°16'26.9"E
Dubai Opera
The Dubai Fountain
Google
Map data ©2018 Google
100 m

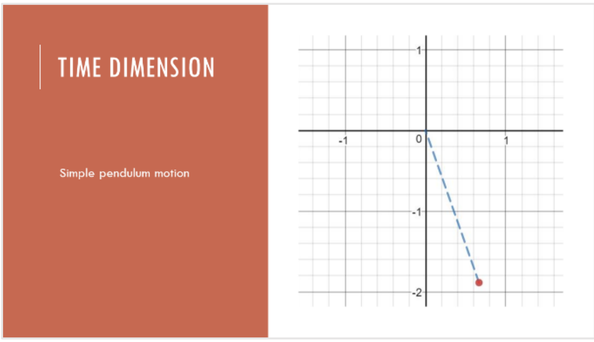

EARTH AS A
POINT

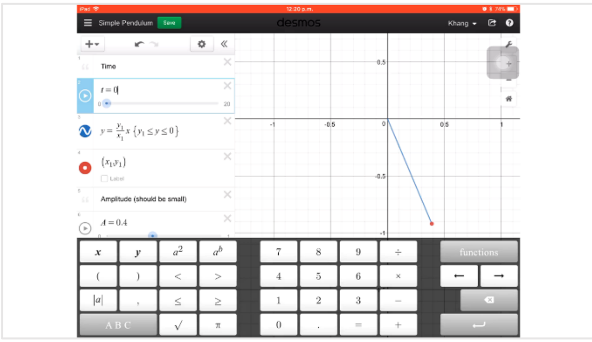

PARAMETER AS A DIMENSION
A parameter is an element of a system that is useful, or critical, when identifying the system, or when evaluating its performance, status, condition, etc.
For our purpose, it is anything that we can assign a value to it. For example:
Temperature
Concentration
Frequency
Population
Probability
Light intensity

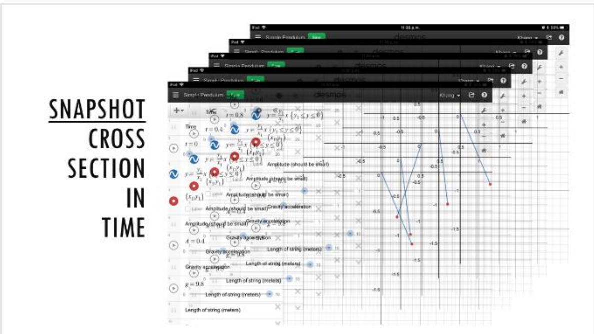

POPULATION
Population density as height dimension.

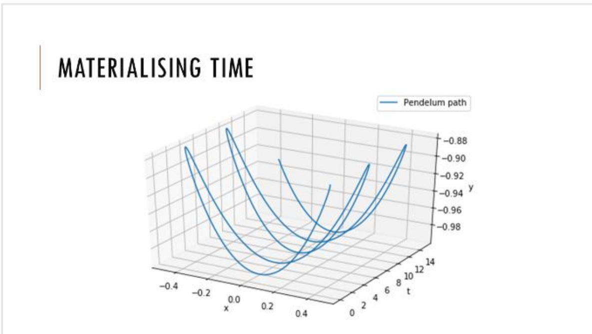

CONTOUR
Mt Everest

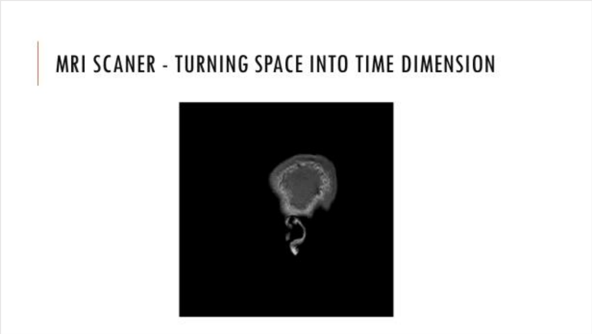

CROSS SECTION

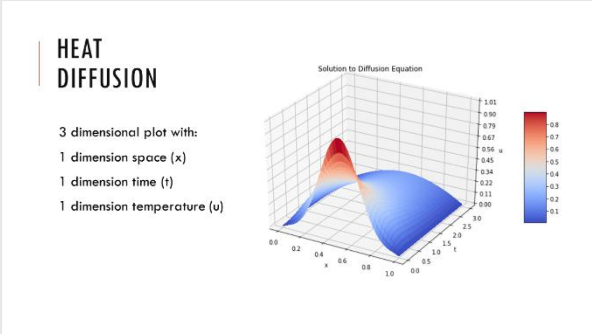

TIME DIMENSION
Simple pendulum motion

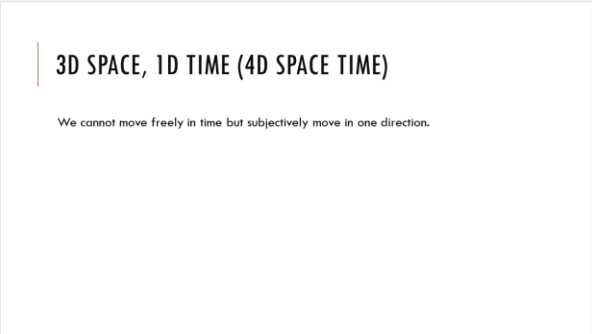

Simple Pendulum
Save
Time
Amplitude (should be small)
functions

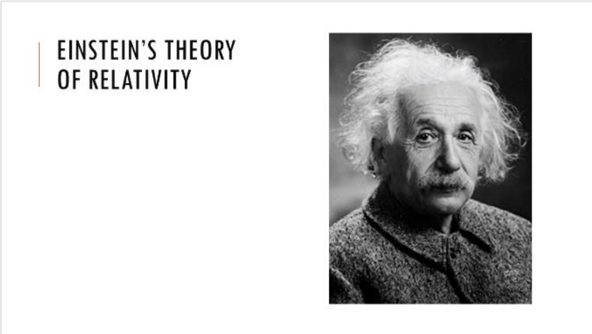

SNAPSHOT
CROSS
SECTION
IN
TIME

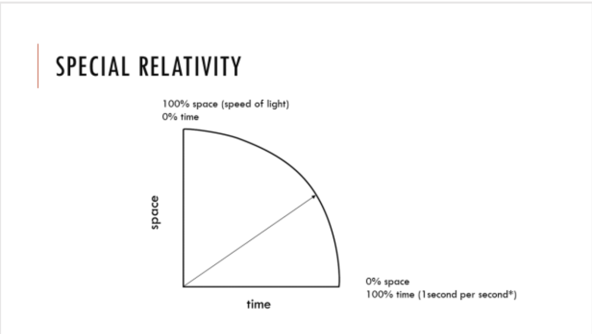

MATERIALISING TIME
Pendelum path

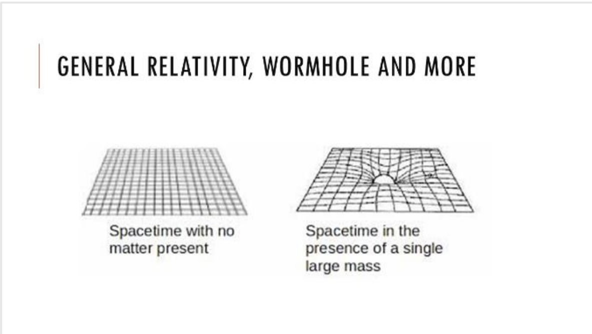

MRI SCANER - TURNING SPACE INTO TIME DIMENSION

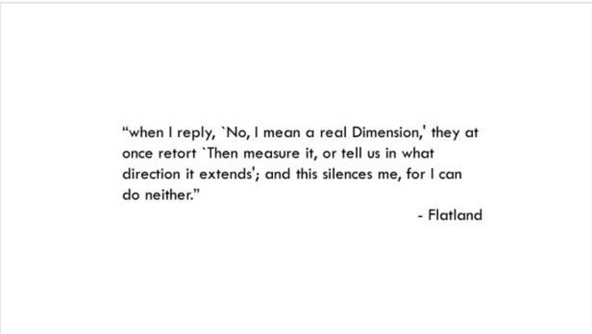

HEAT
DIFFUSION
3 dimensional plot with:
1 dimension space (x)
1 dimension time (t)
1 dimension temperature (u)
Solution to Diffusion Equation

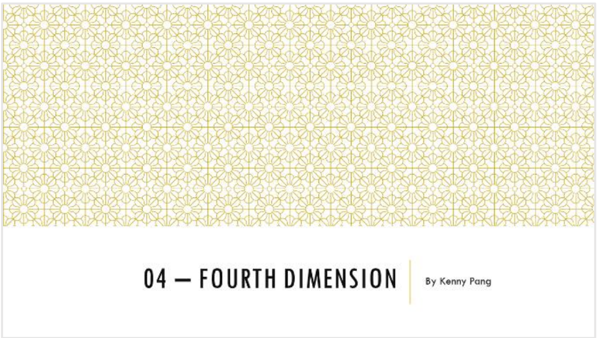

3D SPACE, 1D TIME (4D SPACE TIME)
We cannot move freely in time but subjectively move in one direction.

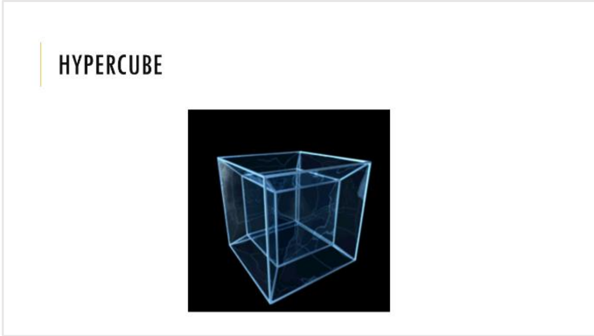

EINSTEIN'S THEORY
OF RELATIVITY

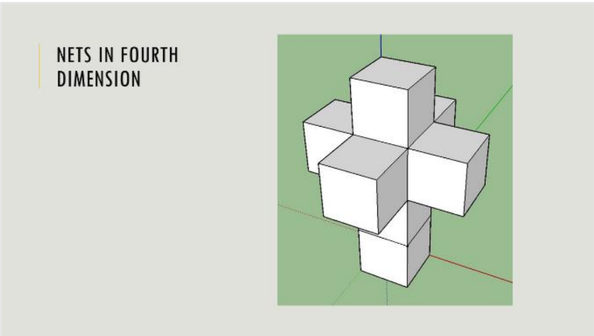

SPECIAL RELATIVITY
100% space (speed of light)
0% time
space
time
0% space
100% time (1second per second*)

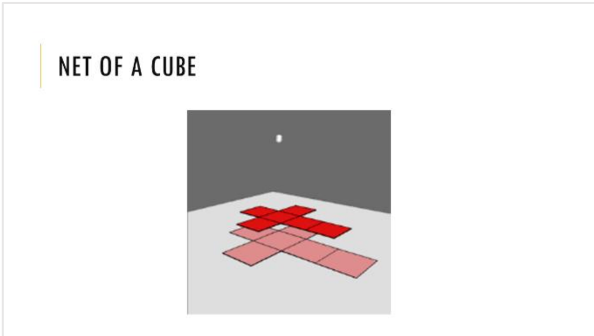

GENERAL RELATIVITY, WORMHOLE AND MORE
Spacetime with no matter present
Spacetime in the presence of a single large mass

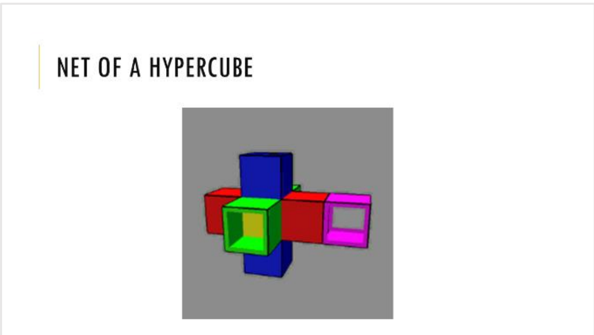

"when I reply, `No, I mean a real Dimension,' they at once retort `Then measure it, or tell us in what direction it extends'; and this silences me, for I can do neither."
- Flatland

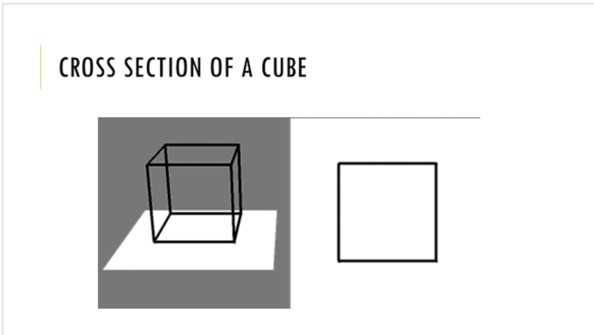

04 – FOURTH DIMENSION
By Kenny Pang

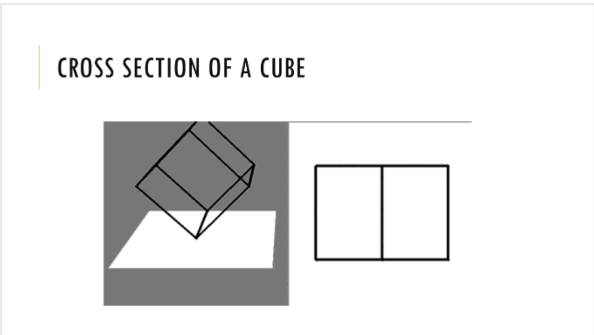

HYPERCUBE

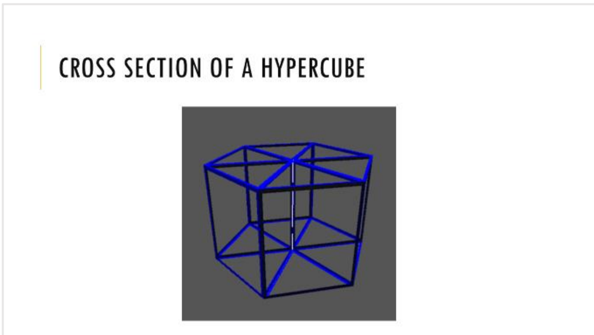

NETS IN FOURTH DIMENSION


NET OF A CUBE

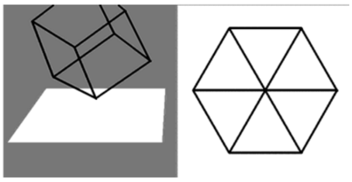

NET OF A HYPERCUBE


CROSS SECTION OF A CUBE

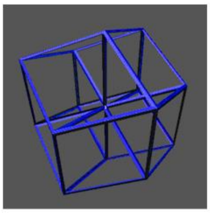

CROSS SECTION OF A CUBE


CROSS SECTION OF A HYPERCUBE

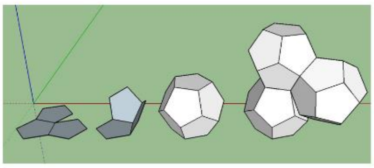
CROSS SECTION OF A CUBE



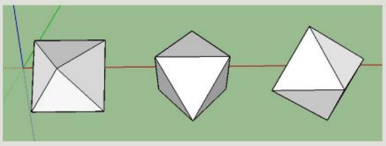
CROSS SECTION OF A HYPERCUBE



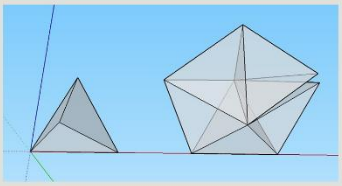
HYPER DODECAHEDRON (120 CELL)



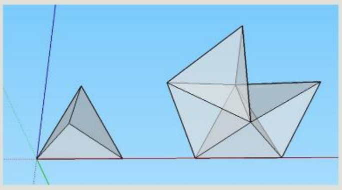
HYPER DIAMOND (24 CELL)



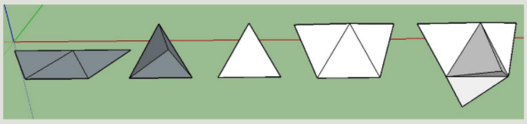
HYPER ICOSAHEDRON (600 CELL)



HYPER OCTAHEDRON (16 CELL)



HYPER TETRAHEDRON (5 CELL)



WHERE IS THE SIXTH COMES FROM?

Joint	Triangle	Square	Pentagon	Hexagon	…
3	Tetrahedron	Cube	Dodecahedron		
4	Octahedron				
5	Icosahedron				

Joint	Tetrahedron	Cube	Octahedron	Dodecahedron	Icosahedron
3	H-Tetrahedron	H-Cube	H-Diamond	H-Dodecahedron	
4	H-Octahedron				
5	H-Icosahedron				

Survey questionnaire

Course evaluation

1. **Year Group**
Mark only one oval.

- Second year
- Third year and transition year

2. **Difficulty of the course**
Mark only one oval.

	1	2	3	4	5	
Extremely easy						Extremely difficult

3. **Picturing 3D object**
Mark only one oval per row.

	Strongly disagree	Disagree	Neutral	Agree	Strongly agree
I can better imagining 3D object					
Learning about nets, polyhedral, and other content in the class helped me develop this skill					
The software used helped me develop this skill					
This skill is useful in my Mathematics class					

4. **Which section of the course you like the most**
Mark only one oval.

- Nets and Geodesic
- Polyhedral and Platonic Solids
- Space and Time Dimension
- Fourth dimension and Regular Polychoron

5. **Overall, how much would you rate the course**
Mark only one oval.

	1	2	3	4	5	
Boring						Enjoyable and interesting

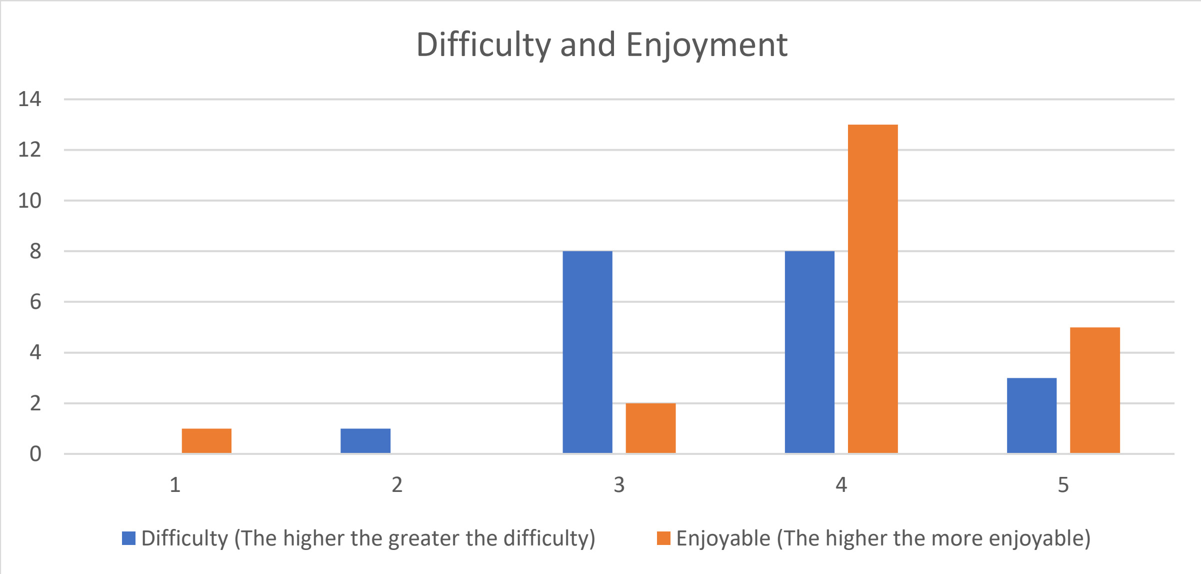
6. **What is your favourite part of the course?**

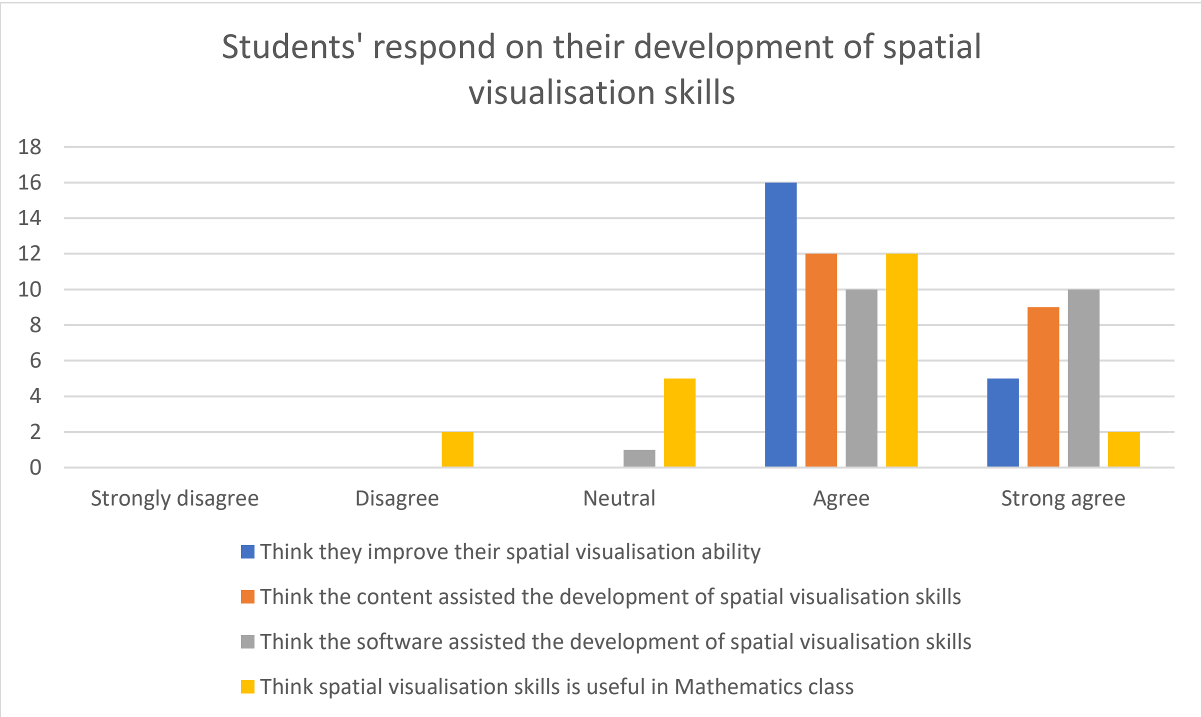
7. **What was your least favourite part of the course?**

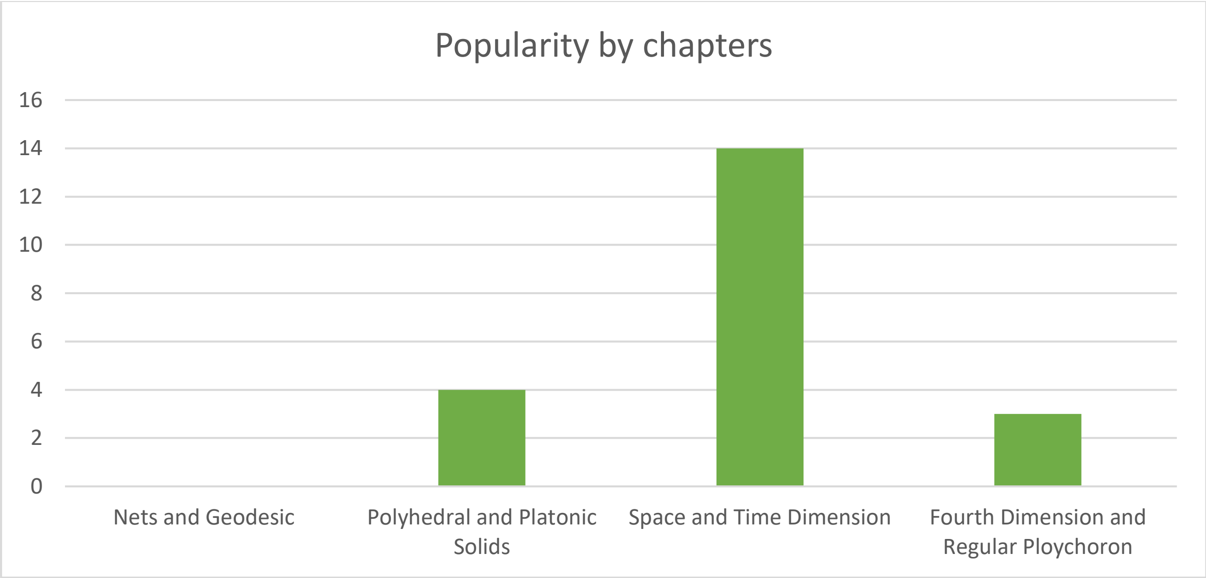
8. **Other comments and suggestions**

Powered by Google Forms

Survey result

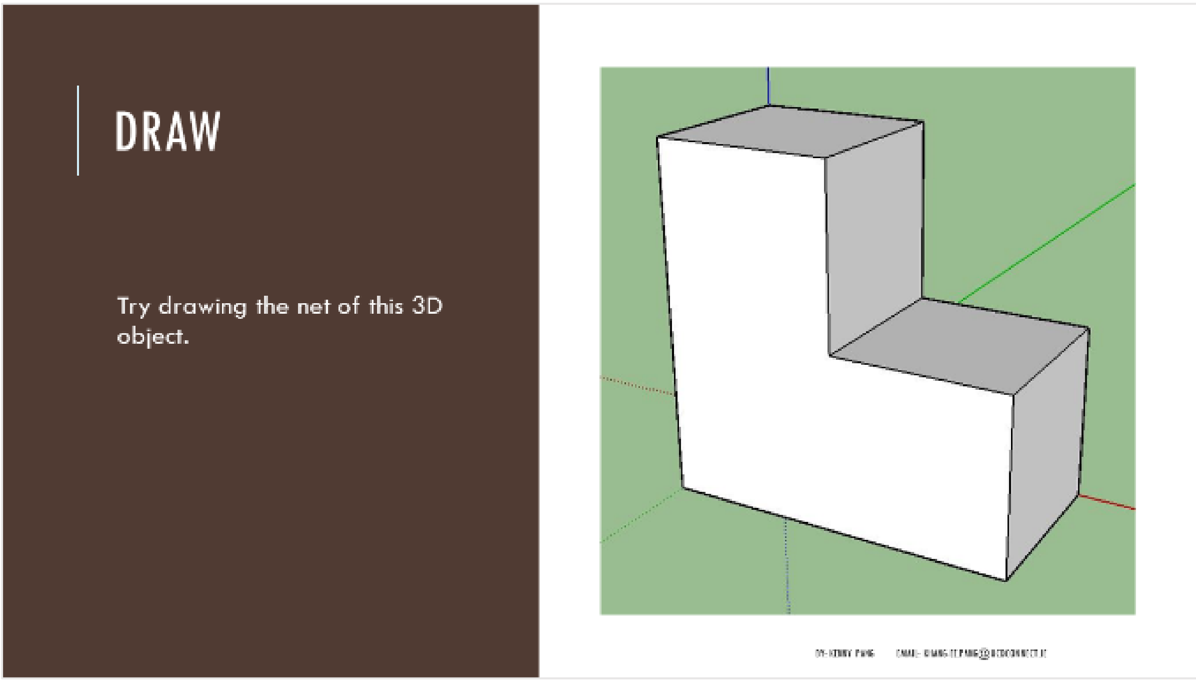




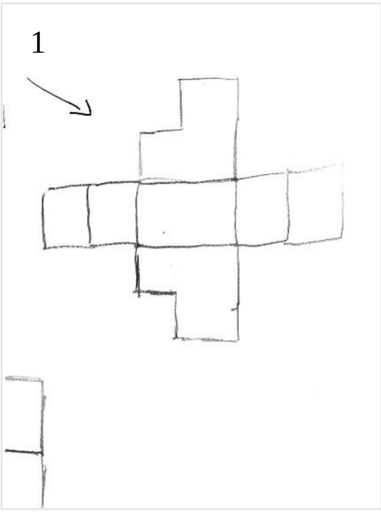

Popularity by chapters
16
14
12
10
8
6
4
2
0
Nets and Geodesic
Polyhedral and Platonic Solids
Space and Time Dimension
Fourth Dimension and Regular Ploychoron

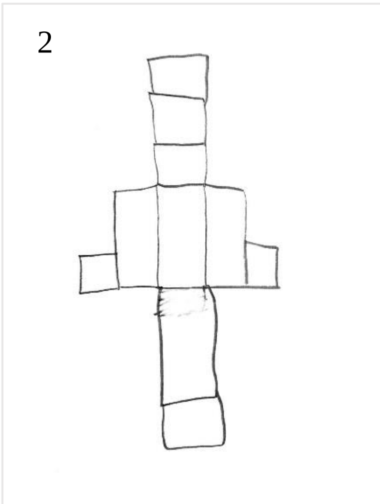
Student number: 14210030

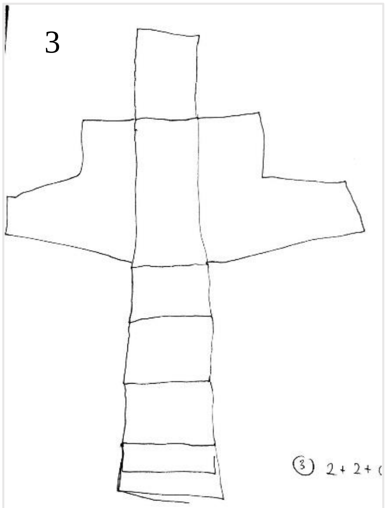
First test



Students' answer







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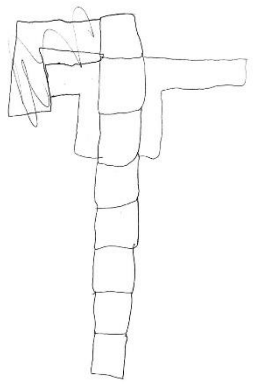

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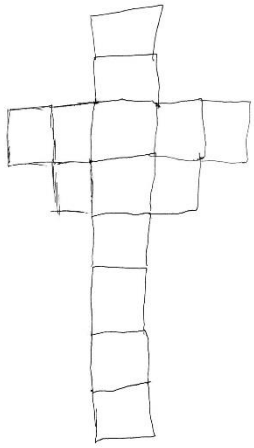

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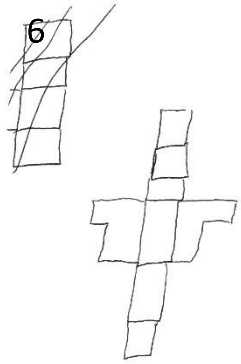

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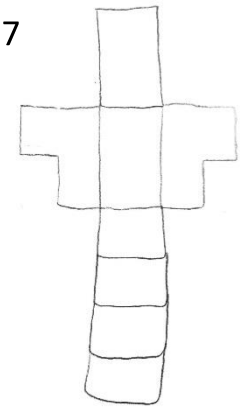

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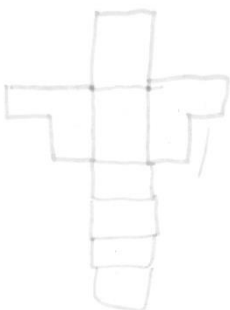

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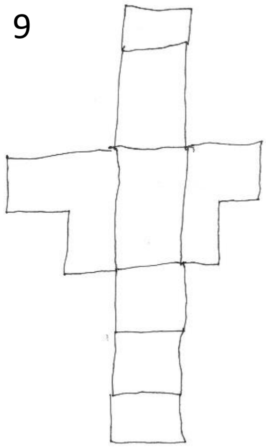

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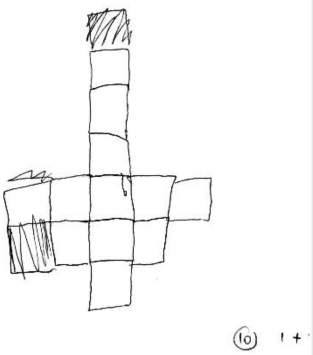

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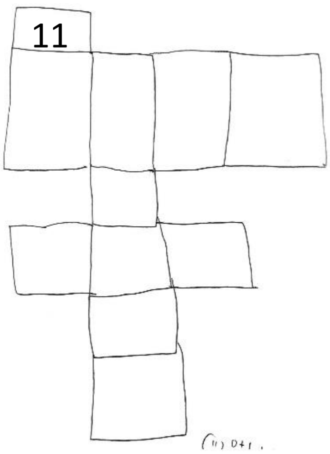

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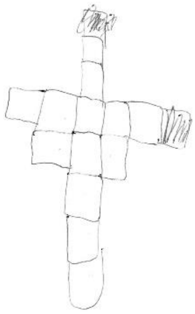

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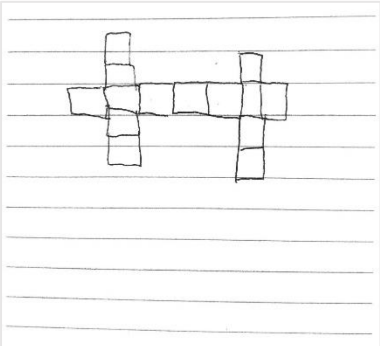

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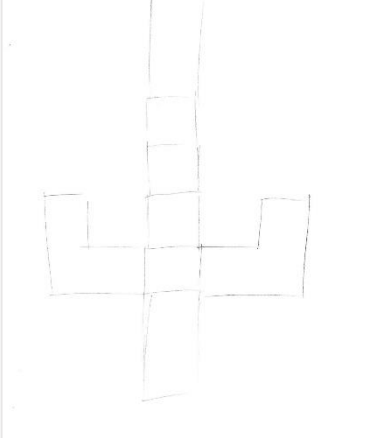

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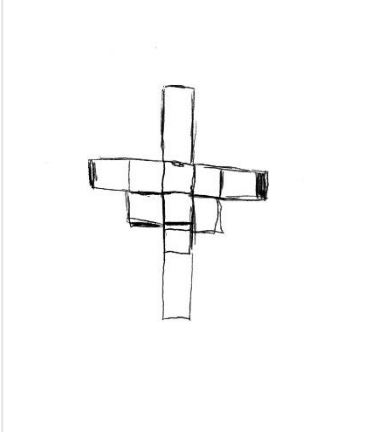

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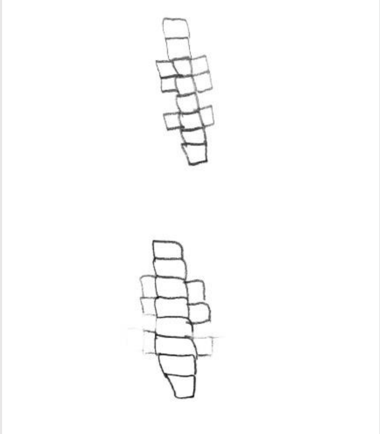

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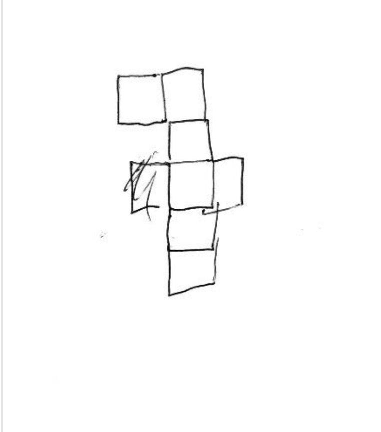

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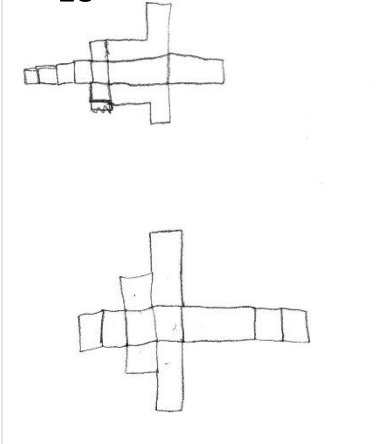

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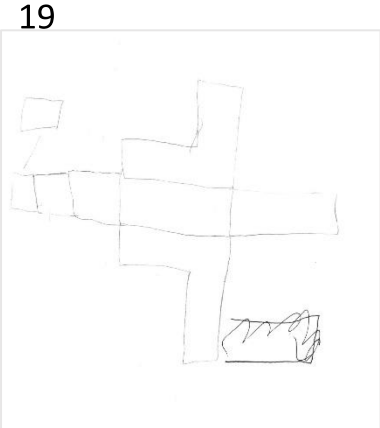

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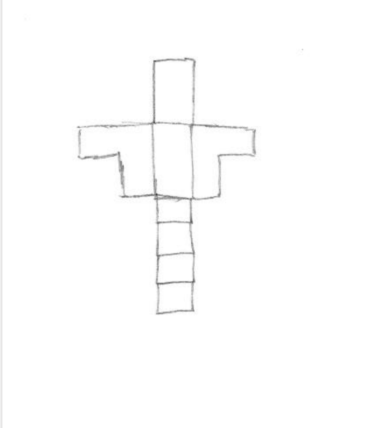


Development questionnaire (second test)

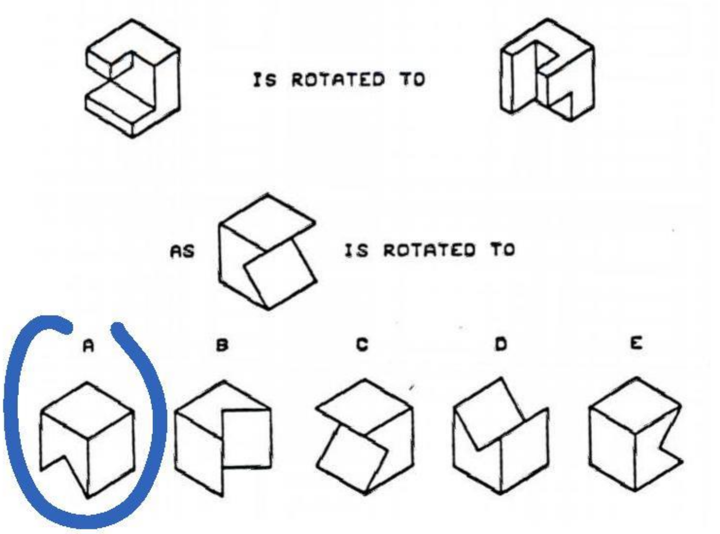
Development questionnaire

Student's Name: ____________________________ Year group: _________________

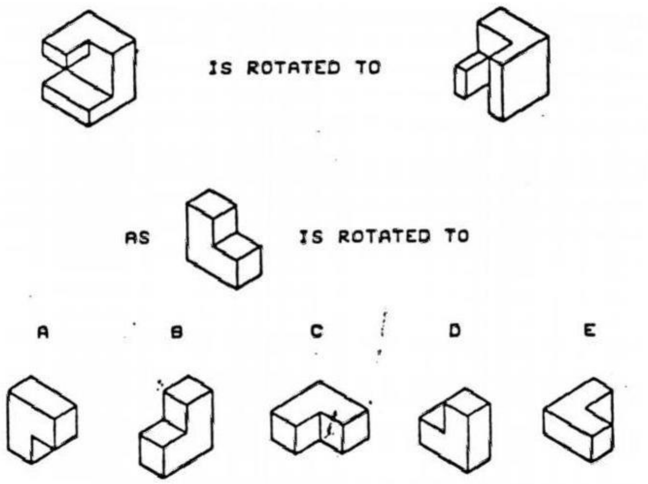
You are to:

1. study how the object in the top line of the question is rotated;
2. picture in your mind what the object shown in the middle line of the question looks like when rotated in exactly the same manner;
3. select form among the five drawings (A, B, C, D, or E) given in the bottom line of the question the one that looks like the object rotated in the correct position.

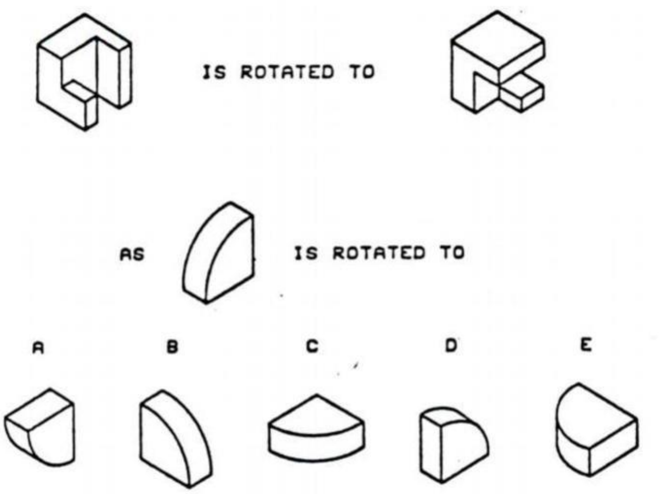
Example



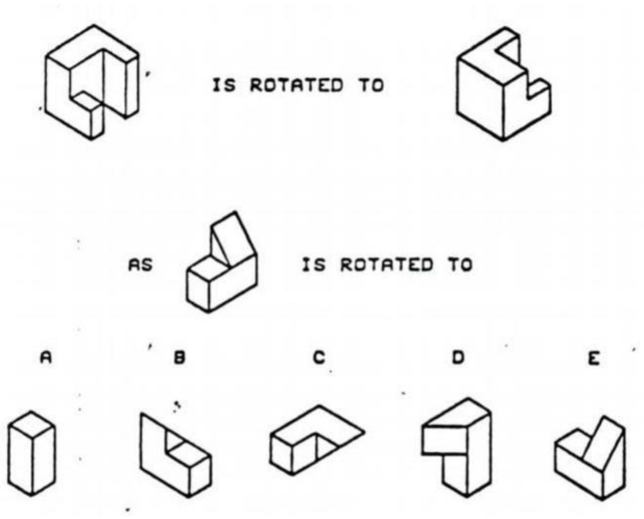
Question 1



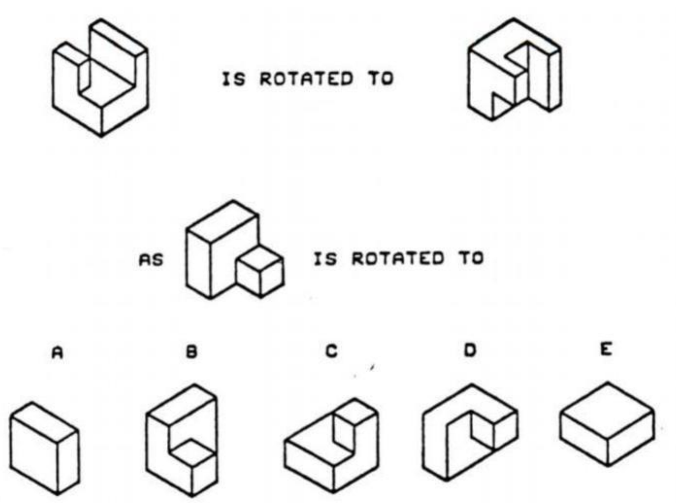
Question 2



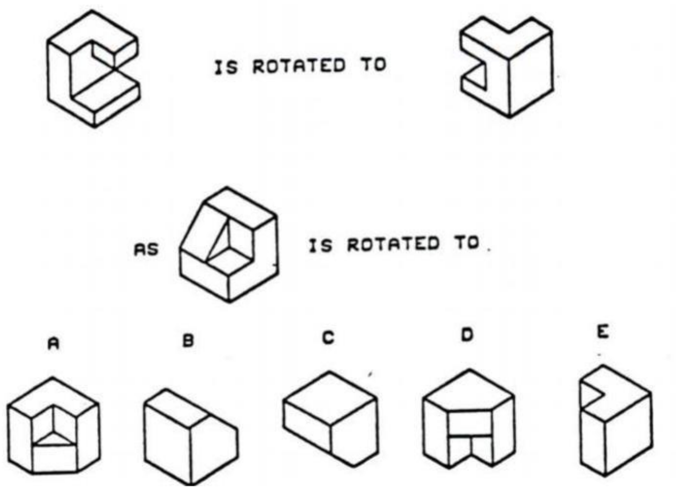
Question 3



Question 4



Question 5



~~ End of questionnaire ~~

Students' answer

Group	Students	Question 1 D	Question 2 A	Question 3 B	Question 4 E	Question 5 B
2nd year	1	D	A	B	E	B
	2	D	A	B	E	B
	3	D	A	B	E	B
	4	D	A	B	E	B
	5	D	D	B	E	B
	6	A	E	E	C	
	7	D	C	B	D	B
	8	D	A	B	E	B
	9	D	A	B	E	B
	10	D	A	B	E	B
	11	D	A	B	E	A
	12	D	A	B	E	B
	13	D	A	B	E	B
3rd and transition year	14	D	A	B	E	B
	15	D	A	B	E	B
	16	D	A	B	A	B
	17	D	A	B	E	B
	18	D	A	B	E	B
	19	D	A	B	E	B
	20	D	A	B	E	B
Number of students answer correctly		19	17	19	17	18