COMP30690: First Assignment

- **Submission:** upload your submission through Brightspace.
  - Your submission is **individual work**. By submitting you implicitly acknowledge that you are familiar with the plagiarism policies of the School of Computer Science.
  - Your answers must be **typed** and in **PDF format**.
  - Programs should be submitted as source code. It is suggested to use Matlab/Octave, but other languages may be used as long as they are free software. **Explicit instructions** about how to run programs must be given in the body of the PDF or in a README file.
  - A submission may include more than one file. Upload all files separately (i.e., main PDF, source code files, README file, etc). Do **not** upload one single zip/tar.gz file.
  - <u>Only one submission attempt is allowed by default</u>, so please double-check your files before submitting them.

Exercise 1: Probability Theory ***[20 marks]***
You have one fair coin and one biased coin, for which the probability of heads is q. You choose one of the two coins uniformly at random, and then flip this coin n times. To your surprise, you get "heads" all n times. Given this piece of information, what is the probability that you actually chose the fair coin?

Exercise 2: Random Variables ***[20 marks]***
A gambler recommends you the following *infallible strategy* for betting on the result of a biased coin which is known to have probability of heads $p = 0.48$:

> "Bet €1 on heads. If heads appears, then take the €1 profit and stop. If heads does not appear and you lose the bet, make additional €1 bets on heads on each of the next two flips of the coin and then stop".

Let X denote the random variable modelling your winnings (profit) when you stop.

a) Find the support and pmf of the random variable X, and compute $\Pr(X > 0)$.

b) Considering the probability that you computed in a), is the gambler's strategy indeed a good winning strategy? Reason your answer.

Exercise 3: Generation of Random Variables *[40 marks]*
Write code to generate m pseudorandom outcomes of a binomial random variable $X \sim \text{Bi}(5, 0.2)$.

You must generate the pseudorandom outcomes in two different ways (i.e., you need to produce two different programs):

1. By using directly the cdf of the binomial random variable.
2. By using the fact that a binomial random variable can be obtained through independent Bernoulli variables.

In both cases, plot the empirical distribution (i.e. normalised histogram) of the outcomes for $m = 10^2$ and $m = 10^4$ versus the theoretical distribution of the random variable (i.e. its pmf) and discuss your results.

Exercise 4: Mutual Information and Entropy *[20 marks]*
Consider two discrete random variables X and Y. We are told that both variables have the same pmf $p(x)$ and support $\mathcal{X}$, but we do not know whether $p_{X,Y}(x, y) = p(x)p(y)$ for all $(x, y) \in \mathcal{X}^2$ or not. We also know that $H(Y) > 0$. If we define the parameter $\lambda = 1 - H(Y|X)\,(H(X))^{-1}$,

a) show that $\lambda = I(X; Y)/H(X)$.

b) show that λ is between 0 and 1.

c) discuss the relationship between X and Y when λ is known to be 0.

d) discuss the relationship between X and Y when λ is known to be 1.