1 Modelling and Computational Aspects of Non-Linear Regression

Suppose you are interested in to make measurements in a specific machine output and based on that infer some quantities that are associated with a specific process. The relation between input and output in the machine are given by $y = a \cdot x + log(a) \cdot x^2 + b \cdot \exp(c \cdot x)$, where x and y are respectively the input and output. Due to the limitations of the equipment you have, all your readings possess some sort of noise. The noise is additive and always non-negative. Use the data provided and filter the noise and estimate the parameters. Model the noise with all these distributions:

- Normal distribution;
- Gamma distribution;
- Chi-squared distribution.

For all the distributions, do the following:

1. Formulate the log-likelihood–the logarithm of the probability density function–for a sample of size N, $x_0, x_1, \ldots, x_{N-1}$;
2. Estimate all the parameters for all distributions using maximum likelihood estimator using local optimization methods, and display them in a table;
3. Which noise model distribution returns the best parameter estimation? In order to answer this question, use metrics that evaluate the estimation, for example, the objective function value or mean squared error. Elaborate on which one you would to pick as 'correct' estimate.
4. Why the can you be sure that this method–maximum likelihood estimator–is safe? And which problems you may find during the estimation problem?

For all the estimation, state the numerical methods used with all its details. Make sure you describe the details. Provide *all the code* organized in a compressed folder (e.g. zip) through email. Failure to submit the codes will result in null marks. All the code in the compressed folder must be described in your lab report. As suggestion, you can set the initial steps for $\begin{bmatrix} a & b & c \end{bmatrix}^\top = \begin{bmatrix} 3 & 2 & 2 \end{bmatrix}^\top$.