ADAPTIVE EQUALIZATION OF A DISPERSIVE COMMUNICATION CHANNEL

In this project you will study the use of the LMS algorithm for adaptive equalization of a linear dispersive channel that produces (unknown) distortion. We assume that the data are all real valued. Figure 1 shows a block diagram of the system to be used for the purpose of carrying out the study. The data source generates the signal $\{a(n)\}$ used for probing the channel while the Gaussian noise source generates white noise $\{v(n)\}$, with zero mean, that corrupts the channel output. These two random-number generators are statistically independent of each other. The adaptive linear equalizer, which is a tap-delay line filter, has the task of correcting for the distortion introduced by the channel in the presence of the additive white Gaussian noise. The signal $\{a(n)\}$, after suitable delay, supplies the desired response $\{d(n)\}$ applied to the adaptive equalizer for computing the estimation error $\{e(n)\}$ sequence. This delay is equal to the time it takes the signal to propagate through the channel (the medium) and the adaptive filter.

The random sequence $\{a(n)\}$ applied to the channel input is in polar form (BPSK – binary phase shift keying), that is, $a(n) = \pm 1$, so the sequence $\{a(n)\}$ has zero mean and variance equal to 1. Five channels, each consisting of three paths, will be investigated in this project. The impulse responses of the channels are given in the table below. The channels differ in the severity of the distortion they introduce. Channel 4 introduces more severe distortions than Channel 1.

	$h(1)$	$h(2)$	$h(3)$
Channel 1	0.2194	1.0	0.2194
Channel 2	0.2798	1.0	0.2798
Channel 3	0.3365	1.0	0.3365
Channel 4	0.3887	1.0	0.3887

The binary transmission system that you will investigate in this project is shown in Figure 1 below.

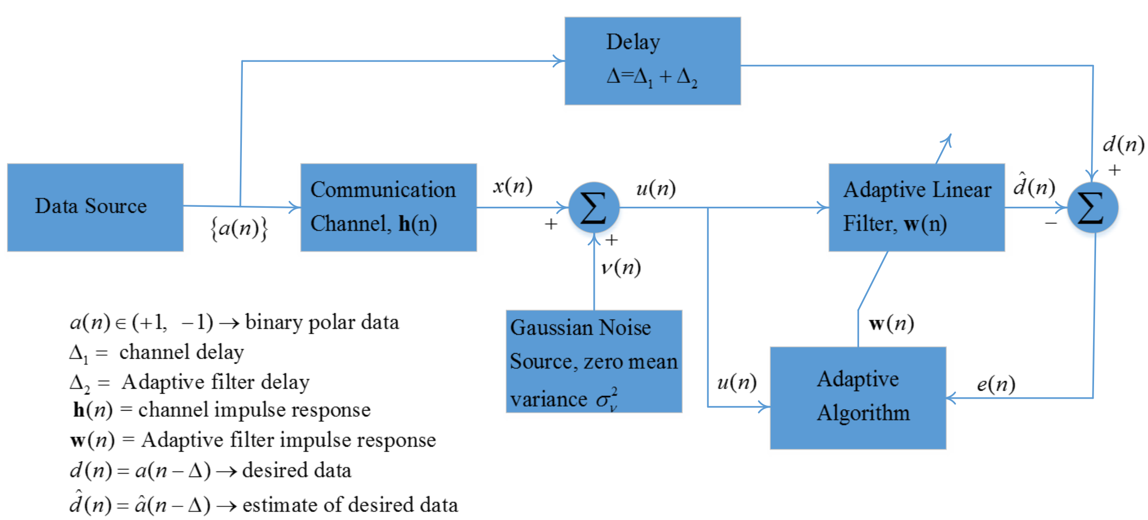

Figure 1 Block diagram for the equalization project

The input to the equalizer filter is given by the convolution sum:

$$u(n) = \mathbf{h}^T \mathbf{x}(n) + v(n) = \sum_{i=1}^{3} h_i x(n-i) + v(n), \quad n = 1, 2, \ldots, N$$

The noise sequence $\{v(n)\}$, has zero mean and variance σ_v^2. From analysis, and because the channel impulse response has only three components, the input autocorrelation matrix is found to have a ***quintdiagonal*** structure of the form:

$$\begin{bmatrix} r(0) & r(1) & r(2) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ r(1) & r(0) & r(1) & r(2) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ r(2) & r(1) & r(0) & r(1) & r(2) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & r(2) & r(1) & r(0) & r(1) & r(2) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & r(2) & r(1) & r(0) & r(1) & r(2) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r(2) & r(1) & r(0) & r(1) & r(2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r(2) & r(1) & r(0) & r(1) & r(2) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & r(2) & r(1) & r(0) & r(1) & r(2) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & r(2) & r(1) & r(0) & r(1) & r(2) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & r(2) & r(1) & r(0) & r(1) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & r(1) & r(0) \end{bmatrix}$$

The individual correlations are given by

$$r(0) = h_1^2 + h_2^2 + h_3^2$$
$$r(1) = h_1 h_2 + h_2 h_3$$
$$r(2) = h_1 h_3$$

You are expected to do the following:

1. **Pre-Experiment Calculations**

For each of the five channels and for noise variance $\sigma_v^2 = 0.0001$, compute the eigenvalues of the matrix $\mathbf{R}$ of dimension $M \times M$, $M = 11$, and fill in the values for the maximum and minimum eigenvalues as well as the eigenvalue spreads in table form as shown below.

	Min Eigenvalue, $\lambda_{\min}$	Max Eigenvalue, $\lambda_{\max}$	Eigenvalue Spread, $\chi = \lambda_{\max} / \lambda_{\min}$
Channel 1			
Channel 2			
Channel 3			
Channel 4			

In this project you are required to solve the equalization problem by recursively computing the filter weights, using the LMS as the adaptive algorithm, and evaluating the response (the mean-square error or learning curve) of the adaptive equalizer, to changes in the eigenvalue spread χ. The filter is supposed to undo the dispersion introduced by the channel.

To assess the performance of the filter, an approximate mean-square error curve is obtained by averaging the instantaneous squared error over K independent runs (or experiments) of the LMS algorithm. The MSE is computed as follows:

$$MSE(n) = \frac{1}{K}\sum_{k=1}^{K} e_k^2(n); \qquad n = 1,2,\ldots,N; \;\; e_k(n) = \text{error for the k}^{th}\text{run}$$

The desired signal $d(n)$, is obtained by delaying the transmitted signal so that the arrival times of $d(n)$ and $a(n)$ are aligned at the filter output. The amount of delay is the time it takes the signal $a(n)$ to propagate through the channel and the filter. The factor N is the number of data samples $\{a(n)\}$, for a single run (or experiment) of the algorithm and K (the outer loop) is the number of independent runs required for the averaging. To ensure that the runs are independent, the seeds used to generate the data and noise samples must be different for each experiment, k. The curves must be plotted, as functions of time n, on the same graph, in semi-log scale, that is the vertical axis must be in the form $MSE(n) = a \times 10^{-k}$, $k = 0,1,2,\ldots$. MATLAB has a semi-log function that does this for you.

2. Effect of Eigenvalue Spread

For the parameters given in the table below, generate and plot the learning curves for Channels 1, 2, 3, and 4 for a fixed step-size parameter μ, as functions of time n, all on one graph. The curves must all be plotted on semi-log scale.

Filter Order	$M = 11$
Step size parameter μ	0.075
Number of data samples N	≥ 500
Number of independent runs (experiments) K	≥ 250
SNR	**40 dB**

In your report, describe your observations and state your conclusions.

3. Effect of Filter Order

For each filter order M and each step size, in the table below, generate, plot and compare the mean-squared errors as functions of time n for Channel 2.

Filter Order	$M = 9$, $M = 11$ and $M = 21$
Step size parameter μ	0.075
Number of data samples N	≥ 500
Number of independent runs (experiments) K	≥ 200
SNR	**40 dB**

Plot 3 MSE curves on one graph (one each for $M = 9$, $M = 11$ and $M = 21$) for $\mu = 0.0375$
In your report, discuss your observations and explain why for some value(s) of filter order M, the MSE curve diverges. Also explain how this problem can be fixed.

4. Effect of Step-size Parameter

To study the effect of step-size parameter, the eigenvalue spread is fixed by focusing on one channel only. For this study, choose Channel 1 and the parameters in the table below. For each of the step-size parameters, generate the approximate MSE curves and plot all three curves on one graph using a semi-log scale.

Filter Order	$M = 11$
Step size parameter μ	0.0125, 0.025, 0.075
Number of data samples N	≥ 1500
Number of independent runs (experiments) K	≥ 250
SNR	**40 dB**

Plot three curves on the same graph (for each of $\mu = 0.0125, 0.025, 0.075$). In your report discuss your observations and state your conclusions.

5. **Comparison of Standard LMS and Normalized LMS**

For this study, choose Channel 2 and the parameters in the table below. Run the standard LMS for the two step-sizes and generate the approximate MSEs as functions of time n. Run the normalized LMS and generate the approximate MSE as a function of time n. Plot all three MSE curves on one graph using a semi-log scale, as functions of time n.

Filter Order	$M = 11$
Step size parameter μ (for standard LMS only)	0.025 and 0.075
Number of data samples N	≥ 1500
Number of independent runs (experiments) K	≥ 250
SNR	**40 dB**

In your report discuss your observations and conclusions.