Assignment 2

Pouyan Keshavarzian
ENEL 671: Adaptive Signal Processing

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Problem 1.

$$R = \begin{bmatrix} 2 & 1 & 0.75 & 0.5 & 0.25 \\ 1 & 2 & 1 & 0.75 & 0.5 \\ 0.75 & 1 & 2 & 1 & 0.75 \\ 0.5 & 0.75 & 1 & 2 & 1 \\ 0.25 & 0.5 & 0.75 & 1 & 2 \end{bmatrix}$$

Filter Order	Eigenvalue Spread	Corresponding Upper Bound, μ
2	3.0	0.5
3	4.2088	0.333
4	5.0396	0.25
5	5.6864	0.2

Table 1: Problem 1 Calculations

You could not use a value close to the upperbound of the second order filter for the fifth order because it exceeds the upper bound therefore the filter would diverge.

Problem 2.

$$P = \begin{bmatrix} 0.5 \\ 0.25 \\ 0.125 \\ 0.0625 \\ 0.03125 \end{bmatrix}$$

The calculated tap-input vectors for their corresponding filter orders are shown below:

$$W_02 = \begin{bmatrix} 0.25 \\ 0 \end{bmatrix} W_03 = \begin{bmatrix} 0.2571 \\ 0.0.0179 \\ -0.0420 \end{bmatrix} W_04 = \begin{bmatrix} 0.2575 \\ 0.0219 \\ -0.0325 \\ -0.0251 \end{bmatrix} W_05 = \begin{bmatrix} 0.2577 \\ 0.0217 \\ -0.0331 \\ -0.0266 \\ 0.0037 \end{bmatrix}$$

Filter Order	MMSE
2	0.8750
3	0.8723
4	0.8714
5	0.8714

Table 2: Problem 2 Calculations

Filter order 4 and 5 give almost an identical MMSE as the filter has converged.

A Matlab Code

```
%% ENEL 671 Assignment 2
% Pouyan Keshavarzian
% ENEL 671
%%
% Define autocorrelation matrix
R = [2 1 0.75 0.5 0.25];
R = toeplitz(R);
for M=2:5
    [V, D]= eig(R(1:M,1:M));
    eigenvalue_spread(M-1)= max(max(diag(D)))/min(min(diag(D)));
    % Property of the autocorelation matrix is that the sum of the
    % eigenvectors is the trace of the matrix
    TraceR(M-1)=sum(diag(D));
end
eigenvalue_spread
UpBoundStepSize=2./TraceR

%% Part 2
% Calculate weights for each filter order

% Autocovarriance matrix p
p = [0.5 0.25 0.125 0.0625 0.03125]';
sigma_squared = 1.0;
% Calculate tap-input vectors for corresponding filter coefficeints
w2 = inv(R(1:2,1:2))*p(1:2);
w3 = inv(R(1:3,1:3))*p(1:3);
w4 = inv(R(1:4,1:4))*p(1:4);
w5 = inv(R(1:5,1:5))*p(1:5);

% Calculate MMSE for corresponding filter coefficeints
MMSE2 = sigma_squared - w2'*p(1:2);
MMSE3 = sigma_squared - w3'*p(1:3);
MMSE4 = sigma_squared - w4'*p(1:4);
MMSE5 = sigma_squared - w5'*p(1:5);

MMSE = [MMSE2 MMSE3 MMSE4 MMSE5]
```