CH3 Concepts and Definitions

Paul Kim

December 10, 2020

1 CH3

1.1 Annuities Basic

$$a + ar + ar^2 + ... + ar^{n-1} = a\frac{1-r^n}{1-r}$$

1. immediate = end of each payment period
   (a) $a_{\overline{n}|i}$ and $s_{\overline{n}|i}$
   (b) "END"
2. due = beginning of each payment period
   (a) $\ddot{a}_{\overline{n}|i}$ and $\ddot{s}_{\overline{n}|i}$
   (b) "BEGIN"

1.2 loans with sightly reduced final payment

1. calculate for pmt
2. recalculate pv with pmt

3. if the pv is greater than loan value , go to 5
4. if the pv is less than loan value, round up the pmt
5. calculate the value of loan at the end period
6. calculate the value of the annuity at the end of the annuity (s)
7. subtract with #6 and #5 in that order (6 should be higher now)
8. subtract the delta with the pmt and that's your last reduced pmt

1.3 perpetuities and dividend model

$$a_{\overline{\infty}|i} = \frac{1}{i}$$
$$\text{double dot} a_{\overline{\infty}|i} = \frac{1}{d}$$

1.

1.4 Outstanding Loan Balance

1. OLB_k = outstanding loan balance right after the k-th pmt
2. prospective: (total PV value of all remaining pmts)
3. retrospective: (Value of the loan at time k) - (total PV value of payments made)

1.5 Non-leveled Annuities

1. ?

1.6 Annuities with Geometric progression pmts

$$a + ar + ar^2 + ... + ar^{n-1} = a\frac{1-r^n}{1-r}$$
$$a + ar + ar^2 + ... = a\frac{1}{1-r}$$

1.7 with Arithmetic progression pmts

P, P+Q, P+2Q, ... P+(n-1)Q where P is the pmt
v = $\frac{1}{1+i}$

$$(I_{P,Q}a)_{\overline{n}|i} = Pa_{\overline{n}|i} + \frac{Q}{i}(a_{\overline{n}|i} - nv^n)$$
$$(I_{P,Q}s)_{\overline{n}|i} = Ps_{\overline{n}|i} + \frac{Q}{i}(s_{\overline{n}|i} - n)$$
$$(I_{P,Q}\ddot{a})_{\overline{n}|i} = P\ddot{a}_{\overline{n}|i} + \frac{Q}{i}(a_{\overline{n}|i} - nv^n)$$
$$(I_{P,Q}\ddot{s})_{\overline{n}|i} = P\ddot{s}_{\overline{n}|i} + \frac{Q}{i}(s_{\overline{n}|i} - n)$$

$$(I_{P,Q}a)_{\overline{\infty}|i} = \frac{P}{i} + \frac{Q}{i^2}$$
$$(I_{P,Q}\ddot{a})_{\overline{\infty}|i} = \frac{P}{d} + \frac{Q}{id}$$

1.8 Annuity Paid Continuously

1. value at t0 of the level annuity with continuous payment to t = n, at a rate of 1 for each period of length = $\int_0^n v dt$

if $a(t) = (1+i)^t$

$$\bar{a}_{\overline{n}|i} = \frac{1-v^n}{\delta} = \frac{1-v^n}{ln(1+i)}$$
$$\bar{s}_{\overline{n}|i} = \frac{(1+i)^n - 1}{\delta} = \frac{(1+i)^n - 1}{ln(1+i)}$$

1. annuity paid continuously at a rate of f(t)
2. amount paid during time [a,b] = $\int_a^b f(t)dt$
3. amount paid during period [t, t+dt] = f(t)dt
4. value at t0 = $\int_0^n v(t)f(t)dt$
5. "annuity paid continuously at constant rate of 1": f(t) = 1
6. "interest is paid continuously at a rate of δ": a(t) = $e^{\delta t}$