Lab_avl : Awful AVL Trees

Week #8 – March 6-8, 2019

Welcome to Lab AVL!
Course Website: https://courses.engr.illinois.edu/cs225/sp2019/assignments/

Overview
In this week's lab you will work on implementing important features for an AVL tree. An AVL tree is a dynamically balancing BST (Binary Search Tree). It maintains a height of at most $O(\lg(n))$ where n is the number of nodes in the BST. This is important since the runtime of searching for an element in a balanced BST is $O(\lg(n))$.

AVL Tree Rotations
In lecture, you learned an AVL tree has four kinds of rotations that it can perform in order to balance the tree: L, R, LR, and RL. Left (L) and right (R) rotations are singular rotations used on "sticks" in order to turn them into "mountains". Left-right (LR) and right-left (RL) rotations are combinations of the previous two rotations that are used to turn "elbows" into "mountains".

Exercise 1: Which type of rotation should be performed on each of these subtrees?

Q1) Right	Left	Left-Right	Right-Left
Q2) Root BF = -2	Root BF = 2	Root BF = -2	Root BF = 2
Q3) Child BF = -1	Child BF = 1	Child BF = 1	Child BF = -1

For each tree:
Q1: What type of rotation fixes each of these trees?
Q2: Identify the balance factor of the root node in each tree.
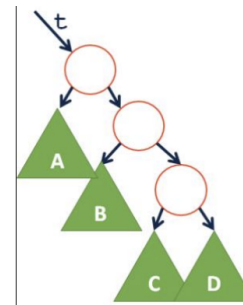Q3: Identify the balance factor of the node below the root in each tree.

Left Rotation
Let's focus on writing the code for a single rotation

Exercise 2: Complete the pseudo-code for a left rotation

Initial structure: Q1: Source code to perform a left rotation

TreeNode *& t = /* point of imbalance */

TreeNode * y = t->right;
t->right = y->left;
y->left = t;
t = y;
Don't forget to update the heights!

Q2: How does a right rotation differ?

Almost the same as the left pseudocode, just swap lefts and rights.

AVL Tree Insertion

Inserting into an AVL tree is very similar to the process of inserting into a BST. The difference between the two is the insertion operation of an AVL tree also rebalances the tree. The pseudocode was given in class, so make sure to look back at your notes for understanding the algorithm of inserting into an AVL Tree.

Exercise 3.1: For the AVL Tree below:
1. Insert the element **29**.
2. After inserting, find the height for each of the nodes in the tree.
3. Identify the lowest point of imbalance: what is its balance factor?
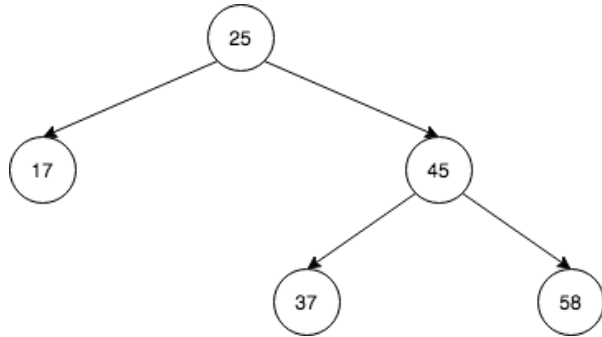
Add 29 as 37's left child.
Heights: 29 -> 0, 58 -> 0, 17 -> 0, 37 -> 1, 45 -> 2, 25 -> 3
Lowest point of imbalance is 25 since height(45) – height(17) = 2

The following nodes are in the elbow: 25, 45, 37

Exercise 3.2: Based on the lowest point and its balance factor, and the balance factor of its child node, what is the type or rotation that needs to be used to rebalance the tree?

Rotation Type: Right-Left

Exercise 3.3: Draw the state of the tree after the rotation is done, and it has been rebalanced.
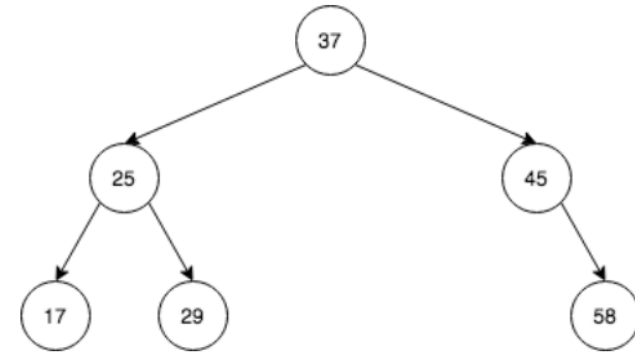
Exercise 3.4: Fill out the **worst-case** running times for the following operations when they are run on an unbalanced BST and an AVL Tree assuming you have access to the root of the tree. There are **n** nodes in the tree.

Operation	Unbalanced BST	AVL Tree
Printing the value of the root node	O(1)	O(1)
Searching for an element	O(h)=O(n)	O(h)=O(lg(n))
Printing the value of each node in the tree	O(n)	O(n)
Finding the node with the largest value	O(h)=O(n)	O(h)=O(lg(n))

In the programming part of this lab, you will:
- Get familiar with AVL Trees
- Practice writing the implementations of rotations
- Implement the functions to rebalance, insert, and remove an AVL Tree
  As your TA and CAs, we're here to help with your programming for the rest of this lab section! ☺