

Risk Premia, Subjective Beliefs, and Forward Guidance*

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Abstract

How can researchers identify monetary shocks and their causal impacts when (i) agents may possess subjective beliefs and (ii) monetary authorities manage current and future interest rates (e.g., forward guidance)? Assuming rational expectations or risk-neutrality trivially enables identification. Without those assumptions, identification of monetary shocks from asset prices hinges on a “Long-Run Neutrality” condition, roughly meaning policy does not affect the compensation for permanent risks. We construct a non-parametric test of the Long-Run Neutrality condition, related to the literature on FOMC announcement effects, and argue that it is violated in the data. Finally, we present some example models in which the Long-Run Neutrality condition is violated, illustrating how this condition is generally distinct from conventional notions of monetary neutrality.

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1 Introduction

How does a monetary authority influence the economy through speeches, forward guidance, and other policies targeting the future? In this paper, we uncover and explore an identification challenge associated with this question. The basic idea is that forward guidance and related policies operate by manipulating investor beliefs, which are not directly observable. Our key question becomes whether or not investor's policy-induced forecast revisions can instead be obtained indirectly by looking at rich enough collections of asset markets. We first show that identification of the various monetary surprises and their impacts requires a Long-Run Neutrality condition that differs from typical notions of monetary policy neutrality. Second, we propose a model-free test of this condition; the evidence suggests Long-Run Neutrality is violated. In other words, our results suggest forward guidance shocks cannot be identified from asset markets.

Multiple monetary instruments. Even in a simplified world without any central bank information advantage and with only conventional tools (e.g., no quantitative easing), monetary authorities impact the economy through several potential channels. Our starting point is that every FOMC meeting can contain up to three categories of shocks:

1. *short-rate shocks*: unexpected changes to the short-term interest rate
2. *forward-guidance shocks*: unexpected modifications to the expected future rate path
3. *uncertainty shocks*: unexpected changes in uncertainty about future rates

For example, when a central bank raises the short-term interest rate (short-rate shock), investors must form beliefs about the persistence of the rate hike (forward-guidance shock), and these investors furthermore must form beliefs about the risk that policy rates rise further in the future (uncertainty shock). Of course, forward-guidance shocks and uncertainty shocks may also occur in isolation without any short-rate shock. We seek the causal impact of each type of policy shock on the economy, which first requires a shock identification procedure.

Challenges to identification. Given this framework, how can the various policy shocks be recovered? For simplicity, imagine we are given a time series of the short-rate shocks. A common approach examines high-frequency changes to Fed Funds futures prices on the FOMC meeting day (Krueger and Kuttner, 1996; Rudebusch, 1998; Kuttner, 2001; Rudebusch, 2002; Bernanke and Kuttner, 2005; Piazzesi and Swanson, 2008). While this

procedure is somewhat controversial, we take it for granted to engage with more novel identification issues.

Turning to forward-guidance and uncertainty shocks, we ask whether they can be recovered from asset price data. Asset markets are a natural arena to explore because of the richness in financial claims covering many horizons (e.g., very far into the future), at many levels of contingency (e.g., isolating specific aspects of the probability distribution), and available at a high frequency. We will discuss survey data as a viable alternative in various parts of the paper, but note for now that surveys do not possess the same richness in horizon, contingency, or frequency.

Consider forward-guidance shocks. A first thought might be to use long-term yields to reveal expectations about future short rates (Expectations Hypothesis). But the preponderance of evidence stands against the Expectations Hypothesis, because of the existence of bond risk premia, and more importantly the time-variation in these risk premia. Changes to the yield curve can only identify shocks to a risk-adjusted expectation of future short rates (for example, the risk-neutral expectation). Currently, no model-free mapping exists between these risk-adjusted expectations and investors' expectations. Assuming this gap away seems like wishful thinking, since unlike short-horizon Fed Funds futures, significant risk premia exist in long-term bonds.

Identification of uncertainty shocks faces an analogous issue, but in distribution space rather than mean space. Even given a full set of options on interest rates of all maturities, all we can identify is a risk-adjusted distribution of future interest rates. Given the existence of time-varying risk premia, it is nontrivial to map this risk-adjusted distribution into investors' subjective distribution. Some approaches have been proposed for separate identification of short-rate shocks versus a second "path" factor encompassing all other monetary shocks ([Gürkaynak et al., 2005b](#); [Swanson, 2021](#)). But interpreting this second factor is challenging without a model that allows us to separate the impacts of beliefs and risk premia. Our framework shows that isolating beliefs is critical.

Identification is not hopeless. Our core set of theoretical results says that forward-guidance and (to a lesser extent) uncertainty shocks can be identified from asset prices if and only if permanent risks and their risk prices are unaffected by monetary policy. That is, if rational expectations and risk-neutrality cannot be assumed to hold, and if we do not write down a fully-specified model of investor preferences and beliefs, then identification requires long-run risk prices to be invariant to monetary policy. We refer to this monetary-invariance condition as *Long-Run Neutrality*. This exact condition is effectively imposed as an identification assumption in the recent papers of [Backus et al. \(2022\)](#) and [Haddad et al. \(2023\)](#), which had different but related goals. We clarify that

this is not a coincidence: identification requires a Long-Run Neutrality assumption.

Our theoretical results connect closely to a broader issue in asset pricing, so-called recovery theory (Ross, 2015; Borovička et al., 2016). Investor beliefs are not revealed by asset prices, because beliefs are co-mingled with other permanent components of marginal utility. In the context of monetary policy, there is a nuance: we do not seek beliefs themselves, but rather shocks to beliefs. And this is why the key assumption of recovery theory, *absence* of a permanent component in marginal utility, is replaced by the *invariance* of such permanent component to monetary policy.

In our paper, we develop a non-parametric test of Long-Run Neutrality. The return of the growth-optimal portfolio in excess of a long-maturity bond identifies the martingale in the pricing kernel (Alvarez and Jermann, 2005). Long-Run Neutrality implies this investment strategy has exactly a zero return on Fed announcements. To repeat, the return should not only be zero on average but zero on each and every announcement. To implement this test, we proxy the growth-optimal portfolio and long-maturity bond with equities and Treasury bonds, respectively, and study their return dynamics near Fed announcements.

Our evidence suggests Long-Run Neutrality is violated. Indeed, equity and long-term bonds behave very differently near Fed announcements. Broadly speaking, Long-Run Non-Neutrality suggests central bank policies may be more powerful, especially through their risk premia effects, than previously understood. For the specific aims of the present paper, Long-Run Non-Neutrality implies that forward-guidance and uncertainty shocks cannot be identified from asset prices alone.

Of course, an existing literature on “announcement effects” (which we cite below) has also studied returns near Fed announcements. What is new here is the tighter connection between our data analysis and our theory. For one, the bonds we investigate are longer-maturity than those commonly studied, an important contribution given our test asks for an arbitrarily long-maturity bond. Second, the existing literature often studies equities and bonds separately and in differing sample periods; we provide a consistent sample to study them jointly. Third, the overwhelming majority of the literature studies average announcement returns, whereas our test requires us to examine other moments beyond the mean. One of our most striking and novel findings is the large announcement volatility of the long equity, short bond investment strategy.

Finally, we discuss structural models in which economic growth and uncertainty are priced sources of risks. A leading example is Bansal and Yaron (2004). If we take these models seriously, our Long-Run Neutrality condition requires both growth and uncertainty to be invariant to monetary policy, *both in the short run and the long run*.

Through the lens of these models, there is an identification paradox: learning the effects of monetary policy requires precisely that monetary policy has no effects.

Literature review. Our framework builds on two pillars. First, we entertain the possibility that market participants possess biased beliefs about future interest rates, asset returns, and monetary policy (Ball and Croushore, 2003; Hamilton et al., 2011; Chun, 2011; Giglio and Kelly, 2018; Cieslak, 2018; Crump et al., 2018; Kryvtsov and Petersen, 2019; d’Arienzo, 2020; Wang, 2021; Xu, 2019; Nagel and Xu, 2022; Bianchi et al., 2022b). Second, we embrace environments in which monetary authorities can manage beliefs about future interest rates (Eggertsson and Woodford, 2003; Werning, 2011; Del Negro et al., 2012; Campbell et al., 2012; McKay et al., 2016), implying the presence of several distinct monetary effects, even ignoring a central bank information advantage (Romer and Romer, 2000; Melosi, 2017; Nakamura and Steinsson, 2018; Cieslak and Schrimpf, 2019; Miranda-Agrippino and Ricco, 2021).¹

While some existing literature does try to measure the impact of forward guidance and other central bank communication, this literature often either extracts a reduced-form “path” factor from long-term bond yields (Gürkaynak et al., 2005b; Swanson, 2021; Altavilla et al., 2019; Lunsford, 2020) or sidesteps the rate path by projecting outcomes on communication directly (Hansen and McMahon, 2016; Leombroni et al., 2021; Gómez-Cram and Grotteria, 2022). By contrast, we seek to identify the shock to investor beliefs about the interest rate path.

In this class of environments, estimating the causal impact of monetary policy requires a Long-Run Neutrality condition that is related to belief recovery theory (Ross, 2015; Borovička et al., 2016; Qin and Linetsky, 2016). Following a literature that decomposes the pricing kernel into permanent and stationary components (Kazemi, 1992; Alvarez and Jermann, 2005; Hansen and Scheinkman, 2009; Bakshi and Chabi-Yo, 2012; Qin and Linetsky, 2017; Corsetti et al., 2023), we develop a model-free test of the Long-Run Neutrality condition in the context of monetary policy.

Our test zooms in on asset price changes around FOMC announcements, which is related to the literature on “announcement effects.” Several authors have argued that equity risk premia are strongly influenced by monetary policy announcements, including both actions and communications (Pearce and Roley, 1985; Rosa, 2011; Savor and Wilson,

¹A large theoretical literature has also considered the effects of monetary policy in environments with distorted beliefs. As we avoid putting too much structure on agents’ beliefs, we do not directly engage with this literature, but some notable examples include Bernanke and Woodford (1997), Evans and Honkapohja (2003), Andolfatto and Gomme (2003), Schorfheide (2005), Milani (2008), Gasteiger (2014), Hommes et al. (2019), and Caballero and Simsek (2022). Caballero and Simsek (2022), in particular, argue that subjective belief dynamics themselves could be the *origin* of monetary policy shocks.

2013, 2014; Lucca and Moench, 2015; Ai and Bansal, 2018; Cieslak et al., 2019; Cieslak and Pang, 2021; Bianchi et al., 2022a,c; Bauer et al., 2023). A related literature examines FOMC announcement effects in government bonds (Ederington and Lee, 1993; Gürkaynak et al., 2005a; Beber and Brandt, 2006; Faust et al., 2007; Hanson and Stein, 2015; Hillenbrand, 2021; Hanson et al., 2021). We connect these literatures by investigating the announcement effect of a particular long-short portfolio, which is the theoretically appropriate object for our purposes.

Finally, to illustrate how strong the Long-Run Neutrality condition can be, we consider a class of structural environments in which long-run growth and uncertainty become priced state variables. A large empirical literature suggests that certain longer-term prospects and news about these prospects matter (McQueen and Roley, 1993; Francis and Ramey, 2005; Beaudry and Portier, 2006; Barsky and Sims, 2011; Schmitt-Grohé and Uribe, 2012; Kurmann and Otrok, 2013; Barsky et al., 2015; Leduc and Liu, 2016; Nakamura et al., 2017; Basu and Bundick, 2017; Schorfheide et al., 2018; Berger et al., 2020; Liu and Matthies, 2022). Models consistent with this evidence imply that persistent shocks to economic growth and uncertainty comprise the permanent component of the pricing kernel (Bansal and Yaron, 2004; Beaudry and Portier, 2004; Bloom, 2009; Bidder and Dew-Becker, 2016; Christiano et al., 2014; Fajgelbaum et al., 2017; Bianchi et al., 2018; Di Tella, 2017; Di Tella and Hall, 2022; Bianchi et al., 2023). A related literature also theorizes and documents the importance of monetary and other policy uncertainty (Baker et al., 2016; Creal and Wu, 2017; Husted et al., 2020; Pastor and Veronesi, 2012; Pástor and Veronesi, 2013; Kelly et al., 2016). In these environments, Long-Run Neutrality implies that monetary policy does not affect the probability distribution of future growth.

2 Motivation: A Simple VAR Example

We begin with a simple example to illuminate the key issues. The point of this model is motivational: it allows us to set up our questions, explain the crux of the identification challenges, and then demonstrate conditions under which some types of identification are possible. In particular, we discuss why it is desirable to obtain investor forecast revisions about the interest rate path, as well as why obtaining these forecast revisions is highly non-trivial. Section 3 substantially generalizes the model and furthermore allows some types nonlinearities to permit the broadest statement of our identification results.

2.1 Model with two monetary actions and subjective beliefs

Consider a three-state model

$$X_t = \begin{bmatrix} g_t \\ r_t \\ f_t \end{bmatrix} = \begin{bmatrix} \text{(demeaned) growth rate} \\ \text{(demeaned) interest rate} \\ \text{forward guidance} \end{bmatrix}.$$

We assume that g_t and r_t are observable, whereas f_t is a latent factor. Suppose the state evolves dynamically according to

$$X_{t+1} = AX_t + B\Delta W_{t+1}, \quad (1)$$

where $\Delta W_{t+1} \sim \text{Normal}(0, I)$ is a 3-dimensional vector of Normal shocks. For the purpose of this example, we will think of the shocks ΔW_{t+1} as containing a “real shock” and two “monetary shocks” that are completely governed by central bank actions—if B were a diagonal matrix, we could assign these labels to the individual shocks, but that is not necessary for our purposes. Equation (1) governs the *objective* state dynamics, which may differ from agents’ *perceived* dynamics that we detail below.

Let us specify the persistence matrix A . Assume

$$A = \begin{bmatrix} a_{gg} & a_{gr} & a_{gf} \\ a_{rg} & a_{rr} & a_{rf} \\ 0 & 0 & a_{ff} \end{bmatrix}. \quad (2)$$

and that A is stable. Note that a_{gr} and a_{gf} capture the effects of short rates and forward guidance on growth. By contrast, a_{rg} captures the “feedback effect” of growth into the interest rate rule. And a_{rf} captures the transmission from forward guidance into the short-rate. In fact, the entire purpose of including f in this system is to add an additional factor governing future short rates. Finally, forward guidance f_t evolves as a univariate AR(1) independently of (g_t, r_t) , a setup that is not necessary to any result but is transparent. We make no assumptions about B .

We allow agents’ beliefs to potentially be non-rational. While we take no stand here, belief distortions could come from multiple sources: pure cognitive biases, imperfect information or attention, finite samples with imperfect priors, etc. To keep things simple, we consider a belief distortion that modifies the persistence of X_t . Under agents’

subjective beliefs

$$X_{t+1} = \tilde{A}X_t + B\Delta\tilde{W}_{t+1}, \quad \Delta\tilde{W}_{t+1} \sim \text{Normal}(0, I). \quad (3)$$

We will assume \tilde{A} , like A , is a stable matrix. The notation $\tilde{\mathbb{E}}$ will stand for the subjective expectation operator for agents in our model, which may or may not coincide with the objective expectation \mathbb{E} .

Thus, agents perceive

$$\Delta\tilde{W}_{t+1} = \Delta W_{t+1} - L_t, \quad \text{where} \quad L_t := B^{-1}(\tilde{A} - A)X_t, \quad (4)$$

to be a standard Normal shock. One interpretation is that the vector L_t represents investors' time-varying degree of optimism.

2.2 The causal effects of monetary policy

Consider the question “what are the causal effects of monetary policy on future growth?” Here, there are two components of policy: short rates and forward guidance.

The *short-rate shock* at time τ defined as

$$z_\tau^r := r_\tau - \tilde{\mathbb{E}}[r_\tau \mid X_{\tau-1}]. \quad (5)$$

In this paper, we take as given the ability to non-parametrically identify the short rate shock from data. In particular, assume the existence of a financial market (e.g., Fed Funds futures) whose price corresponds to $\tilde{\mathbb{E}}[r_\tau \mid X_{\tau-1}]$ at time $\tau - 1$. It is technically appropriate to use the investor expectation $\tilde{\mathbb{E}}$ here, because financial markets reflect investor beliefs. A *forward-guidance shock* at time τ is defined as

$$z_\tau^f := f_\tau - \tilde{\mathbb{E}}[f_\tau \mid X_{\tau-1}]. \quad (6)$$

Unlike the short-rate shock, we do not assume the forward-guidance shock is non-parameterically identified. This will be a key challenge. For completeness, also define the *growth shock* $z_\tau^g := g_\tau - \tilde{\mathbb{E}}[g_\tau \mid X_{\tau-1}]$. Stack the shocks into

$$z_\tau := \left(z_\tau^g, z_\tau^r, z_\tau^f \right)' = B\Delta\tilde{W}_\tau.$$

These shocks are reduced-form in nature, but that is not the key issue. The complication,

instead, comes from the non-rational beliefs: the reduced-form shocks

$$z_\tau = B\Delta W_\tau - (\tilde{A} - A)X_{\tau-1} \quad (7)$$

are related to the true structural shocks ΔW , with a bias that is a function of lagged X .

What are the causal effects of monetary policy on $g_{\tau+t}$? Because we do not have a full structural model of monetary action, we *define* all causal effects by the following IRFs

$$D_h := \frac{\partial}{\partial w'} \mathbb{E}[g_{\tau+h} \mid B\Delta W_\tau = w] = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot A^h \quad (8)$$

The second and third entries of this IRF tell us the effects of the short-rate and forward-guidance shocks. Suppose we seek D_h .

A first convenient result is that, to measure D_h empirically, we may run a regression of future growth $g_{\tau+h}$ onto z_τ , the lagged state $X_{\tau-1}$, and a constant. In other words, we can employ the *subjective shocks* and need not obtain proxies for the objective shocks; this simplification is beneficial because some shocks like z' will be identified from financial market data and therefore reflect investors' beliefs. (In principle, this “local projection” approach, highlighted by [Jordà 2005](#), flexibly estimates the effect of shocks on future g at various horizons. The Jorda approach is considered to be more robust to model misspecification than the alternative that estimates the VAR(1) and computes A^h .) For emphasis, we give this procedure a name:

Procedure 1. *The estimate of b_h from the regression*

$$g_{\tau+h} = a_h + b_h' z_\tau + c_h' X_{\tau-1} + \epsilon_{\tau+h} \quad (9)$$

is an unbiased and consistent estimator of D_h .

Why does Procedure 1 work? An important, and underappreciated, feature is the lagged state control $X_{\tau-1}$: in a distorted-belief world, this control is necessary as it captures the discrepancy between the objective and subjective shocks—see equation (7). Approaches in the literature, instead, have mostly hoped to find objective shocks to monetary policy. If a researcher succeeds in measuring an objective shock, there is no need to control for drivers of belief distortions. But if instead one merely measures a subjective surprise, omitting the lagged states $X_{\tau-1}$ effectively, and perhaps wrongly, assumes rational expectations holds.

A simple numerical example highlights the importance of controlling for $X_{\tau-1}$. We calibrate A , \tilde{A} , and B to roughly match the evidence in [Cieslak \(2018\)](#), who compares the

objective dynamics of employment growth and short rates to the subjective dynamics of these same variables from surveys. Details of this calibration are provided in Appendix A.1. The results for the calibrated A and \tilde{A} imply agents perceive a smaller feedback of growth to future short rates, as well as higher persistences of r and especially of f .

After calibrating, we ask: what is the impact of wrongly assuming that z_τ^r and z_τ^f are objective shocks, instead of perceived investor surprises? Assuming rational expectations corresponds to implementing Procedure 1 without controlling for the lagged state $X_{\tau-1}$. On the other hand, controlling for $X_{\tau-1}$ corrects for belief distortions and therefore yields the correct effect of forward guidance. Figure 1 plots the outcomes of these two regressions. The lines labelled “True IRF” come from correctly implementing Procedure 1, while the lines labelled “Wrong IRF” come from forgetting to control for $X_{\tau-1}$ (and thereby implicitly assuming rational expectations when expectations are biased). One notices a large discrepancy between these measures, so much that even the signs of the effect can be opposite in the two approaches.

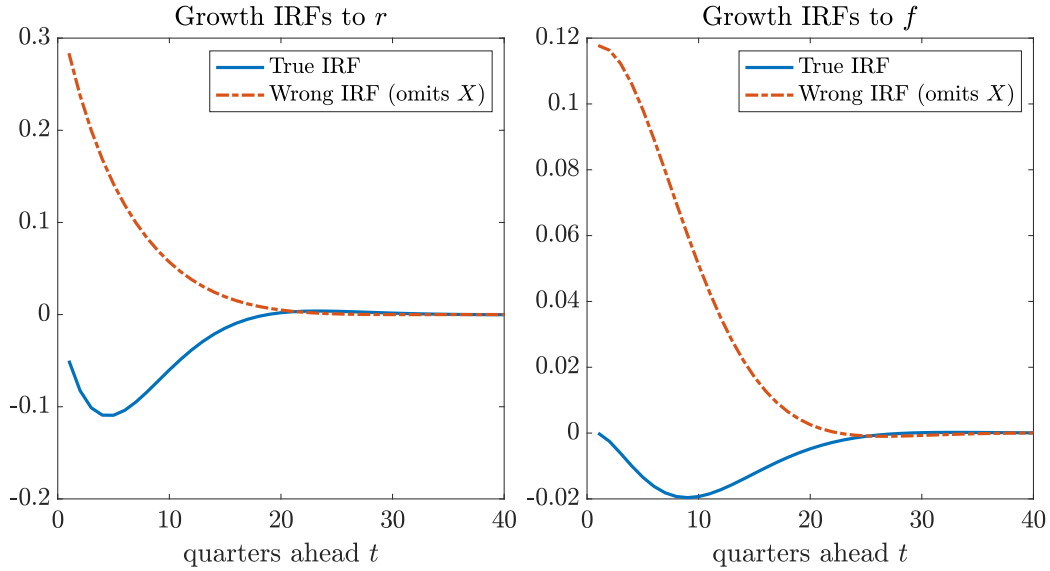


Figure 1. Growth IRFs to forward guidance. Regressions of $g_{\tau+t}$ on the forward-guidance shock z_τ^f with and without the lagged state $X_{\tau-1}$ as a control. The solid curve controls for $X_{\tau-1}$ and coincides with the “True IRF.” The dashed curve omits $X_{\tau-1}$ which yields the “Wrong IRF” and corresponds to assuming rational expectations. The calibrations of A and \tilde{A} are given in Appendix A.1, equation (A.3). The calibration of B is a diagonal matrix with $\sigma_g = 0.0025/4$, $\sigma_r = 0.010/4$, and $\sigma_f = 0.050/4$.

To summarize, the obstacles to implementing Procedure 1 are twofold. First, as mentioned, we need to control for $X_{\tau-1}$ in our estimation. But since X includes the unobservable forward-guidance variable f , we will need a method to recover f . What is this method? Second, we require the shocks z_τ . We will continue to assume that the short-rate shock z_τ^r is observed from financial markets. But how can we recover z_τ^f ?

Given the forward-looking nature of financial markets, let us broach the possibility that asset prices can inform us about both of f and z^f .²

2.3 Identifying forward guidance from asset prices

We need to recover a time series for f , as well as its perceived shock z^f . Since f is primarily about future short-term interest rates, observation of the market's expectation of future short rates should suffice in place of f . A natural place to look for these market beliefs are Treasury bond markets.

The problem with using asset markets is that they do not directly reveal the market's expectation, but rather a risk-adjusted expectation. To address this discrepancy, we will first write down a standard affine term structure model (Duffie and Kan, 1996; Dai and Singleton, 2002; Duffee, 2002; Ang and Piazzesi, 2003) and then impose sufficient structure on the model. In doing so, we will be explicit about which conditions allow us to invert risk adjustments and recover the market expectation.

Pricing model. The asset-pricing model in this example features zero inflation for simplicity, and so the distinction between real and nominal is immaterial here. To the dynamics in Section 2.1, we add the following one-period stochastic discount factor (SDF):

$$\frac{S_{t+1}}{S_t} = \exp \left[-(\bar{r} + r_t) - \frac{1}{2} \pi_t' \pi_t - \pi_t' \Delta W_{t+1} \right], \quad (10)$$

where $\pi_t = \pi_0 + \Pi X_t$ is the time-varying risk price vector. Notice that the SDF is specified under the objective probability measure. This is immaterial and only written this way to conform with the bond pricing literature.

We can also re-write the SDF in terms of the investor measure:

$$\frac{\tilde{S}_{t+1}}{\tilde{S}_t} = \exp \left[-(\bar{r} + r_t) - \frac{1}{2} (\pi_t + L_t)' (\pi_t + L_t) - (\pi_t + L_t)' \Delta \tilde{W}_{t+1} \right]. \quad (11)$$

The variable $\exp[L_t' \Delta W_{t+1} - \frac{1}{2} L_t' L_t] = \frac{S_{t+1}/\tilde{S}_{t+1}}{S_t/\tilde{S}_t}$ changes the probability measure from the objective one to investors' subjective one, while \tilde{S} represents investor marginal utility. Notice that investors' perceived risk prices are $\pi_t + L_t$.

²We do not discuss the growth shock z^g further because its inclusion is already emphasized by the existing literature on the "information effect" of monetary policy. In particular, one should include z^g in Procedure 1, because it may be correlated to monetary surprises z^r and z^f (and so omission of z^g results in an omitted variable bias). For example, if the central bank responds to high-frequency growth information, one needs to control for this growth information to get an appropriate monetary policy instrument (Miranda-Agrippino and Ricco, 2021; Bauer and Swanson, 2023a,b; Andrade and Ferroni, 2021).

In this setting, the risk-neutral dynamics of the state are given by

$$\begin{aligned} X_{t+1} &= A_0^* + A^* X_t + B \Delta W_{t+1}^*, \\ \text{where } A_0^* &:= -B\pi_0 \\ \text{and } A^* &:= A - B\Pi, \end{aligned} \tag{12}$$

where ΔW_{t+1}^* is a Normal shock under the risk-neutral distribution. This framework can be used to solve for bond prices of all maturities. The equilibrium yield-to-maturity for an n -period risk-free zero-coupon bond is given by

$$\begin{aligned} y_t^{(n)} &= \frac{1}{n} \left[\mathcal{B}_0^{(n)} + \mathcal{B}^{(n)} X_t \right] \quad \text{where } \mathcal{B}^{(n)} = (0, 1, 0)(I - A^*)^{-1}(I - (A^*)^n), \\ \mathcal{B}_0^{(n)} &= n\bar{r} + \left(\sum_{i=1}^{n-1} \mathcal{B}^{(i)} \right) A_0^* - \frac{1}{2} \sum_{i=1}^{n-1} \mathcal{B}^{(i)} B B' (\mathcal{B}^{(i)})'. \end{aligned}$$

This solution is standard in the literature.

The challenge: extracting forward guidance. Can we use the model solution to identify f ? Since bond yields are affine in the factors, we should be able to invert for the factor time series, given data on any three maturities. The setting here is even simpler because the growth rate and one-period yield (short rate) are observable. So if we have a single n -period bond ($n > 1$), we can use its yield to obtain the state vector as

$$X_t = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \mathcal{B}^{(n)} \end{bmatrix}^{-1} \begin{bmatrix} g_t \\ r_t \\ n y_t^{(n)} - \mathcal{B}_0^{(n)} \end{bmatrix}.$$

This expression suggests that f can be obtained via data on bond yields and an estimation of the asset-pricing model.

Unfortunately, this identification logic is incorrect in general, as Appendix A.2 explains in more detail. Bond yields themselves do not reveal *particular latent factors*. This is a well-known identification issue in affine term-structure models with latent state variables like our forward guidance variable (Hamilton and Wu, 2012). Without specific knowledge of A , B , or Π , we cannot decide whether the underlying state vector is X_t or some rotation \hat{X}_t of the state, as X and \hat{X} lead to the same pricing implications. This issue goes beyond our particular example. Indeed, the example above represents the “best-case scenario” where we know the true model is a three-factor model with two of the factors observable. Even then, the sole latent factor cannot be identified. The

problem intensifies if the environment is more complex, e.g., if there are additional latent factors driving bond yields or bond risk premia. Therefore, bringing in bond yields generically does not help us identify the causal effects of forward guidance. (As Appendix A.2 discusses, only in a knife-edge case—where forward-guidance is the unique latent variable and where its correlations with all observable states are known—can f be identified from bond yields.)

Surprises to future short rates: A resolution? Given the impossibility of identifying f , hence z^f , let us now ask a humbler question: can our asset-pricing model help us identify belief updates about future short rates? That is, can we obtain

$$z_{\tau}^{r,t} := \tilde{\mathbb{E}}[r_{\tau+t} \mid X_{\tau}] - \tilde{\mathbb{E}}[r_{\tau+t} \mid X_{\tau-1}] = (0, 1, 0) \tilde{A}^t B \Delta \tilde{W}_{\tau} \quad (13)$$

from asset-price data?

Obtaining surprises about future rates is actually good enough for many purposes, for the following reason. Conditional on $X_{\tau-1}$, the correlation between $z_{\tau}^{r,0}$ and $z_{\tau}^{r,t}$ is imperfect:

$$\text{corr}[z_{\tau}^{r,t}, z_{\tau}^{r,0} \mid X_{\tau-1}] = \frac{[\tilde{A}^t B B']_{2,2}}{\sqrt{[\tilde{A}^t B B' \tilde{A}^t]_{2,2} [B B']_{2,2}}}$$

Unless \tilde{A} is a diagonal matrix, which is the uninteresting case where monetary policy has no effects, this correlation will be below one for $t > 0$. Therefore, replacing z_{τ}^f in Procedure 1 with $z_{\tau}^{r,t}$ (for some $t > 0$) allows us to recover all desired objects: the coefficients on z_{τ}^r and $z_{\tau}^{r,t}$ will be the short-rate and forward-guidance effects, respectively. Intuitively, there is a two-factor structure to the short-rate path, and so z_{τ}^r and $z_{\tau}^{r,t}$ pick up different factors. In a richer model with additional latent factors driving short rates, we may want to include an entire collection $(z_{\tau}^{r,t})_{t=1}^T$ of surprises. The fact that these reduced-form surprises to future rates are “good enough” focuses our attention in the remainder of the paper on trying to extract the $z_{\tau}^{r,t}$ surprises, rather than forward guidance per se. For emphasis, we collect this discussion in the following procedure:

Procedure 2. *The estimate of $(b_h^g, b_h^r, b_h^{r,t})$ from the regression*

$$g_{\tau+h} = a_h + b_h^g z_{\tau}^g + b_h^r z_{\tau}^r + b_h^{r,t} z_{\tau}^{r,t} + c_h' X_{\tau-1} + \epsilon_{\tau+h} \quad (14)$$

is an unbiased and consistent estimator of D_h .

To implement Procedure 2, we need a proxy for $z_{\tau}^{r,t}$. If investors are *risk-neutral*, we can recover these surprises. Under risk-neutrality, investor marginal utility features zero

risk-pricing ($\pi_t + L_t = 0$), so the recovered risk-neutral dynamics actually correspond to investors' perceived dynamics. In that case, we can recover the surprises in (13) via $z_t^{r,t} = \mathbb{E}^*[r_{\tau+t} \mid X_t] - \mathbb{E}^*[r_{\tau+t} \mid X_{t-1}]$, given estimates of A^* and B . Intuitively, risk-neutrality allows recovery because it implies an "Expectations Hypothesis" but under the investor subjective belief.

A relaxed version of risk-neutrality also permits shock recovery. Assume

$$\Pi = -B^{-1}(\tilde{A} - A). \quad (15)$$

Condition (15) says that investor perceived risk prices $\pi_t + L_t$ are time-invariant (the coefficient on X_t is zero). While time-invariance may seem quite restrictive, it turns out that it is both necessary and sufficient for identification of $B\Delta\tilde{W}$ in this model.

To understand sufficiency is fairly easy. Substitute (15) into the risk-neutral dynamics to obtain

$$A^* = A - B\Pi = A + BB^{-1}(\tilde{A} - A) = \tilde{A}.$$

If we know A^* , then we know \tilde{A} , which allows us to obtain perceived shocks as $X_{t+1} - \tilde{A}X_t = B\Delta\tilde{W}_{t+1}$. Intuitively, if investor perceived risk prices are constant, then a version of the Expectations Hypothesis holds: long-term bond yields capture investor beliefs about future short-term yields, with a constant shifter. The constant shifter is differenced out when studying belief surprises, rather than belief levels.

Necessity is harder to see. The important fact is that condition (15) is required to make investor-perceived long-run risk prices constant. To see this, we follow the calculations in Backus et al. (2022) to compute the permanent component of the investor SDF \tilde{S} in (11) as

$$\frac{H_{t+1}}{H_t} = \exp \left[-\frac{1}{2} \|\pi_t + L_t - B'v\|^2 - (\pi_t + L_t - B'v) \cdot \Delta\tilde{W}_{t+1} \right],$$

where $v := -[I - (\tilde{A} - B\Pi)']^{-1}(0, 1, 0)'$. In our general environment of Section 3, we explain this permanent component in more detail, and we show that perceived shock identification requires H_{t+1}/H_t to be independent of X_t (under investor beliefs). Taking that result as given, identification thus requires $\pi_t + L_t = \pi_0 + [\Pi + B^{-1}(\tilde{A} - A)]X_t$ to be independent of X_t , which translates to condition (15).

As it turns out, this example sneakily permits looser identification conditions than the more general case covered in Section 3. In particular, we have assumed a very strong structure where *monetary shocks are iid*: they happen every period with the same time-invariant distribution. As our more general model in Section 3 shows, these type of

iid monetary shocks are a knife-edge case without a reasonable intuition. In reality, we expect monetary policy actions to depend on the state of the economy. Outside of this knife-edge case of random monetary interventions, shock identification will require the even stronger condition that H_{t+1}/H_t be *invariant to policy shocks*.

Remark 1 (Risk-neutral shocks). *One limitation of our simple three-factor VAR is that, taking the setup literally, one could get away with using risk-neutral surprises rather than investor’s perceived surprises. In particular, suppose we obtain $z_{\tau}^{r,t,*} := \mathbb{E}^*[r_{\tau+t} \mid X_{\tau}] - \mathbb{E}^*[r_{\tau+t} \mid X_{\tau-1}]$ from bond markets and run a modified version of Procedure 2 with $z_{\tau}^{r,t,*}$ in place of $z_{\tau}^{r,t}$. If the true model has (g, r, f) as the factors, then the modified procedure works—i.e., we recover both the short-rate and forward-guidance effects—with two possible caveats.*

First, while seemingly mundane, it is absolutely critical to control for $X_{\tau-1}$ when running the variant of Procedure 2 with risk-neutral surprises. Controlling for $X_{\tau-1}$ soaks up the wedge between risk-neutral, subjective, and objective beliefs, so one can recover the correct structural effects. The literature often overlooks controls for drivers of belief distortions and risk adjustments; for example, [Gürkaynak et al. \(2005b\)](#) and [Swanson \(2021\)](#), by decomposing the effects of monetary policy on various asset markets, effectively employ risk-neutral surprises, but they do not control for $X_{\tau-1}$. Second, the term structure of investor forecast revisions $(z_{\tau}^{r,t})_{t>0}$ are closer to structural objects of interest than their risk-neutral counterparts $(z_{\tau}^{r,t,})_{t>0}$. If so, measuring $(z_{\tau}^{r,t})_{t>0}$ is preferable. Within this second caveat, we see two specific issues: risk premia effects and multi-dimensional policy.*

Suppose forward guidance has risk premium effects in addition to its traditional impact on future short rates ([Bauer et al., 2023](#)). For instance, signals of higher future rates may lower growth but also reduce risks (like inflation risks in a richer model), with an ambiguous combined impact on long-term bond yields. The risk-neutral surprises $z_{\tau}^{r,t,}$ always contain this joint effect; a regression of economic outcomes onto $z_{\tau}^{r,t,*}$ cannot isolate the pure impact of forward guidance. Depending on the size of risk premia effects, $z_{\tau}^{r,t,*}$ may even take an opposite sign to $z_{\tau}^{r,t}$.³*

In a world of multi-dimensional policy, risk-neutral surprises struggle to correctly attribute the relative impacts of various interventions. To see this, consider that in a general K-factor VAR,

³To take an extreme but enlightening example where risk premia complicate inference, suppose A^* is a diagonal matrix. Then, the risk premium effects in BII offset the off-diagonal elements of A . Bond yields $y_t^{(n)}$ will not reflect forward guidance in this world: the loading $\mathcal{B}^{(n)}$ will have zeros everywhere except for the short-rate entry. In fact, one can verify that $\text{corr}[z_{\tau}^{r,t,*}, z_{\tau}^{r,0,*} \mid X_{\tau-1}] = 1$ when A^* is diagonal, so that $z_{\tau}^{r,t,*}$ provides no additional information relative to the short-rate shock. This offsetting issue is related to the discussion surrounding “unspanned factors” in the term structure literature (e.g., yields may not contain all information about the SDF as in [Cochrane and Piazzesi, 2005](#); [Andersen and Benzoni, 2010](#); [Duffee, 2011](#); [Joslin et al., 2014](#)).

the risk-neutral and investor forecast revisions differ by

$$z_{\tau}^{r,t,*} - z_{\tau}^{r,t} = (0, 1, 0, \dots, 0) \left\{ [(A^*)^t - \tilde{A}^t] B \Delta \tilde{W}_{\tau} + (A^*)^t B [\pi_0 + [\Pi + B^{-1}(\tilde{A} - A)] X_{\tau-1}] \right\}.$$

The second term involving $X_{\tau-1}$ can be managed, assuming one controls for the lagged state as in Procedure 2. But the first term involves the perceived shocks $\Delta \tilde{W}_{\tau}$ and cannot be controlled. To the extent that A^* differs from \tilde{A} , the risk-neutral and investor surprises can load very differently on the underlying shocks. In this world, the risk-neutral surprises cannot help answer questions like: at which horizon is forward guidance most effective? how does uncertainty management compare to traditional forward guidance? how does quantitative easing compare to forward guidance?

3 Identifying Monetary Shocks in a Quasi-Linear World

We now consider the question of whether and how to recover investor surprises about future interest rates. Our setting, based on Hansen and Scheinkman (2009) and Borovička et al. (2016), is Markovian and has complete financial markets. We will work in continuous time, for several reasons. First, continuous time allows us to more naturally delineate between “typical shocks” that occur all the time and “monetary shocks” that occur only at specific dates. Second, we can obtain our results even allowing for some types of nonlinearities in continuous time, which is desirable if we would like to think not only about expected future interest rates but also rate uncertainty.

3.1 General setup

Beliefs. Let the probability measure \mathbb{P} represent investor beliefs. Rational expectations is not assumed: \mathbb{P} may or may not coincide with the true objective probability. We work exclusively in the realm of investors’ subjective beliefs, because our goal when thinking about shock identification is to identify changes in interest rates relative to the market beliefs. Since we will never be referencing the objective probability measure, we will always use (\mathbb{P}, \mathbb{E}) for investor beliefs rather than the Section 2 notation $(\tilde{\mathbb{P}}, \tilde{\mathbb{E}})$.

The econometrician does not know the investor beliefs \mathbb{P} . To state the problem of the econometrician, he wants to learn monetary shocks—which will be policy surprises relative to \mathbb{P} —using only data on asset prices.

States, shocks, and information. There is a stationary n -dimensional economic state X . The evolution of X is perturbed by two types of shocks. First, there are *non-monetary shocks* that occur continuously. Non-monetary shocks are modeled by the increments to

W , which is an n -dimensional Brownian motion under \mathbb{P} . We could have included more of these shocks than state variables, but supposing they are the same number, as in most empirical applications, will streamline our arguments.⁴

Second, there are *monetary shocks* that occur only at specific times. To preserve a stationary and Markovian structure of our economy, we assume these times arrive according to a Poisson process with rate $\lambda(X_{t-})$, which can depend on the state. Whereas monetary announcement dates are deterministic and known in advance, one can think of randomness in these dates as capturing announcements during which some surprises actually occur. Furthermore, during some times of crisis, emergency actions and statements by the central bank can take place. We let M_t be the counter for announcements, so $dM_\tau = 1$ if and only if τ is an announcement date.

At these announcement dates, monetary shocks are modeled by the n -dimensional vector ξ_t . This random variable is independent of W and dictates the jump in the state variable: $X_t - X_{t-} = \xi_t dM_t$. Investors' perceived probability distribution of ξ_t is allowed to depend on the state X_{t-} just prior. For simplicity, assume the mean of ξ_t is equal to zero, so that "expected jumps" are implicitly reflected in the drift of X_t .

Subject to these two types of shocks, the state vector evolves as the jump-diffusion

$$dX_t = \mu(X_{t-})dt + \sigma(X_{t-})dW_t + \xi_t dM_t. \quad (16)$$

The sequence of information sets $(\mathcal{F}_t)_{t \geq 0}$ available to investors is generated by histories of W , M , ξ , and X_0 . In other words, investors observe $(X_t)_{t \geq 0}$. We assume the same information set for the econometrician. (Thus, we sidestep the issue encountered in Section 2 whereby one needs to recover observations of X_t from asset prices.)

Asset prices and the SDF. We assume there exists a stochastic discount factor (SDF) process S whose increment is given by

$$\frac{dS_t}{S_{t-}} = -r(X_{t-})dt - \pi(X_{t-}) \cdot dW_t + \exp[\kappa(X_t, X_{t-})] - 1 - \chi_S(X_{t-})dt, \quad (17)$$

where $S_0 = 1$ and $\chi_S(x)dt$ is the jump compensator.⁵ The variable $r(x)$ denotes the

⁴Borovička et al. (2016) allow for $k > n$ shocks by adding more observables to the states in X .

⁵More formally, let ν denote the random counting measure such that $\nu(\mathcal{B}, [0, t])$ gives the random number of jumps in time interval $[0, t]$ having size in the Borel set \mathcal{B} . We restrict attention to processes with a finite number of jumps in any finite time interval. Then, the compensator is the random measure $\chi(dx' | x)dt$ such that for any predictable function $g(x, t)$, the process $\int_0^t \int_{\mathbb{R}^n} g(x', s) \nu(dx', ds) - \int_0^t \int_{\mathbb{R}^n} g(x', s) \chi(dx' | X_{s-}) ds$ is a martingale. With this notation, we define $\chi_S(x) := \int (\exp[\kappa(x', x)] - 1) \chi(dx' | x)$.

short-term interest rate, while $\pi(x)$ and $\kappa(x', x)$ denote risk prices associated to the non-monetary and monetary shocks (note that $\kappa(x, x) = 0$). Using the SDF, the date- t price of any payoff $f(X_T)$ is

$$\mathbb{E} \left[\frac{S_T}{S_t} f(X_T) \mid X_t \right]. \quad (18)$$

In this environment, [Hansen and Scheinkman \(2009\)](#) show how Perron-Frobenius Theory can be leveraged to obtain a factorization of the SDF as

$$\frac{S_{t+T}}{S_t} = \exp(\eta T) \frac{e(X_t)}{e(X_{t+T})} \frac{H_{t+T}}{H_t}. \quad (19)$$

In (19), $\exp(\eta)$ is a positive eigenvalue of the instantaneous pricing operator; $e(\cdot)$ is the associated positive eigenfunction; and H_t is a martingale under \mathbb{P} . We assume existence of an SDF factorization (19). For the purposes of this section, we also assume the factorization is unique.⁶

Equation (19) decomposes the SDF into a deterministic component, a stationary component, and a permanent component. Some sources of H arising in structural representative-agent models are the permanent component of aggregate consumption or continuation value fluctuations in models with Epstein-Zin preferences and persistent growth or stochastic volatility. We flesh out some examples in Section 5. Using non-parametric methods, [Alvarez and Jermann \(2005\)](#) and [Bakshi and Chabi-Yo \(2012\)](#) argue that H must play a significant role in pricing.

Reduced-form monetary shocks. Monetary actions perturb this environment at an announcement date τ through the shocks $\zeta_\tau = X_\tau - X_{\tau-}$. The new state vector then feeds into current and future interest rates. But for our purposes, we will define monetary shocks directly in terms of their effect on interest rates. First, monetary policy can influence the short-term interest rate, which is given by $r(X_\tau)$. Second, policy can influence the sequence of future interest rates, namely $r(X_{\tau+T})$. Obviously, these future interest rates are random variables: altering future interest rates involves not only modifying the expected future rate path, but potentially also the entire probability distribution of future interest rates. Our reduced-form monetary shocks are defined as follows.

⁶See [Hansen and Scheinkman \(2009\)](#) for sufficient conditions on the existence of such a factorization. In many cases, uniqueness will not hold. If there are multiple SDF factorizations satisfying (19), we follow Proposition 1 of [Borovička et al. \(2016\)](#) in picking the unique one such that X is stationary and ergodic under the probability measure \mathbb{P}^H induced by the martingale H (i.e., defined by $\mathbb{P}^H(A) := \mathbb{E}(1_A H_T)$ for all sets $A \in \mathcal{F}_T$, for any $T \geq 0$). For the purposes of this section, non-uniqueness will not be relevant to the monetary policy questions, which is why we sidestep these issues.

Definition 1. Suppose the central bank intervenes at time τ . The short rate shock is given by

$$z_\tau^0 := r(X_\tau) - \mathbb{E}[r(X_\tau) \mid X_{\tau-}]. \quad (20)$$

The shocks to the expected future short rates are given by

$$z_\tau^T := \mathbb{E}[r(X_{\tau+T}) \mid X_\tau] - \mathbb{E}[r(X_{\tau+T}) \mid X_{\tau-}], \quad T > 0. \quad (21)$$

The shocks to the distribution of future short rates are given by

$$p_\tau^T(r) := \mathbb{P}\{r(X_{\tau+T}) \leq r \mid X_\tau\} - \mathbb{P}\{r(X_{\tau+T}) \leq r \mid X_{\tau-}\}, \quad T > 0. \quad (22)$$

Why do we care about these reduced-form monetary shocks? Motivated by Section 2 (see Procedure 2), regressing future outcomes on both z_τ^0 and z_τ^T , along with controls for the lagged state $X_{\tau-}$ can potentially identify the causal impact of short rates and forward guidance, jointly. If there are more dimensions to forward guidance (e.g., guidance at different horizons), one should include multiple horizons of the expected future short rate shocks $(z_\tau^T)_{T>0}$. If there is guidance about the distribution of future interest rates, one should consider including additional moments of the distributional shocks $(p_\tau^T)_{T>0}$. To run these procedures, we need to identify these reduced-form shocks.

Can an econometrician identify the impacts of the central bank on current and future interest rates? As mentioned in the introduction and Section 2, let us take as given the ability to non-parametrically identify the short rate shock. In particular, we observe the value of $r(X_\tau)$ at time τ , and suppose we also observe $\mathbb{E}[r(X_\tau) \mid X_{\tau-}]$, presumably from a financial market (e.g., Fed Funds futures). Implicitly, this assumes risk prices are sufficiently small for short-horizon interest rates, such that the risk-neutral expectation $\mathbb{E}^*[r(X_\tau) \mid X_{\tau-}]$ coincides with the investor expectation. So let us assume z_τ^0 is observable.

Turning to z_τ^T and p_τ^T , we cannot use the same identification logic as with z_τ^0 . The financial market still allows us to observe the risk-neutral expectations $\mathbb{E}^*[r(X_{\tau+T}) \mid X_\tau]$ and $\mathbb{E}^*[r(X_{\tau+T}) \mid X_{\tau-}]$, but the presence of risk premia embedded in longer-term interest rate futures implies $\mathbb{E}^* \neq \mathbb{E}$ when applied to future interest rates. Similarly, because $\mathbb{P}^* \neq \mathbb{P}$, we cannot expect the financial market to reveal the shock to the entire distribution of future short rates in (22).

What do financial markets reveal? Nevertheless, it turns out that z_τ^T and p_τ^T may sometimes be identified from financial market data. The basic idea, building on [Borovička et](#)

al. (2016), is that the martingale H in the factorization (19) may be used as a change-of-measure from investor beliefs \mathbb{P} to the long-run risk-neutral measure $\hat{\mathbb{P}}$, defined by

$$\hat{\mathbb{P}}(F) := \mathbb{E}[1_F H_t], \quad \forall F \in \mathcal{F}_t. \quad (23)$$

It turns out that asset prices reveal this probability measure, as the next lemma verifies.

Lemma 1. *The econometrician observes $\hat{\mathbb{P}}\{r(X_{\tau+T}) \leq r \mid X_\tau\}$ for every r and every τ, T .*

Except under the very particular degenerate situation $H \equiv 1$, investor beliefs \mathbb{P} will not coincide with the recovered $\hat{\mathbb{P}}$, as explained by Borovička et al. (2016). But our goal is less ambitious. We do not seek \mathbb{P} directly but rather investor surprises or *belief shocks*. As long as the gap, in some sense, between \mathbb{P} and $\hat{\mathbb{P}}$ remains constant before and after monetary policy announcements, we may hope that

$$\hat{\mathbb{E}}[r(X_{\tau+T}) \mid X_\tau] - \hat{\mathbb{E}}[r(X_{\tau+T}) \mid X_{\tau-}] = \mathbb{E}[r(X_{\tau+T}) \mid X_\tau] - \mathbb{E}[r(X_{\tau+T}) \mid X_{\tau-}], \quad (24)$$

and similarly for other moments of $r(X_{\tau+T})$. If equality (24) were to hold, then we would be done: Lemma 1 proves that the left-hand-side is observable, so we will have inferred the belief shocks on the right-hand-side.

The key question is which conditions permit this procedure. As we will see, one critical condition is that policy cannot affect the permanent component of the SDF.

Definition 2. *We say that monetary policy possesses Long-Run Neutrality if*

- (i) *The evolution of $d \log(H_t)$ is independent of the monetary shock dM_t ;*
- (ii) *The evolution of $d \log(H_t)$ is independent of the economic state X_t .*

Definition 2 specifies neutrality in terms of asset markets, via the martingale H . Condition (i) rules out direct effects of policy on the long-run SDF. Condition (ii) rules out indirect policy effects, which can be understood as follows. We are imagining a world in which monetary policy can have real effects and therefore generically affects the state vector X . In that case, if condition (ii) failed, then policy would indirectly affect H_{t+T} by moving X_t . For H to satisfy Definition 2, it must take the form

$$H_t = \exp \left[-\frac{1}{2} \|\beta\|^2 t - \beta \cdot W_t \right] \quad (25)$$

for some n -dimensional vector β . One can interpret β as the constant long-run risk price associated to non-monetary shocks. (Of course, by condition (i), there is a zero long-run risk price for monetary shocks.)

In the next two subsections, we illustrate how Long-Run Neutrality sometimes allows us to identify the shocks in Definition 1. After showing these positive identification results, we will explain how identification fails in some example environments without Long-Run Neutrality.

3.2 Exact identification: linear case

To start, we will make several assumptions such that the entire economy is linear. First, we assume the state dynamics are given by

$$\mu(x) = A_0 + Ax \quad (26)$$

$$\sigma(x) = B, \quad (27)$$

for some $n \times 1$ vector A_0 , and $n \times n$ matrices A and B . Second, we assume that the short-term interest rate r is a linear function:

$$r(x) = \rho_0 + \rho \cdot x, \quad (28)$$

for some constant ρ_0 and some vector ρ . Assuming (28) holds in a linear environment with (26)-(27) is tantamount to an assumption on the evolution of S (for example, the exponential-affine model of Section 2.3 had affine bond yields). Alternatively, one could just think that the short rate r_t is one of the state variables in X_t .

With linear-Gaussian dynamics and a linear interest rate function, we no longer have to think about the uncertainties in future rates (captured by p_τ^T in Definition 1). The variance of the future state vector X_{t+T} , conditional on X_t , is a deterministic function of T . The same holds for all higher moments of X_{t+T} . Still, one wonders whether the forward-guidance shock z_τ^T is identified.

Proposition 1. *Suppose Long-Run Neutrality holds. Consider the linear environment defined by (26)-(28). Then, the forward-guidance shocks $(z_\tau^T)_{T \geq 0}$ are identified from asset price data alone.*

One may be skeptical that belief shocks can be identified at all. Going back to the fundamental identification issues raised by Harrison and Kreps (1979), asset prices do not directly reveal beliefs. More recently, Borovička et al. (2016) argued that beliefs are only revealed if $H_t \equiv 1$ is a degenerate martingale. In our context, how is identification possible under seemingly weaker assumptions? The key simplification is that our environment is linear, whereas these previous papers have tried to argue non-parametrically.

Imposing this stronger assumption on the economic dynamics allows us to weaken the conditions on H for shock identification.

However, even a linear environment is not enough. An additional simplification is that Proposition 1 does not seek beliefs directly, but rather surprises or *belief shocks*. It is easier to recover belief shocks, because they difference out any unobservable level effect in beliefs. Indeed, that is exactly what happens in the proof of Proposition 1.

Let us briefly elaborate on the method to identify z_τ^T . First, by solving an eigenvalue problem, one can use asset prices to recover the long-run risk-neutral probability measure $\hat{\mathbb{P}}$, as demonstrated by Lemma 1. (See also Ross (2015), Borovička et al. (2016), and Qin and Linetsky (2017) for this result more generally). The dynamics of X_t under $\hat{\mathbb{P}}$ are the same as those under \mathbb{P} , less the constant drift $B\beta$:

$$\hat{\mu}(x) = \mu(x) - B\beta = A_0 - B\beta + Ax$$

While β and A_0 are not separately identified, the long-term measure $\hat{\mathbb{P}}$ correctly identifies investors' perceived persistence A . This turns out to be the critical necessary object to compute investors' forecast revisions. By contrast, constant drift distortions like $B\beta$ play no role in these forecast revisions, because investor forecasts just before and just after the monetary announcement are both distorted by the same constant. In other words, the computable object $\hat{\mathbb{E}}[X_{\tau+T} | X_\tau] - \hat{\mathbb{E}}[X_{\tau+T} | X_{\tau-}]$ coincides with the desired investor forecast revision $\mathbb{E}[X_{\tau+T} | X_\tau] - \mathbb{E}[X_{\tau+T} | X_{\tau-}]$.

Ultimately, Proposition 1 is just a generalization of what we observed in our example in Section 2.3. But it is convenient that we can phrase the result in terms of the Long-Run Neutrality condition, which will be the center-piece of our emphasis going forward.

3.3 Approximate identification with stochastic volatility

We continue to assume a linear drift (26) and a linear short rate function (28), but we dispense with homoskedasticity (27). In such a world, the perceived probability distribution of $r(X_{\tau+T})$ becomes non-trivial (i.e., it is not fully characterized by its mean and the horizon T). And so we would ideally like to estimate the uncertainty shocks p_τ^T in addition to the forward-guidance shocks z_τ^T .

To proceed in this more general environment, we need an extra assumption. Roughly speaking, we need to assume that the sources of heteroskedasticity are not priced by the long-run risk-neutral measure. Supposing Long-Run Neutrality holds, so that equation (25) characterizes the permanent component of the SDF, we assume there exists some

n -dimensional vector $\hat{\beta}$ such that

$$\sigma(x)\beta = \hat{\beta} \quad \text{for all } x. \quad (29)$$

In other words, there is a zero in each element of β corresponding to a shock with non-constant volatility. Replacing homoskedasticity assumption (27) with the more general (29), we are still able to identify the forward-guidance shocks but not the uncertainty shocks. Formally, we have the following generalization of Proposition 1.

Proposition 2. *Suppose Long-Run Neutrality holds. Consider the quasi-linear environment defined by (26), (28), and (29). Then, the forward-guidance shocks $(z_\tau^T)_{T \geq 0}$ are identified from asset price data alone.*

The key intuition for Proposition 2 is the same as Proposition 1. Indeed, (29) implies that the drift of X_t under the long-run measure $\hat{\mathbb{P}}$ is

$$\hat{\mu}(x) = \mu(x) - \sigma(x)\beta = A_0 - \hat{\beta} + Ax.$$

As in Proposition 1, investors' perceived persistence A can be inferred from financial data, which is the critical necessary object to compute investors' forecast revisions.

Unfortunately, in the environment considered by Proposition 2, the uncertainty shocks p_τ^T are non-trivial and non-identified. In some applications, we may have a priori reasons to care less about p_τ^T . But in situations where uncertainty matters, we will want to recover p_τ^T .

To make partial progress, we make the following linearity assumption about the form of the state diffusion:

$$\sigma(x)\sigma(x)' = \varsigma_0\varsigma_0' + \sum_{i=1}^n \varsigma_i \text{diag}(x_i) \varsigma_i', \quad (30)$$

where $\text{diag}(x_i)$ is the diagonal matrix with x_i on the main diagonal. The affine approximation in (30) is consistent with standard stochastic volatility models having “square-root dynamics.” With this structure, we can at least identify shocks to *investors' perceived variance* of future interest rates, even if we cannot recover the entire probability distribution of $r(X_{\tau+T})$. (Indeed, one can verify that the same method of proof used in Proposition 3 does not work for third and higher moments.)

Proposition 3. *Suppose Long-Run Neutrality holds. Consider the quasi-linear environment*

defined by (26), (28), (29), and (30). Define the variance surprises

$$v_\tau^T := \text{Var}[r(X_{\tau+T}) \mid X_\tau] - \text{Var}[r(X_{\tau+T}) \mid X_{\tau-}].$$

Then, $(v_\tau^T)_{T \in [0, \tau' - \tau]}$ are identified from asset price data alone, where τ' is the subsequent monetary announcement date after τ .

Together, Propositions 2-3 demonstrate that a forward-guidance shocks and some aspects of uncertainty shocks, at least those pertaining to variances, can be obtained from asset-market data. We require assumptions both on the dynamic evolutions and on the underlying economic model. The key assumption on the dynamics is quasi-linearity, with variance dynamics taking a “square-root” form. The critical economic assumption in all cases is Long-Run Neutrality, along with assumption (29) that volatility shocks feature zero long-run risk prices.

3.4 Non-identification without Long-Run Neutrality

We now provide some examples to illustrate why shock recovery requires Long-Run Neutrality. To provide the best possible chance at achieving identification, let us specialize to the linear setup defined by (26)-(28) in Section 3.2. First, we consider a world where monetary policy affects H directly (violating condition (i) of Definition 2). Second, we consider a world where monetary policy indirectly affects H through its impact on the state vector X (violating condition (ii) of Definition 2). In either environment, monetary policy shocks are generally not identified from asset prices alone.

Direct monetary effects. Consider what happens if dH_t is directly impacted by the monetary shock dM_t . For simplicity, suppose the non-monetary shocks W_t do not impact H_t at all. We will furthermore assume that the jumps in H are log-linear in the jumps in X . The evolution of H_t then takes the form

$$\frac{dH_t}{H_{t-}} = \exp[\zeta \cdot (X_t - X_{t-})] - 1 - \chi_H(X_{t-})dt, \quad (31)$$

for some vector ζ that encodes the long-run risk price of monetary shocks, and where $\chi_H(x)dt$ is the jump compensator that makes H a martingale.

The crux of the identification issue is that monetary interventions that shift X_t are, except in a knife-edge case, dependent on the economic state. In our Markov environment, H_t inherits the shocks to X_t , so state-dependence in monetary shocks translates into state-dependence in H -shocks, which obfuscates the recovery of belief shocks.

To see the problem, use H to again define the long-run probability measure $\hat{\mathbb{P}}$. The relation between the drift of X_t under measures $\hat{\mathbb{P}}$ and \mathbb{P} is⁷

$$\hat{\mu}(x) = \mu(x) + \int (x' - x) \left(\exp[\zeta \cdot (x' - x)] - 1 \right) \chi(dx' | x), \quad (32)$$

where χ is the compensator of the jumps in X_t . The object that is observed from financial data is $\hat{\mu}(x)$. But we would like to recover the persistence matrix A from $\mu(x) = A_0 + Ax$. Such recovery is only possible if the distortion $\int (x' - x) \left(\exp[\zeta \cdot (x' - x)] - 1 \right) \chi(dx' | x)$ is a constant independent of x . This constant case arises if and only if both the arrival rate $\lambda(x)$ and size of monetary surprises ξ_t are state-independent. Such a knife-edge case essentially means *monetary policy acts randomly*.

Indirect monetary effects. Next, consider what happens if dH_t depends on X_t but not dM_t . In this case, rather than the log-normal form (25), H_t takes the form

$$H_t = \exp \left[-\frac{1}{2} \int_0^t \|\beta(X_s)\|^2 ds - \int_0^t \beta(X_s) \cdot dW_s \right] \quad (33)$$

for some non-constant function $\beta(\cdot) : \mathbb{R}^n \mapsto \mathbb{R}^n$. The drift of X_t under the long-run measure $\hat{\mathbb{P}}$ is given by

$$\hat{\mu}(x) = A_0 + Ax - \beta(x)B.$$

Although $\hat{\mu}(x)$ is observable, we cannot separately distinguish between Ax and $\beta(x)B$. When $\beta(\cdot)$ was a constant, we could identify A as the persistence of X_t under $\hat{\mathbb{P}}$. Here, the persistence of X_t differs under $\hat{\mathbb{P}}$ and \mathbb{P} , complicating matters.

Investors' perceived persistence A is the critical determinant of forecast revisions. Indeed, we have

$$\begin{aligned} \mathbb{E}^{X_\tau}[X_T] - \mathbb{E}^{X_{\tau-}}[X_T] &= X_\tau - X_{\tau-} + \mathbb{E}^{X_\tau} \left[\int_0^T \mu(X_{t-}) dt \right] - \mathbb{E}^{X_{\tau-}} \left[\int_0^T \mu(X_{t-}) dt \right] \\ &= X_\tau - X_{\tau-} + \int_0^T A \left(\mathbb{E}^{X_\tau}[X_t] - \mathbb{E}^{X_{\tau-}}[X_t] \right) dt. \end{aligned} \quad (34)$$

Equation (34) is a recursive equation for $\mathbb{E}^{X_\tau}[X_T] - \mathbb{E}^{X_{\tau-}}[X_T]$, but we can only solve it if we know the value of A . Since we cannot infer A , we cannot solve for these forecast revisions.

⁷Formally, under investor beliefs \mathbb{P} , the jumps $\xi_t dM_t$ have conditional mean $\int (x' - x) \chi(dx' | x) dt$. (Of course, we have assumed from the beginning that this conditional mean equals zero, but the argument applies more generally even when this is not the case.) By contrast, under the probability distribution $\hat{\mathbb{P}}$ induced by H , the jumps $\xi_t dM_t$ have conditional mean $\int (x' - x) \exp[\zeta \cdot (x' - x)] \chi(dx' | x) dt$ (c.f., [Kunita and Watanabe, 1967](#), Theorem 6.2). Combining these points leads to formula (32) in the text.

The above suggestive analysis of violating Long-Run Neutrality by either (31) or (33) can be formalized. We have

Proposition 4. *Consider the linear environment defined by (26)-(28). Suppose either (i) dH_t features a contribution from dM_t ; or (ii) the dynamics dH_t depend on X_t . Then, generically, z_τ^T cannot be identified from asset price data.*

Whereas Propositions 1-2 demonstrated the sufficiency of Long-Run Neutrality for identifying forward-guidance shocks in quasi-linear environments, Proposition 4 demonstrates the corresponding necessity result.

4 Testing Long-Run Neutrality

In this section, we construct a simple non-parametric test of Long-Run Neutrality. This test builds on insights by Alvarez and Jermann (2005) and Bakshi and Chabi-Yo (2012) in proxying the permanent and transitory components of the SDF. We then evaluate the non-parametric test using some numbers from existing studies, followed by our own novel evidence.

4.1 A non-parametric test

Roughly speaking, Long-Run Neutrality means that long-run risk premia are invariant to monetary policy. To formalize this, consider two portfolios: (i) an infinite-maturity bond with return $R_{t,t+\Delta}^\infty$; and (ii) the growth-optimal portfolio with return $R_{t,t+\Delta}^*$. The holding period return on the infinite-maturity bond is given by

$$\begin{aligned} R_{t,t+\Delta}^\infty &:= \lim_{T \rightarrow \infty} R_{t,t+\Delta}^T = \lim_{T \rightarrow \infty} \frac{\mathbb{E}[\frac{S_T}{S_{t+\Delta}} \mid X_{t+\Delta}]}{\mathbb{E}[\frac{S_T}{S_t} \mid X_t]} = \exp(-\eta) \frac{e(X_{t+\Delta})}{e(X_t)} \lim_{T \rightarrow \infty} \frac{\mathbb{E}[\frac{1}{e(X_T)} \frac{H_T}{H_{t+\Delta}} \mid X_{t+\Delta}]}{\mathbb{E}[\frac{1}{e(X_T)} \frac{H_T}{H_t} \mid X_t]} \\ &= \exp(-\eta) \frac{e(X_{t+\Delta})}{e(X_t)}, \end{aligned} \quad (35)$$

where the last equality holds if X_t is stochastically stable under the probability measure generated by H , which we implicitly assume (see footnote 6). The link between the long bond and the stationary component of the pricing kernel was originally discovered by Kazemi (1992). On the other hand, the growth-optimal portfolio return $R_{t,t+\Delta}^*$ is defined as investors' expectation of the maximal log return: it is the time- $(t + \Delta)$ measurable return R that maximizes $\mathbb{E}[\log(R) \mid X_t]$ subject to $\mathbb{E}[\frac{S_{t+\Delta}}{S_t} R \mid X_t] = 1$, the solution of which is $R_{t,t+\Delta}^* = \frac{S_t}{S_{t+\Delta}}$ (Bansal and Lehmann, 1997). Putting these results together,

and using the SDF factorization (19), the excess return of the growth-optimal portfolio relative to the infinite-horizon bond is

$$\log(R_{t,t+\Delta}^*) - \log(R_{t,t+\Delta}^\infty) = \log\left(\frac{H_t}{H_{t+\Delta}}\right) \quad (36)$$

over any horizon Δ . The long-short portfolio in (36) was originally studied by [Alvarez and Jermann \(2005\)](#). The result in (36) holds in even more general environments than the one considered here—for instance, in non-Markovian environments ([Qin and Linetsky, 2017](#)). Under condition (i) of Definition 2, the excess return $\log(R_{t,t+\Delta}^*) - \log(R_{t,t+\Delta}^\infty)$ should be identically zero on monetary announcement days. Under condition (ii) of Definition 2, the conditional risk premium $\mathbb{E}[\log(R_{t,t+\Delta}^*) - \log(R_{t,t+\Delta}^\infty) \mid X_t]$ should be time-invariant.

Equation (36) suggests a test: one can examine high-frequency changes in $R_{t,t+\Delta}^*$ and $R_{t,t+\Delta}^\infty$ around monetary announcements to detect the policy impact on H . As long as investor beliefs are not singular with respect to the objective probability, invariance of H to policy under investor beliefs is equivalent to invariance under the objective measure, justifying this test. (By contrast, it is harder to test the time-invariance of $\mathbb{E}[\log(R_{t,t+\Delta}^*) - \log(R_{t,t+\Delta}^\infty) \mid X_t]$, because it could be so under investor beliefs but not under the objective measure.) For example, if we suppose $R_{t,t+\Delta}^\infty$ is well-approximated by returns on 30-year Treasuries, and $R_{t,t+\Delta}^*$ is well-approximated by stock market returns, then Long-Run Neutrality says that monetary policy impacts the stock market and 30-year Treasuries in an identical way.

4.2 Existing announcement effect evidence

Following [Alvarez and Jermann \(2005\)](#), suppose $R_{t,t+\Delta}^\infty$ is well-approximated by returns on long-term US Treasuries, and $R_{t,t+\Delta}^*$ is well-approximated by US stock market returns. Then, by piecing together existing evidence from various sources, we can shed light on the portfolio in (36). Together, this collection of existing evidence suggests that H responds to monetary policy.

First, stock returns are significantly higher than long-term bond returns around FOMC meeting days. In particular, [Lucca and Moench \(2015\)](#) show (for 1994–2011) that the SPX return was on average 33 bps higher in the 24 hours before the FOMC announcements, relative to other days. By contrast, [Hillenbrand \(2021\)](#) shows (for 1989–2021) that 30-year Treasuries returns were approximately 13.8 bps to 18.6 bps higher per

day in a 3-day window around the FOMC announcements, relative to other days.⁸ This evidence suggests that the average returns on stocks and long-term bonds differ both in magnitudes and the timing around the FOMC announcements.

Although our focus is primarily on monetary announcements, it is worth reviewing evidence from a more comprehensive set of macroeconomic announcements. The broader announcement literature has argued that other macro announcements also induce asset price responses, suggesting that the mechanisms generating announcement premia around macroeconomic and policy announcements can be related. Using a much longer sample (1958-2009), [Savor and Wilson \(2013\)](#) report returns on stocks and 30-year Treasury bonds to be respectively 11.5 bps and 4.5 bps higher on announcement days. Additionally, [Savor and Wilson \(2014\)](#) show in a similar sample (1964-2011) that the CAPM beta of 30-year Treasuries is 0.14 on announcement days, whereas Long-Run Neutrality predicts it should be 1.

A parallel strand of research examines asset responses to monetary policy surprises rather than average announcement returns. This literature relies on surprises identified from high-frequency short-term interest rate changes around FOMC announcements. For example, [Gürkaynak, Sack and Swanson \(2005b\)](#) study high-frequency responses to monetary surprises (during 1990–2004), finding that a 25 bp surprise rate cut leads to 1% SPX return but only a 0.32% 10-year Treasury return. Using an updated 1988–2019 sample, [Bauer, Bernanke and Milstein \(2023\)](#) find even stronger effects in stocks, with a 10 bps surprise cut associated with a 1% SPX return, although they do not study long-term riskless bonds.

4.3 New evidence on Long-Run Neutrality

While suggestive, the evidence from the extant literature does not speak directly to the Long-Run Neutrality of the Fed-driven news. The samples vary substantially in length, choice of events windows, and are often inconsistent between equities and bonds, with little direct evidence on the behavior of long-short equity-bond portfolios. Importantly, most studies document average asset-specific announcement effects, ignoring higher moments. Below, we use a consistent sample for equities and long-duration bonds, compare results for various windows around FOMC announcements, and investigate other moments of equity-bond portfolios beyond the mean effect.

⁸We impute this range for the 30-year bond average return using evidence in [Hillenbrand \(2021\)](#) that 30-year Treasury yields decline between 0.46 bps to 0.62 bps more per day in the 3-day window surrounding FOMC meetings. We use the duration-approximation $\log(R_{t,t+\Delta}^T) \approx -T(y_{t+\Delta} - y_t)$, $T = 30$.

Following much of the literature, we consider scheduled FOMC decision announcements. The FOMC meets eight times per year on a pre-announced schedule.⁹ Since 1994, the FOMC has published statements of the policy decisions, and since 2011, the statements have been followed by press conferences by the Fed Chair, initially every other announcement, and starting in 2019, after each announcement. Until 2011, statements were released at 14:15 ET. The time changed in 2011, alternating between 12:30 and 14:15 ET, depending on whether the meeting was followed by a press conference. Currently, statements are published at 14:00 ET, and the press conference is held at 14:30 ET.

We obtain price data at a one-minute frequency on the E-mini S&P 500 futures and the Treasury bond (T-bond) futures from TickData.com. Our main focus is on the Treasury futures with a 30-year T-bond as the underlying, the longest maturity available. We refer to this contract as the 30-year T-bond futures, recognizing that the actual delivery can take place in bonds with maturities of 15 years and above.¹⁰ Our high-frequency sample starts in September 1997 when the E-mini S&P 500 futures contract was introduced and runs through December 2023, covering 210 scheduled FOMC announcements and 70 press conferences.

We consider event windows from 24 hours before, narrowly around, and up to 24 hours after the FOMC decision announcements and press conferences. Table 1 summarizes the distribution of log returns of E-mini futures in excess of 30-year T-bond futures in different windows. The results in Panel A show that equities have done particularly well in the 24 hours before the FOMC announcement, earning on average 26 bps higher returns than bonds during the 1997–2023 sample. This result is consistent with, albeit somewhat weaker than, the original [Lucca and Moench \(2015\)](#) finding based on the 1994–2011 sample. While on average equities performed similarly to bonds in the narrow window of ± 15 minutes around the announcements, they underperformed bonds by about 20 bps in the 24 hours after the announcement. Panel B summarizes returns around press conferences, showing a broadly similar pattern.

The narrow event windows are particularly informative given that the FOMC-driven news is plausibly the main source of variation in asset prices at those times. The equity-bond portfolio returns show a volatility of nearly 60 bps in the ± 15 minutes window of the decision announcement and 40 bps in the ± 15 around the start of the press conference, which cover the opening remarks by the Chair. For comparison, the volatility is

⁹In 2020, one scheduled meeting was canceled.

¹⁰In January 2009, the CME introduced a new 30-year Treasury bond futures contract, called “Ultra,” which requires delivery of a bond with at least 25-year maturity. At that time, the range of eligible maturities for the original or “classic” 30-year T-bond futures was adjusted from a 15–30-year range to a 15–25-year range. Our current analysis focuses on the classic 30-year contract.

Panel A. Monetary policy decision announcements						
	N	Mean	SE(mean)	SD	Skew	Kurtosis
(−24h, −15m)	210	26.0	8.2	118.2	1.8	18.6
(−15m, +15m)	210	−1.3	4.1	59.5	−1.5	11.3
(−15m, +24h)	210	−20.1	12.3	178.2	−0.8	6.0

Panel B. Press conferences						
	N	Mean	SE(mean)	SD	Skew	Kurtosis
(−24h, −15m)	70	19.8	9.1	76.1	0.0	3.7
(−15m, +15m)	70	9.4	4.7	39.1	1.2	5.5
(−15m, +60m)	70	2.4	8.2	68.4	0.2	4.3
(+60m, +24h)	70	−43.5	19.2	160.5	−1.0	5.7

Table 1. Summary statistics. The table reports summary statistics for log returns on S&P500 E-mini futures minus log returns on 30-year Treasury bond futures in various windows around scheduled monetary FOMC decision announcements and press conferences. A futures “return” in window $(t, t + \Delta)$ is defined as $F_{t+\Delta}/F_t$, where F denotes the futures price. The sample covers FOMC meetings from 1997:09 through 2023:12, with press conferences introduced in 2011.

27 bps in the ± 15 -minutes window around 14:00 ET on all other days in the 1997:09–2023:12 sample, i.e., less than half of that around the FOMC decision announcements. The insignificant average ± 15 -minutes announcement return in Table 1 thus masks a sizeable time variation in returns around announcements. To illustrate that variation, Figure 2 plots the cumulative equity-bond portfolio returns obtained by summing the ± 15 window decision announcement returns across announcements. The cumulative returns show a persistent downward trend in the first half of the sample, reaching -1004 bps at the November 2009 meeting, and from then onward a persistent upward trend through the end of our sample. Thus, equities underperformed long-term bonds on FOMC announcements from the late 1990s through the first rounds of the quantitative easing implemented after the global financial crisis but outperformed long-term bonds afterward through the most recent period.

While speculative, let us mention a theory that highlights time-varying Fed *credibility* as a possibility for the reversal shown in Figure 2. A long literature beginning with Barro and Gordon (1983) explores the role of a central bank’s reputation in modulating the efficacy of its policy. In a recent contribution, Bhattarai et al. (2023) argue that quantitative easing programs can increase central bank commitment power to keep interest rates low, which amplifies the power of forward guidance. This mechanism was particularly relevant after 2008, when the binding zero lower bound made the combination of

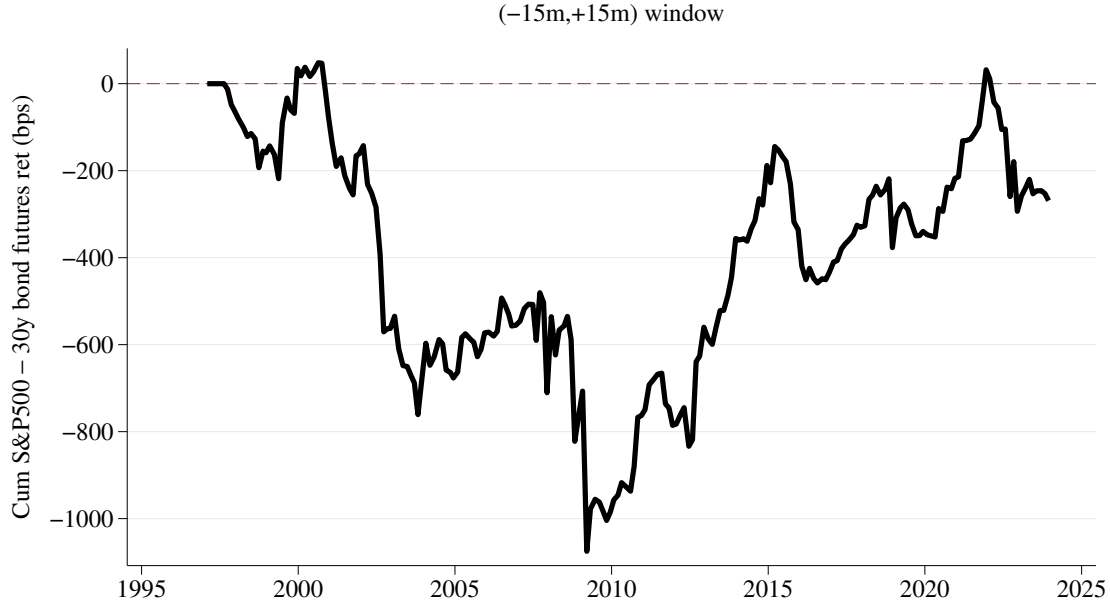


Figure 2. Cumulative returns in a narrow window around FOMC policy announcements. The figure plots cumulative log returns on a long-short portfolio of S&P 500 E-mini futures in excess of 30-year Treasury bond futures. The returns are calculated from -15 to $+15$ minutes around scheduled FOMC announcements. The sample period covers 210 meetings from 1997:09 through 2023:12.

quantitative easing and forward guidance the relevant policy tools.

If the Long-Run Neutrality condition holds, the FOMC-driven news should move equity and T-bond returns one for one, implying that a regression of T-bond returns on equity returns should have a slope coefficient (beta) and an R^2 both equal to one. Figure 3 shows that these predictions are rejected in the data. Bond-equity betas are statistically different from one across various windows around decision announcements and press conferences. While the ± 15 -minute betas are positive, they are significantly below one, reaching 0.32 for decision announcements and 0.22 for press conferences, with R^2 of 9.9% and 20.1%, respectively. Outside narrow windows, the evidence against Long-Run Neutrality strengthens further, with even lower R^2 and betas becoming negative.

To assess how often the Long-Run Neutrality condition could potentially hold in our sample, we compute high-frequency betas and R^2 using realized covariances and variances of one-minute equity and T-bond returns in the ± 15 window around decision announcements. The histograms in Figure 4 indicate that 90% of announcements feature a beta below 0.5 and an R^2 below 0.42, again suggesting that the Long-Run Neutrality most of the time remains violated.

One concern with the results so far is that the bond underlying the 30-year T-bond futures contract is a poor proxy for the R^∞ return, given that the effective duration of the underlying is about 15 years on average. Therefore, we perform additional analy-

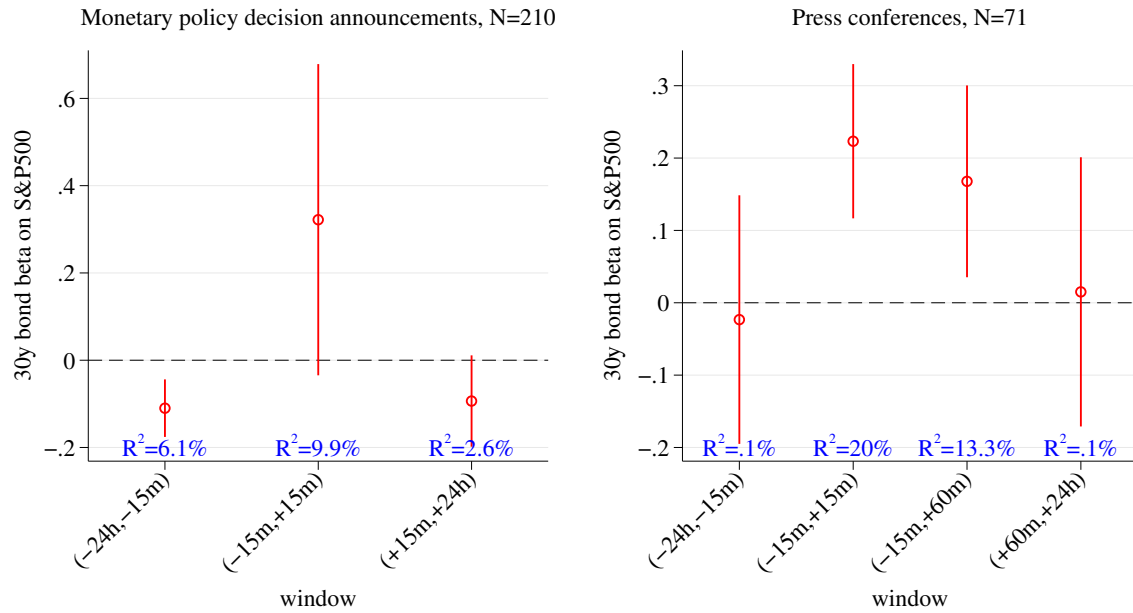


Figure 3. Bond betas. The figure presents betas of 30-year Treasury bond futures returns on S&P 500 E-mini futures returns in different windows around scheduled FOMC announcements and press conferences. Robust 95% confidence intervals are included.

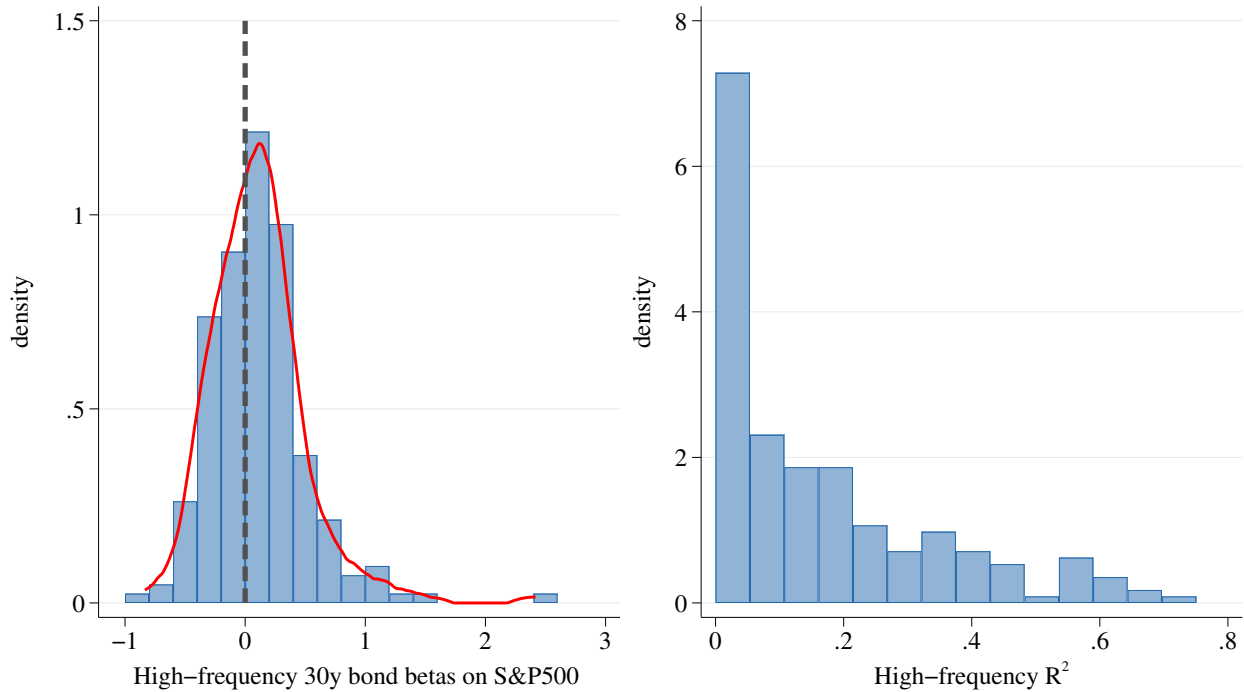


Figure 4. High-frequency bond betas. The figure presents the distribution of high-frequency realized betas of the 30-year Treasury bond futures returns on S&P 500 E-mini futures returns. For each ± 15 -minutes window around scheduled FOMC decision announcements, the beta and R^2 are calculated from realized covariances and variances of one-minute returns.

sis using daily zero-coupon yield curve data regularly updated by the Federal Reserve (Gurkaynak et al., 2007). The longest reported zero-yield maturity is 30 years, which is also the maximum maturity issued by the US Treasury. To proxy for the market portfolio, we use the daily CRSP market return from Ken French’s website. We construct the equity-bond portfolio as the log return on the CRSP market return in excess of the log return on the 30-year zero coupon bond.

Figure 5, left panel, displays the cumulative return on the equity-bond portfolio in the three-day window (days $-1, 0, +1$) around scheduled FOMC announcements. The sample starts in 1994, when the FOMC began releasing public statements. The choice of the window is motivated by the finding in Hillenbrand (2021) that Treasury bonds earn essentially all returns in the three days surrounding the announcement. The right panel in Figure 5 presents cumulative returns disaggregated by day $-1, 0, +1$ around the announcement. To assess the magnitudes, we juxtapose the cumulative FOMC window returns against the cumulative returns on all other days and scale each by the total number of days in the respective sample. There are 717 days falling in the three-day FOMC window and 7108 days falling outside. The last observation along each trajectory represents the sample average return and is reported on the graph.

The return trajectories indicate that, in economic terms, the equity-bond portfolio has earned larger (in absolute value) returns in the FOMC window than on all other days, consistent with the idea that FOMC-driven news is associated with deviations from the Long-Run Neutrality condition. At the same time, the disaggregated results in the right panel of Figure 5 reveal a complex interpretation of the directional effect of the FOMC news in three-day announcement windows, suggesting that the equity-bond portfolio undergoes regime-like shifts and can switch sign over time and across specific days. In particular, most of the deviations from neutrality appear in years 2008-2012, but oppositely on announcement day versus surrounding days. The evidence suggests that the permanent component of marginal utility falls substantially on announcement days but rises substantially the day before and after.

The balance of our evidence on the log excess return $\log(R_{t,t+\Delta}^*) - \log(R_{t,t+\Delta}^\infty)$ suggests that monetary Long-Run Neutrality is violated. This violation is visible in the mean return, in line with the existing literature, but the stronger violations appear in higher moments: the long-short portfolio displays significant time-variation, and its two legs co-move weakly in windows surrounding FOMC announcements. Beyond documenting these higher moments, our analysis also contributes by studying various time windows and return frequencies. We uncover several nuanced patterns deserving of further investigation, such as the reversal around 2009 of the long-short portfolio’s performance in

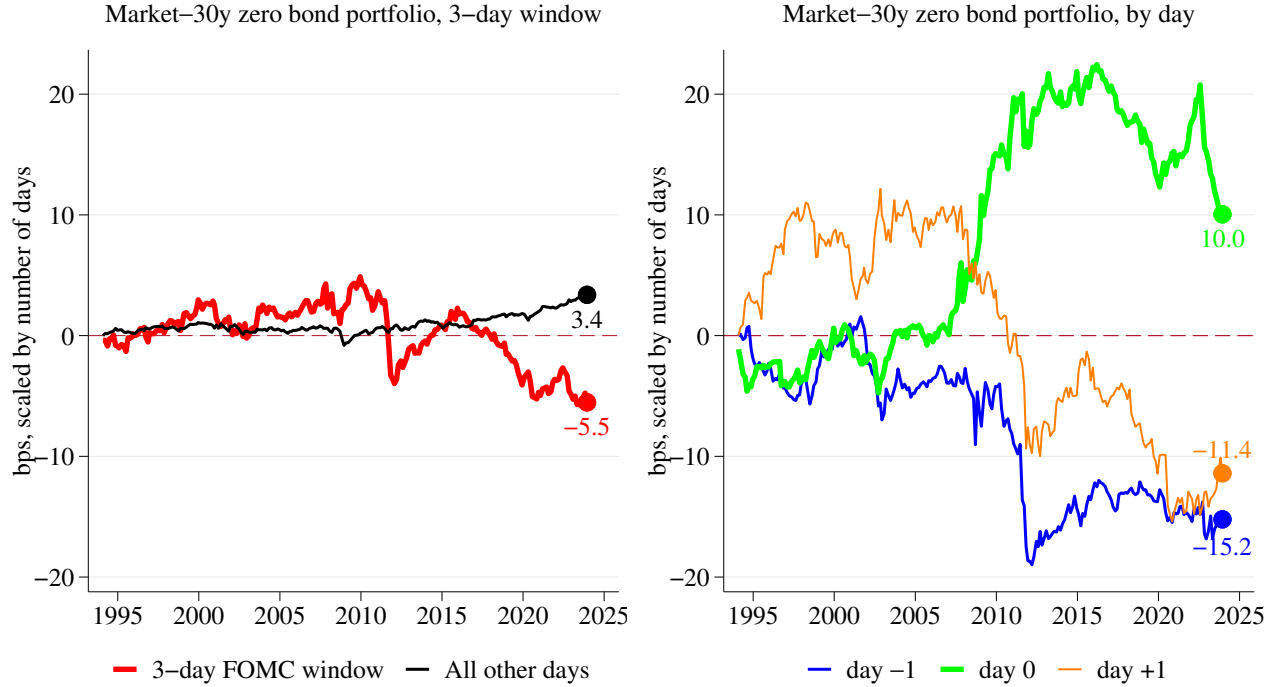


Figure 5. Market-bond portfolio returns, daily frequency. The figure presents cumulative daily close-to-close log returns on the market portfolio in excess of the 30-year zero-coupon Treasury bond. The three-day FOMC window comprises days -1 , 0 , and $+1$ around the scheduled FOMC announcements. Cumulative returns are expressed in basis points and scaled by the number of days in a given sample. The sample runs from 1994:01 through 2023:12. There are 717 days in the three-day FOMC window (239 on each of days $-1, 0, +1$) and 7108 on all other days outside the three-day FOMC window.

narrow windows around FOMC, and the opposite performance on announcement day versus the surrounding days.

5 Examples of H : Interpreting Long-Run Neutrality

We present some example economies in which the SDF S features a permanent component H . In each example, we discuss what is meant, economically, by the Long-Run Neutrality statement “monetary policy does not affect H .” Thus, we can evaluate the stringency of conditions that allow identification of monetary policy shocks. The examples in this section are based on [Bansal and Yaron \(2004\)](#), with related analysis in [Hansen and Scheinkman \(2009\)](#) and [Borovička et al. \(2016\)](#). Generalizing these economies to explicitly include monetary policy is an interesting avenue for future research.

5.1 Long-run risk model

Suppose aggregate consumption has the trend-stationary dynamics

$$\log C_{t+1} = \log C_t + \alpha \cdot (X_{t+1} - X_t),$$

where the state vector X_t follows a stationary VAR(1):

$$X_{t+1} = A_0 + AX_t + B\Delta W_{t+1}, \quad \Delta W_{t+1} \sim \text{Normal}(0, I).$$

If investors have subjective beliefs, the same form of these equations hold in reality and under investor beliefs, but with alternative values of A_0 .

Suppose the representative investor has recursive preferences as in [Kreps and Porteus \(1978\)](#) and [Epstein and Zin \(1989\)](#), and unitary elasticity of intertemporal substitution (EIS). The investor's continuation value satisfies the following recursion:

$$V_t = (1 - \beta) \log C_t + \beta \frac{\log \mathbb{E}_t[\exp((1 - \gamma)V_{t+1})]}{1 - \gamma},$$

where $\gamma > 1$ denotes the investor's coefficient of relative risk aversion and β is the subjective discount factor. Guess that the solution is $V_t = A_0 t + v_0 + v \cdot X_t$, for some constant v_0 and vector v . In that case, one can show that the solution is

$$v = (I - A')^{-1} \alpha \quad \text{and} \quad v_0 = (1 - \beta)^{-1} [\beta A_0 + \frac{1}{2} (1 - \gamma) v' B B' v].$$

In this model, the SDF is given by

$$\frac{S_{t+1}}{S_t} = \beta \frac{C_t}{C_{t+1}} \frac{\exp((1 - \gamma)v \cdot X_{t+1})}{\mathbb{E}[\exp((1 - \gamma)v \cdot X_{t+1}) \mid X_t]}$$

Given that consumption is a trend-stationary process, the permanent component of the SDF is clearly given by the third piece, i.e.,

$$\frac{H_{t+1}}{H_t} = \frac{\exp((1 - \gamma)v \cdot X_{t+1})}{\mathbb{E}[\exp((1 - \gamma)v \cdot X_{t+1}) \mid X_t]} = \exp \left[-\frac{1}{2} (1 - \gamma)^2 v' B B' v + (1 - \gamma) v' B \Delta W_{t+1} \right].$$

In this model, if monetary policy shocks do not affect H , then there are two possibilities. One trivial possibility is that $\gamma = 1$ corresponding to log utility, which rules out priced growth-rate shocks. In that case, Long-Run Neutrality corresponds to the conventional wisdom that monetary policy does not have a permanent effect on the consumption

level, which is hard-wired in this example with trend-stationary consumption.

Alternatively, supposed growth-rate shocks are priced. Then, letting $B^{(i)}$ denote the i th column of B , Long-Run Neutrality requires $v'B^{(i)} = 0$ for every shock $\Delta W_{t+1}^{(i)}$ that can be impacted by monetary policy. For example, if $\Delta W_{t+1}^{(1)}$ is the short-rate shock, and $\Delta W_{t+1}^{(2)}$ is a shock corresponding to forward guidance, then a requirement for identification is $v'B^{(1)} = v'B^{(2)} = 0$. But since the elements of $v = (I - A')^{-1}\alpha$ are generically non-zero, the requirement implies that $B^{(i)} = 0$. In words, identification requires that *growth, in both the short and long run, is invariant to monetary policy.*

5.2 Stochastic-volatility model

Consumption has the following dynamics, with stochastic volatility:

$$\begin{aligned}\log C_{t+1} &= \log C_t + g + \sqrt{X_t} \Delta W_{t+1}^{(1)} \\ X_{t+1} &= \mu + a(X_t - \mu) + \sigma \sqrt{X_t} \Delta W_{t+1}^{(2)}, \quad \Delta W_{t+1} \sim \text{Normal}(0, I),\end{aligned}$$

where $a < 1$. As above, the representative investor has Epstein-Zin utility with unitary EIS. In this model, that means that the SDF takes the form:

$$\frac{S_{t+1}}{S_t} = \beta \exp \left[-g - \gamma X_t - \frac{1}{2} X_t \left(\left(\frac{\gamma}{\gamma-1} \sigma \kappa \right) \right)^2 - \sqrt{X_t} \left(\frac{\gamma}{\gamma-1} \sigma \kappa \right) \cdot \Delta W_{t+1} \right],$$

where $\kappa < 0$ is the larger root of the quadratic equation $\frac{1-\gamma}{2} \sigma^2 \kappa^2 + \log(\beta) \kappa - \frac{1}{2} \gamma = 0$.

In this environment, it is easy to verify that the stationary component of the SDF is characterized by the eigenfunction $e(x) = \exp(vx)$, where v is a root of the quadratic equation $0 = \frac{1}{2} \sigma^2 v^2 - [(\gamma - 1) \sigma^2 \kappa + (1 - a)]v - \gamma$ (the choice of the root is to ensure the dynamics of X_t are stable under the measure induced by the resulting H_t). Consequently, the permanent component of this SDF is

$$\frac{H_{t+1}}{H_t} = \exp \left[-\frac{1}{2} \left(\left(\frac{\gamma}{\gamma-1} \sigma \kappa - \sigma v \right) \right)^2 X_t - \sqrt{X_t} \left(\frac{\gamma}{\gamma-1} \sigma \kappa - \sigma v \right) \cdot \Delta W_{t+1} \right],$$

Imagine we are not in the knife-edge case where $(\gamma - 1) \kappa = v$. Then, identification of monetary shocks requires that *monetary policy does not affect uncertainty*, since uncertainty affects H . For example, identification requires output growth volatility and stock market volatility be invariant to monetary actions.

5.3 Discussion and take-aways

The preceding models clarify that Long-Run Neutrality may be a severe assumption. Through the lens of these models where news shocks are priced, monetary authorities who influence the probability distribution of future real outcomes will often violate Long-Run Neutrality. Even more broadly, the dynamics in the models above are specified in terms of investor beliefs. So even if the monetary authority merely conveys information about growth to the public without impacting it in a true causal sense, Long-Run Neutrality will be violated.

An additional take-away from both of these models is that Long-Run Neutrality and conventional notions of monetary neutrality can differ. In the first model, consumption is trend-stationary, so monetary policy can never impact long-term consumption, even though it could impact H . In the second model, volatility is stationary, so long-horizon consumption is fully determined by the level shock; thus, monetary policy could affect H through volatility without affecting long-term consumption. On the other hand, one could imagine studying a conventional long-run risk setting (where consumption grows over time and has an AR(1) conditional expected growth rate) without Epstein-Zin preferences; if monetary policy affects the growth rate in such a setting, Long-Run Neutrality will hold while conventional neutrality will fail. Our Long-Run Neutrality simply differs from conventional notions. (This is why, for example, [Backus et al. \(2022\)](#) use both conventional neutrality and Long-Run Neutrality as distinct identification assumptions in their term structure model.)

6 Final remarks

If researchers do not want to impose rational expectations or risk-neutrality, how can they use asset prices to recover beliefs of investors? The current frontier of knowledge says this problem has no general solution, except in the degenerate case where the pricing kernel features no permanent shocks. But if a researcher more humbly seeks to identify *shocks to investor beliefs*, then identification is possible under weaker conditions.

We explore such shock identification in the context of monetary policy that can affect current and future interest rates. In quasi-linear environments (either linear or with stochastic volatility of the “square-root” form), shock identification is possible provided a *Long-Run Neutrality* condition holds: policy must not affect variables that permanently shift the pricing kernel. Unfortunately, evidence from monetary announcements—from the existing literature and our own data analysis—suggests Long-Run Neutrality is vi-

olated. Furthermore, in some popular structural models featuring priced news about growth and uncertainty, Long-Run Neutrality is equivalent to saying monetary policy does not affect the real economy. Through the lens of these models, identification of monetary policy effects relies paradoxically on monetary policy having no effects.

Three questions come to mind. First, how can researchers identify investor forecast revisions when asset prices do not? The best option, in our view, is to leverage survey data on future interest rates, inflation, and the like. Analysis of survey evidence has been a fruitful and growing area of research, and we see promise in connecting these survey data with more structural models of monetary policy. Our framework sheds light on how such survey data should be included in regressions with short-rate shocks to estimate the effects of monetary policy jointly.

Second, what really are monetary shocks? Our approach follows most of the empirical literature in measuring shocks as interest rate forecast errors. Several potential theoretical explanations exist for these forecast errors: e.g., true randomization in rate setting; uncertain and time-varying interest rate rules; signalling future prospects via policy; belief disagreements among central bankers and investors. Further developing these monetary shock microfoundations seems like an important research direction, to understand whether and how monetary policy has long-term impacts or not (e.g., what do these stories imply about H), as well as the appropriate procedure for estimating monetary IRFs.

Third, one can generalize our model to many types of policies that either make promises or operate through beliefs about the future. To identify the effects of these interventions, one needs an analogous long-run neutrality condition, and this is testable by investigating the log excess return $\log(R_{t,t+\Delta}^*) - \log(R_{t,t+\Delta}^\infty)$ at times of intervention. Which other interventions besides monetary policy also impact permanent risks?

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Appendix:

Risk Premia, Subjective Beliefs, and Forward Guidance

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February 23, 2024

A Additional discussion pertaining to Section 2

A.1 Numerical example

To get a sense for the consequences of misspecification, we provide a numerical example that is roughly calibrated to the evidence in Cieslak (2018). After calibrating, we illustrate how misspecification wrongly assuming rational expectations impacts the estimated IRF of growth to a forward-guidance shock. The result is Figure 1 in the main text.

To calibrate, we mimic Cieslak (2018) and run the following two regressions in the model:

$$r_{t+j} = \beta_0^{(j)} + \beta_g^{(j)} g_t + \beta_r^{(j)} r_t + \text{residual} \quad (\text{A.1})$$

$$\tilde{\mathbb{E}}_t[r_{t+j}] = \tilde{\beta}_0^{(j)} + \tilde{\beta}_g^{(j)} g_t + \tilde{\beta}_r^{(j)} r_t + \text{residual} \quad (\text{A.2})$$

where $\tilde{\mathbb{E}}_t[r_{t+j}]$ denotes subjective expectations of interest rates, obtained from the Blue Chip Financial Forecasts. The results of these regressions, in the data, are presented in the bottom row of Table A.1.

	A. Dependent variable r_{t+j}				B. Dependent variable $\tilde{\mathbb{E}}_t[r_{t+j}]$			
	$j = 1 \text{ quarter}$		$j = 4 \text{ quarters}$		$j = 1 \text{ quarter}$		$j = 4 \text{ quarters}$	
	$\beta_g^{(1)}$	$\beta_r^{(1)}$	$\beta_g^{(4)}$	$\beta_r^{(4)}$	$\tilde{\beta}_g^{(1)}$	$\tilde{\beta}_r^{(1)}$	$\tilde{\beta}_g^{(4)}$	$\tilde{\beta}_r^{(4)}$
Model:	0.195	0.902	0.547	0.493	0.145	0.952	0.215	0.746
Data:	0.180	0.880	0.650	0.500	0.100	0.930	0.120	0.840

Table A.1. Short-rate forecasts in the model and data. Regressions of future short rates (Panel A) and survey-based expectations of future short rates (Panel B) on current growth and short rates (g_t, r_t). The calibration of B is a diagonal matrix with $\sigma_g = 0.0025/4$, $\sigma_r = 0.010/4$, and $\sigma_f = 0.050/4$. For the “Data” row, Cieslak (2018) proxies r_t by the Federal Funds Rate (with survey expectations obtained from the Blue Chip Financial Forecasts) and g_t by employment growth.

We set A and \tilde{A} in the model to roughly match these empirical results. We calibrate

$$A = \begin{bmatrix} 0.90 & -0.05 & 0 \\ 0.50 & 0.75 & 0.05 \\ 0 & 0 & 0.80 \end{bmatrix} \quad \text{and} \quad \tilde{A} = \begin{bmatrix} 0.90 & -0.05 & 0 \\ 0.45 & 0.80 & 0.05 \\ 0 & 0 & 0.99 \end{bmatrix}. \quad (\text{A.3})$$

The comparison between our model and the data is presented in Table A.1. The fit, while not perfect, is good enough to illustrate our main points here. Relative to the econometric transition matrix, agents perceive a smaller feedback of growth to future short rates, as well as higher persistences of r and especially of f . The true and perceived dynamics of short rates in the model, plotted in Figure A.1, illustrate the discrepancy between A and \tilde{A} , especially the greater perceived persistence of the forward-guidance shock.

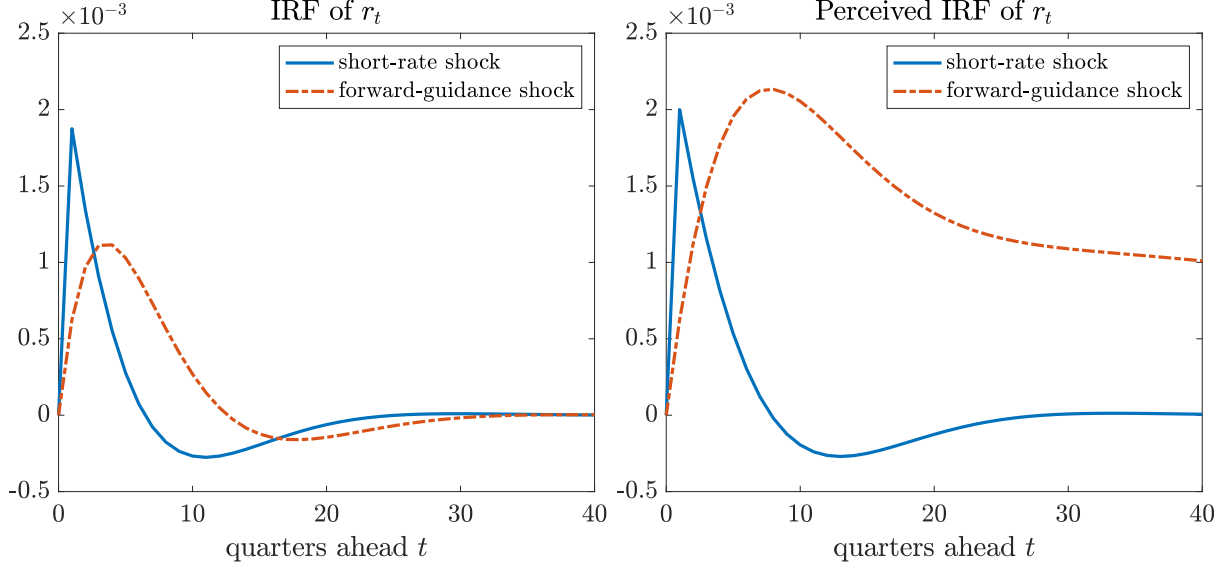


Figure A.1. IRFs of short rates to the two monetary shocks. The left panel plots the true IRFs. The right panel plots the perceived IRFs. The calibrations of A and \hat{A} are given in (A.3). The calibration of B is a diagonal matrix with $\sigma_g = 0.0025/4$, $\sigma_r = 0.010/4$, and $\sigma_f = 0.050/4$.

A.2 Identifying latent factors from term structure models

We consider the three-factor VAR (1), paired with the SDF (10). In this model, the forward guidance variable f is latent. One would hope that asset prices, paired with the pricing model, allow us to “invert bond yields” for the latent factor, but this is not the case as we explain here.

For example, consider using the transformed state variable $\hat{X}_t = UX_t$, where

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ u_g & u_r & 1 \end{bmatrix}.$$

While the subsequent argument works for any U , this particular example acknowledges that (g, r) are observable and lets the latent factor be a linear combination of forward guidance with the observables. At the same time, suppose the risk price vector is $\hat{\pi}_t = \hat{\pi}_0 + \hat{\Pi}\hat{X}_t$, where $\hat{\pi}_0 = \pi_0$ and $\hat{\Pi} = \Pi U^{-1}$. Then, the SDF is identical to the original specification (i.e., $\hat{\pi}_t = \pi_t$) and the risk-neutral dynamics of $X_t = U^{-1}\hat{X}_t$ are identical to the original specification.¹¹

¹¹Indeed, the physical dynamics of $\hat{X} := UX$ are $\hat{X}_{t+1} = (UAU^{-1})\hat{X}_t + (UB)\Delta W_{t+1}$, so the risk-neutral dynamics for \hat{X} are

$$\hat{X}_{t+1} = -UB\hat{\pi}_0 + (UAU^{-1} - UB\hat{\Pi})\hat{X}_t + (UB)\Delta\hat{W}_{t+1}^*$$

where $\Delta\hat{W}_{t+1}^* := \Delta W_{t+1} + \hat{\pi}_t$ is the risk-neutral shock. If $\hat{\pi}_0 = \pi_0$ and $\hat{\Pi} = \Pi U^{-1}$, then risk prices are

$$\hat{\pi}_t = \hat{\pi}_0 + \hat{\Pi}\hat{X}_t = \pi_0 + \Pi U^{-1}UX_t = \pi_t.$$

As a result, the risk-neutral shocks coincide: $\Delta\hat{W}_{t+1}^* = \Delta W_{t+1}^*$. Thus, the risk-neutral dynamics of \hat{X} are

$$\hat{X}_{t+1} = UA_0^* + UA^*U^{-1}\hat{X}_t + (UB)\Delta W_{t+1}^*$$

These dynamics imply the same risk-neutral dynamics for X_t displayed in (12). One can also easily show that, as a consequence, the solution for equilibrium bond yields is invariant to the choice of u_g and u_r .

To address the generic non-identification of latent factors, we must effectively make some assumption about U . A canonical approach starting with [Hamilton and Wu \(2012\)](#) assumes the state vector \hat{X} is chosen such that its risk-neutral persistence is a diagonal matrix. This can be done by implicitly picking U^{-1} as the matrix of eigenvectors of $A^* = A - B\Pi$, since UA^*U^{-1} is the risk-neutral persistence of \hat{X} . But after making this choice, only $\hat{X}_t = UX_t$ is recovered from bond yields, not X_t itself. Without additional assumptions, bond yields do not reveal *particular latent factors*.

What can be done in light of the challenges to observing f ? On the one hand, we are still able to extract \hat{X} from bond yields, a state vector which spans the same space as X . This is good enough for the purpose of controlling for the past state: controlling for $\hat{X}_{\tau-1} = UX_{\tau-1}$ in Procedure 1 instead of $X_{\tau-1}$ will lead to identical inference for b_h . Thus, we may still recover the desired IRF in principle. On the other hand, failure to specifically recover f prevents us from constructing its shock z^f , which prevents us from actually implementing Procedure 1 as stated. We must necessarily exclude the forward-guidance shock z_τ^f and so cannot measure any forward-guidance effect, at least not in this way.¹²

Although it is not our focus, excluding z_τ^f may also bias estimates of the causal impact of short-rate shocks. Indeed, note that $\text{Cov}[z_\tau^r, z_\tau^f \mid X_{\tau-1}] = (0, 1, 0)BB'(0, 0, 1)'$. So if we run Procedure 1 without z_τ^f , any correlation between short-rate and forward-guidance shocks will be impounded into the coefficient on z_τ^r .

What solves some issues in the simple baseline model is an assumption that forward-guidance shocks are orthogonal to short-rate and growth shocks. In other words, if we assume that the third row of B is proportional to $(0, 0, 1)$, then we can recover f from bond yields. This is because the unique matrix U that preserves this orthogonality is $U = I$, and so $\hat{X} = X$ is uniquely pinned down by yields. However, even armed with this orthogonality assumption, we re-encounter difficulties as soon as there are additional latent factors present. Examples of such problematic environments would be if forward guidance was described by a multi-factor structure, or if monetary policy managed either uncertainty or risk premia in addition to future short rates. In such case, f is once again unidentified without additional assumptions.

¹²A similar critique applies to the VAR approach in place of the Jorda projection. If we fully observed X , then we would not need to run Jorda projections; we could estimate the VAR directly via time series regressions, obtain A , and then construct the desired IRF A^h . But if we obtain only $\hat{X} = UX$, we would not recover A and would therefore obtain an incorrect IRF.

B Proofs

We first prove Lemma 1 regarding what is recovered from asset price data. Then, we prove Propositions 1-4.

PROOF OF LEMMA 1. Given the Markovian environment, the asset prices in (18) can be represented by a family of pricing operators $(Q_t)_{t \geq 0}$ as

$$[Q_t f](x) = \mathbb{E}[S_t f(X_t) \mid X_0 = x]. \quad (\text{B.1})$$

The operator Q_t is the t -period pricing operator for any claim f that is a function of the Markov state. In all that follows, we assume we observe Q_t (i.e., this is what is meant by “asset price data” in a complete market environment).

Now, solve the eigenvalue problem

$$[Q_t e](x) = \exp(\eta t) e(x). \quad (\text{B.2})$$

By the Perron-Frobenius theory, $\exp(\eta) > 0$ is a positive eigenvalue of $\lim_{t \rightarrow 0} t^{-1} Q_t$, and its associated eigenfunction e is strictly positive. Given Q_t is observable, we thus can infer e and η from data.¹³

After recovering these objects, we may construct

$$H_t := \exp(-\eta t) S_t \frac{e(X_t)}{e(X_0)}. \quad (\text{B.3})$$

Of course, S is not directly observable in data, and so neither is H , but the important point is that the H in (B.3) is the same one in the factorization (19) by construction. Note that H_t is a strictly positive martingale since

$$\mathbb{E}[H_T \mid \mathcal{F}_t] = \frac{\exp(-\eta T)}{e(X_0)} \mathbb{E}[S_T e(X_T) \mid \mathcal{F}_t] = \frac{\exp(-\eta T)}{e(X_0)} \exp(\eta(T-t)) e(X_t) S_t = H_t,$$

by (B.2). Although the construction of $\hat{\mathbb{P}}$ in (23) depends on the unobservable H , note that

$$\begin{aligned} \hat{\mathbb{P}}\{r(X_{\tau+T}) \leq r \mid X_\tau\} &= \mathbb{E}\left[\frac{H_{\tau+T}}{H_\tau} 1_{\{r(X_{\tau+T}) \leq r\}} \mid X_\tau\right] = \mathbb{E}\left[\exp(-\eta T) \frac{S_{\tau+T}}{S_\tau} \frac{e(X_{\tau+T})}{e(X_\tau)} 1_{\{r(X_{\tau+T}) \leq r\}} \mid X_\tau\right] \\ &= \frac{\exp(-\eta T)}{e(X_\tau)} [Q_T \hat{e}](X_\tau). \end{aligned}$$

Note that $\hat{e}(x) := e(x) 1_{\{r(x) \leq r\}}$ is a computable payoff as a function of x . Since η , e , and Q_T are all also observable, we can observe $\hat{\mathbb{P}}\{r(X_{\tau+T}) \leq r \mid X_\tau\}$ from asset price data. \square

PROOF OF PROPOSITION 1. This proposition is implied by Proposition 2, since the constant diffusion condition (27) implies the condition (29). \square

¹³In a discrete-time model, it would suffice to study the instantaneous pricing operator Q_1 , since the law of iterated expectations allows us to apply Q_1 in succession t times in order to obtain Q_t . In continuous time, the analogous operator is the instantaneous pricing operator $\lim_{t \rightarrow 0} Q_t/t$.

PROOF OF PROPOSITION 2. Let $m_t^x := \mathbb{E}^x[X_t]$ denote the (investor-perceived) conditional mean of X_t , starting from point x . By applying Itô's formula to X_t , we have that m_t^x solves the differential equation (since the compensated monetary shock has zero mean)

$$\frac{d}{dt}m_t^x = \mathbb{E}^x[\mu(X_t)]$$

subject to the initial condition $m_0^x = x$. Specializing to the linear drift from (26), the ODE becomes

$$\frac{d}{dt}m_t^x = \mathbb{E}^x[A_0 + AX_t] = A_0 + Am_t^x$$

This ODE is affine, and the solution takes the well-known form

$$m_t^x = \exp(At) \left[x + \int_0^t \exp(-As) A_0 ds \right].$$

We may then compute

$$m_t^{X_\tau} - m_t^{X_{\tau-}} = \exp(At)(X_\tau - X_{\tau-}).$$

(The interpretation of $\exp(At)$ is as the Taylor series $\sum_{k=0}^{\infty} \frac{t^k}{k!} A^k$.)

Finally, using assumption (29), we have that the drift of X_t under $\hat{\mathbb{P}}$ is $\hat{A}_0 + \hat{A}X_t$, where $\hat{A}_0 = A_0 - \hat{\beta}$ and $\hat{A} = A$. Since \hat{A} is observable (by Lemma 1), we have that $A = \hat{A}$ is also observable. Therefore, $m_t^{X_\tau} - m_t^{X_{\tau-}}$ is observable for all t . By assumption (28), we have obtained $z_\tau^T = \rho \cdot (m_\tau^{X_\tau} - m_\tau^{X_{\tau-}})$. \square

PROOF OF PROPOSITION 3. Let $m_t^x := \mathbb{E}^x[X_t]$ and $V_t^x := \mathbb{E}^x[(X_t - m_t^x)(X_t - m_t^x)']$ denote the conditional mean and variance of X_t , starting from point x . A standard result on SDEs (e.g., Chapter 5.5 of Särkkä and Solin, 2019) is that

$$\frac{d}{dt}V_t^x = \mathbb{E}^x[\mu(X_t)(X_t - m_t^x)'] + \mathbb{E}^x[(X_t - m_t^x)\mu(X_t)'] + \mathbb{E}^x[\sigma(X_t)\sigma(X_t)'].$$

This equation holds at times t that are non-announcement dates. Specializing to the linear drift from (26) and the square-root assumption on the diffusion (30), we obtain

$$\begin{aligned} \frac{d}{dt}V_t^x &= \mathbb{E}^x[(A_0 + AX_t)(X_t - m_t^x)'] + \mathbb{E}^x[(X_t - m_t^x)(A_0 + AX_t)'] + \mathbb{E}^x[\zeta_0\zeta_0' + \sum_{i=1}^n \zeta_i \text{diag}(u^{(i)} \cdot x) \zeta_i'] \\ &= A\mathbb{E}^x[X_t(X_t - m_t^x)'] + \mathbb{E}^x[(X_t - m_t^x)X_t']A' + \zeta_0\zeta_0' + \sum_{i=1}^n \zeta_i \text{diag}(u^{(i)} \cdot m_t^x) \zeta_i' \\ &= AV_t^x + V_t^x A' + \zeta_0\zeta_0' + \sum_{i=1}^n \zeta_i \text{diag}(u^{(i)} \cdot m_t^x) \zeta_i', \end{aligned}$$

where $u^{(i)}$ is the i th elementary vector. Subject to the initial condition $V_0^x = [0]_{n \times n}$, this ODE for V_t^x is a Riccati equation, for which the solution has the well-known form

$$V_t^x = \int_0^t \exp(A(t-s)) \left[\zeta_0\zeta_0' + \sum_{i=1}^n \zeta_i \text{diag}(u^{(i)} \cdot m_t^x) \zeta_i' \right] \exp(A'(t-s)) ds.$$

Again, this solution holds for any time t prior to the next monetary surprise. We may then compute

$$V_t^{X_\tau} - V_t^{X_{\tau-}} = \int_0^t \exp(A(t-s)) \sum_{i=1}^n \zeta_i \text{diag}[u^{(i)} \cdot (m_s^{X_\tau} - m_s^{X_{\tau-}})] \zeta_i' \exp(A'(t-s)) ds.$$

By Proposition 2, the object $m_s^{X_\tau} - m_s^{X_{\tau-}}$ is observable. In addition, under assumption (29), we have $A = \hat{A}$ observable. Hence, $V_t^{X_\tau} - V_t^{X_{\tau-}}$ is observable. This is enough, since by assumption (28), we have $v_\tau^T = \rho'(V_t^{X_\tau} - V_t^{X_{\tau-}})\rho$. \square

PROOF OF PROPOSITION 4. Suppose, leading to contradiction, that z_τ^T is identified by asset price data. The same procedure also identifies

$$\hat{z}_\tau^T := \hat{\mathbb{E}}[r(X_{\tau+T}) | X_\tau] - \hat{\mathbb{E}}[r(X_{\tau+T}) | X_{\tau-}], \quad T > 0, \quad (\text{B.4})$$

where the probability measure $\hat{\mathbb{P}}$ is defined in (23). Indeed, Proposition 2 of Borovička et al. (2016) says that the observable asset prices can be obtained by formula (18) using either (i) probability measure \mathbb{P} and SDF S , or (ii) probability measure $\hat{\mathbb{P}}$ and SDF \hat{S} , where

$$\hat{S}_t := S_t \frac{H_0}{H_t}. \quad (\text{B.5})$$

Therefore, the same asset price data that identify z_τ^T also identify \hat{z}_τ^T .

Since both z_τ^T and \hat{z}_τ^T are identified by the same procedure on asset prices, their values must be identical:

$$\begin{aligned} \mathbb{E}\left[\frac{H_{\tau+T}}{H_\tau} r(X_{\tau+T}) | X_\tau\right] - \mathbb{E}\left[\frac{H_{\tau+T}}{H_{\tau-}} r(X_{\tau+T}) | X_{\tau-}\right] \\ = \mathbb{E}[r(X_{\tau+T}) | X_\tau] - \mathbb{E}[r(X_{\tau+T}) | X_{\tau-}]. \end{aligned} \quad (\text{B.6})$$

Using the fact that (B.6) holds for all X_τ and $X_{\tau-}$, we must have

$$\mathbb{E}[H_T r(X_T) | X_0 = x] = \mathbb{E}[r(X_T) | X_0 = x] + \alpha(T), \quad (\text{B.7})$$

where $\alpha(T)$ may depend on T but is independent of x . Recall $r(x) = \rho_0 + \rho \cdot x$. Without loss of generality, let us assume that ρ has non-zeros in all of its entries. It must therefore generically be the case that

$$\mathbb{E}[H_T X_T | X_0 = x] = \mathbb{E}[X_T | X_0 = x] + \epsilon(T), \quad (\text{B.8})$$

where $\epsilon(T)$ is an n -dimensional vector independent of x .

Now, under both hypotheses (i) and (ii) of the Proposition, it must generically be the case that dH_t depends on X_{t-} . (Generically, because hypothesis (i) can be consistent with $dH_t \perp X_{t-}$ in the knife-edge case that the announcement arrival rate $\lambda(x) \equiv \lambda$ is constant and the probability distribution of monetary shocks ξ_t is independent of X_{t-} .) We may summarize both cases by writing the drift of X_t under $\hat{\mathbb{P}}$ as

$$\hat{\mu}(x) = \underbrace{A_0 + Ax}_{\mu(x)} + \Gamma(x), \quad (\text{B.9})$$

where $\Gamma(x)$ is a non-constant function of x . Using (B.9), compute

$$\begin{aligned}\mathbb{E}^x[H_T X_T] - \mathbb{E}^x[X_T] &= \hat{\mathbb{E}}^x[X_T] - \mathbb{E}^x[X_T] \\ &= A \int_0^T \left(\hat{\mathbb{E}}^x[X_t] - \mathbb{E}^x[X_t] \right) dt - \int_0^T \hat{\mathbb{E}}^x[\Gamma(X_t)] dt\end{aligned}\tag{B.10}$$

Guess that (B.8) holds, and replace in (B.10) to get

$$\epsilon(T) = A \int_0^T \epsilon(t) dt - \int_0^T \hat{\mathbb{E}}^x[\Gamma(X_t)] dt.\tag{B.11}$$

Clearly, the last term in (B.11) generically depends on x , contradicting the guess that $\epsilon(T)$ is independent of x . Thus, z_τ^T is not identified. \square