Assignment 4 *INF5620-Peter*

Peter, peterek, peterek@student.matnat.uio.no

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1 Exercise 8: Compute with a non-uniform mesh

1.1 problem a)

Derive a linear system for the problem

$$-u''(x) = 2 \tag{1}$$

on $\Omega = [0, 1]$ with $u(0) = 0$ and $u(1) = 1$, using P1 elements and a non-uniform mesh. Vertices has coordinates $x_0 = 0 < x_1 < x_2 .... < x_{N_n-1} = 1$, and the length of cell number e is $h_e = x_{e+1} - x_e$.
Considering equation 1 as its Gelerkin formulation:

$$(-u'', v) = (2, v)$$

where v is a test function with similar properties as u. Using integration by parts to obtain:

$$-u'v\Big|_0^1 + (u', v').$$

Now considering following equation, which is a linear system:

$$(u', v') = (2, v).$$

Express $u = \Sigma_{j=0}^{N} c_j \psi_j$ to obtain

$$(\psi_i', \psi_j')c_i = (2, \psi_i),$$

which can be written as $A_{i,j}c_i = b_i$.

We are now looking for elementwise computations of the A matrix and the b vector. For the element matrix, we start by splitting up the integral over Ω into a sum of contributions from each element:

$$A_{i,j} = \int_\Omega \psi_i' \psi_j' dx = \Sigma_e A_{i,j}^e,$$

$$A_{i,j}^e = \int_{\Omega^e} \psi_i' \psi_j' dx.$$

The right hand side of the linear system, vector b, is also computed elementwise:

$$b_i = \int_\Omega 2\psi_i dx = \Sigma b_i^e, b_i^e = \int_{\Omega^e} 2\psi_i dx.$$

For elementwise computations, let r and s denote local node numbers corresponding to the global node numbers $i = q(e, r)$ and $j = q(e, s)$ so that the caclulation for each element is:

$$A^e = \int_\Omega \psi_{q(e,s)}' \psi_{(e,s)}' dx$$

$$b_i^e = \int_{\Omega^e} 2\psi_{q(e,r)} dx.$$

Instead of computing the integrals over some element $\Omega^e = [x_L, x_R]$ in the physical coordinate system, let $X \in [-1, 1]$ be the domain to map physical coordinates into. A linear mapping from X to x can be written:

$$x = \frac{1}{2}(x_L - x_R) + \frac{1}{2}(x_L + x_R)X,$$

where $h = (x_L + x_R)$. To integrate on the reference element, change the variable from x to X :

$$\phi(X) = \psi_{q(e,r)}(x(X)).$$

This makes the

$$ax^2 + bx + c = 0$$

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