Assignment 3 *INF5620-Peter*

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1 Exercise 2: Compute the deflection of a cable with sine functions

1.1 problem a)

The exact solution of the scaled problem

$$u''(x) = 1 \tag{1}$$

with boundary conditions $u(0) = 0$ and $u'(1) = 0$ has the the solution of the form:

$$u(x) = \frac{1}{2}x^2 - x \tag{2}$$

Because of symmetry condition at $x = 1$, the domain is halved, so we have the domain $\Omega = [0, 1]$

1.2 problem b)

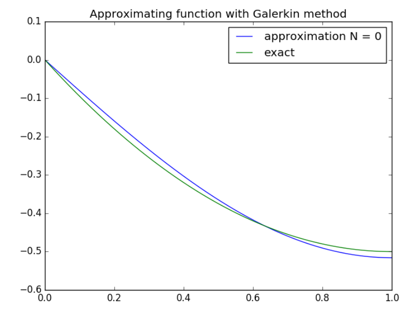
A possible function space is spanned by $\psi_i = \sin((2i+1)\pi x/2), i = 1, ..., N$. These functions fulfill the necessary condition $\psi_i(0) = 0$ and $\psi_i'(0) = 0$ such that both boundary conditions are fulfilled by the expansion $u = \Sigma_j^N c_j \psi_j$.

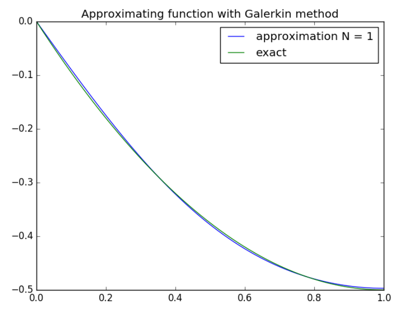
The error in the maximum deflection at $x = 1$ when only one basis function is used ($N = 0$):

$$f(1) - c_0\psi_0(1) = 0.0160245509312$$

1.3 problem c)

Visualizing the solutions in b) for $N = 0, 1, 20$





modified_sine_functions_020.png

1.4 problem d)

Choosing now a simpler basis, namely "all" sine functions $\psi_i = \sin((i+1)\pi x/2)$. These functions does not fulfill the condition $\psi_i(0) = 0$ and $\psi_i'(0) = 0$. For $i = 1, 3, 5, ..., 2N + 1$ we have that $\psi_i'(0) = \pi, 2\pi, 3\pi, ...$
The approximation wil not improve the solution compared with those in b).

1.5 problem e)

Symmetry condition at $x = 1$ is dropped, so the domain is extended to $\Omega = [0, 2]$ such that it covers the entire scaled physical cable. Problem now reads

$$u'' = 1, x \in (0, 2), u(0) = u(2) = 0.$$

The solution turns out to be exactly the same as for the halved domain

$$u(x) = \frac{1}{2}x^2 - x$$

Basis from d) fulfills boundary conditions as $\psi_i(0) = \psi_i(2) = 0$.

entire_domain_000.png

entire_domain_001.png

entire_domain_020.png

2 exercise 5: Compute the deflection of a cable with 2 P1 elements

Basis function for 2 P1 elements of equal length h

$$m(T) = \begin{cases} 0 & x < x_i \\ (x - x_{i-1})/h & x_{i-1} \leq x < x_i \\ 1 - (x - x_i)/h & x_i \leq x < x_{i+1} \\ 0 & x \geq x_{i+1} \end{cases} \tag{3}$$

cable_2P1_modified.png

(a) modified for $u(0) = 0$

cable_2P1_excluded.png

(b) excluded endpoint

Figure 1: deflection of a cable with 2 P1